General properties of ocean waves

(waves 101)

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Short course on ocean waves

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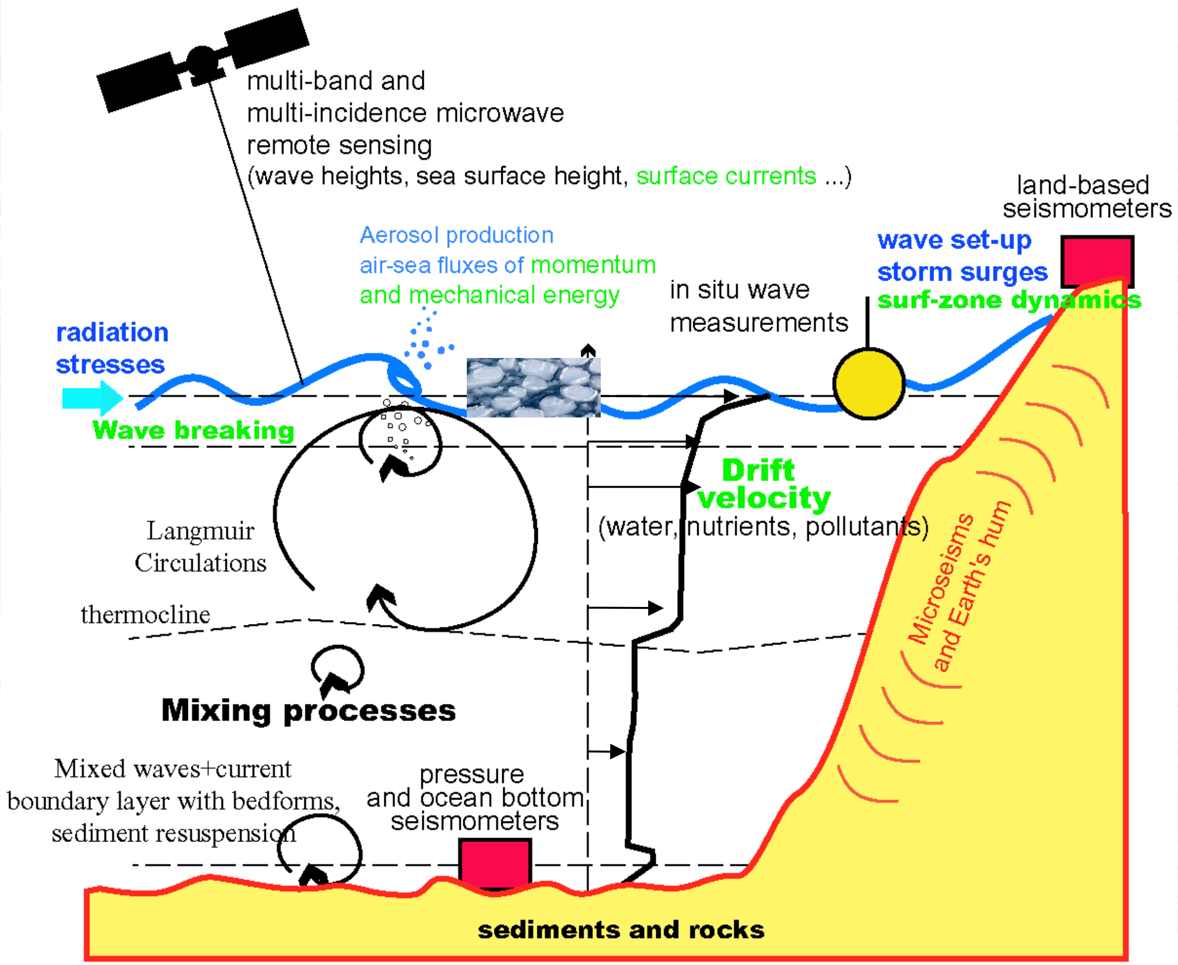
Why waves? A key rôle at Earth System interfaces…

- **air-sea**
  - **Land-sea**
  - **Ocean-ice**
- **ocean-crust**

+ engineering...



Outline

1. **What are waves : waves and « sea state »**
2. **Time and space scales… and wave spectra**
3. **measuring waves in situ**
4. **Wave propagation : effects of depth & currents**

Material : « Waves in geosciences »

ftp://ftp.ifremer.fr/ifremer/ww3/COURS/

waves_in_geosciences_2016.pdf

1

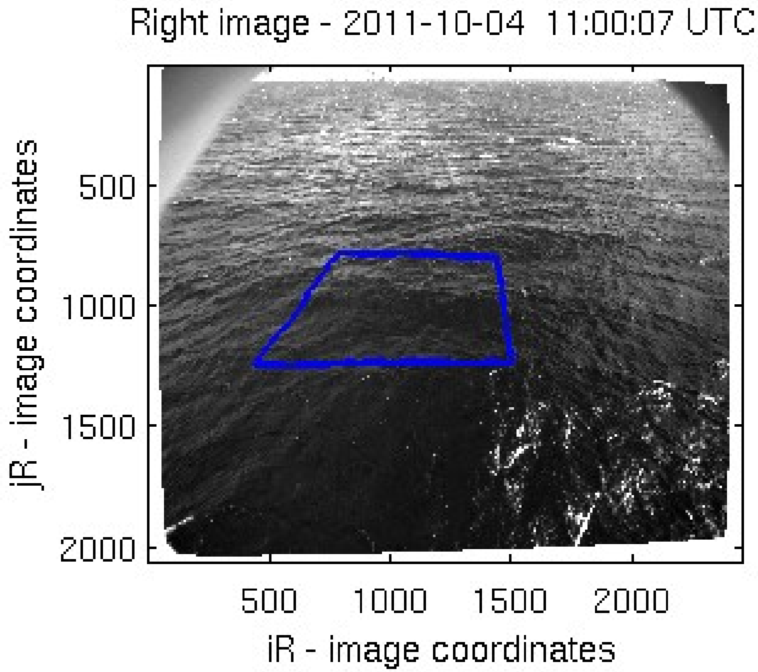
What are waves ?

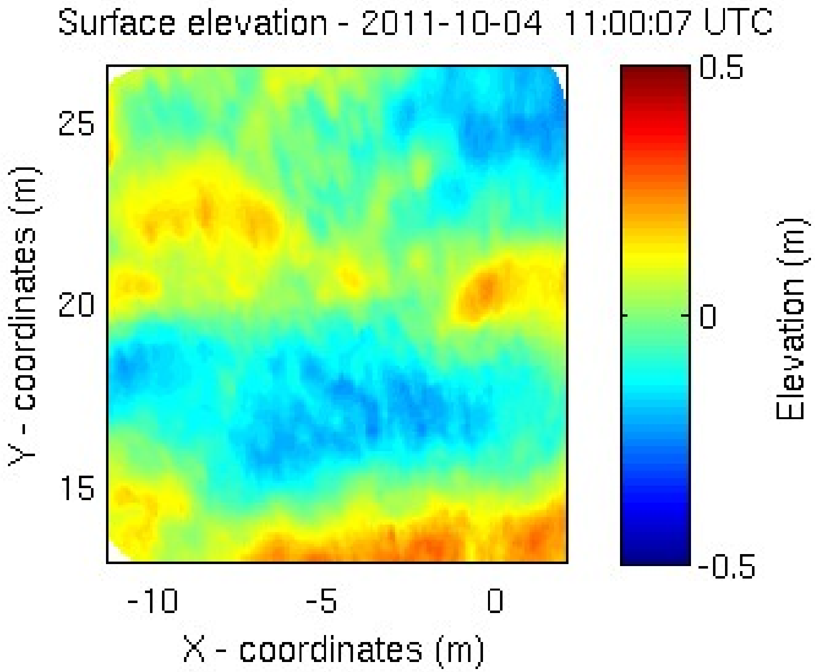
Waves and « sea state »
Wave energy and Stokes drift

What are waves ?

We think of waves as the large crests we see…

the red blob that moves : here is a wave.



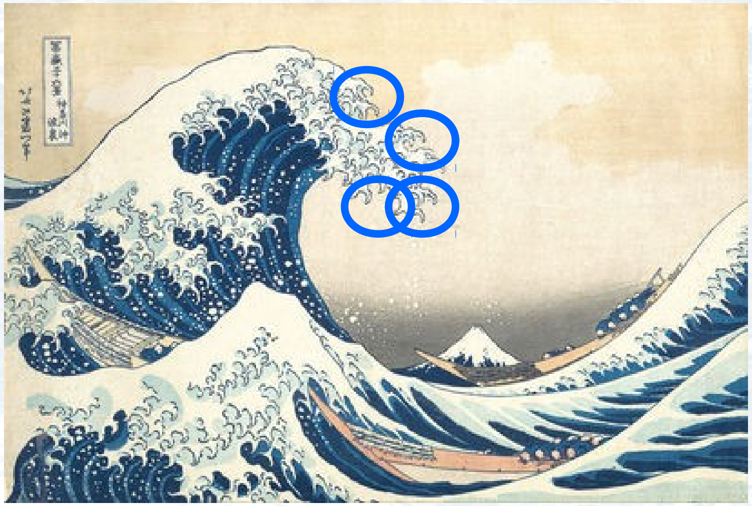


What are waves ?

But there are also short « wavelets » riding on the largest waves
This collection of waves, long and short, tall and small is…
the « sea state »

A typical distance between 2 crests is the **wavelength L** .

The time between 2 crests passing a fixed point is the **wave period T**



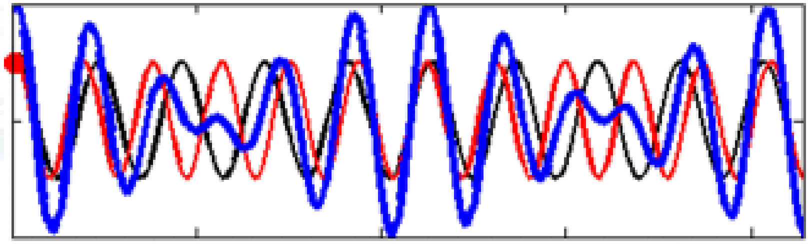
L and T are related by the Dispersion relation.

In deep water this is $L = g/(2\pi)\, T^2$: T = 10 s → L = 150 m

In general : $L \tanh(2\pi D / L) = g/(2\pi)\, T^2$
or $\sigma^2 = gk \tanh(kD)$

The phase speed is $C = L / T$
The group speed is different $C_g = d\sigma / dk$

In deep water $C_g = C/2$



Wave motion : elevation → energy

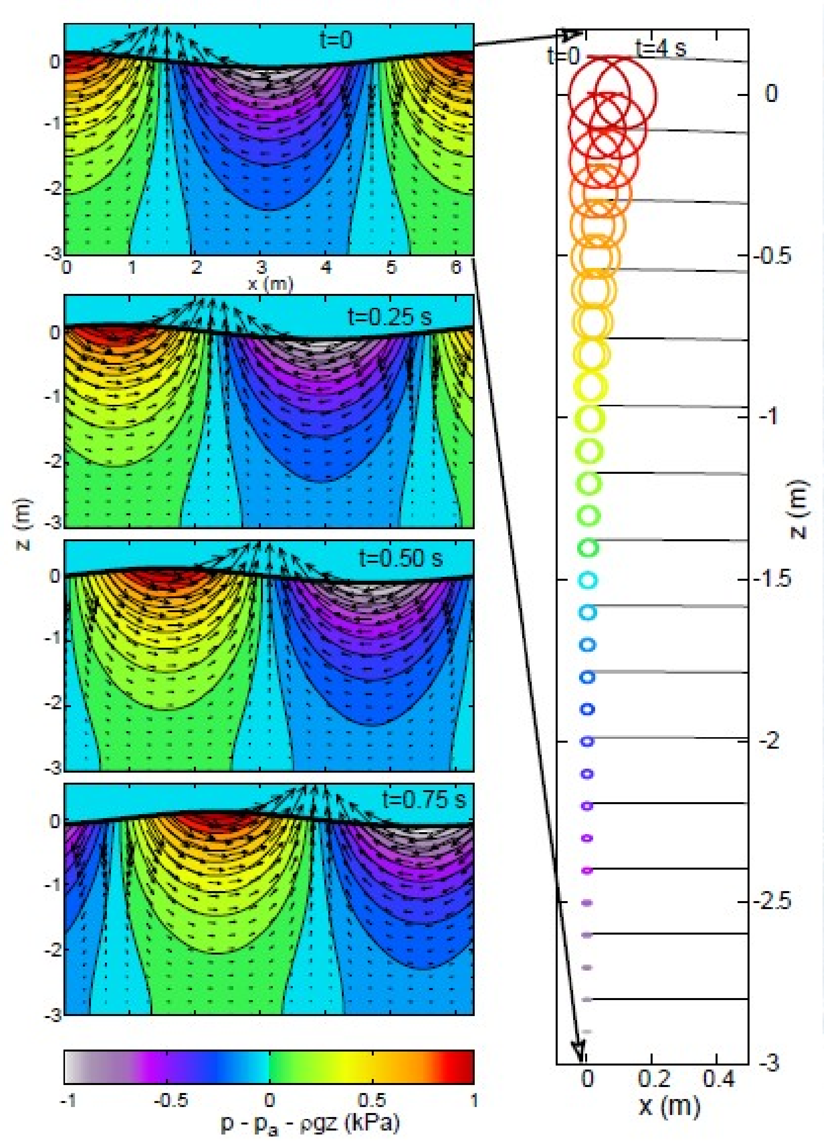
Surface elevation :
$\zeta = a \cos(k \cdot x - \sigma t)$

Velocity
$u = \sigma a \cos(k \cdot x - \sigma t) \frac{\cosh(kz + kh)}{\sinh(kD)}$

Energy : potential + kinetic

$$E_t = E_c + E_p = \frac{1}{2}\rho_w g a^2 = \rho_w g E$$

E = < $\overline{\left(\zeta - \overline{\zeta}\right)^2}$ >, variance of surface elevation in m^2



Wave motion : Stokes drift

E = < $\overline{\left(\zeta - \overline{\zeta}\right)^2}$ >

Drift velocity

$$\mathbf{U}_s = \sigma \mathbf{k} E \frac{\cosh(2kz+2kh)}{\sinh^2(kD)}$$

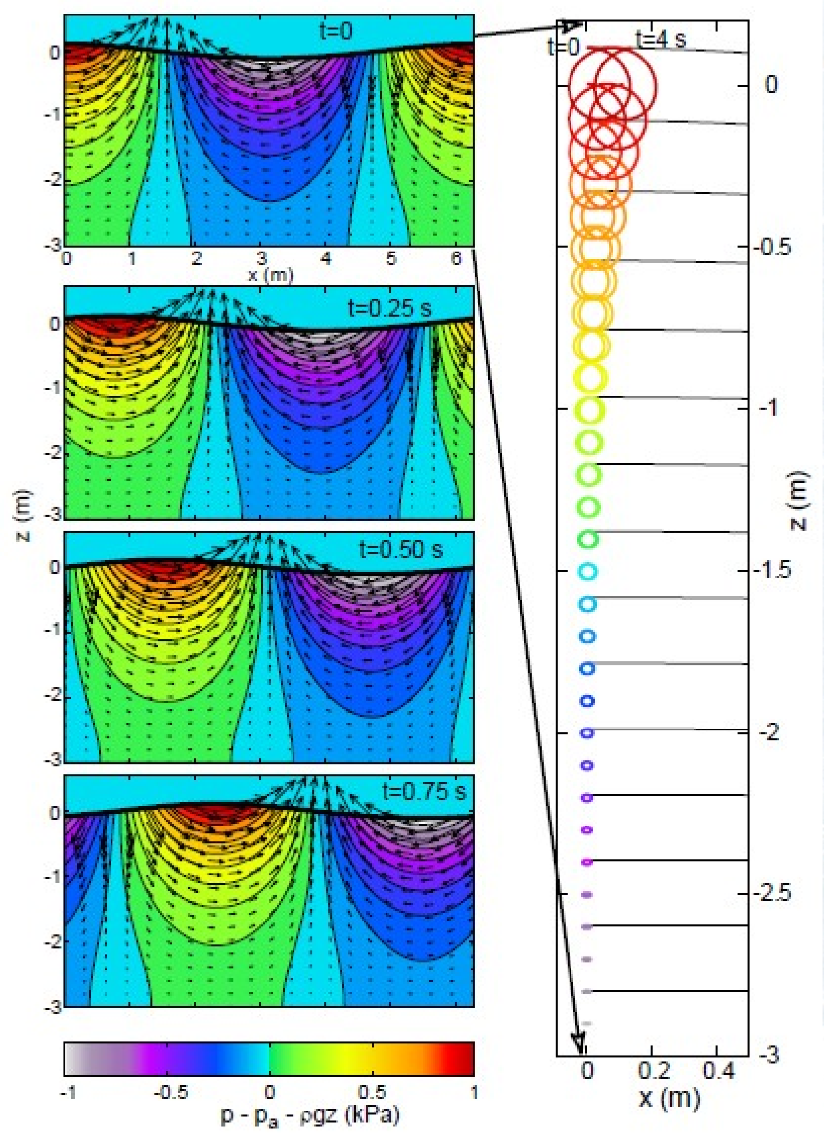
Surface value ~ 1.5 % of wind

In deep water :

$$\mathbf{U}_s = 2\sigma \mathbf{k} E \mathrm{e}^{2kz}$$

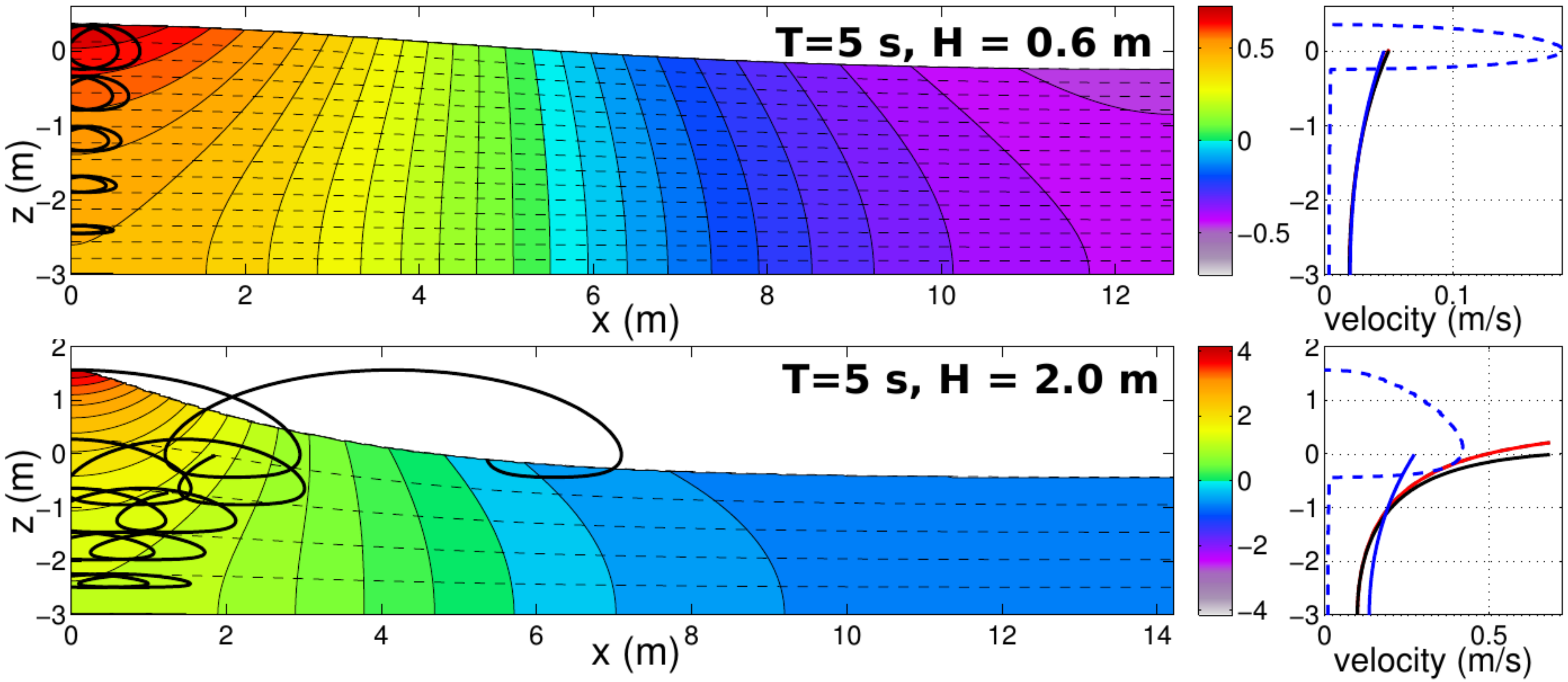
For all depths, transport is :

$$\mathbf{M}^w = \rho_w \int_{-h}^{0} \mathbf{U}_s \mathrm{d}z = \rho_w g \frac{a^2}{2C} = \frac{E_t}{C}$$



Wave motion : Stokes drift

Examples in shallow water, D = 3 m.



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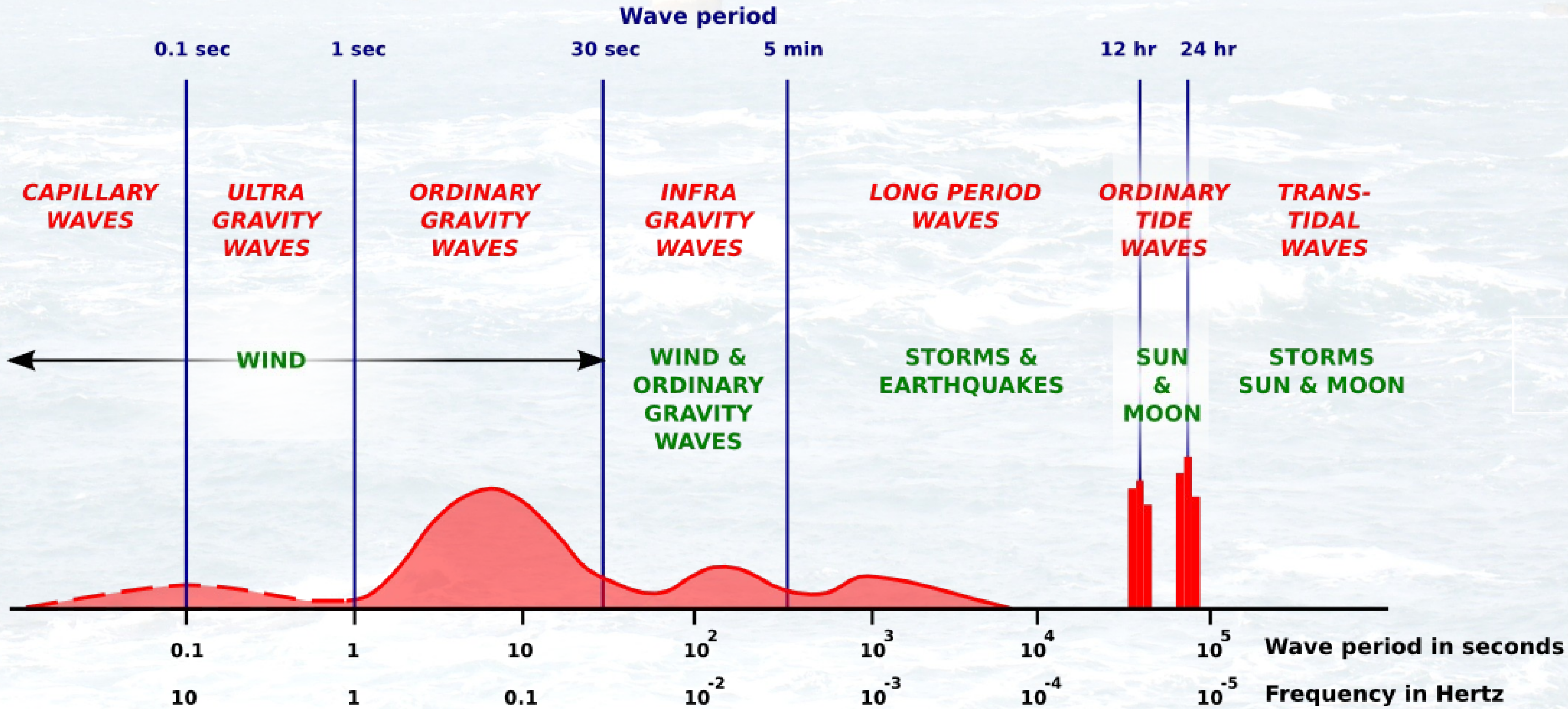
Time and space scales
… and wave spectra

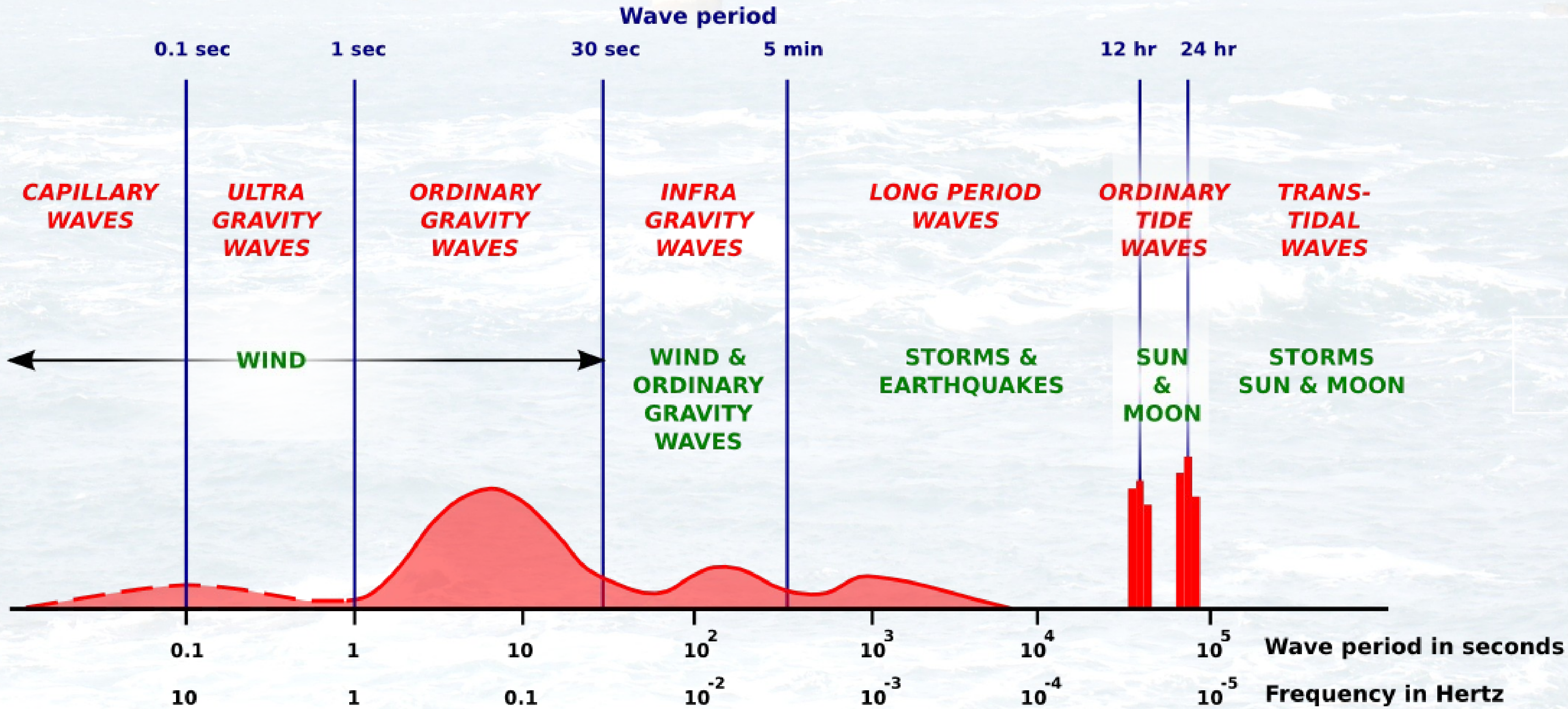
Time scales : general context



adapted from Walter Munk (1950)

Time scales : general context

Order of magnitude

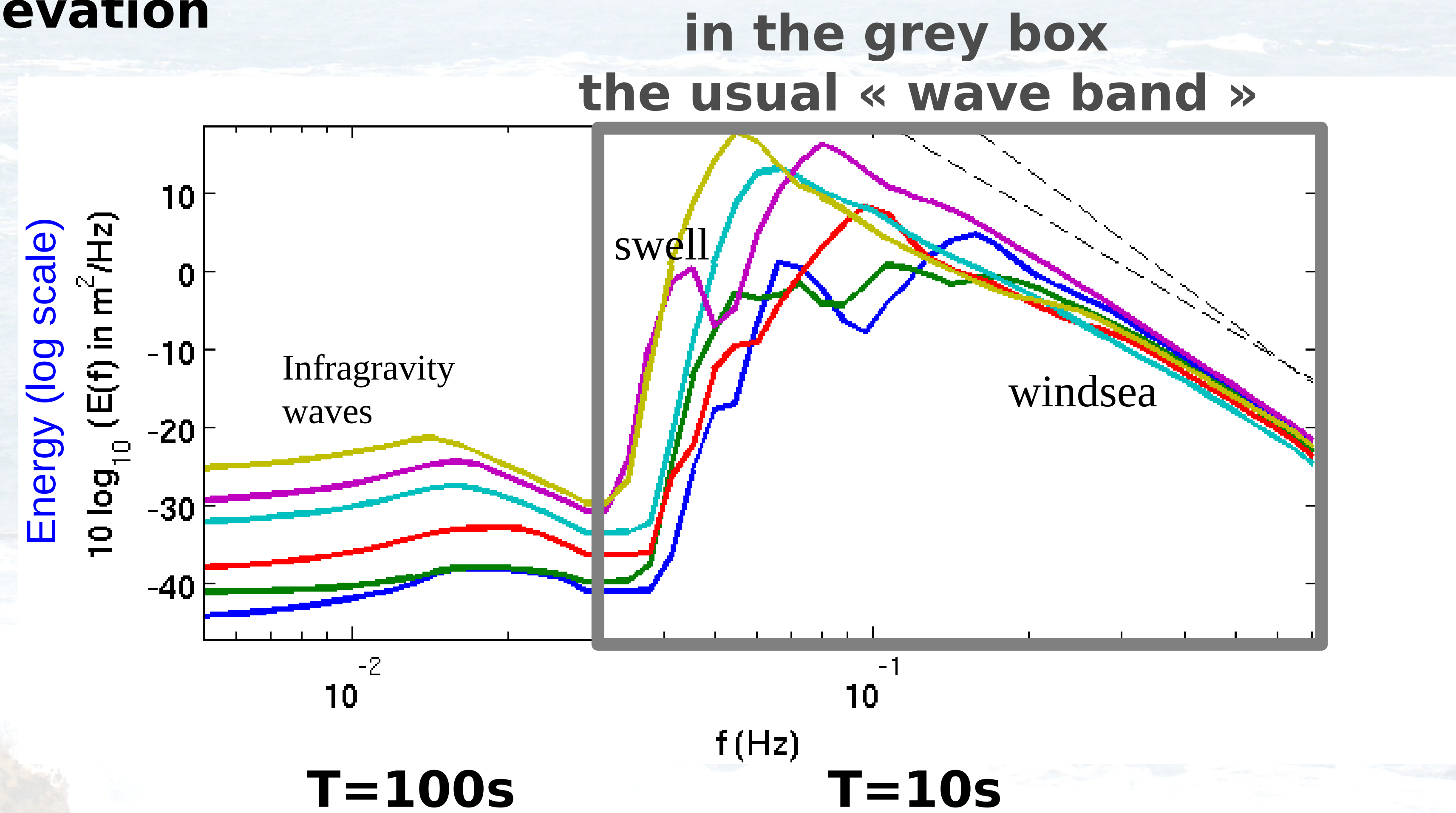


1.5 m 150 m 200 km Wavelength

(in the case of free waves)

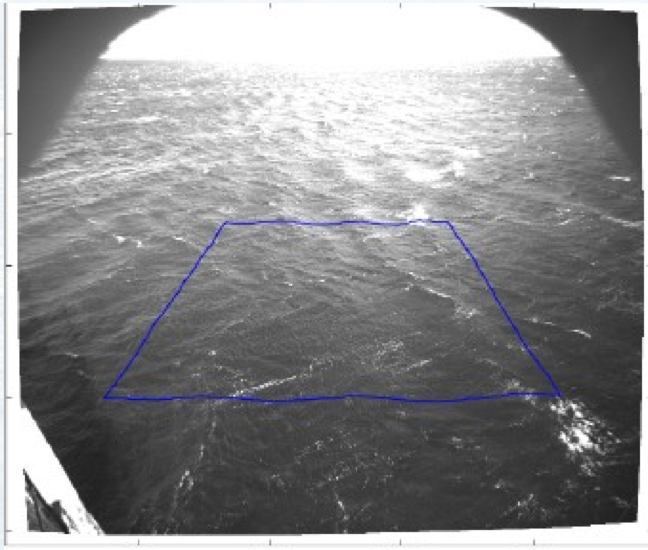
Time scales and frequency spectrum

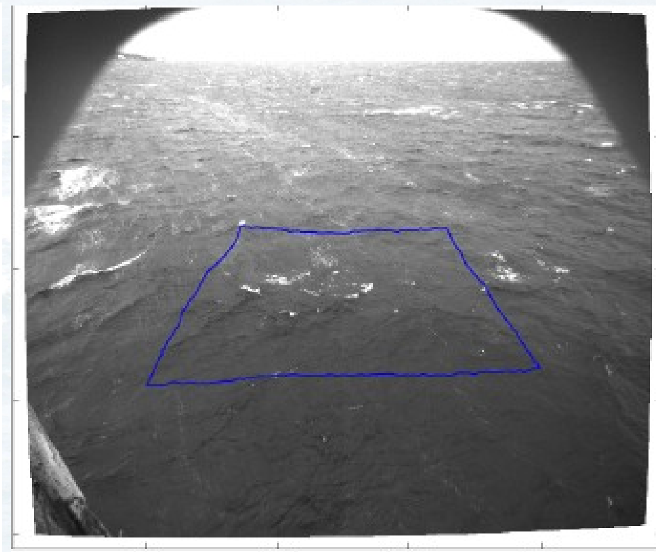
Some examples of power spectrum of surface elevation

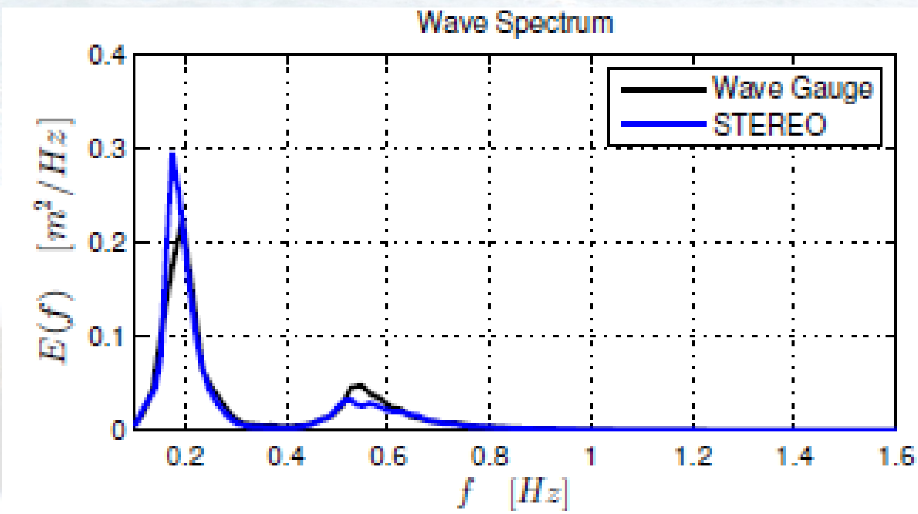


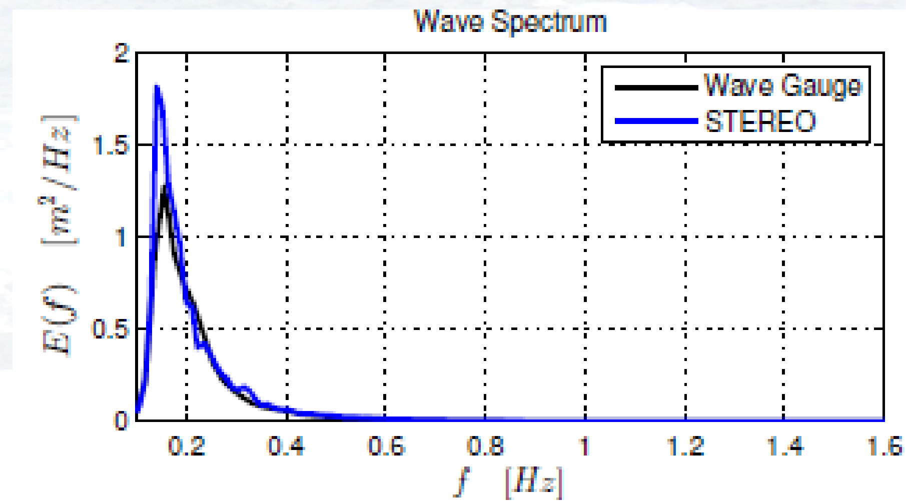
Time scales and frequency spectrum

Here are 2 examples, 1 day apart, Katsiveli platform, Black Sea

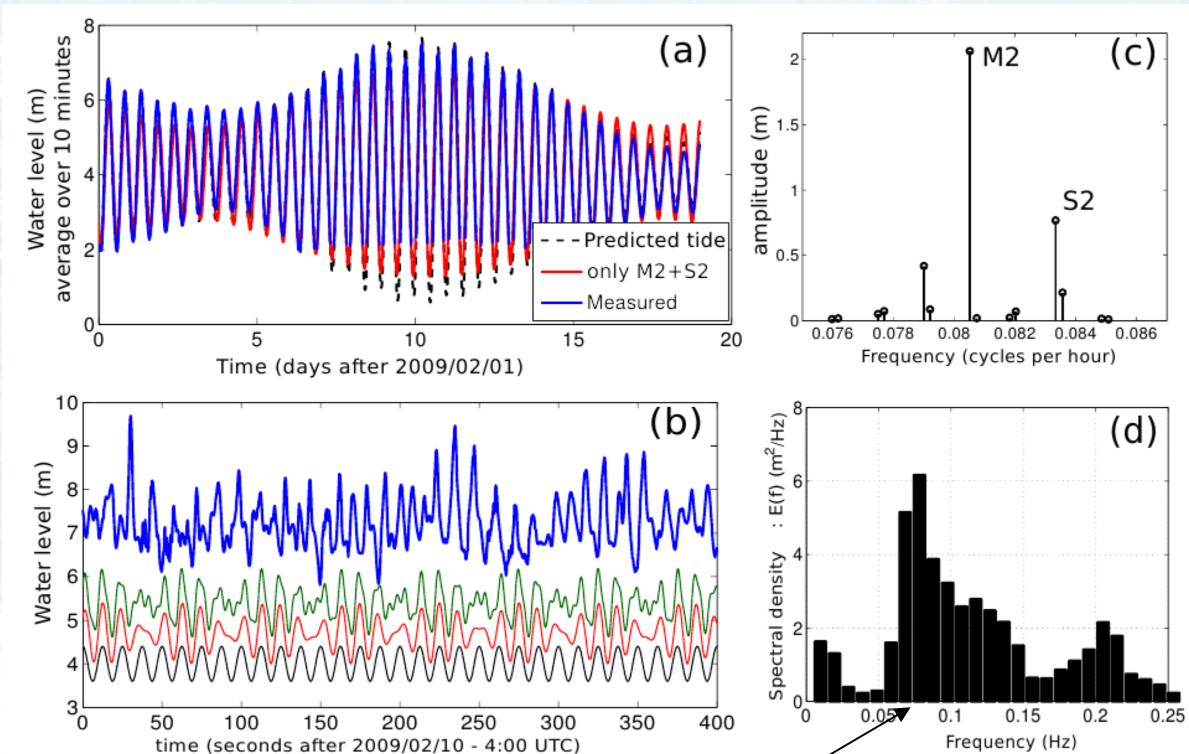








The spectrum is ...
a distribution of the **variance** across **scales**
For example **E** **f=1/T**



Tidal signal : discrete spectrum

Wave signal : continuous spectrum

E = sum (E(f) * df), **area in black** = variance of elevation

Definition of significant wave height : **Hs = 4 √E**

The 2D frequency-direction spectrum

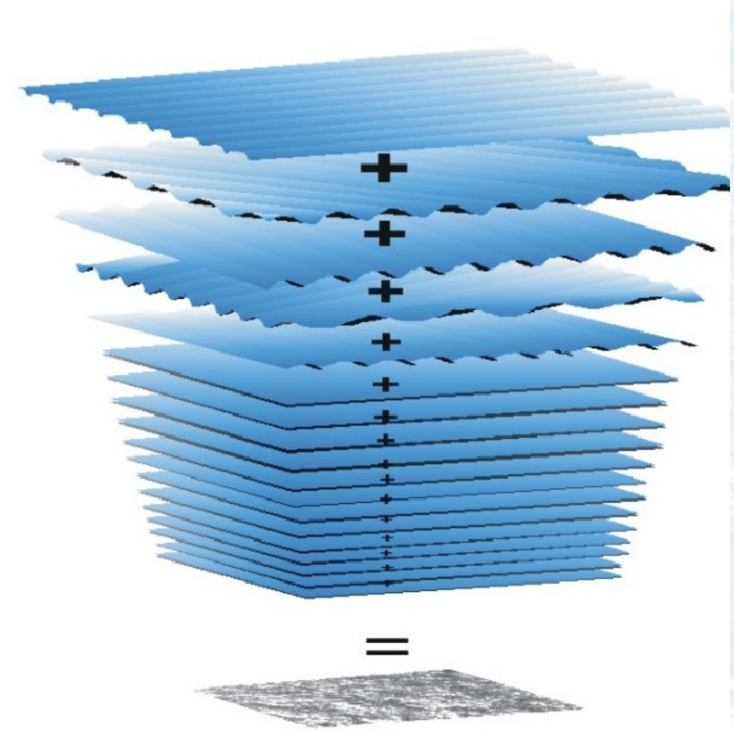
Because ζ is a function of x,y,t the complete spectrum is ζ(kx,ky,f)

For linear waves in homogeneous medium : 2D only

$$\zeta_m(x,y,t) = \sum_{i=1}^{N}\sum_{j=1}^{M} a_{m,i,j}\cos(2\pi f_i t - k_i\cos(\theta_j)x - k_i\sin(\theta_j)y + \Theta_{0,m,i,j})$$

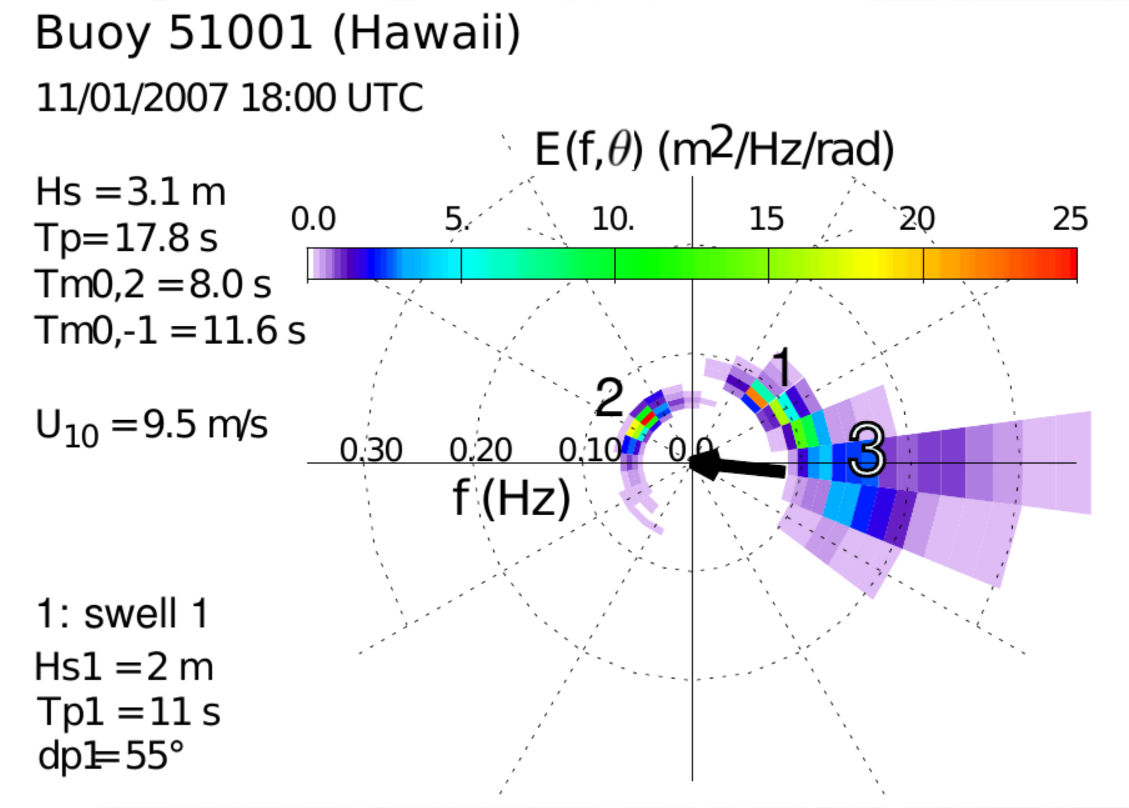
How much « energy » in each band of frequencies and directions ?

$$E(f,\theta) = \lim_{\Delta f\to 0}\lim_{\Delta\theta\to 0}\frac{1}{\Delta f\Delta\theta}\left\{\frac{1}{2}\rho g a_{i,j}^2\right\}$$



The 2D frequency-direction spectrum

Example of 2D spectrum :



This is what is computed in a model like WAVEWATCH III

→ watch animation

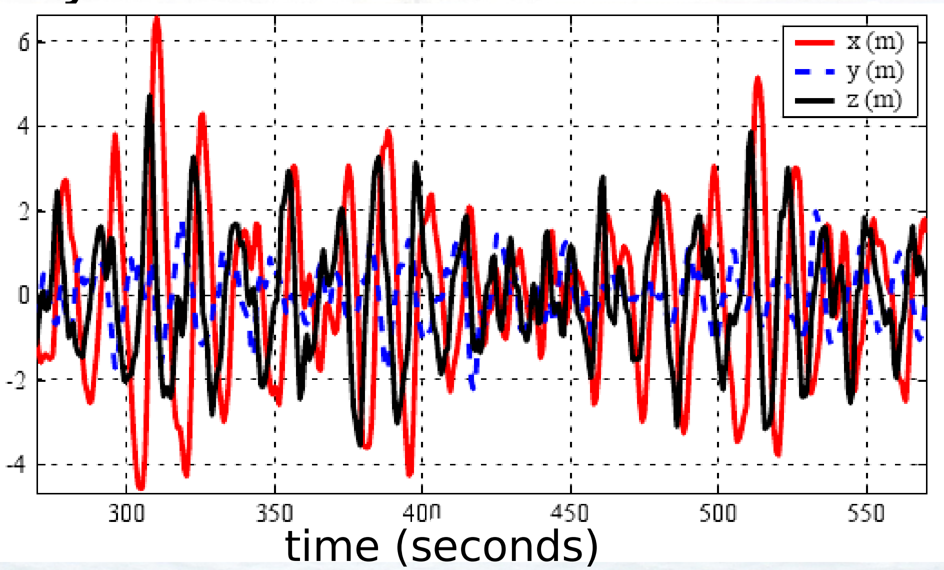
3

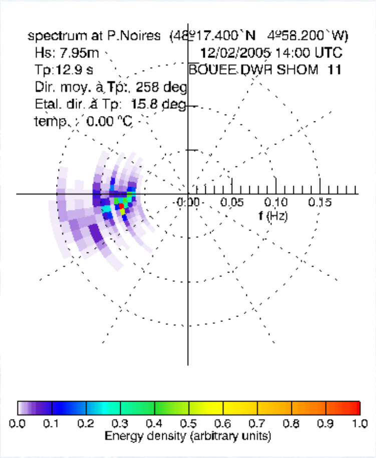
Measuring waves

Measuring $E(f, \theta)$?

With a single buoy ?

A datawell buoy

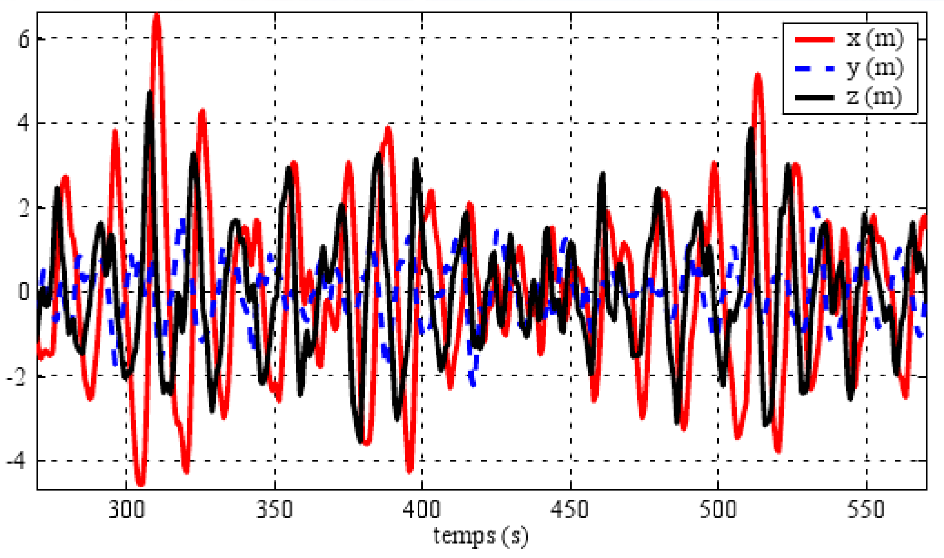




Corresponding $E(f, \theta)$
Using MEM (Maximum Entropy Method)

How does it work ?

Measuring $E\left(f,\theta\right)$?



Directional buoy → 3 time series :
x, y and z displacements

E(f) is spectrum of z : Czz(f)
Spectrum of x : Cxx(f)
Spectrum of y : Cyy(f)

Now we can also « mix » x, y and z to make co-spectra :
Cxz(f)
Cyz(f)
Cxy(f)

Co-spectra are the specta of the correlations, they have an amplitude and a phase

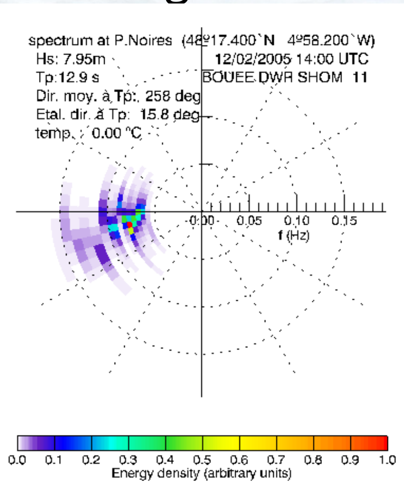
Measuring $E\left(f,\theta\right)$?

So we have 6 parameters for each frequency.
1 is the « check ratio » : (Cxx + Cyy)/Czz
5 define the directional distribution : the « first 5 »

E(f), a1(f), b1(f), a2(f), b2(f) ... or equivalently
E(f), th1(f), sth1(f), th2(f), sth2(f)

How do you get 36 numbers (or more) from just 5 ? ...
... statistical estimator.

- Most widely used : Max. Entropy Method (Lygre & Krogstad, 1986)
  - Makes the narrowest peaks that fit the data
  - Is able to detect 2 peaks at same frequency



From elevation to other parameters… and back : modulation transfer functions (MTFs)

Using linear wave theory, we can use $E\left(f,\theta\right)$ to determine other properties : velocities, slopes, pressure …

Conversely, spectra of other properties – when dominated by linear waves – can be used to estimate $E\left(f,\theta\right)$

$$A = M\zeta \quad \rightarrow \quad E_A(f,\theta) = |M|^2 E(f,\theta)$$

$$\zeta = a\cos\Theta,$$

$$\mathbf{u} = a\frac{\mathbf{k}}{k}\sigma\frac{\cosh\left(kz+kh\right)}{\sinh\left(kD\right)}\cos\Theta,$$

$$w = a\sigma\frac{\sinh\left(kz+kh\right)}{\sinh\left(kD\right)}\sin\Theta,$$

$$p = \overline{p}^H + \rho_w g a\frac{\cosh\left(kz+kh\right)}{\cosh\left(kD\right)}\cos\Theta$$

without currents currents :

$$\sigma^2 = gk\tanh\left(kD\right)$$

Warning : current effects

$E(f, \theta)$, as estimated from buoy data,

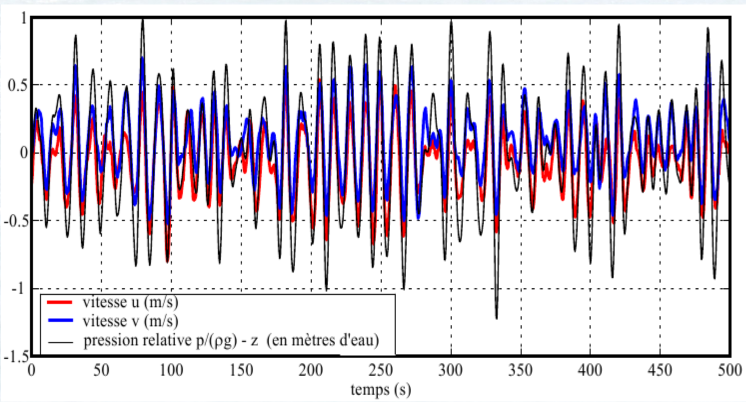
is different from $F(f_r, \theta)$ coming out of WAVEWATCH,

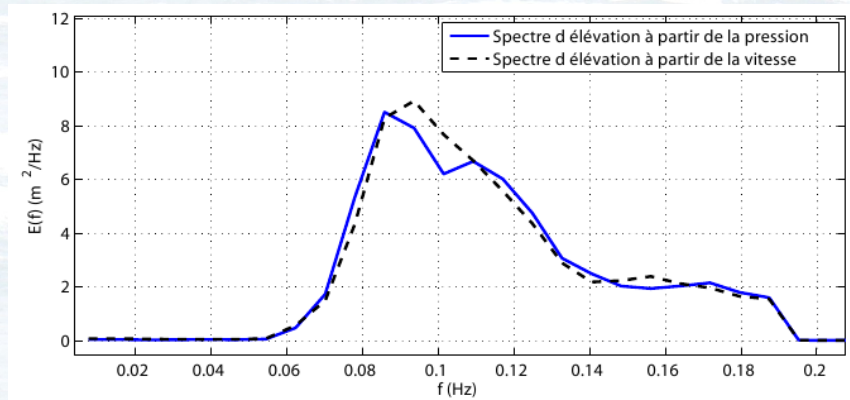
because $\omega = \sigma + \mathbf{k} \cdot \mathbf{U}$

ω = 2 pi / f σ = 2 pi /f_r

Measurin$E\left(f,\theta\right)$?

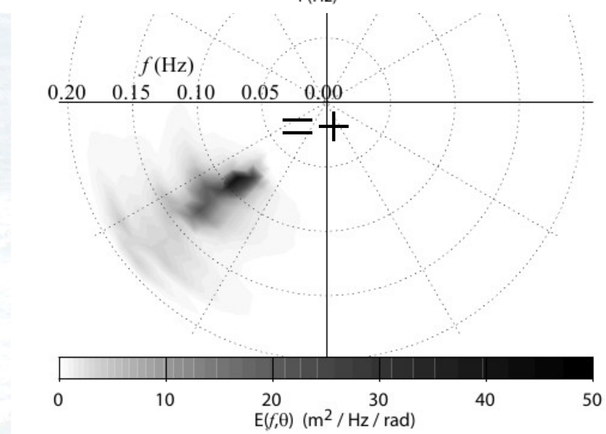
Using bottom pressure and velocity : again 6 spectra + co-spectra





$$\mathbf{u} = a\frac{\mathbf{k}}{k}\sigma\frac{\cosh\left(kz+kh\right)}{\sinh\left(kD\right)}\cos\Theta$$

$$p = \overline{p}^{H} + \rho_w g a\frac{\cosh\left(kz+kh\right)}{\cosh\left(kD\right)}\cos\Theta$$



Summary

1) **2D wave** spectrum represents distribution of wave « energy » among linear waves

2) **2D spectrum** gives statistics of the full 4D wave field, assuming :
- Linear waves
- Irrotational motion
- Homogeneous wave field

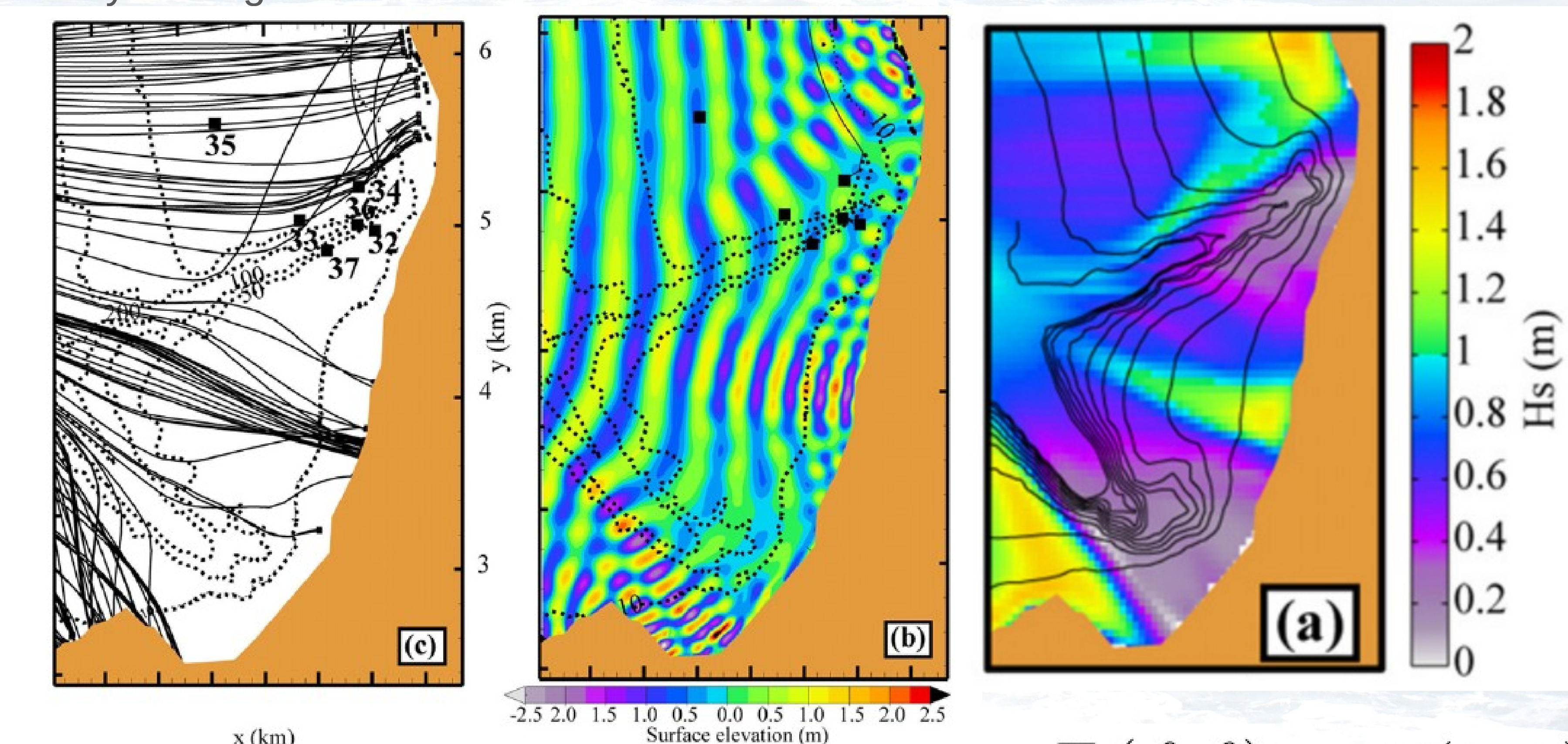
3) Buoys do not measure a full **2D spectrum**, but only 5 parameters at each freq.

4) linear wave theory gives simple transfer function from, e.g. bottom pressure to surface elevation : this is why you can (or can't) measure waves with a bottom pressure recorder.

Additional remark : non-homogeneity

You can look at wave spectra evolution at scales ***shorter*** than a wavelength :
This example is from the WISE Group (2007)

Ray-tracing MMSE model SWAN or WW3



However, in that case, there is no simple link between $E(f,\theta)$ and $F(k,\theta)$