A Comparison of Two Methods for Determining Wave Heights from a Discus Buoy with a Strapped-Down Accelerometer

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I. INTRODUCTION

On August 29, 2005 the eye of Hurricane Katrina passed 49 nm to the west of a 3-m discus buoy moored in the Mississippi Sound. Buoy motions were measured with a strapped down 6 degree of freedom accelerometer, a 3-axis magnetometer, and a survey grade GPS receiver from which wave measurements were made. On September 12, 2008 the eye of Hurricane Ike passed over the top of a 2.25-m discus buoy moored on the Texas Continental Shelf in the vicinity of the Flower Garden Banks National Marine Reserve. Buoy motions were measured with a strapped down 6 degree of freedom accelerometer and a 3-axis magnetometer. Both buoys operated for the entire storm and provided a continuous record of the rapidly evolving sea state.

Computing wave heights from any accelerometer record begins by first recognizing the sensor experiences an offset due to gravity, then by implementing a method to remove the measurement of gravity from the data. Any orientation of the accelerometer that is not vertical complicates the situation by putting a component of gravity into each of the three axes. Even an internally gimbaled accelerometer must remove gravity from its acceleration measurements. The raw data records from the strapped-down, 3-axes linear accelerometers on both buoys allowed us to investigate two methods for removing the gravity offset, including how it is typically done for a strapped down 1D accelerometer. Compared to the method that mathematically gimbals the accelerometer output to an earth-referenced vertical acceleration, the strapped down 1D (deck-relative) acceleration overestimated the wave heights by 57% for Katrina and by 27% for Ike. The sustained heel of the buoy, due to wind forcing on the superstructure, is deemed to be a likely cause of the bias. This bias is subsequently confirmed in tests on a Hippy 40 and three solid-state accelerometers conducted on the Ocean Wave Instrumentation Facility at the National Data Buoy Center in Stennis Space Center, MS.

The purpose of this paper is to describe the instrument setup of the buoys, the data obtained, and the methods used to process the accelerometer data into significant wave heights and periods. We will describe wave records during hurricanes Katrina and Ike and use this data to show the potential for the heel of a discus buoy to bias the calculated wave heights from a fixed, one-axis accelerometer.

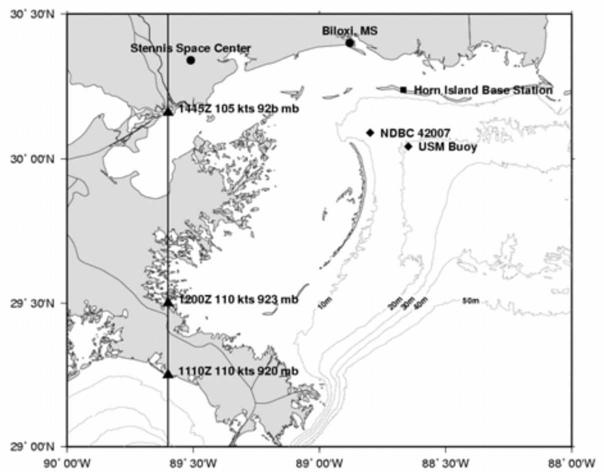

Figure 1. Location of the 3-m discus buoy in the Mississippi Sound and the path of Hurricane Katrina on August 29, 2005.

II. INSTRUMENTATION

Both buoys were independently built by the Geochemical and Environmental Research Group (GERG) at Texas A&M University. The 3-m buoy's system design, electronics, and sensor integration are fully described in [1] & [2], which describes the instrument setup of the buoy, the motion sensor data obtained, and the four methods used to process the accelerometer and compass data into wave heights and periods. The 2.25-m buoy's design and instrumentation is generally similar to that of the 3-m buoy, with the exception of an upgrade in the model of the accelerometer. The data processing methods are identical.

Both buoys were equipped with a Crossbow accelerometer and a Honeywell HMR compass to measure three-dimensional motion. The 3-m buoy also had a Novatel OEM4-g2 GPS for measuring vertical motion, which is described in [2] and is not the subject of this paper, though we note that it confirms what we present here. A PC104 UNIX-based central computer directed the sampling strategy and saved the raw data to an onboard hard drive, which were retrieved after the buoys were recovered for routine maintenance.

The 3-m buoy used an older Crossbow IMU 400CC series accelerometer and the 2.25-m buoy used a newer Crossbow IMU 440 series accelerometer. Both accelerometers are solid state measurement systems designed to measure the linear acceleration along three orthogonal axes and the rotation rates around the same three orthogonal axes. The unit was not gimbaled, but was mounted (strapped-down) inside the system controller housing within the instrument well of the buoy. The instrument has an update rate of greater than 100 Hz, but was sub-sampled to 4 Hz. The sub-sampled data were time stamped by the buoy's central computer as the raw data were saved to the data base. The major difference in the two accelerometers was the orientation of the positive *z*-axis; in the 400 series it is oriented up, contrary to IEEE convention, but in the newer 440 series it is oriented down. Given proper care of the signs, the non-standard orientation of the 400 series does not affect the final results.

The Honeywell HMR 3300 digital compass is a solid state 3-axis, magnetometer-based compass that uses an internal two-axis accelerometer for enhanced operation. This electronically gimbaled compass gives accurate headings even when the compass is tilted at 60˚. The compass is capable of data rates up to 8 Hz, but was sub-sampled to 4 Hz. The sub-sampled heading, pitch and roll data were time stamped by the central computer after the data were acquired. Additional details of the Crossbow accelerometer and the Honeywell compass are found in [2].

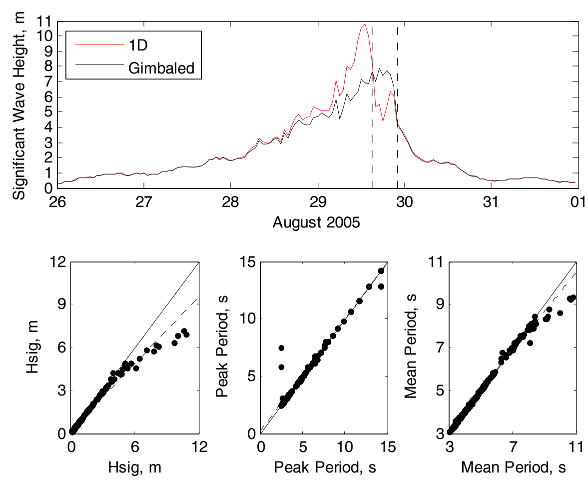

Figure 2. The top panel shows the time series of the FFT spectra determined significant wave heights for the 1D accelerometer versus the mathematically gimbaled accelerometer for the period from August 26th through September 1st 2005 during the passage of Katrina. The vertical dotted lines denote the time period the buoy was moving. The bottom panel shows the scatter plot of the significant wave height, peak period, and mean period for the 1D accelerometer (horizontal axes) and the mathematically gimbaled accelerometer (vertical axes). The line of perfect agreement is shown as a solid line and the least squares linear fit as a dotted line.

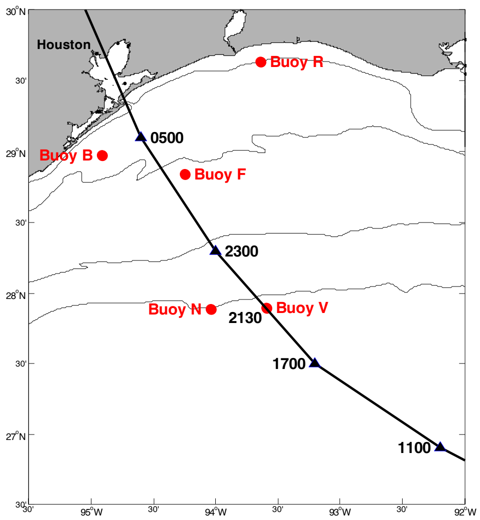

Figure 3. The track of Hurricane Ike across the Texas shelf from 1000 UTC on September 12, 2009 to 2200 UTC on September 13, 2008. The location of the Texas Automated Buoy System (TABS) buoys are shown in red. The 2.25 m discus buoy, designated as buoy V, recorded a local minimum in wave heights at 2130 UTC on September 12th as the eye passed over the buoy. The 10-, 20-, 50- and 100-m depth contours are shown. All times are in UTC.

III. PROCESSING

The specific details of how the accelerometer data were processed into significant wave height, peak period and mean period are discussed at length in [2]; we emphasize the methods are identical for both buoys. In brief, the removal of the effects of gravity from the accelerometer data is not a trivial step. Since we know an accelerometer responds with an upward acceleration in reaction to gravity pulling it down, the method for removing the component of gravity along each of the sensor's three axes is crucial, and particularly so when the buoy is heeled over during the wave ensemble time.

The 3-axis linear accelerometer data of the Crossbow allowed us to investigate several different methods for estimating and then removing gravity. Of the five possible correction methods presented in [2], here we used the deck relative or mast acceleration (Method I) because it is used for many strapped-down 1D accelerometers, and the earth-referenced vertical acceleration (Method IV), because it is mathematically similar to what a gimbaled accelerometer such as a Hippy 40 would measure. Method I, which we henceforth will refer to as the *1D accelerometer*, calculates the vertical acceleration time series by assuming the pitch and roll of the buoy are small enough so that gravity only contaminates the *z*-axis of the accelerometer. Gravity is then removed by simply subtracting *g* from the measurements, giving the deck-relative acceleration. This method leads to an underestimation of the waves when the mean of the buoy's tilt is near zero, but when the buoy has a sustained heel caused by wind forces on the superstructure, conditions expected during storms, it can lead to an overestimation of the wave heights. Method IV, which we henceforth will refer to as the *mathematically gimbaled accelerometer*, uses the accelerations from all three axes and the pitch and roll information to obtain the true earth-referenced vertical accelerations of the buoy. This correctly removes gravity from each of the three axes and it is not affected by the buoy heel.

Once corrected for gravity, the vertical acceleration time series data were then processed to remove outliers; followed by a Kalman filter to remove instrument and process noise. The acceleration spectra of the filtered data were calculated with a Fast Fourier Transform (FFT) of the vertical displacement data. The details of the segmenting and windowing are described in [2]. A frequency domain filter was applied to the acceleration spectra in order to remove low frequency noise, also described in [2]. The acceleration spectra were then converted to the displacement spectra. The significant wave height, peak period and mean wave period were determined from the displacement spectra.

IV. RESULTS

Hurricane Katrina

On August 29, 2005 at approximately 1400 UTC the eye of hurricane Katrina passed 49 nautical miles (nm) to the west of a 3-m discuss buoy operated by

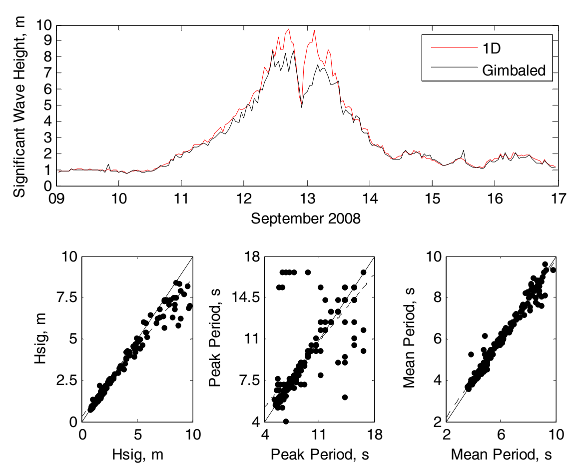

Figure 4. The same as Fig. 2, except for the 2.25 m buoy for the period from September 9th through September 17th 2008 during the passage of Ike.

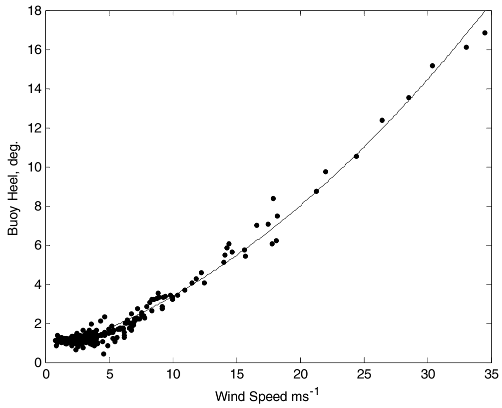

Figure 5. The relationship between wind speed and buoy heel measured by the 3-m discus buoy during the passage of Katrina. The solid line is the parabolic best fit.

the Central Gulf of Mexico Ocean Observing System. The buoy was moored in a depth of 19 m and its location is shown in Figure 1. The buoy and mooring, with an 8500 lb. in air concrete anchor, were moved slightly to the northeast during the final approach of Katrina, but following the hurricane's landfall, and from what would appear to be a direct result of the storm surge relaxation, the buoy was relocated to the southeast during the eight hours from 1500 to 2300 UTC. Currents were calculated to have reached nearly 5 knots. The buoy's onboard accelerometer and magnetometer operated continuously throughout the storm. The buoy experienced a maximum heel of 17^o during Katrina that was directly recorded by the magnetometer and indirectly measured by the pitch and roll rate sensors of the accelerometer. The heel is defined as the 20-minute average of the buoy's instantaneous tilt. The instantaneous tilt of the buoy caused by the changing slope of the waves was considerably greater, sometimes reaching nearly 45^o.

Figure 2 shows a comparison of significant wave heights for the 1D accelerometer versus the mathematically gimbaled accelerometer. There is virtually no visual difference in wave heights less than 3 m, but there is a marked difference in larger wave heights. Figure 2 also shows (bottom panel) the scatter plots for significant wave height, peak period and mean period. This comparison specifically excludes any data recorded during the time period the buoy was moving. The linear least squares fit of significant wave heights has a slope of 0.779, an intercept of 14.7 cm, and an r^2 correlation of 0.969. The r^2 correlation for the peak period is 0.985; the slope of the least squares linear fit is 0.976. The r^2 correlation for the mean period is 0.989; the slope of the least squares linear fit is 0.919. The major difference between the two methods is the total energy in the spectra, but because the peak period and mean period are nearly identical the spectral shapes are self-similar.

Hurricane Ike

On September 12, 2008 at approximately 2130 UTC the eye of hurricane Ike passed over the top of a 2.25-m discus buoy operated by the Texas Automated Buoy System. The buoy was moored in a depth of 89 m. The location of the buoy and the track of hurricane Ike are shown in Figure 3. The buoy's onboard accelerometer and magnetometer operated continuously throughout the storm and provided real-time wave information. The buoy experienced a maximum heel of 17^o during Ike that were directly recorded by the magnetometer and indirectly measured by the pitch and roll rate sensors of the accelerometer.

Figure 4 shows a comparison of significant wave heights for the 1D accelerometer versus the mathematically gimbaled accelerometer. There is virtually no visual difference in wave heights less than 3 m, but there is a noticeable difference in larger wave heights. In the eye, where the wind speed dropped from 24.5 to 11.5 m s^{-1} and the buoy heel decreased from 17^o to only 4^o, the wave heights are identical. Figure 4 also shows (bottom panel) the scatter plots for significant wave height, peak period and mean period. The linear least squares fit of significant wave heights has a slope of 0.844, an intercept of 23.6 cm, and an r^2 correlation of 0.970. . The r^2 correlation for the peak period is only 0.616; the slope of the least squares linear fit is 0.804. The r^2 correlation for the mean period is 0.971; the slope of the least squares linear fit is 0.952. Obviously the major difference between the two methods is the total energy in the spectra, but unlike the Katrina spectra we are also seeing differences in the peak frequency.

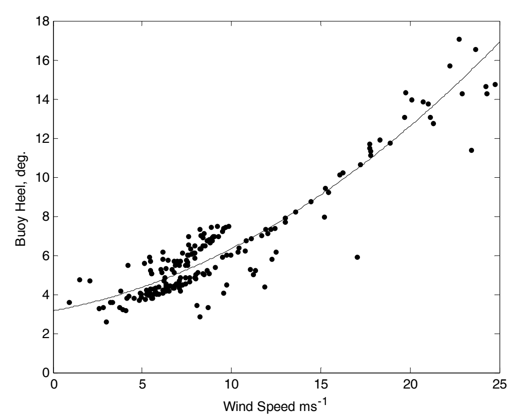

Figure 6. Same as Fig. 5, except for the 2.25-m discus buoy during the passage of Ike.

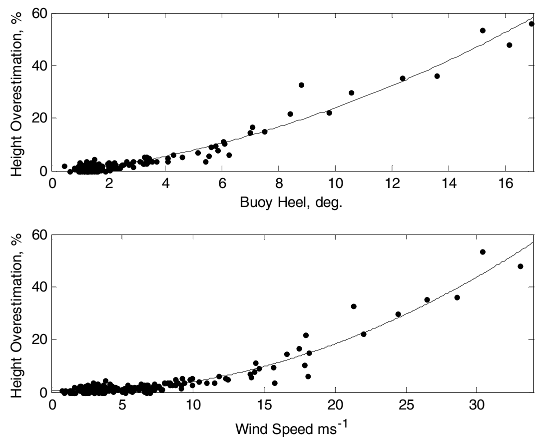

Figure 7. The empirical relationship between the heel of the buoy, the wind speed, and the percent over prediction in wave heights for the 3-m discus buoy during the passage of Katrina. The solid line is the parabolic best fit.

V. DISCUSSION

The fact that the wind acting on a buoy's superstructure causes it to heel should be of no surprise [3]. Figure 5 shows the empirical relationship between the 10-min averaged wind speed recorded by the buoy and the buoy's heel for the 3-m discus buoy that experienced Katrina. Figure 6 shows the same for the 2.25 m discus buoy that experienced Ike. Both buoys experienced a maximum heel of only 17^{o}. Because of the greater inertia and righting moment of the 3-m buoy, it required a 35 m s^{-1} wind to achieve the same tilt as the 2.25 m buoy did in a 25 m s^{-1} wind. Of greater interest is the relationship between wind speed, buoy heel and the difference in wave heights between the 1D accelerometer and the mathematically gimbaled accelerometer. These relationships are shown in Figures 7 and 8 for Katrina and Ike respectively.

Figures 7 and 8 can be summarized in two points:

- the larger the heel, the greater the deviation in wave heights
- the shallower the water depth the greater deviation in wave heights, but only when the buoy is heeled.

A simple means of understanding the first point can be explained with an accelerometer that reads a negative one g (*z*-axis down) when the accelerometer is stable. When installed on a pitch and roll buoy, a constant heel results in a positive offset of the accelerometer's *z*-axis component of gravity. The standard conversion to a deck relative acceleration, i.e., add one g to the *z*-axis measurements, simply means that the heel induced offset is seen as a DC upward acceleration superimposed on an AC component. When the heel is large, the amplitude of the AC component can lead to an over-estimation of the wave heights.

Recent testing at the National Data Buoy Center's (NDBC) Ocean Wave Instrumentation Facility (OWIF), courtesy of Mr. Theodore "Ted" Mettlach, illustrates this explanation. The OWIF is a Ferris-wheel like test assembly that was designed in the 70's for the pre-deployment testing of accelerometer packages such as Datawell's Hippy 40, but is now being used to also evaluate solid state accelerometers. The OWIF consists of a motor-driven one-meter, lever arm that rotates a platform in a circular motion. The one meter amplitude corresponds to a theoretical significant wave height of 2.83 m. The period of rotation can be controlled manually from 4.5 to 45 s. Like the passenger compartment on a Ferris wheel, the platform remains horizontal, unless it is mechanically forced by cam action to tilt as it rotates. This tilt simulates the changing surface slope of a wave.

In this test series, a Hippy 40 and three solid state accelerometers were bolted to a plate with a 20^{o} heel that was in turn bolted to the platform. In this manner we simulated a buoy heel expected for high winds. An extensive range of periods and tilt angles were simulated for the sole purpose of testing sensors and verifying the kinematics describing the OWIF motion. Here we show a period of 15 s and a tilt angle of 8^{o}. A one-m amplitude Stokes wave with a 15 second period would have a maximum slope of 4.1^{o} so this experimental set-up represents unrealistic conditions. Nonetheless, it clearly illustrates our explanation that the amplitude of the AC component can lead to an over-estimation of the wave heights when the buoy is heeled. Figure 9 shows the vertical acceleration for the internally gimbaled Hippy 40, after *g* is removed from the measurements, and the

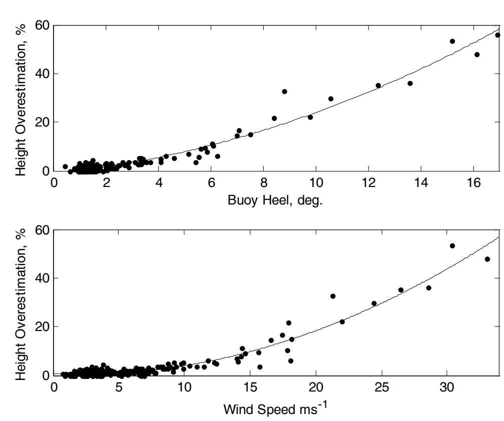

Figure 8. Same as Fig. 7, except for the 2.25-m discus buoy during the passage of Ike.

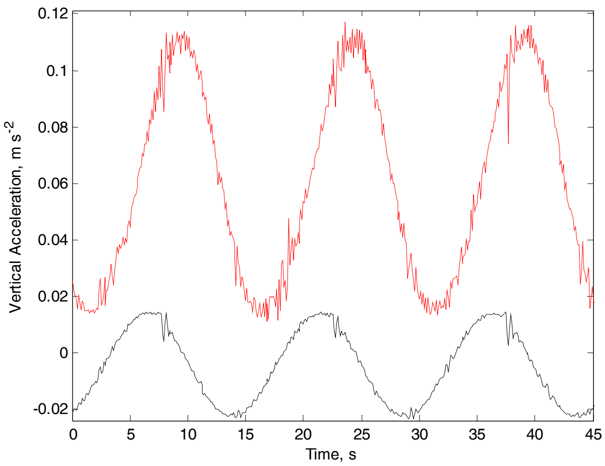

Figure 9. The measured vertical acceleration for the OWIF simulation run of a one-m amplitude wave with a period of 15 s, a maximum wave slope angle of 8°, and a buoy heel of 20°. The Crossbow 1D accelerometer is shown in red and the Hippy 40 is in black.

1D acceleration computed from the *z*-axis of the strapped-down Crossbow IMU 440. The large amplitude of the 1D acceleration is a direct result of the 20° heel and the 8° tilt. The corresponding significant wave height for the Hippy 40 is 2.75 m and for the Crossbow, 7.36 m. This is after the time series is demeaned, removing the DC component. After the data for the Crossbow has been mathematically gimbaled (see Figure 10), the Crossbow and Hippy measure the same vertical acceleration. The significant wave height for the Crossbow is 2.72 m.

The second point, that the overestimation of wave heights is a function of water depth as well as heel,

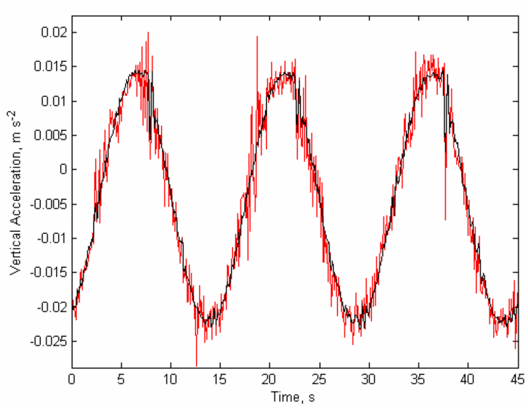

Figure 10. The measured vertical acceleration for the OWIF simulation run of a one-m amplitude wave with a period of 15 s, a maximum wave slope angle of 8°, and a buoy heel of 20°. The mathematically gimbaled Crossbow accelerometer is shown in red and the Hippy 40 is in black.

can be explained as a simple consequence of shoaling. The surface slope of a wave increases as the wave enters shallower water, but in deeper water the slope is less. The typical method of removing gravity from a strapped-down, 1D accelerometer is affected by the magnitude of the tilt, the less the tilt the less the over-estimation.

VI. CONCLUSIONS

We conclude with three points:

- A strapped-down, solid state accelerometer can give comparable wave heights and periods to an internally gimbaled accelerometer if the orientation of the strapped-down accelerometer is properly accounted for.

- Using a pitch-roll buoy with a 1D strapped-down accelerometer to measure individual, large wave events in shallow water could lead to overestimated wave heights.

- Deep water discus buoys with large diameter hulls and strapped-down 1D accelerometers are largely unaffected by this bias. On the other hand, shallow water, smaller diameter buoys with a strapped-down 1D accelerometer can be significantly affected.

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