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Contributions to the wave-mean momentum balance in the surf zone



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ABSTRACT

Mean (wave-averaged) cross-shore flow in the surfzone has a strong vertical variation. Good understanding and prediction of this mean velocity profile is of crucial importance, as it determines the advective transport of constituents, such as sediment, and consequently the coastal morphological evolution. Most modeling systems for coastal hydrodynamics and morphodynamics do no resolve the wave motion, and wave-current coupling is a challenging topic. This paper investigates stresses and forces that control mean surfzone hydrodynamics based on detailed wave flume velocity measurements above a fixed sloping bed including two breaker bars. The data show that the vertical distribution of normal stress below the wave trough level is fairly uniform. At the same time, the data suggest that a significant part is concentrated between the wave trough and crest level. Furthermore, it is concluded that the horizontal radiation stress gradients and the vertical shear stress radients can be of the same order of magnitude in the vicinity of the breaker bar. Although usually ignored in 3D mean flow modeling systems, the wave trough level could be reasonably well predicted using the classical [16] expression, accounting for the contribution between wave crest and trough. The model of [39] reproduces the main trends in the wave Reynolds stresses above the bottom boundary layer.

1. Introduction

Mean (wave-averaged) surfzone hydrodynamics are strongly affected by the presence of waves. Waves generate currents through mean transport of mass and momentum.

The mean and depth-integrated horizontal momentum transport caused by the waves only is known as radiation stress (see e.g. [16,28]). For a uniform coast the cross-shore variation in the cross-shore component of the radiation stress tensor is responsible for the setdown and setup of the mean water level. The cross-shore gradient of the longshore radiation stress component is the driving force for longshore currents.

The cross-shore radiation stress gradient is not uniformly distributed over the water depth under breaking waves; it is higher near the surface. The opposing pressure due to the water level gradient has a (nearly) uniform vertical distribution. This results in a seaward waveaveraged current near the seabed (undertow) and onshore flow higher in the water column in the inner surfzone (see e.g. [19]).

The mean vertical fluxes of horizontal momentum have a turbulence contribution, the turbulent Reynolds stress, and a direct contribution due to the wave orbital motion, also known as wave Reynolds stress.

The wave Reynolds stress can yield a non-zero mean value when the horizontal and vertical orbital motions are not exactly 90° out of phase due to bed friction, bed slope or wave breaking effects (see e.g. [6,39,12]). This wave-averaged shear stress leads to a small near-bed mean current (wave boundary streaming) that is generally onshore-directed [15].

This process acts opposite to the net current generated in a turbulent bottom boundary layer by a velocity-skewed or acceleration-skewed oscillation (wave shape streaming). This near-bed current is generated by a non-zero wave-averaged turbulent stress, due to the different characteristics of the time-dependent turbulence during the on- and offshore phase of the wave [35,21].

Due to the above-described effects, the mean horizontal current within the surfzone has a strong variation in the vertical direction. Better understanding of this mean current profile is of crucial importance for a better understanding and prediction of the advective transport of constituents, such as suspended sediment, and consequently the coastal morphological evolution.

Most modeling systems for ocean and coastal hydrodynamics and morphodynamics (e.g. Delft3D and ROMS) do no resolve the wave

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motion, and wave-current coupling is a challenging topic. Many theoretical approaches and implementations have been proposed for this (see for a review [2]).

In this paper we will investigate the stresses and forces that control mean surfzone hydrodynamics based on detailed wave flume velocity measurements above a fixed sloping bed including two breaker bars [3]. This paper distinguishes itself from other experimental studies (e.g. [24,33,34,30,31,7,8,38]) by the focus on the controlling forces, the level of detail of the measurements and the inclusion of breaker bars in the bed profile. An important aim of this paper is to provide insight in the contributions to the momentum balance that should be accounted for in 3D coastal modeling systems.

The paper is organized as follows. Section 2 presents the mean momentum balance. The experimental set-up, measurements and data-processing are described in Section 3. Section 4 discusses the experimental results. The discussion and conclusions are presented in Section 5.

2. Mean momentum balance

2.1. Depth-dependent

We can decompose the velocities and pressure in a turbulent, orbital and wave-mean part, for example

$$u = \overline{u} + u' = \langle u \rangle + \widetilde{u} + u' \tag{1}$$

for the horizontal velocity where --- means averaging over the turbulent timescale and $\langle ... \rangle$ over the wave timescale. We can then derive the wave-averaged 2DV momentum equation in the horizontal x-direction (see e.g. [19]):

$$\frac{\partial}{\partial x} [\rho \langle u \rangle^2] + \frac{\partial}{\partial z} [\rho \langle u \rangle \langle w \rangle] = -\frac{\partial \langle p \rangle}{\partial x} - \rho \frac{\partial}{\partial x} [\langle \overline{u'^2} \rangle + \langle \widetilde{u}^2 \rangle] - \rho \frac{\partial}{\partial z} [\langle \overline{u'w'} \rangle + \langle \widetilde{u}\widetilde{w} \rangle]$$
(2)

in which ρ is the water density, *u* the velocity in *x*-direction, *w* the velocity in z-direction, p is pressure and g the acceleration due to gravity. This equation ignores temporal variation, viscous stresses and other body forces than gravity.

[28] derived the following expression for the wave-averaged pressure:

$$\langle p \rangle = \rho g \left(\langle \zeta \rangle - z \right) - \rho \langle \widetilde{w}^2 \rangle - \rho \langle \overline{w'}^2 \rangle \tag{3}$$

by vertical integration of the 2DV momentum equation in z-direction. $\langle \zeta \rangle$ is the wave-averaged water surface elevation. This expression ignores the contribution due to $-\rho \langle w \rangle^2$ and the wave-mean of the horizontal derivative of the vertical integral of shear stresses, as these are generally small. If we combine Eqs. (2) and (3) we get:

$$\frac{\partial}{\partial x}(\rho\langle u\rangle^2) + \frac{\partial}{\partial z}(\rho\langle u\rangle\langle w\rangle) = -\rho g \frac{\partial\langle \zeta\rangle}{\partial x} + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{xz}}{\partial z}$$
(4)

with

$$\sigma_{xx} = -\rho(\langle \widetilde{u}^2 \rangle + \langle \overline{u'^2} \rangle - \langle \widetilde{w'^2} \rangle - \langle \overline{w'^2} \rangle)$$
(5)

the mean normal stress and

 $\tau_{xz} = -\rho\left(\langle \widetilde{u}\widetilde{w} \rangle + \langle \overline{u'w'} \rangle\right)$

the mean shear stress.

2.2. Depth-integrated

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In case of mild surface and bedslope, the time-averaged, depthintegrated momentum equation reads (see e.g. [28]):

$$\rho \frac{\partial}{\partial x} \left\langle \int_{-d}^{\zeta} u^2 dz \right\rangle = -\rho g h \frac{\partial \langle \zeta \rangle}{\partial x} - \frac{\partial S_{xx}}{\partial x} + \langle R_x^s \rangle - \langle \tau_{bx} \rangle$$
(7)

in which z = -d is the bed level, $h = (\langle \zeta \rangle + d)$ the mean water depth, $\langle R_x^s \rangle$ the mean stress at the surface in *x*-direction and $\langle \tau_{bx} \rangle$ the mean bed shear stress in x-direction. S_{xx} is the radiation stress, i.e. the excess flux of momentum due to the presence of waves (including turbulent contributions):

$$S_{xx} = \left\langle \int_{-d}^{\zeta} (\rho \widetilde{u}^2 + \rho \overline{u'^2} + p) dz \right\rangle - \frac{1}{2} \rho g h^2$$
$$= \left\langle \int_{-d}^{\zeta} (\rho \widetilde{u}^2 + \rho \overline{u'^2} - \rho \widetilde{w'^2} - \rho \overline{w'^2}) dz \right\rangle + \frac{1}{2} \rho g \langle \eta^2 \rangle$$
(8)

using the time-dependent variant of Eq. (3) and with $\eta = (\zeta - \langle \zeta \rangle)$ the water level variation due to wave motion.

[16] derived the following expression for the radiation stress (without turbulence) using linear wave theory for \tilde{u} , \tilde{w} and η , and ignoring higher order terms $(O(kh)^3$, with k the wave number):

$$S_{\rm xx,LHS} = \left(2n - \frac{1}{2}\right)E\tag{9}$$

in which $n = c_g/c$ with c_g the wave group celerity and *c* the wave celerity, and with $E = 1/8\rho g H^2$ the wave energy with H the wave height.

We can see the similarity between Eqs. (8) and (5). The difference appears in the second term on the RHS of Eq. (8) which is the hydrostatic pressure contribution due to the presence of waves:

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$$\left\langle \int_{\langle \zeta \rangle}^{\zeta} \rho g \left(\zeta - z \right) dz \right\rangle = \left\langle -\frac{1}{2} \rho g \left(\zeta - z \right)^2 \Big|_{\langle \zeta \rangle}^{\zeta} \right\rangle = \left\langle \frac{1}{2} \rho g \left(\zeta - \langle \zeta \rangle \right)^2 \right\rangle$$
$$= \frac{1}{2} \rho g \langle \eta^2 \rangle = \frac{1}{2} E \tag{10}$$

using linear wave theory. There is thus a substantional pressure contribution to the radiation stress which takes place above the wave trough level (see also [11,17,13]). This contribution is not present in Eq. (5) as the expression for the wave-average pressure, Eq. (3), does not include it.

Energy dissipated during the breaking process is generally assumed to be first converted into organised vortices (the surface roller) before being dissipated into small-scale, disorganised turbulent motions [5]. The roller transports mass and momentum, and exerts a shear stress to the water below, affecting wave setup and undertow [26,27,11]. The expression of [16] does not account for the roller contribution.

3. Wave flume experiments

3.1. Experimental set-up

The experiments were carried out in the 40 m long, 0.8 m wide and 1.05 m deep wave flume of Delft University of Technology [3]. The fixed bed profile was based on a natural beach and included two breaker bars with a trough in between (see Fig. 1). The bed was built up with a fill of sand and a mortar toplayer, which was smoothened to reduce bed roughness. The Nikuradse bed roughness was estimated to have a value of about 0.5 mm. The still water level was at 0.75 m above the flume bottom.

In this paper we study data from two irregular wave (JONSWAP spectrum) conditions: 1B and 1C. Table 1 shows the experimental conditions, including the surf similarity parameter (also known as the Iribarren number) ξ defined as:

$$\xi = \frac{\tan\beta}{\sqrt{\frac{H}{L}}} \tag{11}$$

where $\tan \beta$ is the beach slope and L the wave length. We have calculated ξ using the offshore slope of the first breaker bar (0.054), the offshore (spectral) significant wave height $(H_{\rm m0,off})$ and the wave length following linear wave theory using the offshore spectral peak

(6)



Fig. 1. (a) Measured wave heights, (b) bed level and still water level. z=0 corresponds to the flume bottom.

Table 1Overview of wave conditions.

Condition	$H_{\rm m0,off}$ (m)	$T_{\rm p,off}$ (s)	ξ(-)	Breaker type
1B	0.206	2.03	0.31	Spilling
1C	0.103	3.33	0.71	Weakly plunging

period $(T_{p,off})$.

In the experiments spilling waves were observed for Test 1B and weakly plunging waves for Test 1C, which is in line with the criterion of [1] based on the surf similarity parameter.

Fig. 1a shows the significant wave heights based on the measured spectrum. It shows the strong decrease of wave height just onshore from the top of the breaker bar related to wave breaking. Wave shoaling can only be observed for Test 1C, which included lower and longer waves.

3.2. Measurements and data-processing

One experimental run consisted of multiple repeated wave series. A single series contained about 100 waves, corresponding to 157 s for Test 1B and 245 s for Test 1C. The repetition allows for determination of the turbulent velocities using the ensemble averaging technique.

The water surface elevations were measured at 20 Hz with wave gauges with an inner distance of 1 m in the shoreface and 0.2 m in the surfzone. The u and w flow velocities were measured with laser-Doppler velocimeters (LDVs) with a measuring frequency of 100 Hz and measurement volume of about 0.1 mm³, capturing small-scale turbulence. The horizontal resolution of the LDV measurements was about 0.2 m in the surfzone, and the vertical resolution varied between 1 mm in the wave boundary layer to 1 cm in the remainder of the water column.

The turbulent component was computed by subtracting the ensemble-averaged velocity from the total velocity:

$$u'_{ij} = u_{ij} - \overline{u}_i = u_{ij} - \frac{1}{N} \sum_{j=1}^N u_{ij}$$
(12)

where i the time counter, j the wave series counter and N the number of

$$\widetilde{u}_i = \overline{u}_i - \langle u \rangle = \overline{u}_i - \frac{1}{M} \sum_{i=1}^M \overline{u}_i$$
(13)

wave series. The mean and orbital component then follow from:

with M the number of measurement values within one wave series. The mean turbulent stresses presented below are the average of the mean values per wave series. The number of wave series was 11 which is relatively low to distinguish orbital from turbulent velocities. For example, [32] suggest values of 20 and 40, respectively. [25] studied the root-mean-square deviations in the ensemble-averaged flow quantities under irregular breaking waves calculated with successively increasing number of realizations. With 10 realizations, the root-mean-square deviations were 5% for the horizontal velocity, and 15% for the vertical velocity and turbulence intensity. This paper discusses wave-averaged quantities (averaged over the wave-series), which will decrease deviations. This gives an indication of the accuracy of the stresses presented in this paper.

The velocity measurements above the (average) wave trough level were discarded, as these data were too much affected by air bubbles related to wave breaking.

4. Experimental results

4.1. Undertow

Fig. 2 shows the undertow at 4 positions around the first breaker bar (located at x=20.9 m) as a function of the dimensionless vertical co-ordinate:

$$\varsigma = \frac{\zeta_{meas}}{h} - 1 \tag{14}$$

with z_{meas} the vertical measurement level above the local bed, such that $\varsigma = 0$ corresponds to the mean water level and $\varsigma = -1$ to the local bed level. The location of the vertical profiles with respect to the bed level are indicated in Fig. 3.

This figure shows that the undertow is stronger for Test 1B compared to 1C, related to the higher wave height. A "belly-shape" undertow profile can especially be observed at x=21.37 m and x=22.37 m for Test 1B, which is generated by the onhore roller force exerted at the surface. The positive wave-mean velocities close to the bed at x=21.37 m for Test 1B indicate onshore-directed Longuet-



Fig. 2. Vertical profiles of undertow at four cross-shore positions around the first breaker bar (crest located at x=20.9 m).



Fig. 3. Mean normal stress for (a) Test 1B and (b) Test 1C. The solid black line indicates the mean wave trough level; the black dots indicate the locations of the vertical profiles presented in other figures. z=0 corresponds to the flume bottom.



Fig. 4. Vertical profiles of mean normal stress at four cross-shore positions around the first breaker bar (crest located at x=20.9 m). The lines denote predictions with the original (solid lines) and adjusted (dashed lines) formula of [16] for Test 1B (black) and 1C (red). The LH64 radiation stress predictions are converted to normal stress by dividing by the water depth. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Higgins boundary layer streaming. For a further discussion of the undertow and turbulent kinetic energy reference in made to [3].

4.2. Mean normal stress

Fig. 3 shows the mean normal stress computed using Eq. (5), interpolated in the horizontal (with a 1 cm resolution) and vertical direction (with a 1 mm resolution). Fig. 4 shows vertical profiles of the normal stress.

The figures show the gradual increase in normal stress towards the first breaker bar, followed by a rapid decrease. The same cross-shore variation occurs around the second breaker bar. The normal stress appears to be fairly uniformly distributed over the water depth, except for Test 1B close to the crest of the most offshore-located breaker bar.

The normal stresses are higher for the more energetic 1B case. Shoreward from the first breaker bar (x > 21 m) the difference in wave height between the two cases is not so strong, as it is mainly controlled by the local water depth (see Fig. 1). This is reflected in similar normal stress values.

Fig. 5 shows that the horizontal orbital velocity term, $-\rho \langle \tilde{u}^2 \rangle$, is the dominant contributor to the radiation stress for Test 1B. The same goes for Test 1C (not shown here). Even close to the breaker bar, the turbulence contribution to the radiation stress, $-\rho \langle \overline{u'^2} \rangle$, is minor. This is in agreement with observations by [24] for a plane sloping beach (i.e. without breaker bar) with spilling and plunging breakers of the same

order of magnitude as presented here. It also means that the pressure contributions, expressed through $\rho \langle \widetilde{w^2} \rangle$ and $\rho \overline{\langle w'^2} \rangle$, are relatively small.

The fact that the normal stresses reach a maximum close to the bed and then decrease towards the bed (visible at x=19.67 and 20.71 m) is a bottom boundary layer effect. As the horizontal orbitial velocity term is the dominant contributor to the normal stress, these stresses reflect the typical vertical profile of the horizontal orbital velocity with a nearbed overshoot (see e.g [36]).

Fig. 4 also includes the mean normal stress computed using the expression derived by [16] (LH64) based on linear wave theory, Eq. (9). We have converted the radiation stress [N/m] to normal stress $[N/m^2]$ by dividing by the water depth, i.e. assuming a uniform vertical distribution, and we have changed the sign in line with the stress definitions in Eqs. (4) and (7). This analytical expression does not include the turbulent contributions to the radiation stress, but this is not too important since Fig. 5 shows that the horizontal orbital velocity contribution is dominant. Furthermore, the LH64 model does not account for the roller contribution. However, this mainly takes place above the wave trough level for which we have no measurements (see [18]).

The LH64 model captures the cross-shore variation and the difference between Test 1B and 1C in a qualitative sense. According to Eq. (10) there is a considerable pressure contribution to the radiation stress above the wave trough level, which was not measured. We have extracted this contribution $(\frac{1}{2}E)$ from the LH64 prediction,



Fig. 5. Vertical profiles of mean normal stress for Test 1B (bar crest located at x=20.9 m). Open circles: total, filled triangles: contribution due to the horizontal orbital velocity.



Fig. 6. Mean shear stress for (a) Test 1B and (b) Test 1C. The solid black line indicates the mean wave trough level; the black dots indicate the locations of the vertical profiles presented in other figures. z=0 corresponds to the flume bottom.



Fig. 7. Vertical profiles of different contributions to mean shear stress for Test 1B (bar crest located at x=20.9 m). The green lines denote predictions with the model of [39].(For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

q = kh

before converting the radiation stress to normal stress. Fig. 5 shows that the normal stresses below the wave trough level can be reasonably well reproduced with this adjusted LH1964 formula.

As has been shown by [24] and [10], the radiation stress is reduced by non-linear effects. According to the relation proposed by [10], the ratio of the non-linear radiation stress to non-linear wave energy density at the four presented locations is approx. 10-15% lower than the linear expression, Eq. (9). This could help explain the general overprediction of the radiations stress below the wave trough with the LH64 model.

4.3. Mean shear stress

Fig. 6 shows the mean normal stress computed using Eq. (6), interpolated in the horizontal (with a 1 cm resolution) and vertical direction (with a 1 mm resolution). Figs. 7 and 8 show the vertical profiles of the different contributions to the mean total shear stress.

These figures include a comparison with the analytical model of [39] for the wave Reynolds stress:

$$-\rho \langle \widetilde{u}\widetilde{w} \rangle = \frac{GE}{h} \Biggl\{ \frac{\partial h}{\partial x} \Biggl[1 - \frac{1}{(1+G)} \frac{q}{\tanh q} \frac{(z+h)}{h} \Biggr] \\ - \frac{G}{(1+G)^2} (1-q \tanh q) \frac{(z+h)}{h} \Biggr] \dots + \frac{f_w |U_b|}{2c} \Biggl[\cosh Q - \frac{cQ}{c_g \sinh 2q} \Biggr] \\ - \frac{B^3 k H}{2\pi} \frac{(z+h)}{h} \Biggr\}$$
(15)

 $G = \frac{2q}{\sinh 2q} \tag{16}$

$$Q = k(z+h) \tag{18}$$

(17)

 $f_{\iota\nu}$ the wave friction factor (computed with the equation of [29] with k_s =0.5 cm corresponding to the smooth concrete flume bottom), $|U_b|$ the horizontal orbital velocity amplitude at the bed, and *B* is an empirical breaker coefficient of O(1) (we took *B*=0.5). This solution is valid for the interior flow region, i.e. far above the wave bottom boundary layer.

The expression is based on potential wave theory for a sloping bed, incorporating wave bottom dissipation and wave breaking effects. It is a generalization of the previous solutions of [11,9] and [22]. The RHS terms of Eq. (15) represent bed slope, bottom friction and wave breaking effects, respectively.

The figures show that the model of [39] agrees reasonably well with the measured wave Reynolds stresses. The computed stresses are dominated by the bed slope effect. Therefore, the sign of the wave Reynolds stress changes from negative before the breaker bar (upward sloping bed, $\frac{\partial h}{\partial x} < 0$) to positive after the breaker bar (downward sloping bed, $\frac{\partial h}{\partial x} > 0$). This is also observed in the measurements. For these cases, friction effects are negligibly small, as the bed was very smooth. The wave Reynolds stresses are also not very much affected by wave breaking. This effect would have been stronger if a coefficient B=1instead of B=0.5 would have been taken, but the generally tendencies remain the same. Similarly, [8] found a reasonable good match between wave Reynolds stresses measured above a sloping bed (with-

where



Fig. 8. Vertical profiles of different contributions to mean shear stress for Test 1C (bar crest located at x=20.9 m). The green lines denote predictions with the model of [39].(For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

out breaker bar) and the model of [39] using B=0.7.

The turbulent Reynolds stress, $-\rho \langle \overline{u'w'} \rangle$, is generally postive and smaller than the wave Reynolds stress, supporting what was found by [7,8]. The wave Reynolds stress is thus clearly not negligible compared to the turbulent Reynolds stress. However, for the mean flow it is the vertical gradient of the shear stress (force) that matters. This will be further discussed in the next section.

The vertical gradient in turbulent Reynolds stress generally suggests a downward transport of turbulence from the water surface. This is supported by the laboratory experiments by [7,8] and field observations by [23]. These measurements show that the turbulent Reynolds stress is largest higher up in the water column, associated to wave breaking. At x=21.37 m for Test 1B (Fig. 7c) we can also observe boundary layer effects with the largest turbulent Reynolds stress close to the bed.

4.4. Forces

The horizontal gradient of the mean horizontal momentum flux $(\rho \langle u \rangle^2)$ and the vertical gradient of the mean vertical momentum flux $(\rho \langle u \rangle \langle w \rangle)$ should balance the setup/setdown, the horizontal gradient of the normal stress (σ_{xx}) and the vertical gradient in the shear stress (τ_{xz}) , as expressed in Eq. (4).

The forces derived from the data show much variation due to the use of spatial derivatives in combination with measuring uncertainties (possibly related to the limited spatial resolution). This especially applies to the vertical derivative of the shear stresses. To avoid this to a certain degree, a second-order polynomial is fit to the shear



Fig. 9. Relative vertically-averaged force contributions.

stresses, and the gradient is determined from the slope of the fitted line. To determine the horizontal derivatives, the values are first interpolated in horizontal and vertical direction (see Figs. 3 and 6).

Fig. 9 shows the relative importance of the vertically-averaged, $\langle ... \rangle_{z_2}$, force contributions around the breaker bar, e.g.

$$f_{wf} = \frac{\left\langle \left| \frac{\partial \sigma_{xx}}{\partial x} \right| \right\rangle_{z}}{\left\langle \left| \frac{\partial \sigma_{xx}}{\partial x} \right| \right\rangle_{z} + \left\langle \left| -\rho \frac{\partial \langle \widetilde{u} \widetilde{w} \rangle}{\partial z} \right| \right\rangle_{z} + \left\langle \left| -\rho \frac{\partial \langle \overline{u' w'} \rangle}{\partial z} \right| \right\rangle_{z}}$$
(19)

for the horizontal gradient of the mean horizontal momentum flux ('wave force').

The figure shows that the vertical shear stress gradient $(\partial \tau_{xz}/\partial z,$ called the 'shear force') around the breaker bar is larger than the wave force. The consequence hereof is that this shear force should be included in 3D mean flow modeling. Furthermore, the vertical wave Reynolds stress gradient $(-\rho \partial \langle \tilde{u} \tilde{w} \rangle / \partial z)$ is an important contributor to the shear force. So when including the shear stress in 3D nearshore circulation modeling, the wave Reynolds stress should be accounted for. This concerns the distribution over the full water column (see Figs. 7 and 8), and not only in the near-bed wave boundary layer as is currently done in, for example, the Delft3D modeling system [37]. This is in line with the conclusions of [4] and [20].

5. Discussion and conclusions

This paper investigated stresses and forces that control mean surfzone hydrodynamics based on detailed measurements of velocities below the wave trough level in wave flume experiments. It involved two irregular wave conditions; a spilling and a weakly plunging breaker. The sloping bed profile was fixed and included two breaker bars.

The main conclusions are:

- 1. The normal stress below the wave trough level is fairly uniformly distributed over the water column. The horizontal orbital velocity contribution, $-\rho \langle \tilde{u}^2 \rangle$, is dominant. Comparison with the classical analytical expression of [16] suggests that a significant part of the normal stress is concentrated between the wave trough and crest level.
- 2. The wave Reynolds stress, $-\rho \langle \tilde{u}\tilde{w} \rangle$, is an important contribution to the total shear stress. Its sign changes from negative before the breaker to positive thereafter. At the same time the vertical gradient changes from postive to negative. The wave Reynolds stresses are reasonably well predicted using the analytical model of [39].
- 3. Apart from the horizontal normal stress gradients, the vertical shear stress gradients are important in the force balance for the breaker zone.

These forces, together with the eddy viscosity, control the vertical distribution of the mean flow in the surfzone. 3D mean flow modeling systems like Delf3D (see [14]) and ROMS (see [13]) require parameterizations for the wave forces as these are not explicitly accounted for. Based on the data-analysis presented in this paper we suggest that reliable undertow predictions can only be achieved by 1) including a vertical distribution of the normal stress with larger values between the wave crest and trough level, and 2) accounting for the wave Reynolds stress distribution over the full water column, e.g. using the model of [39].

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References

- J.A. Battjes, Surf similarity, in: Proceedings of the 14th Coastal Engineering Conference, Copenhagen, Denmark, 1974, pp. 466–480
- [2] A. Bennis, F. Ardhuin, F. Dumas, On the coupling of wave and three-dimensional

circulation models:choice of theoretical framework, practical implementation and adiabatic tests, Ocean Model. 97 (40) (2011) 260-272.

- [3] M. Boers, Surf zone turbulence, (Ph.D. thesis), Delft University of Technology, The Netherlands, 2005.
- [4] M. Boers, J. Van de Graaff, Profile change of a sheet flow dominated beach, in: Proceedings Coastal Dynamics Conference, Gdansk, Poland, 1995, pp. 257–268.
- [5] E.D. Christensen, D.J.R. Walstra, N. Emerat, Vertical variation of the flow across the surf zone, Coast. Eng. 45 (2002) 169–198.
- [6] A.G. Davies, C. Villaret, Eulerian drift induced by progressive waves above rippled and very rough beds, J. Geophys. Res. 104 (C1) (1999) 1465–1488.
- [7] F. De Serio, M. Mossa, Experimental study on the hydrodynamics of regular breaking waves, Coast. Eng. 53 (2006) 99–113.
- [8] F. De Serio, M. Mossa, A laboratory study of irregular shoaling waves, Exp. Fluids 54 (2013) 1536.
- [9] H.J. De Vriend, N. Kitou, Incorporation of wave effects in a 3D hydrostatic mean current model, Report H-1295, Delft Hydraulics, The Netherlands, 1990
- [10] R.G. Dean, C.J. Bender, Static wave setup with emphasis on damping effects by vegetation and bottom friction, Coast. Eng. 53 (2006) 149–156.
- [11] R. Deigaard, J. Fredsøe, Shear stress distribution in dissipative water waves, Coast. Eng. 13 (1989) 357–378.
- [12] W.M. Kranenburg, J.S. Ribberink, R.E. Uittenbogaard, S.J.M.H. Hulscher, Net currents in the wave bottom boundary layer: on waveshape streaming and progressive wave streaming, J. Geophys. Res. 117 (F03) (2012) 005.
- [13] N. Kumar, G. Voulgaris, J.C. Warner, Implementation and modification of a threedimensional radiation stress formulation for surf zone and rip-current applications, Coast. Eng. 58 (2011) 1097–1117.
- [14] G.R. Lesser, J.A. Roelvink, J.A.T.M. Van Kester, G.S. Stelling, Development and validation of a three-dimensional morphological model, Coast. Eng. 51 (2004) 883–915.
- [15] M.S. Longuet-Higgins, Mass transport in water waves, Philos. Trans. R. Soc. 345 (1953) 535-581.
- [16] M.S. Longuet-Higgins, R.W. Stewart, Radiation stresses in water waves: a physical discussion, with applications, Deap-sea Res. 11 (1964) 529–562.
- [17] G.L. Mellor, The depth-dependent current and wave interaction equations: a revision, J. Phys. Oceanogr. 38 (2008) 2587–2596.
- [18] K.C.C.J. Neessen, Analysing depth-dependence of cross-shore mean-flow dynamics in the surf zone, Master's thesis, University of Twente, The Netherlands, 2012
- [19] P. Nielsen, Coastal Bottom Boundary Layers and Sediment Transport, World Scientific, Singapore, 1992.
- [20] P. Nielsen, Z.-J. You, Eulerian mean velocities under non-breaking waves on horizontal bottoms, in: Proceedings International Conference on Coastal Engineering, Sydney, Australia, 1996, pp. 4066–4078.
- [21] J.S. Ribberink, A.A. Al-Salem, Sheet-flow and suspension of sand in oscillatory boundary layers, Coast. Eng. 25 (1995) 205–225.
- [22] F.J. Rivero, A.S. Arcilla, On the vertical distribution of (ãŵ), Coast. Eng. 25 (1995) 137–152.
- [23] B.G. Ruessink, Observations of turbulence within a natural surf zone, J. Phys. Oceanogr. 40 (2010) 2696–2712.
- [24] M.J.F. Stive, H.G. Wind, A study of radiation stress and set-up in the nearshore region, Coast. Eng. 6 (1982) 1–12.
- [25] N.J. Sultan, F.C.K. King, Experimental study of undertow and turbulence intensity under irregular waves in the surf zone, in: Proceedings of the 2nd International Symposium on Ocean Wave Measurement and Analysis, New Orleans, USA, 1993, pp. 602–613.
- [26] I.A. Svendsen, Wave heights and set-up in a surf zone, Coast. Eng. 8 (1984) 303–329.
- [27] I.A. Svendsen, Mass flux and undertow in a surf zone, Coast. Eng. 8 (1984) 347–365.
- [28] I.A. Svendsen, Advanced Series on Ocean Engineering, Introduction to Nearshore Hydrodynamics Volume 24, World Scientific, Singapore, 2006.
- [29] D.H. Swart, Offshore sediment transport and equilibrium beach profiles, Delft Hydr. Lab. Publ. 131, Delft Hydraulics, The Netherlands, 1974
- [30] F.C.K. Ting, Laboratory study of wave and turbulence velocities in a broad-banded irregular wave surf zone, Coast. Eng. 43 (183-208) (2001).
- [31] F.C.K. Ting, Laboratory study of wave and turbulence characteristics in narrowband irregular breaking waves, Coast. Eng. 46 (291-313) (2002).
- [32] F.C.K. Ting, J.T. Kirby, Observation of undertow and turbulence in a laboratory surf zone, Coast. Eng. 24 (51-80) (1994).
- [33] F.C.K. Ting, J.T. Kirby, Dynamics of surf-zone turbulence in a strong plunging breaker, Coast. Eng. 24 (177-204) (1995).
- [34] F.C.K. Ting, J.T. Kirby, Dynamics of surf-zone turbulence in a spilling breaker, Coast. Eng. 27 (131-160) (1996).
- [35] J. Trowbridge, O.S. Madsen, Turbulent wave boundary layers:2. second-order theory and mass transport, J. Geophys. Res. 89 (C5) (1984) 7999–8007.
- [36] D.A. Van Der A, T. O'Donoghue, A.G. Davies, J.S. Ribberink, Experimental study of the turbulent boundary layer in acceleration-skewed oscillatory flow, Journal of Fluid Mechanics, 68, 4, 2011, pp. 251–283.
- [37] D.J.R. Walstra, J.A. Roelvink, J. Groeneweg, Calculation of wave-driven currents in a 3d mean flow model, in: Proceedings of the 27th International Conference on Coastal Engineering, Sydney, Australia, 2000.
- [38] H. Yoon, D.T. Cox, Large-scale laboratory observations of wave breaking turbulence over an evolving beach, J. Geophys. Res. 115 (C10007) (2010).
- [39] Q. Zou, A.J. Bowen, A.E. Hay, Vertical distribution of wave shear stress in variable water depth:theory and observations, J. Geophys. Res. 111 (2006) 1–17.