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IMPROVED MODELLING OF NONLINEAR FOUR-WAVE INTERACTIONS IN SHALLOW WATER

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Abstract: A finite depth version of the Discrete Interaction Approximation (DIA) has been developed and implemented in the SWAN model. This modification of the DIA makes the presently used depth-scaling obsolete. The capabilities of the finite depth DIA have been compared with results from an exact technique for the calculation of the nonlinear transfer rate. Firstly, the nonlinear transfer rate was computed for a JONSWAP spectrum in deep and shallow water. Secondly, two growth curves have been computed for a shallow lake with a constant depth of 5 m and 2 m. The results of the computations indicate that for mean kh-values larger than 1.3 no effects are noticeable. Only when kh<1.3 the finite depth DIA yields different results. This leads to small changes in wave period and spreading measures.

INTRODUCTION

The present generation of full spectral discrete wave prediction models is based on the concept that each physical process can be modelled with a separate source term. In deep water source terms for wind input, whitecapping dissipation and nonlinear four-wave interactions are active. As the waves enter shallow water the source terms for bottom friction, depth-induced waves breaking as well as nonlinear three-wave interactions become important. Formulations of these source terms are often based on a combination of theoretical studies and analysis of field and laboratory measurements. Despite these efforts many different formulations for most of these source terms exist and no generally accepted formulation for each of these source terms exists.

The only source term for which a closed theoretical framework exists is the one describing the nonlinear four-wave interactions, which was proposed by Hasselmann (1962). The computation of the theoretical expression of the nonlinear four-wave

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interactions is very time consuming because its formulation contains a 6-dimensional integral. Due to these computational requirements it is not used in operational wave prediction models. To overcome this obstacle, Hasselmann et al. (1985) derived an approximation of the full expression. This approximation is known as the Discrete Interaction Approximation (DIA) and it initiated the development of the present day third generation wave prediction models like WAM (WAMDI, 1988), WAVEWATCH (Tolman, 1991), SWAN (Booij et al., 1999) and TOMAWAC (Benoit et al., 1996).

Depth effects on the nonlinear transfer rate can be incorporated in the full theoretical expression by using the finite depth dispersion relation and the finite depth interaction coefficient. Theoretical and numerical studies show that finite depth affects the transfer rate in various ways. Firstly, the overall magnitude increases as the water becomes shallower. Secondly, the frequency and directional distribution of the transfer rate change. In the DIA, however, finite depth effects are crudely schematised using a simple scaling law in which only the magnitude changes while the shape remains unchanged.

The last years it has become evident that the DIA is not able to properly represent the nonlinear transfer function for deep and shallow water (cf. Van Vledder et al., 2000). Consequently, it distorts the source term balance in a wind wave spectrum. To overcome the shortcomings of the DIA, coefficients in the source terms for wind input and whitecapping dissipation in WAM-type models are heavily tuned to compensate for the mismatch in the DIA. This situation hampers the further development of other source terms as long as the DIA is used to investigate source terms with such a numerical wave model.

The need to replace the DIA by better approximations has been widely accepted in the wave modelling community and various authors have proposed extensions or modifications to the original DIA (cf. Hashimoto and Kawaguchi, 2001, and Van Vledder 2001). However, until now these efforts are only aimed at improving the deep water transfer rate. In this paper attention is given to improve the modelling of the nonlinear four-wave interactions in shallow water. To that end a finite depth version of the DIA has been derived. This modification makes the currently used depth scaling obsolete. Its basic features will be illustrated by comparisons with exact solution techniques for a JONSWAP spectrum and some shallow water growth curves experiment.

NUMERICAL MODELLING OF WIND WAVES

The spatial and temporal evolution of the wave field is conveniently described by the wave action balance equation. In flux form this equation is given by:

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(c_{g,x}N\right)+\frac{\partial}{\partial y}\left(c_{g,y}N\right)+\frac{\partial}{\partial \theta}\left(c_{\theta}N\right)+\frac{\partial}{\partial k}\left(c_{k}N\right)=S \qquad (1)$$

in which $N=N(\sigma,\theta,x,y,t)$ is the wave action density spectrum, $c_{g,x}$ and $c_{g,y}$ are the group velocities in x-and y-direction, and c_θ and c_k are the spectral propagation velocities. The growth, decay and redistribution of wave action is given by the source term S. The source term S is considered to be the sum of individual source terms, each representing a specific physical process:

$$S = S_{wind} + S_{wcap} + S_{nl4} + S_{fric} + S_{brk} + S_{nl3} \tag{2}$$

In this expression S_{wind} is the wind input term, S_{wcap} whitecapping dissipation, S_{nl4} nonlinear four-wave interactions. In shallow water additional source terms are active; S_{fric} energy decay by bottom friction, S_{brk} energy decay by breaking waves as well as S_{nl3} nonlinear three-wave interactions. The nonlinear interaction terms only redistribute wave action within the spectrum. Descriptions of these source terms can be found in papers describing particular wave models like WAVEWATCH and SWAN. For the purposes of this paper, the basic equations of the methods for computing the nonlinear four-wave interactions are repeated.

NONLINEAR FOUR-WAVE INTERACTIONS

Nonlinear wave-wave interactions between pairs of four wave components play an important role in the evolution of wind-generated waves (cf. Young and Van Vledder, 1993). Hasselmann (1962) developed the theoretical framework for these interactions and he formulated an integral expression for the computation of these interactions, which is known as the Boltzmann integral for surface gravity waves. Hasselmann (1962) found that a set of four waves, called a quadruplet, could exchange energy when the following resonance conditions are satisfied:

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_3 + \boldsymbol{k}_4 \tag{3}$$

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \tag{4}$$

in which ω_j the radian frequency and $\boldsymbol{k}_j$ the wave number (j=1,..,4). The linear dispersion relation relates the frequency and the wave number:

$$\omega^2 = gk \tanh(kh) \tag{5}$$

Here, g is the gravitational acceleration and h the water depth. Hasselmann (1962) describes the nonlinear interactions between wave components in a quadruplet in terms of their action density n_i, where $n(k_i)=E(\boldsymbol{k}_i)/\omega_i$. The rate of change of action density at a wave number $\boldsymbol{k}_1$ due to all quadruplet interactions involving $\boldsymbol{k}_1$ is given by:

$$\begin{aligned}\frac{\partial n_1}{\partial t} = &\iiint G\left(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4\right) \times \delta\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4\right) \times \delta\left(\omega_1 + \omega_2 - \omega_3 - \omega_4\right) \\ &\times \left[n_1 n_2 \left(n_3 + n_4\right) - \left(n_1 + n_2\right) n_3 n_4\right] d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4\end{aligned} \tag{6}$$

where $n_i=n(\boldsymbol{k}_i)$ is the action density at wave number $\boldsymbol{k}_i$ and G is the coupling coefficient, which is a complicated function of the four wave numbers involved in an interaction. Deep and finite depth expressions for this coefficient have been given by Hasselmann (1962) and Herterich and Hasselmann (1980). The delta functions in (6) ensure that contributions to the integral only occur for quadruplets that satisfy the resonance conditions. They also ensure conservation of energy, action and momentum.

The computation of the Boltzmann integral is rather complicated and very time consuming since it requires the solution of a 6-dimensional integral. In the numerical evaluation of Eq. (6) thousands of possible wave number configurations need to be evaluated to determine the nonlinear transfer rate for a particular wave spectrum. Numerical integration techniques for the Boltzmann integral have been developed by Hasselmann and Hasselmann (1981), Masuda (1980) and Resio et al. (2001). The latter use a technique based on methods developed by Webb (1978) and Tracy and Resio (1982). These techniques are referred to as exact methods because they are able to solve the Boltzmann integral to any prescribed degree of accuracy. Because of the computational requirements of these methods it is (still) not feasible to include a full solution of the Boltzmann integral in operational wave models.

THE DISCRETE INTERACTION APPROXIATION

In contrast to exact methods, the Discrete Interaction Approximation considers only one wave number configuration, and its mirror image. In this configuration the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are equal and the other two wave numbers have different magnitude and direction. Their frequencies are related via the parameter λ such that their configuration is uniquely determined:

$$\begin{aligned} f_1 &= f_2 = f \\ f_3 &= (1+\lambda)\, f = f^+ \\ f_4 &= (1-\lambda)\, f = f^- \end{aligned} \tag{7}$$

Here, the superscripts + and – are used to emphasise the link with previously reported notations of the DIA. The directions of the wave numbers $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$, relative to the direction of the wave numbers $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, follow from Eq. (7) and the resonance conditions (3) and (4). In the DIA proposed by Hasselmann et al. (1985) $\lambda=0.25$, leading to the angles $\theta_1=\theta_2=0°$, $\theta_3=\theta^+=+11.48°$ and $\theta_4=\theta^{--}=33.56°$.

The DIA source term describes the rate of change of energy density in all four (in fact three independent) wave numbers involved in an interaction. The corresponding energy densities are denoted by $E=E(f,\theta)$, $E^+=E(f^+,\theta^+)$ and $E^-=E(f^-,\theta^-)$, and the contributions to the corresponding transfer rates are denoted by δS_{nl}, δS^+_{nl} and δS^-_{nl}. The functional form of the DIA source term is given by:

$$\begin{pmatrix} \delta S_{nl} \\ \delta S_{nl}^+ \\ \delta S_{nl}^- \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} C_{nl4}\, g^{-4} f^{11} \left(E^2 \left(\frac{E^+}{(1+\lambda)^4} + \frac{E^-}{(1-\lambda)^4} \right) - 2E \frac{E^+ E^-}{\left(1-\lambda^2\right)^4} \right) \tag{8}$$

in which C_{nl4} is a scale parameter. In the WAM and SWAN models $C_{nl4}=3 \times 10^7$.

To compute the nonlinear quadruplet source term for a discrete wave spectrum, equation (8) is applied to all spectral bins, taking E as the energy density at this bin. The positions of the other two interacting bins with energy densities E^+ and E^- are determined relative to the central bin E. In general, the locations (f^+,θ^+) and (f^-,θ^-) of the wave numbers $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$ in the spectral grid do not coincide with discrete spectral grid points. To obtain the energy density at these points one can apply bi-linear interpolation between the four surrounding grid points or one may take the energy density at the nearest grid point. In the computational procedure Eq. (8) is also applied to its mirror image, obtained by reversing the signs of the angles θ_3 and θ_4.

In the computational procedure of the DIA special attention is given to wave number configurations that cross the boundaries of the spectral grid. Periodicity is assumed to take care of the directional boundaries. Further, in the case that the frequency f^- is lower than the lowest discrete frequency f_{min}, the corresponding energy density E^- is set to zero, and in the case the frequency f^+ is higher than the highest discrete frequency a parametric decay of energy densities in the spectral tail is assumed, usually given by:

$$E(f) = E(f_{\max})\left(\frac{f}{f_{\max}}\right)^p \quad \text{for} \quad f \geq f_{\max} \tag{9}$$

in which p is the power of the spectral tail. Additional quadruplets in the spectral tail need to be accounted for to ensure that all possible interactions between wave numbers in the prognostic range of the spectral grid and in the spectral tail are included. This is achieved by extending the spectral grid towards higher frequencies, such that the bin E^- with frequency f^- is located just at or just above the highest model frequency f_{max}. Application of Eq. (8) produces the rate of change of energy density at the interacting wave numbers. These rates are distributed among the four surrounding spectral bins using the same (interpolation) weights as used for the determination of the energy density at these wave numbers.

In WAMDI (1988) a simple method was proposed to include finite depth effects on the nonlinear transfer rate. Firstly, the nonlinear transfer rate is computed assuming deep water. Secondly, the resulting transfer rate is multiplied with a constant factor R. This factor is a function of the dimensionless water depth $\tilde{k}h$, and constant for all spectral components of the spectrum. To enhance model robustness in the case of arbitrarily shaped spectra, the mean wave number is computed in a special way as (cf. Tolman, 1991):

$$\tilde{k} = \left(\frac{1}{E_{tot}}\int_0^{2\pi}\int_0^{\infty}\frac{1}{\sqrt{k}}E(f,\theta)\,df d\theta\right)^{-2} \tag{10}$$

with E_{tot} the total wave variance. The nonlinear finite depth transfer rate is computed as:

$$S_{nl4}^{h}(f,\theta) = S_{nl4}^{\infty}(f,\theta)\times R(x) \tag{11}$$

in which $x = 0.75\tilde{k}h$ and where the function *R(x)* is given by:

$$R(x) = 1 + \frac{5.5}{x}\left(1 - \frac{6x}{7}\right)\exp\left(-\frac{5x}{4}\right) \tag{12}$$

To avoid numerical instabilities, the value of *R* is limited to at most 5. This parameterisation of the depth scaling is based on an analysis of results of computations for JONSWAP spectra on deep and finite depth with the wave model EXACT-NL (Hasselmann and Hasselmann, 1981). The functional behaviour of Eq. (12) is shown in Figure 1. As can be seen finite depth effects are only relevant for $\tilde{k}h < 3$, and an increase in magnitude of the nonlinear transfer rate only occurs when $\tilde{k}h < 1$.

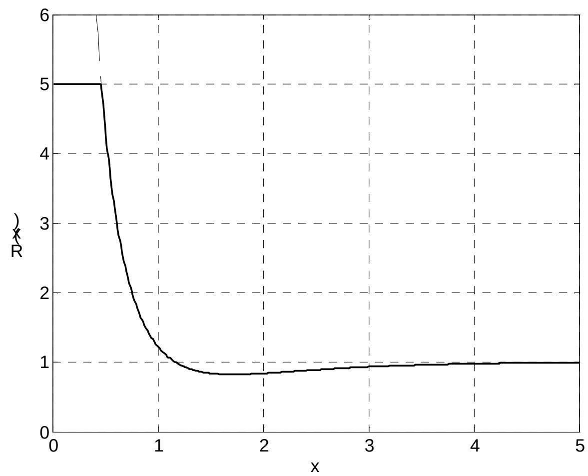

Fig. 1. Parameterisation of depth scaling in the DIA

COMPARISONS OF NONLINEAR TRANSFER RATE FOR DEEP AND SHALLOW WATER

To illustrate some of the shortcomings of the DIA a comparison was made of the nonlinear transfer rate computed with an exact method and with the DIA. The exact nonlinear transfer rates were computed with WRT method, developed by Webb (1978), Tracy and Resio (1982), and Resio et al. (2001), and rewritten by the first author. To that end the same deep water JONSWAP spectrum was used as in Hasselmann et al. (1985), viz. a JONSWAP spectrum with f_p=0.1 Hz, α=0.0175, γ=3.3 and a $cos^2(\theta)$-directional spreading. The result is shown in Figure 2.

From Figure 2 it is evident that the DIA has the following deficiencies. The negative lobe is over-predicted. A relatively large positive lobe exists at about twice the peak frequency, which does not result from the exact method. Moreover, the frequency of the first zero-crossing of the transfer rate is much higher than the peak frequency, whereas the exact computation predicts its position to be located at the peak frequency.

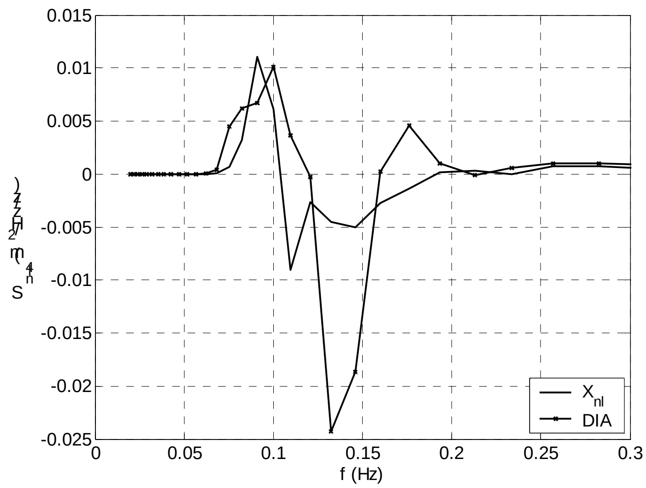

Fig. 2. Comparison of nonlinear transfer rate computed with an exact method (solid line) and with the DIA (solid line with crosses) for a deep water JONSWAP spectrum with f_p=0.1 Hz, α=0.0175, γ=3.3 and a $cos^2(\theta)$-directional spreading.

In shallow water finite depth effects change the magnitude and shape of the nonlinear transfer rate. This is illustrated in Figure 3 on the basis of exact computations for a mean JONSWAP spectrum with a peak frequency of 0.1 Hz in deep and shallow water with a depth of 10 m. This figure also shows the scaled nonlinear transfer rate for which $\tilde{k}h \approx 0.78$, resulting in a multiplication factor of $R = 3.2$.

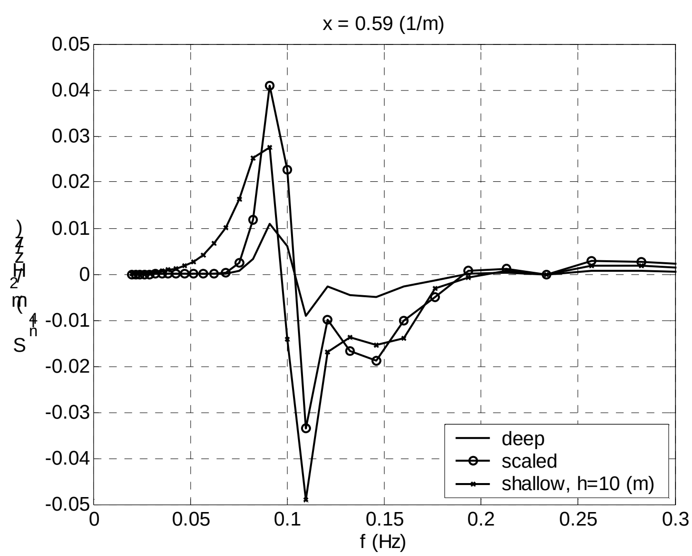

Fig. 3. Nonlinear transfer rates for a deep water JONSWAP spectrum with f_p=0.1 Hz, α=0.0175, γ=3.3 and a $cos^2(\theta)$-directional spreading. Results for deep water, a depth of 10 m and the scaled nonlinear transfer rate.

An important feature of finite depth effects on the nonlinear transfer rate is that the first positive lobe shifts towards lower frequencies and becomes wider than the scaled deep water transfer. This may lead to a faster shift of wave energy towards lower frequencies in shallow water in comparison with the scaled nonlinear transfer rate and to higher wave period measures.

THE FINITE DEPTH DISCRETE INTERACTION APPROXIMATION

To improve the depth behaviour of the DIA, the DIA was re-derived while keeping all finite depth terms in the equations. A detailed derivation of the finite depth DIA can be found in Van Vledder and Rasmussen (2002) and its main result is repeated here. The starting point for the derivation of the finite depth DIA is the principle of detailed balance formulated by Hasselmann (1966). This principle states that the change of wave action per unit time of each wave number involved in a resonant interaction is equal. To take advantage of this principle the Boltzmann integral is written in a symmetrical form:

$$\begin{pmatrix} \Delta n_1 \\ \Delta n_2 \\ \Delta n_3 \\ \Delta n_4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \frac{G}{4} \delta\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4\right) \delta\left(\omega_1 + \omega_2 - \omega_3 - \omega_4\right) P d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{k}_3 d\boldsymbol{k}_4 \Delta t \qquad (13)$$

in which P is the wave action product term $P=n_1n_2(n_3+n_4)-n_3n_4(n_1+n_2)$. Eq. (13) has not the most convenient form to integrate with respect to the original wave numbers $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$. Following Hasselmann and Hasselmann (1981), and Rasmussen (1995, 2002). Eq. (13) can be written as:

$$\begin{pmatrix} \Delta n_1 \\ \Delta n_2 \\ \Delta n_3 \\ \Delta n_4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \frac{G}{4} \left| J \right| P d\boldsymbol{k}_1 d\boldsymbol{k}_{2t} d\boldsymbol{k}_3 \Delta t \qquad (14)$$

in which $\boldsymbol{k}_{2t}$ is a tangential component of the wave number vector $\boldsymbol{k}_2$. J is the Jacobian of the transformation from Eq. (13) to Eq. (14). It is given by:

$$J = \left| \boldsymbol{c}_{g2} - \boldsymbol{c}_{g4} \right|^{-1} \qquad (15)$$

in which $\boldsymbol{c}_{g,i}$ is the group velocity vector of wave number vector $\boldsymbol{k}_i$. Next, a number of additional transformations are made to replace the change of wave action for a given wave number into the rate of change of energy density for a given frequency and direction. In addition the DIA assumption $\boldsymbol{k}_1 = \boldsymbol{k}_2$ is used. Details of this derivation will be given in Van Vledder and Rasmussen (2002). The result is the basic expression of the finite depth DIA:

$$\begin{pmatrix} \delta S_{nl} \\ \delta S_{nl}^{+} \\ \delta S_{nl}^{-} \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} C_{nl4}^{'} G \left| JJ' \right| f^3 \times$$
$$\times \left[\left(\frac{c_g E}{f k} \right)^2 \left\{ \left(\frac{c_g^+ E^+}{f^+ k^+} \right) + \left(\frac{c_g^- E^-}{f^- k^-} \right) \right\} - 2 \left(\frac{c_g E}{f k} \right) \left(\frac{c_g^+ E^+}{f^+ k^+} \right) \left(\frac{c_g^- E^-}{f^- k^-} \right) \right] \qquad (16)$$

in which $E=E(f,\theta)$ and J' an additional Jacobian:

$$J' = \frac{k_1 k_3}{c_{g1} c_{g2} c_{g2t}} \tag{17}$$

The main differences with the original DIA are the inclusion of the group velocities, frequencies and wave numbers in the product term, the explicit expression of the interaction coefficient G and the scaling term JJ' arising from the Jacobian's. Another difference with the DIA is that the shape of the interacting wave number configuration is depth dependent. This latter effect is illustrated in Figure 4. The upper left panel of this figure shows the standard DIA configuration with $\lambda=0.25$ for a central frequency of 0.0705 Hz, resulting in the wave numbers $k_1=k_2=0.02$ (1/m). The other panels show the modification of the DIA configuration with decreasing depth. The wave number magnitudes increases and their directions become more aligned.

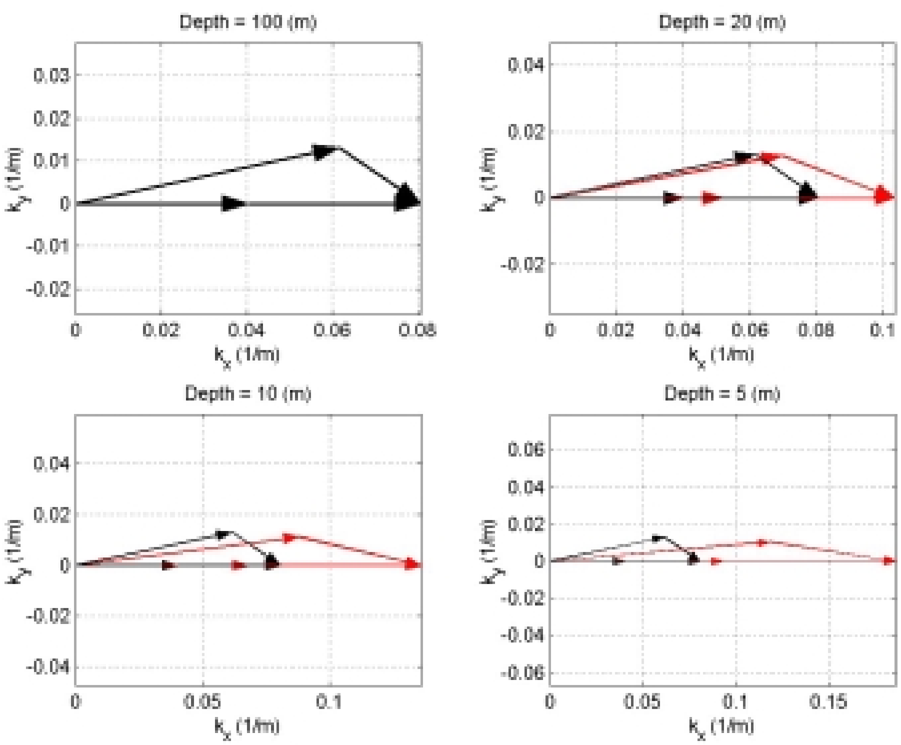

Fig. 4. Modification of the DIA wave number configuration with decreasing depth, for a central frequency of 0.0705 Hz and the water depths of 100 m, 20 m, 10 m and 5 m.

For deep water, all terms in Eq. (16) can be written in terms of frequencies and the gravitational acceleration g, such that the original DIA formulation (8) is obtained. The performance of the finite depth DIA is illustrated in Figure 5, in which the nonlinear transfer rate was computed for a mean JONSWAP spectrum with a peak frequency of 0.1 Hz, in deep water and in shallow water with a depth of 10 m.

Figure 5 shows that the finite depth DIA is able to reproduce some of the basic features of the changes of the nonlinear transfer rate in shallow water as inferred from exact computations. The overall magnitude increases, the first positive lobe shifts towards lower frequencies, and the first zero-crossing is closer to the peak frequency of the spectrum. Despite these improvements, the position of the negative lobe along the frequency axis is still much too high and the second positive lobe is also still too high.

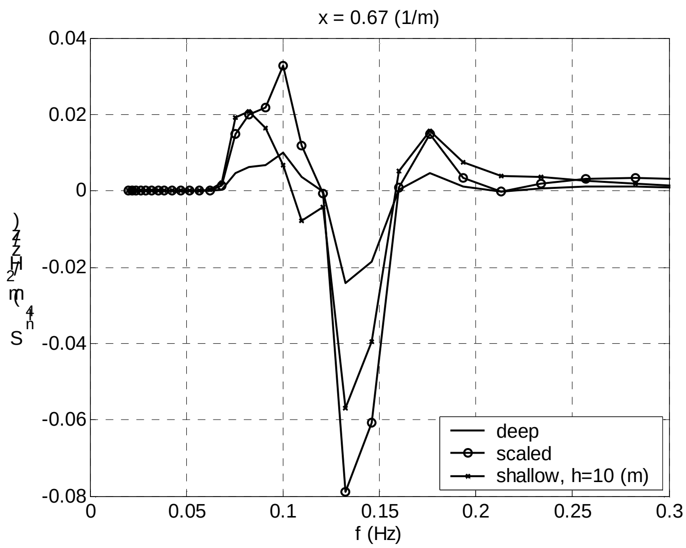

Fig. 5. Nonlinear transfer rate computed with the finite depth DIA for a deep water JONSWAP spectrum with f_p=0.1 Hz, α=0.0175, γ=3.3 and a $cos^2(\theta)$-directional spreading. Results for deep water, a depth of 10 m and the scaled nonlinear transfer rate.

IMPLICATIONS FOR WAVE MODELLING

To assess the implications of an improved modelling of the nonlinear four-wave interactions in shallow water the finite depth DIA was implemented in a test version of the SWAN model (Booij et al., 1999), version 40.11. Two test cases were defined, representing a shallow lake with a constant water depths of 5 and 2 m. These situations refer to typical RIZA problems (cf. Bottema et al., 2002). Wave model computations were made with the modified SWAN model in one-dimensional mode, a wind speed of 25 m/s and the source terms for bottom friction and wave breaking activated. The triad source term was deactivated to avoid numerical problems with this version of the SWAN model. The growth curves for the significant wave height H_s, mean wave period T_{m01}, directional spreading σ and spectral narrowness κ for the 5 m case are shown in Figure 6. The results in this figure show that the explicit inclusion of shallow water effects in the DIA hardly affects the results. Only the frequency spectra become more peaked. These results are not surprising since the $\tilde{k}h$ value at the end of the fetch is about 1.72 resulting in a scale factor of 0.91. Although the waves are depth limited, the water is not deep in terms of kh-values. The results for the 2 m case are shown in Figure 7, indicating that the finite depth DIA gives slightly lower wave periods, narrower directional distributions and somewhat wider frequency spectra. Inspection of the numerical results showed that at the end of the fetch the $\tilde{k}h$ value at the end of the fetch is about 1.21 resulting in a scale factor of 1.45.

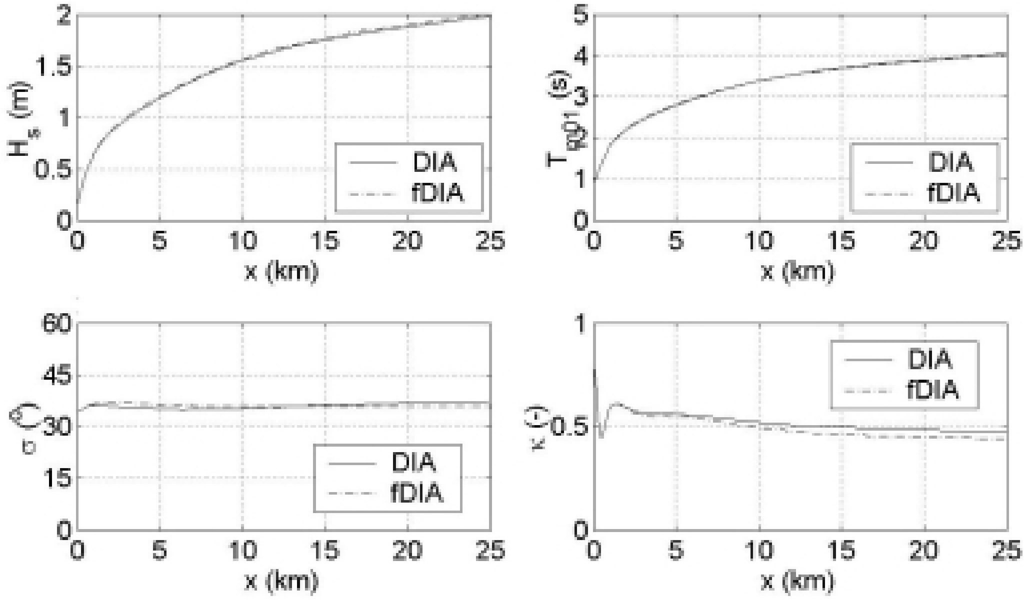

Fig. 6. Growth curves computed with the standard DIA (solid line) and finite depth DIA (fDIA, dashed line) for a constant water depth of 5 m and a wind speed of 25 m/s using the modified SWAN model.

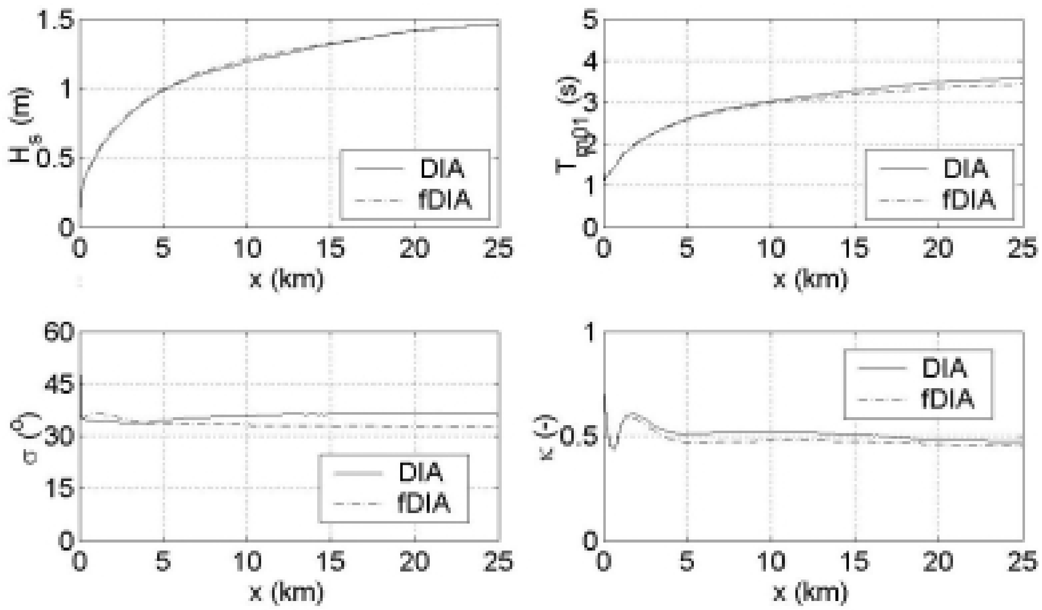

Fig. 7. Growth curves computed with the standard DIA (solid line) and finite depth DIA (fDIA, dashed line) for a constant water depth of 2 m and a wind speed of 25 m/s using the modified SWAN model.

DISCUSSION

The results of the computations for the academic test spectrum indicate that the finite depth DIA leads to a better representation of shallow water effects on the nonlinear transfer rate compared to the present depth scaling. Still, a mismatch in the nonlinear transfer rate exists in comparison with results of exact computations. This is probably due to the fact that only one wave number configuration has been used in these computations.

The results of the academic growth curves indicate that the 5 m depth shallow lake is not shallow in terms of $\tilde{k}h$ values, although the growth is depth-limited in terms of H_s/h values. This is one of the reasons why the wave measurements described in the companion paper (Bottema et al., 2002) are not only interesting from an operational point of view, but also from a physical point of view. Only for the 2 m case depth effects become noticeable. The fact that the mean wave period decreases is surprising, but this may be due to the fact that only one wave number configuration was used. The decrease in directional spreading is probably due to the fact that the resonant wave number vectors become more aligned in shallow water, thus limiting the amount of energy transferred to off-wind directions. Investigation of the spectral shapes, related source terms and more detailed comparison with the results of exact computation are needed to fully understand these results.

The finite depth version of the DIA is only a first step in bridging the gap between the fast but inaccurate DIA and the accurate but time consuming exact methods for computing the nonlinear transfer rate in deep and shallow water. Two other methods are suggested to further improve the DIA. The first method is to include more wave number configurations. Such multiple DIA's have been presented by Hashimoto and Kawagushi (2001) and Van Vledder et al. (2000). A basic shortcoming of these methods is that only a limited set of wave number configurations is considered. Therefore, a more general extension of the DIA with generally shaped wave number configurations is needed (Van Vledder, 2001).

CONCLUSIONS

The finite depth version of the DIA makes the presently used depth scaling obsolete. The finite depth DIA is able to account for some shallow water effects on the nonlinear transfer rate. Results of the computations indicate that for typical shallow lake situations the inclusion of finite depth effects in the DIA yields slightly different results only when $\tilde{k}h < 1.3$. Additional and more complicated tests are needed to assess the implications of the finite depth DIA. The potential benefits of the finite depth DIA are probably obscured by the mismatch in shape of the nonlinear transfer rate. This is due to the fact that only one wave number configuration has been used. Therefore, additional and arbitrarily shaped wavenumber configurations are needed to further improve the DIA.

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