Detecting aliased tidal errors in altimeter height measurements

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Abstract. A simple statistic is derived for quantifying the potential for the aliasing of tidal errors in a given linear estimate of sea surface height constructed from altimeter data. The existence of M_2 tidal constituent errors in Geosat data processed in the traditional way (i.e., with orbit errors removed using least squares fits to 1 cycle per revolution sinusoids) which are of sufficient magnitude to alias into apparently westward propagating ocean features is demonstrated by artificially inducing aliasing. The aliasing statistic presented here responds clearly to the induced aliasing and to actual aliasing caused by real data dropouts in the Geosat data. The potential for aliasing M_2 tidal errors is shown to vary with latitude depending on the time interval between ascending and descending ground tracks near the location of interest. The methods developed here are applied to Geosat data from the northeast Atlantic to demonstrate the presence of M_2 tidal error aliasing in those data.

1. Introduction

The fact that ocean tidal signals alias into altimetric sea surface height (SSH) measurements is well known [Parke et al., 1987; Jacobs et al., 1992] and is the reason that model-based estimates of the ocean tides must be removed from altimeter data as a primary data processing step. The Schwiderski [1980] and Cartwright-Ray [Cartwright and Ray, 1990; Cartwright et al., 1991] tidal models generally used to correct altimetric data are summarized by Ray [1993]. Because of the limited quantity and nonoptimal geographical distribution of high-quality in situ tidal records in the open ocean [e.g., Cartwright and Ray, 1990; Ray, 1993], it is difficult to obtain reliable error statistics for state-of-the-art tide models. It is generally acknowledged that the presently available tide models may be uncertain by 10 cm or more over large areas of the ocean [e.g., Wagner, 1991; Ray, 1993]. The existence of significant tide errors in the Geosat data is supported by the work of Jacobs et al. [1992]. They presented an elegant method for estimating and removing tidal model errors that relies upon the aliasing characteristics of Geosat. The potential for residual tidal signal aliasing into SSH data must therefore be recognized.

Despite the large amplitude of this important geophysical correction, little attention is generally paid to tidal errors in oceanographic applications of altimeter data [Jacobs et al., 1992, 1993, are notable exceptions]. The prevailing attitude seems to be a blind faith either that residual errors in the model tide corrections are

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Paper number 94JC00568. 0148-0227/94/94JC-00568\$05.00 smaller than the signal of interest or that the tidal errors average to near zero in fields of SSH constructed from spatially and temporally averaged altimeter data. In large part this casual concern for tidal errors has developed as a consequence of the large number of successful altimeter studies in which there is little or no evidence that tidal errors are a major concern. Most of these studies have focused on mesoscale variability in regions where the amplitudes of energetic eddies are much larger than tidal errors. Moreover, until recently, most applications of altimeter data have used simplistic short-arc polynomial corrections to mitigate the effects of orbit errors [see Chelton and Schlax, 1993 for a discussion]; to the extent that errors in tide models are spatially coherent, many of the residual tidal errors are removed along with the orbit error by these short-arc polynomial corrections. Short-arc orbit error corrections are adequate for mesoscale studies, but more sophisticated methods that retain the large-scale sea level signals of interest and focus specifically on orbit errors are necessary for studies of basin-scale sea level variability. Ray [1993] has cautioned that the inaccuracy of tide models will become much more apparent as altimeter data are applied to studies of large-scale variability.

The objectives of this study are to present both a technique for quantifying the potential for tidal aliasing and examples in which oceanographic interpretation of altimeter data is seriously compromised by errors in presently available tide models. This analysis was motivated in part by the study by *Jacobs et al.* [1992], which is one of the few published altimetric studies of large-scale sea level variability that explicitly addresses the problem of tidal aliasing. They showed that for the Geosat 17-day exact repeat orbit, the aliased M_2 tidal constituent has a nearly annual period (317 days) and is manifested spatially as a westward propagating signal whose wavelength in kilometers and phase speed in centimeters per second are approximately $834 \cos(\theta)$ and $3\cos(\theta)$, respectively, where θ is latitude. As this frequency and wavelength fall very close to the dispersion curve for the first baroclinic-mode annual Rossby wave, this aliased tidal signal might easily be misinterpreted as evidence for westward propagating Rossby waves.

It is shown here that aliasing of M_2 tidal error has serious implications for numerous recent publications citing evidence for Rossby waves based on analysis of Geosat data. In a typical analysis of this phenomenon, the phase speeds of Rossby waves are deduced from time-longitude plots of SSH along a given latitude (so-called Hovmöller diagrams) and their corresponding wavenumber-frequency power spectral density plots. The time-longitude plots are constructed from a uniform space-time grid of smoothed estimates of SSH derived from the irregularly sampled altimeter data. These estimates are usually obtained by applying some type of linear smoother to the raw SSH data. Examples include simple space-time averaging [e.g., Tokmakian and Challenor, 1993; Matthews et al., 1992], loess smoothing [Chelton et al., 1990; Matano et al., 1993], or objective analysis [e.g., White et al., 1990]. These estimates incorporate data from both ascending and descending ground tracks, which complicates the characterization of the resulting tidal aliasing.

As in most other previous studies of tidal aliasing in altimeter data, Jacobs et al. [1992] treat the problem as one of simple undersampling of the errors of each tidal constituent with a sampling interval equal to the repeat period of the satellite. The formalism for determining the frequency aliased by a specific tidal constituent for a particular repeat period is given by Parke et al. [1987]. Such a treatment effectively considers ascending or descending data in isolation and ignores the reduced aliasing that is possible when data from ascending and descending tracks in the vicinity of the SSH estimation point are combined. The aliasing resulting from the more complicated, irregular sampling pattern imposed by combining data from ascending and descending ground tracks is quantified by the method presented here. Following a brief discussion in section 2 of tide model errors in Geosat data, a statistic for determining the potential for tidal aliasing in irregularly sampled altimeter data is presented in section 3. Several examples that demonstrate the utility of this statistic are presented in section 4.

2. Tide Model Errors in Geosat Data

Before TOPEX/Poseidon geophysical data records (GDR) became available in mid-1993, data from the first 2 years of the Geosat Exact Repeat Mission (ERM) were the primary resource for altimetric studies of ocean processes. An unfortunate aspect of the many applications of the Geosat ERM data is the bewildering array of corrections and processing methods applied to the GDR data by individual investigators. Because of the small magnitude of many of the ocean signals of interest and the relatively large magnitude of some of the corrections applied to the data (tidal, orbit error, wet tropospheric and electromagnetic bias, to name a few), it is often difficult to compare the results of studies of the same region that were done using data with dissimilar corrections or correction methods. Ocean tide corrections made to Geosat data are a case in point.

The model tide values supplied with the original geophysical data records processed with the orbits computed by the Naval Astronautics Group (the NAG GDRs), dating prior to October 23, 1988, were subject to two errors that are described by Doyle et al. [1989] and Cheney et al. [1991]. Because their existence is not widely known, these tidal errors are summarized here. First, the Q1 model constituent lacked a term for the mean longitude of lunar perigee Po. Second, nodal variation terms for all 11 model constituents were omitted. The difference between the corrected and uncorrected tide models is approximately 1.8 cm rms over the open ocean. For the M_2 constituent in particular, the second error can amount to nearly 4% of the M₂ amplitude and cause small errors in the phase (P. L. Woodworth, personal communication, 1993). These errors were corrected in the latest version of the Geosat data, the GEM-T2 GDRs distributed on CD-ROMs [Cheney et al., 1991].

Another source of tide error in both the NAG and the GEM-T2 GDRs corrected with the Schwiderski model is the lack of a correction for the elastic loading tide. For the M_2 constituent this can amount to an error of several centimeters [*Ray and Sanchez*, 1989].

Some investigators have used the tide model of *Cartwright and Ray* [1990] to correct Geosat data. While this model is not subject to any of the problems disscused above and includes the elastic loading tide, there is evidence that it may be influenced by the large orbit errors contaminating all of the Geosat data (D. B. Chelton and M. G. Schlax, 1 cpr variability in TOPEX data: Orbit error or tidal error? submitted to *Journal of Geophysical Research*, 1994).

The Geosat data used here were corrected for tides by the Schwiderski tide model values from the NAG GDRs but incorporated the GEM-T2 orbits. Orbit error corrections were applied as described by *Chelton and Schlax* [1993].

3. A Simple Statistic for the Potential for Tidal Aliasing

Let h(x, y, t) denote the residual SSH after the usual instrumental, environmental, orbit height, and tidal corrections have been applied [e.g., *Chelton*, 1988]. Consider a single tidal constituent with frequency f. Denote the error in the tidal model used to correct hby the real part of the complex quantity $\epsilon(x, y, t) =$

$$h(x, y, t) = \eta(x, y, t) + \operatorname{Re}\left[\epsilon(x, y, t)\right], \qquad (1)$$

where η represents the entirety of the ocean signal not containing the tidal error component.

An estimate of SSH at the point (x_0, y_0, t_0) resulting from the application of a linear smoother to n data near the estimation point is

$$\hat{h}(x_{0}, y_{0}, t_{0}) = \sum_{j=1}^{n} \alpha_{j} h(x_{j}, y_{j}, t_{j})$$

$$= \sum_{j=1}^{n} \alpha_{j} \eta(x_{j}, y_{j}, t_{j}) + \sum_{j=1}^{n} \alpha_{j} \operatorname{Re} \left[\epsilon(x_{j}, y_{j}, t_{j}) \right], \quad (2)$$

where the smoother weights α_j depend on both the location of the estimate and the particular smoothing algorithm used [e.g., Schlax and Chelton, 1992; Buja et al., 1989].

The second term in (2) represents the contribution of the error in the tidal model to the estimate. Assuming that the amplitude and phase of the tidal error do not vary rapidly over the region from which the data making up the smoothed estimate are drawn, we may write the tidal error contribution as

$$\operatorname{Re}\left[\hat{\epsilon}(x_{0}, y_{0}, t_{0})\right] \equiv \sum_{j=1}^{n} \alpha_{j} \operatorname{Re}\left[\epsilon(x_{j}, y_{j}, t_{j})\right]$$
$$= \operatorname{Re}\left[\sum_{j=1}^{n} \alpha_{j} \epsilon(x_{j}, y_{j}, t_{i})\right]$$
$$= \operatorname{Re}\left[\operatorname{E}(x_{0}, y_{0})\hat{A}(f)\right], \quad (3)$$

where

$$\hat{A}(f) = \sum_{j=1}^{n} \alpha_j \exp(-2\pi i f t_j).$$
(4)

Equation (3) provides an upper bound for the magnitude of the aliased tide error as

$$|\operatorname{Re}\left[\hat{\epsilon}(x_0, y_0, t_0)\right]| \le |\operatorname{E}(x_0, y_0)||\hat{A}(f)|.$$
 (5)

In the absence of detailed knowledge about the amplitude and phase of the tidal error $E(x_0, y_0)$, the term $|\hat{A}(f)|$ is a measure of the potential for the presence of tidal aliasing in a given estimate of SSH. In practice, the magnitude of $|\hat{A}(f)|$ generally ranges from zero when there is no possibility for aliasing of the tidal error to 1 when the potential exists for 100% of the tidal error to be aliased. Depending on the distribution of data and the exact form of the tidal error, the tidal error might actually be amplified; that is, $|\hat{A}(f)|$ can exceed unit value. If $|\hat{A}(f)|$ is small, then the magnitude of the aliased tidal error will be small as well, regardless of the amplitude or phase of $E(x_0, y_0)$. If $|\hat{A}(f)|$ is large, then the possibility of tidal error aliasing cannot be discounted, although the amount of tidal error aliasing will depend upon the amplitudes and relative phases of the terms in (3). When $|\hat{A}(f)|$ is not small, the investigator is obliged to expend some effort to determine the nature of the tidal error term $E(x_0, y_0)$ and the magnitude of the resulting aliased tidal error to make a convincing case that the signal in the data is of oceanographic origin and not aliased tidal errors.

The aliasing statistic |A(f)| may also be derived using the equivalent transfer function formalism of Schlax and Chelton [1992] [also Chelton and Schlax, 1994]. The frequency content of an estimate derived from a linear smoother is quantified by equivalent transfer function for that estimate:

$$\hat{P}(x_0, y_0, t_0; s_x, s_y, f) = \sum_{j=1}^n \alpha_j \exp(2\pi i (s_x x + s_y y - f t_j)),$$

where s_x and s_y are the wavenumbers in the x and y directions, respectively. The aliasing statistic presented here is just the modulus of \hat{P} at zero wavenumbers, expressing the assumption leading to (5) that the amplitude and phase of the tidal error do not change rapidly in space.

When applying the aliasing statistic derived in this section, the fundamental assumption leading to (5) must be borne in mind, namely, that the tidal error term E(x, y) does not change rapidly in space. This assumption may not be valid in certain regions where the structure of the tide is complex (e.g., over continental shelves).

4. Examples

In this section, four examples of the use of the aliasing statistic $|\hat{A}(f)|$ are presented. The Geosat data used in this section were processed using the procedures described by *Matano et al.* [1993], with orbit errors removed as described by *Chelton and Schlax* [1993]. The smoothed SSH estimates are the result of applying a quadratic loess smoother with half-spans of 4° in latitude and longitude and 30 days to residual SSH data that were previously smoothed along track to remove unwanted variablility with scales shorter than 100 km. All estimates were made at the crossovers of ascending and descending tracks at 17-day intervals.

4.1. Inducing Apparent Westward Propagation through M_2 Tidal Error Aliasing

Figures 1a and 1b show time-longitude and power spectral density plots of smoothed SSH between 20°W and 0°W along latitude 38.9°S in the southeastern Atlantic. These figures show only weak evidence of westward propagation. The corresponding plot of the alias-



Figure 1. (a) Time-longitude section of smoothed sea surface height (SSH) along latitude 38.9° S. The contour interval is 0.05 m. (b) Power spectral density corresponding to Figure 1a smoothed to provide 18 degrees of freedom. The contour interval is 0.1 m²/cpd per cycle per degree longitude. The point to which the M₂ tide is aliased by Geosat is marked by an asterisk. (c) Values of the aliasing statistic $|\hat{A}(f)|$ for the smoothed SSH in Figure 1a. The contour interval is 0.2.

ing statistic $|\hat{A}(f)|$ for these SSH estimates (Figure 1c) exhibits low (≤ 0.1) values for most of the timelongitude section, with some localized areas of high values during periods of Geosat data dropouts. According to (5), there is thus little aliased tidal error in most of these estimates, regardless of the magnitude and phase of the error in the M_2 tidal model used to correct these data.

Figures 2a and 2b are time-longitude and power spectral density plots of the same estimates constructed from only the data along ascending tracks. In this case there is clearly an apparent westward propagation centered in frequency-wavenumber space on the M₂ tidal alias (as calculated by Jacobs et al. [1992]). The corresponding plot of the aliasing statistic (Figure 2c) shows values uniformly greater than or equal to 1, indicating that any tidal error present may, depending on its complex phase relative to that of A(f), be completely aliased into the smoothed SSH estimates constructed from ascending track data only. Tidal error aliasing does in fact occur in this instance and is manifested at this latitude as the westward propagating signal of 5- to 10-cm amplitude evident in Figure 2a with wavenumber 0.00157 km^{-1} and frequency 0.00315 s^{-1} (Figure 2b), corresponding to a phase speed of 2.3 cm/s.

These figures are conclusive evidence for the presence in this region of significant errors at the M₂ tidal frequency in the processed Geosat data analyzed here. Moreover, the magnitude of the tidal error observed here agrees with the results of *Jacobs et al.* [1992, Figure 6a], who used the corrected Schwiderski model supplied with the GEM-T2 GDRs. They calculated M₂ tidal errors in this region with amplitudes ranging from 4 to 9 cm. This tidal error is obscured in SSH estimates constructed from combined ascending and descending track data because the time interval between ascending and descending ground tracks at this particular latitude is favorable for weak aliasing. This condition is reflected by the small values of $|\hat{A}(f)|$ in Figure 1a (see also section 4.3).

In this example of induced aliasing, the worst possible case has been presented: that of a region with only data from ascending or descending tracks. This is an important point, as in many regions (e.g., off the west coast of California, in the Malvinas Basin, and in the Aghulas Return Current), loss of either ascending or descending track data for extended periods was common for Geosat because of problems with attitude control [*Ch*eney et al., 1988, Figure 2]. Aliasing of M₂ tidal errors is thus a serious concern in these regions. The evidence for westward propagating baroclinic Rossby waves off the California coast in particular, as presented by, for example, *White et al.* [1990], *Matthews et al.* [1992], and *Kelley et al.* [1993], is compromised by the almost total loss of descending track data in this region.

4.2. Data Dropouts in the Agulhas Return Current Region

One of the difficulties encountered when interpreting data contaminated by tidal error aliasing is the variation of the degree of the aliasing over the space-time region where SSH estimates are made. The aliasing statistic $|\hat{A}(f)|$ can provide some guidance in such sit-



Figure 2. (a) Time-longitude section of smoothed SSH along latitude 38.9° S, determined using only data from ascending tracks. The contour interval is 0.05 m. (b) Power spectral density corresponding to Figure 2a smoothed to provide 18 degrees of freedom. The contour interval is 0.5 m²/cpd per cycle per degree longitude. The point to which the M₂ tide is aliased by Geosat is marked by an asterisk. (c) Values of the aliasing statistic $|\hat{A}(f)|$ for the smoothed SSH in Figure 2a. The contour interval is 0.1.

uations. This is demonstrated here using Geosat data from the Agulhas Return Current region, where systematic dropouts from ascending tracks for extended periods of time were common. In the absence of detailed information on the amplitude and phase of the error in the tidal model used here, we will use the *Schwider*ski [1980] model M₂ tidal values as sampled by Geosat directly as an example of the efficacy of the aliasing statistic.

Figure 3 is a time-longitude section of the M_2 tidal constituent between 19°E and 44°E along 38.9°S, which coincides approximately with the axis of the Agulhas Return Current. These estimates were made in the manner described above except that the model M₂ tide values were substituted for the actual Geosat data. The aliasing of the tide into apparently westward propagating features is clear in certain portions of the plot. An outstanding feature of this plot is the interruption of the bands of the aliased tidal signal at times between 0 and 70 days and between 280 and 430 days. This behavior is explained by the periodic nature of the data dropouts in this region and is made clear by the shading in Figure 3. which represent estimates for which $|A(f)| \ge 0.7$. Areas of Figure 3 that display strong aliased westward propagation coincide with areas of large values of $|\hat{A}(f)|$, corresponding to periods of missing data along ascending ground tracks. The horizontal bands in Figure 3 that interrupt the aliased tidal signal correspond to areas with low |A(f)|, corresponding to periods of little data loss.

4.3. Global Patterns of M_2 Tidal Aliasing in Geosat Data

The aliasing statistic provides a global picture of locations where the potential for aliasing of tidal errors is high. For the estimates made here, there is a welldefined relation between the value of $|\hat{A}(f)|$ and the relative tidal phase difference between the ascending and descending track samples at the crossover point (Figure 4). This relationship is simple to understand if the estimates are considered weighted averages of the data. If the ascending and descending tracks sample the tidal error at relative phases that are separated by nearly one half of the tidal period, then the tidal error will aver-



Figure 3. Time-longitude section of smoothed values of the M₂ tide along latitude 38.9°S. The contour interval is 0.1 m. The gray areas denote smoothed estimates for which $|\hat{A}(f)| \ge 0.7$.

age nearly zero in the estimate. If, on the other hand, the sample times are at nearly the same relative phase, then the averaging will not be as effective at mitigating the tidal error.

The interval between ascending and descending track sample times depends only upon latitude. Given the relation expressed in Figure 4, this dependence will hold for $|\hat{A}(f)|$ as well. The variation of $|\hat{A}(f)|$ with latitude is shown in Figure 5 for the M_2 tidal constituent. These values of the aliasing statistic were derived assuming that there were no data dropouts; Figure 5 thus presents an optimistic assessment of the latitudinal variation of the potential for tidal aliasing. (It should be noted that $|\ddot{A}(f)|$ in general depends upon both the spatial and temporal locations of the estimate. The smoothing parameters used here were selected using the method described by Chelton and Schlar [1994] so that the mean squared errors of the SSH estimates do not depend on the time at which the estimate is made. As a result, the aliasing statistic does not vary with the time of the estimate.)

It is evident from Figure 5 that the potential for the aliasing of M_2 tidal errors in Geosat data fluctuates rapidly with latitude, depending on the time interval between ascending and descending ground tracks. Tidal error aliasing will be small along 38.9° S, regardless of the amplitude and phase of the M_2 tidal error. This is the reason that 38.9° S was selected for the examples in the previous sections; at that latitude, the effects of data dropouts on the aliasing of M_2 tidal errors will be most apparent.

4.4. Tidal Error Aliasing in Geosat Data from the Azores Frontal Region

In a recently published study of Geosat data from the northeast Atlantic, *Tokmakian and Challenor* [1993, Figures 6 and 7] show a number of time-longitude



Figure 4. $|\dot{A}(f)|$ versus the difference between ascending track sample time and descending track sample time expressed as the relative phase difference with respect to the M₂ tidal period. The aliasing statistic values are for quadratic loess estimates at crossover locations using the smoothing parameters given in the text.



Figure 5. $|\hat{A}(f)|$ versus latitude for the loess estimates at crossover locations using the smoothing parameters given in the text. Asterisks mark the latitudes and corresponding values of $|\hat{A}(f)|$ for the time-longitude sections in Figure 6.

and wavenumber-frequency spectral density plots that present clear evidence of westward propagation. In two such plots, covering the longitude range 45° W to 5° W at latitudes 30° N and 35° N, they report the existence of westward propagating signals whose wavenumberfrequency characteristics are compatible with the dispersion relation for baroclinic Rossby waves. At a third latitude, 40° N, a westward propagating signal is present but is not consistent with the hypothesis that it is the result of a Rossby wave.

Tokmakian and Challenor [1993] acknowledge that the signals that they observe are very close to those to be expected from tidal error aliasing, but they argue that the observed signals are not aliased tides by noting that they achieve the same results when using either the Schwiderski [1980] model as supplied with the NAG GDRs or the Cartwright and Ray [1990] model. They further argue that it is highly unlikely that either of these models could be in error by an amount consistent with the 10-cm magnitude of the westward propagating signal that they observe. Given that Jacobs et al. [1992] calculate M_2 tide errors of approximately 8 cm in this region and given the cautionary remarks of Ray [1993] about the inaccuracies of the presently available tidal models, it is well to examine the nature of the westward propagating signals observed by Tokmakian and Challenor [1993] using the aliasing statistic presented here.

Figure 6 shows a series of four time-longitude plots made at latitudes 29.84°N, 32.65°N, 35.29°N, and 38.98°N. These latitudes were selected for two reasons. First, they are near those used by Tokmakian and Challenor, and second, they are the latitudes of crossovers whose inherent M₂ aliasing potentials vary dramatically (Figure 5). Each plot in Figure 6 consists of contours of the smoothed estimates of SSH and a gray overlay that marks estimates for which the potential for tidal error aliasing is high ($|\hat{A}(f)| \ge 0.7$). The SSH estimates



Figure 6. Time-longitude sections of smoothed SSH along latitudes (a) 29.84°N, (b) 32.65°N, (c) 35.29°N, and (d) 38.98°N. The contour interval is 0.05 m. Missing estimates are not contoured. The gray areas denote smoothed estimates for which $|\hat{A}(f)| \ge 0.7$.

making up these plots were constructed as weighted averages of nearby observations as described by Matano et al. [1993]. Since Tokmakian and Challenor used simple spatial averaging and the Cartwright-Ray tide model, the plots presented here differ in detail from theirs, but the essential features are the same in both sets of plots. The time-longitude sections along 29.84°N and 35.29°N (Figures 6a and 6c, respectively) show both strong westward propagating signals and uniformly high values of |A(f)|, the latter in accordance with Figure 5. The corresponding wavenumber-frequency power spectral density plots (not shown) both have dominant peaks centered on the M_2 alias. In contrast, the timelongitude sections along 32.65°N and 38.98°N (Figures 6b and 6d, respectively) provide weaker visual evidence of a westward propagating signal. The gray shading indicates that the estimates of SSH at these latitudes are generally much less prone to aliasing than are those in Figures 6a and 6c, again in agreement with Figure 5. The power spectral densities (not shown) for these latter figures still show some energy near the M_2 alias, but the magnitudes of these signals are much less than those at latitudes 29.84°N and 35.29°N.

The latitudinal variations of both the spectral energy at the M₂ alias and |A(f)| are quantified in Figure 7. Figure 7a shows estimates of the power spectral density near the M_2 alias for each of the four latitudes that are formed as the average of the four points in the raw wavenumber-frequency spectrum nearest to the M₂ alias. Figure 7b shows the variation of the potential for M₂ aliasing with latitude. The dashed curve is reproduced from Figure 5 and shows the ideal variation of $|\ddot{A}(f)|$ with latitude (i.e., the value of $|\ddot{A}(f)|$ when there are no data dropouts). The solid curve is the average value of $|\hat{A}(f)|$ from the actual Geosat data for the SSH estimates in each time-longitude plot. The correspondence between the spectral energy near the M_2 alias and $|\hat{A}(f)|$ is clear in Figure 7. At the latitudes where aliasing of any existing M_2 tidal error is expected, energetic westward propagation at the M2 alias is observed. Conversely, at latitudes where little M_2 aliasing is expected, there is low signal energy at the M_2 alias.

Since 32.65°N and 38.98°N are latitudes at which there is little potential for tidal error aliasing, the method used in section 3.1 for inducing aliasing may be applied there. Time-longitude sections were constructed



Figure 7. (a) Variation with latitude of the power spectral density near the M_2 alias for the timelongitude sections of Figure 6. The 90% confidence intervals for the 8 degrees-of-freedom spectral estimates are indicated by the vertical lines. The asterisks show the spectral density values obtained at latitudes 32.65° N and 38.98° N when only data from descending tracks are used. (b) The average values of $|\hat{A}(f)|$ over each of the time-longitude sections of Figure 6 (solid curve) and the variation of $|\hat{A}(f)|$ with latitude when there are no data dropouts (dashed curve).

using only data from descending tracks, and the corresponding estimates of the power spectral density near the M_2 alias were made as described above. For the time-longitude section at 32.65°N, the power spectral density increased from a value of 1.5 when all data were used to a value of 9.7 when only data from descending tracks were used. At 38.98°N the corresponding increase was from 2.1 to 9.7. These elevated spectral values (shown by the asterisks in Figure 7a) are close to the spectral density values obtained at the M_2 alias for the neighboring time-longitude sections at 29.84°N and 35.29°N, where the potential for tidal aliasing is large for estimates constructed from combined ascending and descending track data. The amplitude of the apparent westward propagation is in the 5- to 10-cm range.

Thus at two latitudes at which the use of all of the data precludes the aliasing of any M_2 tide error, we have induced such aliasing by using the data only from descending tracks. The amplitude of the westward propagation so induced is very nearly the same as the amplitude of westward propagation observed at nearby latitudes where M₂ tidal aliasing will occur even if data from ascending and descending tracks are used, given the presence of errors in the M_2 model constituent. If the westward propagation observed in the time-longitude sections were truly the result of Rossby waves, the observed magnitude of the propagation manifested in the Geosat data would not change simply because we chose to use only ascending or descending track data. It is difficult to imagine any signal other than that from aliased M₂ tidal error that would appear at precisely the correct location in wavenumberfrequency space and behave in the manner predicted by the aliasing statistic.

It is significant that the magnitude of the aliased M_2 signal observed here is close to that calculated by Jacobs et al. [1992] and indicates that the tidal error that we observe is not due solely to the erroneous implementation of the Schwiderski model discussed in section 2. Furthermore, since the results obtained with the NAG GDR version of the Schwiderski tide model apparently agree well with those obtained using the Cartwright and Ray model [Tokmakian and Challenor, 1993], the M_2 tidal error observed here is not just the result of failure to account for the elastic loading tide but is indicative of other errors that are independent of which tide model is used.

The results presented in this section do not discount the possibility that Rossby waves exist in the Azores Frontal Region or that they might exhibit themselves in the Geosat data. However, this analysis suggests that at least some of the westward propagation observed in the Geosat data from this region is the result of aliasing errors in the model M_2 tidal constituents used to correct the Geosat data. These results underscore the caution that must be exercised when interpreting the presence of westward propagating signals and their latitudinal variation as evidence for the presence of Rossby waves.

5. Conclusion

A simple statistic that warns of the potential for tidal error aliasing in a given linear estimate of SSH has been developed. This statistic, which has been shown to be the modulus of the equivalent transfer function of the estimate at zero wavenumbers, quantifies the extent to which any tidal errors may be incorporated into the estimate. The aliasing statistic $|\hat{A}(f)|$ can be calulated with ease for any tidal constituent and any altimeter orbital configuration. Attention has been restricted here to the M_2 tide and data from Geosat because of the similarity between the characteristics of aliased M_2 tidal error and annual Rossby wave propagation for the Geosat orbit.

The latitudinal dependence of the potential for M_2 tidal aliasing has been calculated for Geosat for the ideal case in which there are no data dropouts. As illustrated by the examples in sections 4.2 and 4.4, the aliasing statistic can change when there are significant data dropouts. The aliasing characteristics vary rapidly with latitude and depend on the time interval between ascending and descending tracks near the location of interest.

The aliasing statistic has been used here to study the latitudinal variation of westward propagating signals observed in Geosat data from the northeastern Atlantic. Because the magnitude of signals occurring at the M_2 alias varies in a manner consistent with that predicted by the aliasing statistic, it is highly probable that at least some of the westward propagation observed in this region is due to tidal error aliasing. At latitudes where there is little aliasing and correspondingly weak westward propagation, it was possible to induce aliasing and strong apparent westward propagation by selectively omitting data from ascending tracks when constructing smoothed SSH fields. The ability to do so provides conclusive evidence for the presence in the Geosat data of westward propagating signals whose genesis is aliasing of M_2 tidal errors.

The formalism presented here offers a method for quantifying the accuracy of tides in altimeter data. In regions where the aliasing statistic |A(f)| is small for SSH estimates constructed from both ascending and descending track data, the magnitude of tidal errors can be determined by consideration of the ascending or descending data in isolation, which results in SSH estimates with a large |A(f)|. The examples of this application of the aliasing statistic in sections 4.1 and 4.4 determined that after the Schwiderski [1980] tidal correction has been applied and orbit errors have been removed in the traditional manner, the M₂ tidal constituent is in error by 5-10 cm in the southeast Atlantic and the northeast Atlantic and confirms the results of Jacobs et al. [1992] in this regard. A systematic application of this technique to the global ocean along each latitude for which $|\hat{A}(f)|$ is small in Figure 5 would complement the tidal error analysis of Jacobs et al. [1992] and provide a great deal of information about the accuracies of tidal corrections in altimeter data. Work in progress is attempting to unravel how much of the tidal error can be attributed to errors in the particular tide model used and how the tidal errors are affected by orbit error corrections.

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