UNCERTAINTIES IN SEASONDE CURRENT VELOCITIES

Belinda Lipa
Codar Ocean Sensors
125 La Sandra Way, Portola Valley, CA 94028
blipa@pogo.com

Abstract – This paper describes the methods used to derive uncertainties in SeaSonde radial and total velocity vectors. Studies of baseline deviations are used to illustrate/validate the results. Recommendations are made for further work leading to reduction of the uncertainties.

INTRODUCTION

A measurement is incomplete without an estimate of its uncertainties. SeaSonde current vector output now includes uncertainties in both radial and total velocity vectors. This paper describes how current vector uncertainties are derived.

A single SeaSonde unit measures the component of the velocity radial to the radar. Most uncertainty in these radial vectors is due to spatial and temporal variations of the current field and use of non-optimal analysis parameters. Spatial uncertainty results because there can be many different current velocities present in the radar scatter patch due to horizontal shear; these velocity values are calculated during analysis and are averaged to produce the value output for that location. The uncertainty is estimated by calculating the standard deviation of all the velocities that fall on a particular location. This spatial uncertainty usually increases with distance from the radar as the size of the radar scatter patch increases proportionally with range. Uncertainty can also arise from variations in the current velocity field over the duration of the radar measurement, and from assumptions and simplifications that are made during the analysis process. The uncertainty due to these effects is estimated by calculating the standard deviation of radial velocities resulting from analysis of successive short spectral averages.

The total current vector is obtained by least-squares fitting to the radial vectors from two or more radar sites on a grid that extends over the joint coverage area. The uncertainties in the total velocities follow from those in the radials using standard linear error propagation, which includes the effects of radial uncertainties as well as the geometry.

We illustrate and validate our results using data obtained along the baseline joining two radar sites. Ideally, radial velocity components from the two sites are equal and opposite at each point on the baseline; in practice they are not, due to data imperfections. Baseline deviations give a quantitative measure of the degree of imperfection and can be used to test various hypotheses.

The rest of the paper is structured as follows: Section A. describes sources of radial velocity uncertainties and their calculation. Section B describes the derivation of the total velocities and their uncertainties. Section C describes tests along a baseline between two Long Island Sound SeaSondes.

A: UNCERTAINTIES IN RADIAL VECTORS

ii) *Sources of uncertainty*

Assuming that the radar is operating correctly, we can identify the following sources of uncertainty in the radial velocities:

(a) Variations of the radial current component within the radar scattering patch.

(b) Variations of the current velocity field over the duration of the radar measurement.

(c) Errors/simplifications in the analysis. For example these may include the use of incorrect antenna patterns and analysis parameters, and errors in empirical first-order line determination.

(d) Statistical noise in the radar spectral data**.**

(2) Computation of radial velocity uncertainties

The radar analysis proceeds as follows: In the first step, from analysis of a 10-minute voltage spectral average, we collect all the radial vectors falling on a given radar cell, as defined by range

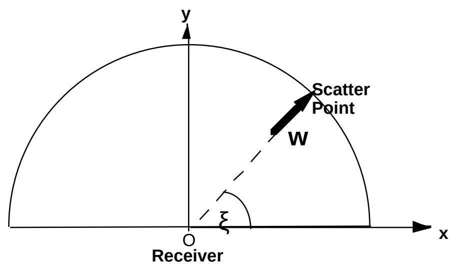

Fig. 1: Geometry of a backscatter radar

from the radar and a finite angular increment (usually 5°) around a central azimuth angle. We then calculate the mean and standard deviation of these vectors. We believe that this uncertainty is due to mainly horizontal velocity shear within the radar cell (Source (a) above) and will refer to the standard deviation as the 'spatial uncertainty' in what follows. Support for this hypothesis will be given in Section C.

In step 2, successive 10-minute radial files are merged over an hour to produce the final output result, which consists of the mean radial velocity and the standard deviation of the mean, obtained from the 10-minute radial velocities at each location. This uncertainty results from the remaining sources (b) – (d) above.

The spatial uncertainty is not at present included in the standard Seasonde output but is available in the unmerged radial files. It is not included because the output radial current vectors represent the area average of the radial velocities over the radar scatter patch; the corresponding uncertainty is the standard deviation of this mean, which is calculated from successive time samples. Thus even if the spatial unceratintties are high, the current maps may look much the same from time to time, as they represent spatial averages.

SECTION B. UNCERTAINTIES IN TOTAL VELOCITIES

Firstly, we define a rectangular grid covering the joint coverage area with cell size typically 1.5 x 1.5 km. In practice, all radial components that fall within a combining circle of defined radius (typically 2km) are combined to form a single total current vector at the grid point. We define these m components as having magnitudes w_i and inclinations ξ^i for i=1, 2, ...m, if the component is produced by backscatter, it is normal to the circular range cell (see Fig. 1). We define a total velocity vector with components U and V along the x, y axes, which are determined by minimizing the sum of weighted deviations given by:

$$S = \sum_{i=1}^{m} \frac{\left(w_i - U\cos(\xi^i) - V\sin(\xi^i)\right)^2}{(\Delta w_i)^2} \qquad (1)$$

where the weights Δw_i are the uncertainties in the radial velocities. The solutions to this minimization are given by:

$$U = \frac{\sum_{i=1}^{m}\frac{w_i\cos(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\sin^2(\xi^i)}{(\Delta w_i)^2} - \sum_{i=1}^{m}\frac{w_i\sin(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2}}{\sum_{i=1}^{m}\frac{\cos^2(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\sin^2(\xi^i)}{(\Delta w_i)^2} - \left(\sum_{i=1}^{m}\frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2}\right)^2} \qquad (2)$$

$$V = \frac{\sum_{i=1}^{m}\frac{w_i\sin(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\cos^2(\xi^i)}{(\Delta w_i)^2} - \sum_{i=1}^{m}\frac{w_i\cos(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2}}{\sum_{i=1}^{m}\frac{\cos^2(\xi^i)}{(\Delta w_i)^2}\sum_{i=1}^{m}\frac{\sin^2(\xi^i)}{(\Delta w_i)^2} - \left(\sum_{i=1}^{m}\frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2}\right)^2} \qquad (3)$$

with the corresponding variances obtained from linear error propagation (see for example Brandt: 'Statistical and Computational Methods in Data Analysis'). When a data matrix W is defined in terms of a parameter matrix P through a linear transformation matrix T i.e.

$$W = TP \qquad (4)$$

then the covariance matrix C_P is defined in

$$C_P = \left(T' C_W^{-1} T\right)^{-1} \qquad (5)$$

terms of the covariance matrix of W, C_W by the relation

In our case, P is a (2 x1) matrix with components u, v (the components of the total velocity vector) and W is the matrix containing the radial velocities from the different radar sites at that geographical location.

Ignoring correlations between radial velocities measured by the different radars, the variances in U and V and their covariance are given by:

$$\langle \Delta U^2 \rangle = \frac{\displaystyle\sum_{i=1}^{m} \frac{\sin^2(\xi^i)}{(\Delta w_i)^2}}{\displaystyle\sum_{i=1}^{m} \frac{\sin^2(\xi^i)}{(\Delta w_i)^2} \sum_{i=1}^{m} \frac{\cos^2(\xi^i)}{(\Delta w_i)^2} - \left(\sum_{i=1}^{m} \frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2} \right)^2} \quad (6)$$

$$\langle \Delta V^2 \rangle = \frac{\displaystyle\sum_{i=1}^{m} \frac{\cos^2(\xi^i)}{(\Delta w_i)^2}}{\displaystyle\sum_{i=1}^{m} \frac{\sin^2(\xi^i)}{(\Delta w_i)^2} \sum_{i=1}^{m} \frac{\cos^2(\xi^i)}{(\Delta w_i)^2} - \left(\sum_{i=1}^{m} \frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2} \right)^2} \quad (7)$$

$$\langle \Delta U \Delta V \rangle = \frac{\displaystyle\sum_{i=1}^{m} \frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2}}{\displaystyle\sum_{i=1}^{m} \frac{\sin^2(\xi^i)}{(\Delta w_i)^2} \sum_{i=1}^{m} \frac{\cos^2(\xi^i)}{(\Delta w_i)^2} - \left(\sum_{i=1}^{m} \frac{\sin(\xi^i)\cos(\xi^i)}{(\Delta w_i)^2} \right)^2} \quad (8)$$

For the case of two backscatter units along the x axis, it follows from (7) that the variance in the component V along the baseline between the sites becomes infinite. This must be true because all radial velocity components are parallel to the baseline and therefore the input data provides no information on the perpendicular component of the total velocity vector.

This calculation is similar to the that termed 'geometrical dilution of precision'. The latter however assumes that errors in the radial velocities are constant, which of course they are not; see for example: K.-W. Gurgel, 'Shipborne measurements of surface current fields by HF radar', L'Onde Electrique, September-October 1994, Vol. 74, No. 5.

SECTION C: BASELINE ANALYSES

This section describes analysis of radial velocities measured at points along the baseline between radar sites at Montauk and Misquamicut. The baseline lies over about 40km of open ocean. The current velocity and horizontal velocity shear increase close to Montauk, as the current swirls around the Montauk point. Ideally, radial vectors from the two sites would be equal and opposite at a point on the baseline, so the magnitude of their sum represents a measure of imperfection in the data. In Subsection 1) we describe the data from the individual radars, and in Subsection 2) we give results of tests based on baseline deviations.

1) Radial data and their uncertainties

In Fig. 2, we plot for the two sites at points along the baseline the rms radial velocity, together with the mean spatial uncertainty and the standard deviation of the mean velocity. Values are obtained by averaging hourly data over the period of a week. Ideally, the rms speeds from the two sites would be identical. Clearly however, close to Montauk, the Montauk speed is much larger than for Misquamicut and the Misquamicut spatial uncertainty becomes almost as large as the speed itself. We can explain this as follows: (a) there is a large horizontal velocity shear close to Montauk (b) the radar cell size is large as it is distant from the radar. Therefore when the velocities from the radar cell are averaged in the first step described in Section A2, there is a lot of averaging-down, yielding a value much lower than the extreme value. In contrast, at this location, the Montauk radar cell size is small, as it is close to the radar, and there is much less averaging-down, leading to a higher velocity value and a lower spatial uncertainty.

2) Baseline deviations

We define the mean-square baseline deviations at points on the baseline as

$$B = <(v_1 + v_2)^2> \quad (9)$$

Where v_1, v_2 are the Montauk, Misquamicut radial velocities and the average is taken over time (in our case, the period November 11 to November 17, 2000). Ideally B is zero; its magnitude provides a useful parameter to aid in making analysis decisions. We now give two examples of this.

i) The effect of spectral averaging time on accuracy.

In Fig. 3 (upper), the rms baseline deviations from equation (9) ($\sqrt{B}$) are plotted versus distance along the baseline for 1-hour and 10-minute spectral averages. If the baseline deviations were due to statistical noise, one would expect the 1-hour results to be a factor of $\sqrt{6}$ lower than the 10-minute deviations. As these curves are almost identical, it can be concluded that statistical noise is not a significant contributor. We note that the baseline deviations increase as Montauk is approached, corresponding to the divergence between the velocities from the individual sites shown in Fig. 2.

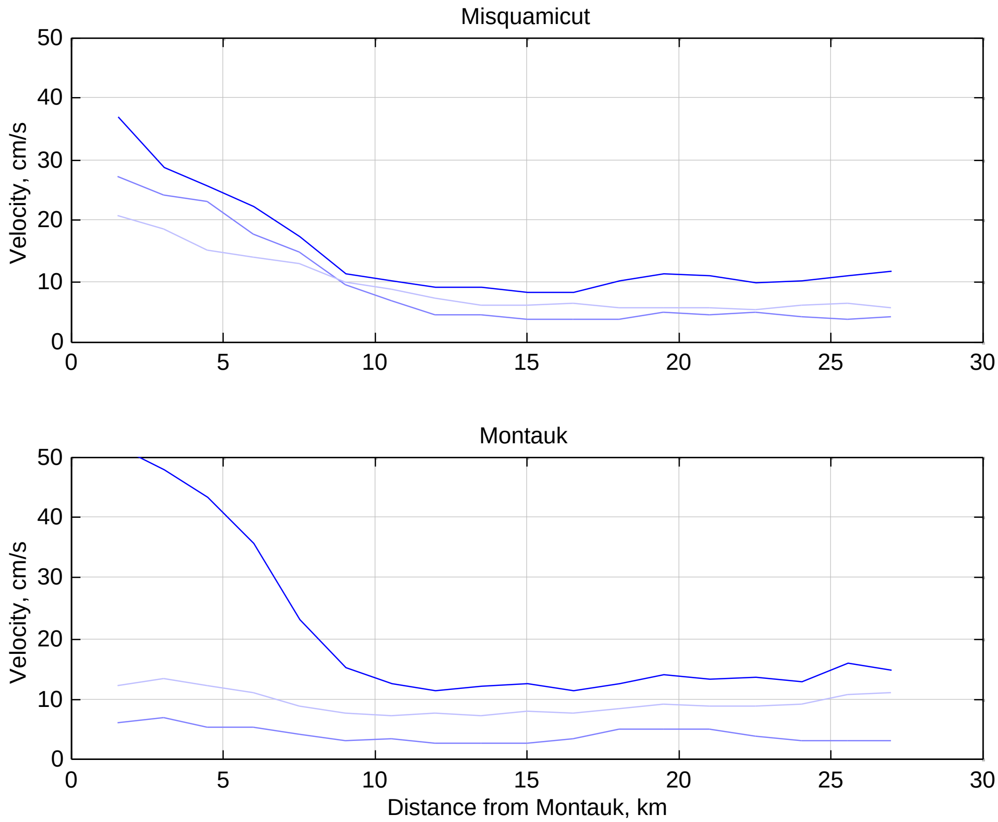

Fig. 2: RMS radial velocities and their uncertainties for Misquamicut (upper) and Montauk (lower) obtained by averaging over SeaSonde hourly results obtained alone the baseline between the sites from November 11 to November 17, 2000. Solid line: (dark) rms radial velocity; dashed line: (medium) averaged spatial uncertainty; dotted line (light): averaged standard deviation in the mean velocity.

ii) Comparison of baseline deviations and spatial uncertainties

We now show how baseline deviations can be used to validate our uncertainty calculations. Expanding about the ideal velocities gives:

$$v_1 = v_{1\,ideal} + \Delta v_1 \;;\; v_2 = v_{2\,ideal} + \Delta v_2 \quad (10)$$

Substituting (10) into (9), and using the fact that the sum of $v_{1\,ideal}$ and $v_{2\,ideal}$ is zero, gives the following expression for the mean-square baseline deviation:

$$B = < (v_{1\,ideal} + v_{2\,ideal} + \Delta v_1 + \Delta v_2)^2 > \text{ or}$$
$$B = \sigma^2(v_1) + \sigma^2(v_2) + 2\rho(v_1, v_2)\,\sigma(v_1)\,\sigma(v_2) \quad (11)$$

where σ is the rms value and $\rho(v_1,v_2)$ is the correlation coefficient. Ignoring correlations between v_1 and v_2, it follows the rms deviations obey the approximate relation:

$$\sqrt{B} \approx (\sigma^2(v_1) + \sigma^2(v_2))^{1/2} \quad (12)$$

Fig. 3 (lower) shows the two curves representing the right and left side of (12), v_1, v_2 are the Montauk, Misquamicut radial velocities and the average is taken over time and where spatial uncertainties only are incuded on the right-hand. The good agreement between the curves indicates that the large baseline deviations are explained by the disparity in the sizes of the radar cels for the two sites, together with the large horizontal velocity shear.

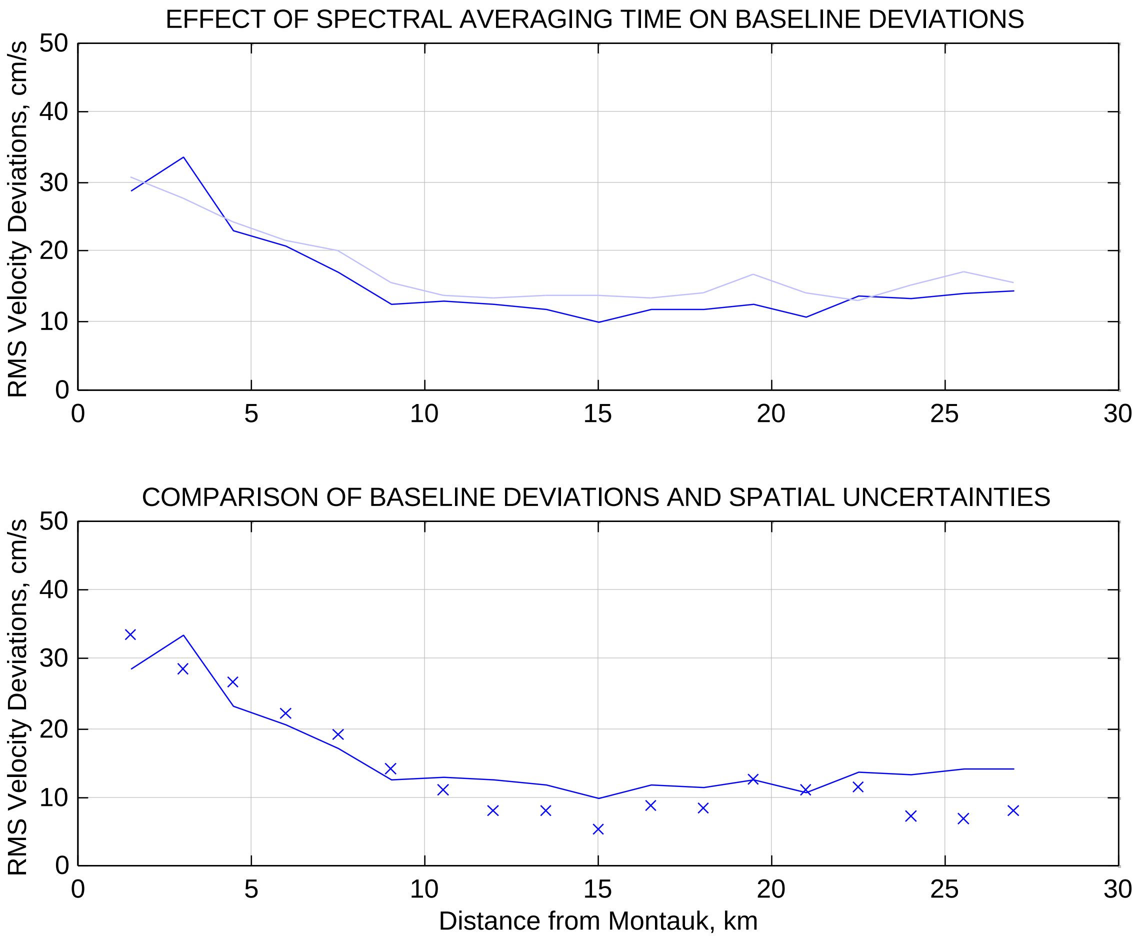

Fig. 3: BASELINE TESTS
Upper: : Comparison of the rms baseline deviations for one-hour spectral averages (solid line) and 10-minute spectral averages (dotted line)
Lower: Comparison of the rms baseline deviations (solid line) and the rms sum of the spatial uncertainties (crosses)

CONCLUSIONS

In conclusion: we have described the uncertainties inherent in SeaSonde current measurements: many of these are familiar: statistical uncertainties, analysis errors and so on. There is an additional source of uncertainty common to all systems that make polar measurements: as the area covered by the measurement increases with range, so does the span of values contained within the area. This should be taken into consideration whenever measurements with a different footprint are being combined or compared. We have shown in this paper that large baseline deviations can occur when velocities obtained from large and small radar cells are compared, even though the central points of the cells coincide on the ocean surface. Similarly errors may occur when radial vectors corresponding to very different cell sizes are combined to form a total velocity vector, as the velocity from the larger cell size is more area-averaged. This will be significant in regions of high current shear.

When comparing Seasonde and ADCP measurements, it must be born in mind that the SeaSonde gives an area measurement and the ADCP gives a point measurement; it is only in regions of low current shear and / or a small radar cell size that results from the two instruments can be expected to be in good agreement. When making such a comparison, the SeaSonde spatial uncertainties should be examined – if they are high at the ADCP location, one can not/should not expect good agreement.

This paper presents evidence from baseline tests that indicate that the spatial standard deviation in SeaSonde radial velocity measurements is a measure of the horizontal velocity shear in the radar cell; However, this hypothesis awaits experimental verification, for example by comparison with simultaneous output of two or more ADCPs in the same radar scatter patch.

In future studies, the effects on SeaSonde results of reducing the angular and radial size of the radar cell will be examined. In addition, we will be studying the effect of reducing the measurement duration on the radial velocity uncertainties. The result is not immediately obvious, as there is a tradeoff: reducing the measurement time will increase the uncertainty due to statistical noise, and reduce the uncertainty due to variation of the ocean current pattern.

At present, uncertainties in the total vector components do not include contributions from spatial uncertainties in the radial vectors. These need to be included, because, as for the baseline deviations, significant errors result when radial vectors corresponding to radar cells of different sizes are combined to form a total velocity vector.

Finally, it is clear that a complete calculation of total vector uncertainties must include the uncertainties in the radial velocities as well as the effects of geometry. Calculations that include only the latter (commonly termed ‘geometrical dilution of precision’) are necessarily incomplete.

Recommendations for future work:

1. Verification that SeaSonde spatial uncertainties represent horizontal current velocity shear using ADCPs located in a single radar cell.

2. Reduction of SeaSonde spatial uncertainties by reducing the size of the radar scatter patch.

3. Studies to determine whether SeaSonde uncertainties can be reduced by decreasing the time duration of a measurement.

4. Inclusion of spatial uncertainties in the total vector output.

Acknowledgements: *Many thanks to David Kaplan, for his critical reading of this manuscript and many valuable suggestions, to Don Barrick and David Ullman for helpful discussions and to David Ullman for providing the data used in the baseline analyses.*