Theoretical Model of the Thermocline in a Freshwater Basin

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ABSTRACT

Two striking empirical facts represent a starting point of the discussion: (i) vertical temperature profile in a thermocline, a region of supercritical stability adjoining a mixed layer, proves to be self-similar in a series of laboratory experiments and, though with much less accuracy, in natural water reservoirs; (ii) comparison of the experimental data on kinematic heat flux Q and vertical temperature gradient $\partial T/\partial z$ shows that the effective heat conductivity $K \equiv -Q/(\partial T/\partial z)$ is much higher than the molecular one, and, rather unexpectedly, increases with the increase of $|\partial T/\partial z|$, in contrast with the well-known inverse dependence of K on $|\partial T/\partial z|$, which takes place in weakly stable shear flows.

A theoretical model of a regime under consideration is proposed based on the hypothesis on governing parameters of intermittent turbulence generated on the background of strongly stable stratification by breaking of internal waves and turbulence/wave interactions, the heat transfer equation and the balance equation for turbulent kinetic energy. The solution of the problem of a propagating-wave type coincides with the empirical approximation of a dimensionless, self-similar temperature profile only in the special case of pronounced deepening of the mixed layer. This explains the fact of much better accuracy of similarity representation of the temperature profile in laboratory thermoclines. Indeed, all known experiments just dealt with development of a thermocline under the condition of a deepening mixed layer.

The model contains two dimensionless constants whose values are found by means of comparison of the solution with the results of laboratory experiments on penetrative convection. Using these constants, a numerical simulation of the thermocline in laboratory experiments by Deardorff and Willis on shear currents in an annulus appears to be quite realistic. This result confirms applicability of the model to different types of laboratory thermoclines.

1. Introduction

In vertical structure of oceans, seas, lakes, ponds—of all natural water bodies and manmade reservoirs with the free surface subjected to atmospheric effects—two essentially different layers are observed. First, the upper quasi-homogeneous well-mixed layer, and second, the underlying thermocline, a layer characterized by an abrupt increase of density with increasing depth, i.e., by a sharply stable stratification. Similarly, the lower atmospheric layer may be generally divided into two essentially different parts: the near-surface, well-mixed layer and the capping temperature inversion, which may be called an atmospheric thermocline. Richardson numbers in a thermocline considerably exceed critical values. Therefore, well-developed turbulence cannot be permanently maintained here due to mean-flow instability. Nevertheless, measurements prove that turbulence really exists in the thermocline, not fully developed but intermittent, localized in separate irregularly scattered spots, which form a so-called microstructure (Fedorov 1976; Kraus 1977; Monin and Ozmidov 1981). Each turbulence spot has a limited life, but degenerated spots are replaced by new ones, so there is always a multitude of such spots. Its summary action results in transport of heat, salt, and buoyancy much more effective than due to molecular heat conductivity and diffusion. The main unsolved problem connected with the thermocline is how to calculate this transport and to explain the energetics of intermittent turbulence.

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2. Self-similarity of temperature profile in thermocline

For calculations of the thermal regime of oceans, seas, lakes, and ponds the following expression for the vertical profile of temperature T is widely used:

$$T = \begin{cases} T_s, & \text{at } 0 \leqslant z \leqslant h \\ T_s - (T_s - T_a)\vartheta(\zeta), & \text{at } h \leqslant z \leqslant h + \Delta h. \end{cases} \quad (1)$$

Here, z is the depth; h is the thickness of the mixed layer (the temperature is actually constant with depth within this layer, so it may be assumed to be equal to the water surface temperature, T_s); Δh is the thickness of the thermocline; T_a is the temperature at the depth $z = h + \Delta h$; and ϑ is a function of dimensionless variable

$$\zeta = (z - h)/\Delta h, \quad (2)$$

satisfying the following boundary conditions:

$$\vartheta(0) = 0, \quad \vartheta(1) = 1. \quad (3)$$

Figure 1 illustrates schematically the temperature profile that corresponds to such an expression. If the function ϑ is known, this profile is completely characterized by four parameters: T_s, T_a, h, Δh, which may be functions of time t and horizontal coordinates x and y. Certainly, instead of T_a one can use the temperature difference across the thermocline $\Delta T = T_s - T_a$ as a characteristic parameter.

The use of the model of a well-mixed (and therefore temperature-uniform) upper layer in natural water basins was initiated by Kraus and Turner (1967). The self-similarity concept of the temperature profile in a thermocline was introduced by Kitaigorodskii and Miropolsky (1970). They suggested the expression

$$\vartheta = \frac{8}{3}\zeta - 2\zeta^2 + \frac{1}{3}\zeta^4$$

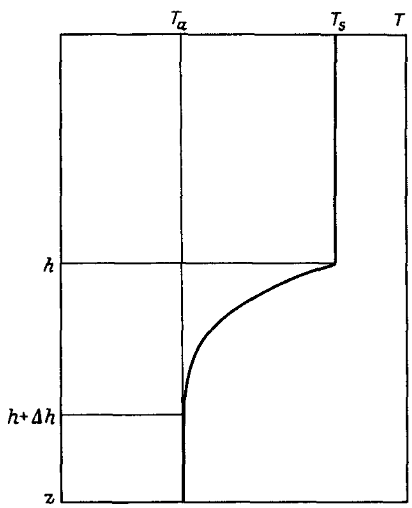

FIG. 1. Schematic temperature profile in the mixed layer and thermocline.

and confirmed it as a first approximation by processing data from measurements in the oceanic active layer.

Laboratory experiments (Linden 1975; Wyatt 1978) confirmed the concept of self-similarity of thermocline much better than field measurements.

The simplest reasonable a priori expression for the function ϑ was derived by Arsenyev and Felzenbaum (1977). They used the third-order polynomial whose four coefficients are determined from the boundary conditions (3) and naturally looking conditions of conjugation at the lower boundary of a thermocline. In case of homogeneity of the underlying layer, the latter conditions have the form $\vartheta'(1) = \vartheta''(1) = 0$. These boundary conditions result in the formula

$$\vartheta = 1 - (1 - \zeta)^3, \quad (4)$$

which rather slightly differs from the formula of Kitaigorodskii and Miropolsky against the background of data scatter.

The subsequent processing of oceanic data (Miropolsky et al. 1970; Reshetova and Chalikov 1977) revealed so great a scatter of points on the empirical curves $\vartheta(\zeta)$ that the concept of thermocline self-similarity became doubtful. Tamsalu (1982) rehabilitated this idea to a certain degree on the phenomenological basis. He noted that the form of the function $\vartheta(\zeta)$ depends considerably on the behavior of the mixed layer and identified two alternative situations that differ sharply from one another: (i) growth of the mixed layer ($\dot{h} \equiv dh/dt > 0$) and (ii) its steady state or collapse ($\dot{h} \leqslant 0$). These cases exhibit different self-similarity, and Mälkki and Tamsalu (1985) obtained two expressions for ϑ from processing Baltic Sea data:

$$\vartheta = \begin{cases} 1 - (1 - \zeta)^3, & \text{at } \dot{h} > 0 \quad (5a) \\ 1 - 4(1 - \zeta)^3 + 3(1 - \zeta)^4, & \text{at } \dot{h} \leqslant 0. \quad (5b) \end{cases}$$

Equation (5a) coincides with Eq. (4), but Eq. (5b) has essentially different form in which the mixed layer interfaces the thermocline smoothly. Thus, the sharpening of the temperature gradient at $z = h + 0$ (i.e., something similar to a discontinuity at the lower boundary of the mixed layer) was confirmed only in the case of its growth. The data from Lake Ladoga and Lake Sevan confirmed the existence of two types of self-similar temperature profiles in a thermocline (Zilitinkevich 1991).

The theoretical explanation of a possible thermocline self-similarity during the mixed-layer growth was given in simultaneously published papers of Turner (1978) and Barenblatt (1978). These authors noted that the heat transfer equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} K_H \frac{\partial T}{\partial z}, \quad (6)$$

where K_H is the turbulent heat conductivity, may have the following self-similar solution of a propagating-wave type in the region $z > h(t)$ at $\dot{h} > 0$:

$$T(z, t) = T(z'), \quad (7)$$

where $z' = z - h(t)$, which is easily reduced to the form of Eqs. (1) and (2).

Barenblatt supposed K_H to be constant, while Turner examined both this case and one other, more interesting case:

$$K_H = -(l^2\dot{h}/\Delta T)\partial T/\partial z, \quad (8)$$

where l is a length scale.

When a thermocline is developing in a two-layer fluid (i.e., when the underlying layer is homogeneous) the following boundary conditions must hold:

$$T = T_s \quad \text{at} \quad z = h; \quad T = T_a, \quad \partial T/\partial z = 0 \quad \text{at} \quad z = h + \Delta h. \quad (9)$$

Only two of these boundary conditions are usually satisfied, because the heat transfer equation is of second order with respect to z. The extra one allows the thermocline thickness Δh to be determined.

It is easy to see that, at T_a = const and with two additional restrictions that T_s = const and l = const, the solution of the problem (6), (8), and (9) belongs to the same class as Eq. (7) and can be written as a second line of Eq. (1), where

$$\vartheta = 1 - (1 - \zeta)^2, \quad \Delta h = 2l. \quad (10)$$

If T_s varies slowly with time, such that

$$dT_s/dt \ll 2\dot{h}\Delta T/\Delta h, \quad (11)$$

Eq. (10) is no longer exact but an approximate solution. If the condition (11) is not valid, neither exact nor approximate self-similarity of the temperature profile in a thermocline exists.

In support of his formula (8) Turner refers to the unpublished paper by Gill and Trefethen, who compared the experimental data on kinematic heat flux Q and vertical temperature gradient $\partial T/\partial z$. They came to the conclusion that the effective heat conductivity, $K_H \equiv -Q/(\partial T/\partial z)$, in a thermocline is, first, much higher than the molecular heat conductivity and, second, increases when $-\partial T/\partial z$ increases (in contrast to the well-known inverse dependence of K_H on $-\partial T/\partial z$, which takes place at subcritical Richardson numbers, i.e., in weakly stable shear flows). Assuming that $K_H \propto -\partial T/\partial z$, Turner apparently added the factor $l^2\dot{h}/\Delta T$ on the right-hand side of his expression for K_H so that it acquires the proper dimensionality.

3. Governing parameters of wave-generated intermittent turbulence

The direct dependence of K_H on $-\partial T/\partial z$ in the thermocline could be the result of the following mechanism, which seems to be the only possible one. The disturbances at the lower boundary of the mixed layer generate internal gravity waves, which propagate in different directions and cause the transfer of kinetic energy downwards. The occurrence of breaking waves manifests itself in generation of turbulence spots; i.e., waves expend a part of their energy for maintenance of intermittent turbulence. It is precisely the same mechanism that was implied in the work of Turner. Nevertheless, in Turner's formula for K_H the buoyancy parameter $\beta = ga_T$ (where g is gravitational acceleration and a_T is thermal expansion coefficient) was missed. It obviously must be taken into account. Indeed, if $a_T = 0$ (and consequently, $\beta = 0$), there would be no density stratification associated with temperature stratification, and therefore no buoyancy effect on turbulence and no internal gravity waves, i.e., nothing similar to considered phenomenon.

Thus, the list of parameters determining the intermittent turbulence in a thermocline must include the buoyancy parameter β along with the temperature gradient $\partial T/\partial z$ and the length scale l [in which capacity Zilitinkevich et al. (1988) assumed the amplitude of disturbances at the mixed layer–thermocline interface, since it determines a typical amplitude of generated waves]. Using these three parameters, the following formula for the effective heat conductivity K_H is obtained from dimensional arguments:

$$K_H = C_* l^2 N, \quad (12)$$

where C_* is a dimensionless constant, $N = (-\beta\partial T/\partial z)^{1/2}$ is the buoyancy frequency. This expression for K_H can be presented in a traditional form: $K_H \propto le^{1/2}$, where e is the statistically averaged turbulent kinetic energy per unit of mass. Indeed, if we apply our dimensional analysis to derive the expression for e, then

$$e = (lN)^2, \quad (13)$$

where the omitted dimensionless constant is included into the definition of l. Hence, $K_H \propto le^{1/2}$ is equivalent to Eq. (12).

Exactly the same expression for kinetic energy within the turbulence spot was obtained by Ozmidov (1983). It is worth mentioning also that the formula $l = N^{-1}e^{1/2}$, which immediately follows from Eq. (13), coincides with the expression for the depth of penetration of vertically moving fluid particle with the energy e into the stratified layer with the buoyancy frequency N (Zeman 1975; Zeman and Tennekes 1977).

Now we consider the development of a thermocline in a two-layer fluid system on the basis of Eqs. (6) and (12) with the boundary conditions (9). The propagating-wave–type solution of the problem is

$$\vartheta = 1 - (1 - \zeta)^3, \quad \Delta h = 3C_*^{2/3}l^{4/3}(\beta\Delta T)^{1/3}\dot{h}^{-2/3}. \quad (14)$$

The former formula coincides with the polynomial approximation (4) and with the empirical formula (5a) at $\dot{h} > 0$.

Using (7), Eq. (6) may be written in the following form:

$$\dot{h} dT/dz' = dQ/dz'. \tag{15}$$

It follows from Eq. (15) that in case of a propagating-wave–type solution, the vertical profiles of temperature T and of heat flux Q in the layer $h < z < h + \Delta h$ are similar; that is, the thermocline and the turbulent entrainment layer coincide. Therefore, we can identify the half thickness of thermocline $\frac{1}{2}\Delta h$ with the vertical scale l for disturbances at the external boundary of the mixed layer (cf. Deardorff et al. 1980; Zilitinkevich 1987). Then the second formula (14) is reduced to

$$\Delta h/h = 2l/h = \frac{16}{27} C_*^{-2} (\beta \Delta T h)^{-1} \dot{h}^2. \tag{16}$$

Equation (16) was verified by Zilitinkevich et al. (1988) using the data of laboratory experiments on development of the mixed layer and thermocline in case of (i) drift current in an annulus (Kreiman 1982) and (ii) penetrative convection (Deardorff et al. 1980). Although the turbulization mechanisms were essentially different in these experiments, both sets of points follow one and the same curve:

$$\Delta h/h = 7600(\beta \Delta T h)^{-1} \dot{h}^2 + 0.33 \quad \text{at} \quad (\beta \Delta T h)^{-1} \dot{h}^2 > 10^{-5}. \tag{17}$$

The empirical Eq. (17) evidently agrees with Eq. (16) at sufficiently high values of $\dot{h}$, which gives the estimate $C_* = 9 \times 10^{-3}$. Note that the limitation on $\dot{h}$ given in Eq. (17) is of the same nature as Eq. (11) and corresponds to applicability of the propagating-wave–type solution.

4. Energy balance of turbulence

Turbulence energetics was not considered explicitly in section 3. Actually, the turbulence length scale l was taken to be depth-constant. Such a hypothesis can probably serve as reliable approximation, but only in case of a comparatively thin thermocline associated with very intensive deepening of the mixed layer. In a more general model, both basic characteristics of turbulence, viz., the length scale l and kinetic energy e, must be considered as unknown depth-dependent variables. We derive such a model by supplementing the heat transfer equation with the turbulent energy balance equation

$$\partial e/\partial t = -\partial F/\partial z - \beta Q - \epsilon, \tag{18}$$

where Q is the kinematic heat flux (βQ is the rate of turbulent energy loss for overcoming the buoyancy forces), ϵ is the viscous dissipation rate of the energy, and F is the vertical energy flux. We close the equations in a traditional way by means of the concept of turbulent exchange coefficients for heat, K_H, and kinetic energy, K_E:

$$Q = -K_H \partial T/\partial z, \quad F = -K_E \partial e/\partial z, \tag{19}$$

and the Kolmogorov (1942)–Heisenberg (1948) hypothesis:

$$\frac{K_H}{C_H} = \frac{K_E}{C_E} = l e^{1/2}, \quad \epsilon = C_\epsilon \frac{e^{3/2}}{l}, \tag{20}$$

where C_H, C_E, and C_ϵ are dimensionless constants.

Finally, we use the expression (13), which reflects the specific features of turbulence in case of extremely strong stability. It can be interpreted now as the definition of the turbulence length scale l.

The system (6), (13), (18)–(20) is closed. With respect to the vertical coordinate, it is of the second order relative to temperature T and kinetic energy e. Therefore, two boundary conditions are required for T and e, say, prescribing them at the upper and lower boundaries of the thermocline:

$$T = T_s \text{ at } z = h, \quad T = T_a \text{ at } z = h + \Delta h, \tag{21}$$

$$e = e_h = (l_h N_h)^2 \text{ at } z = h, \quad e = 0 \text{ at } z = h + \Delta h, \tag{22}$$

where e_h is expressed in terms of the amplitude of turbulent disturbances and the buoyancy frequency at the mixed layer–thermocline interface.

5. Analytical solution

Let the mixed-layer depth h be a monotonically increasing function of time. Our system of equations (6), (13), (18)–(20) is written in the form

$$\beta \frac{\partial T}{\partial t} = -C_H \frac{\partial}{\partial z} eN, \tag{23}$$

$$(C_H + C_\epsilon)^{-1} \frac{\partial e}{\partial t} = C_e \frac{\partial}{\partial z}\left(\frac{e}{N}\frac{\partial e}{\partial z}\right) - eN, \tag{24}$$

where C_H, C_ϵ, and $C_e = C_E(C_H + C_\epsilon)^{-1}$ are dimensionless constants.

We search for the solution of Eqs. (23) and (24) of a propagating-wave type in the half space $z > h$ (i.e., $\Delta h \rightarrow \infty$). Then, the dependence of unknown variables on z and t is expressed through a self-similar coordinate $z' = z - h(t)$ [cf. Eq. (7)], and the following expressions hold true for partial derivatives:

$$\frac{\partial}{\partial z} = \frac{d}{dz'}, \quad \frac{\partial}{\partial t} = -\dot{h} \frac{d}{dz'}, \tag{25}$$

where $\dot{h} > 0$. Such a solution is exact if T_s, T_a, e_h, and $\dot{h}$ do not change with time, and is the approximate one if they do change, but not very quickly.

Substituting (25) into Eq. (23), then integrating it over z' and using the boundary conditions (21) and (22), we obtain

$$T - T_a = Q/\dot{h} = (C_H/\beta\dot{h})eN. \quad (26)$$

Then, the heat flux Q_h and the buoyancy frequency N_h at the thermocline upper boundary are

$$Q_h = \dot{h}(T_s - T_a), \quad N_h = \frac{\beta\dot{h}(T_s - T_a)}{C_H e_h}. \quad (27)$$

According to (25), the left-hand sides of Eqs. (23) and (24) take the form

$$\beta\frac{\partial T}{\partial t} = \dot{h}N^2, \ (C_H + C_\epsilon)^{-1}\frac{\partial e}{\partial t} = -(C_H + C_\epsilon)^{-1}\dot{h}\frac{de}{dz'}. \quad (28)$$

In this way, the problem is reduced to solution of a system of two ordinary differential equations relative to $e(z')$ and $N(z')$, taking prescribed values of these variables at $z' = 0$ and satisfying the condition $e \to 0$ at $z' \to 0$.

The following sets of parameters are considered as known: T_s, $\dot{h}$, e_h, T_a, or, according to (22a), (27b), T_s, $\dot{h}$, l_h, N_h. Using the second one, we determine the following dimensionless variables:

$$\xi = z'/(C_e^{1/2}l_h), \quad \lambda = l/l_h, \quad \nu = N/N_h. \quad (29)$$

Then, Eqs. (23) and (24) become

$$\frac{d}{d\xi}\lambda^2\nu^3 = -E_*\nu^2, \quad (30)$$

$$\frac{d}{d\xi}\left(\lambda^2\nu\frac{d}{d\xi}\lambda^2\nu^2\right) = \lambda^2\nu^3 - \frac{C_H}{C_e(C_H + C_\epsilon)}E_*\frac{d}{d\xi}\lambda^2\nu^2, \quad (31)$$

where E_* is the dimensionless entrainment rate:

$$E_* = \frac{C_e^{1/2}}{C_H}\frac{\dot{h}}{l_h N_h}. \quad (32)$$

The second term in the right-hand side of Eq. (31) reflects nonstationarity of the turbulent energy budget, which is considered usually as being of minor importance. Neglecting this term, we obtain

$$\frac{d}{d\xi}\left(\lambda^2\nu\frac{d}{d\xi}\lambda^2\nu^2\right) = \lambda^2\nu^3. \quad (33)$$

As to Eq. (30), it is clear that the term $E_*\nu^2$ cannot be neglected, no matter how small E_* is.

The boundary conditions for λ and ν are

$$\lambda = \nu = 1 \quad \text{at} \quad \xi = 0, \quad (34)$$

$$\lambda^2\nu^2 \to 0 \quad \text{at} \quad \xi \to \infty. \quad (35)$$

The solution of the problem (30), (33), (34), (35) has the form

$$\lambda = \eta^3\exp(\sqrt{6}E_*/\eta - \sqrt{6}E_*),$$

$$\nu = \eta^{-2}\exp(\sqrt{6}E_* - \sqrt{6}E_*/\eta), \quad (36)$$

$$\int_\eta^1 \eta'^2\exp(\sqrt{6}E_*/\eta')d\eta' = \frac{1}{\sqrt{6}}\exp(\sqrt{6}E_*)\xi, \quad (37)$$

where $\eta^2 = (\lambda\nu)^2$ is the dimensionless energy.

According to (37), η is a monotonically decreasing function of ξ at any $E_* > 0$. Behavior of λ and ν is more complicated. The dimensionless turbulence length scale λ is a monotonically increasing function of ξ at $E_* \geq \sqrt{2/3}$, and, at $E_* < \sqrt{2/3}$, it first decreases, reaching a minimum at the depth $\xi = \xi_\lambda$, and only then increases unlimitingly. The dimensionless buoyancy frequency ν is a monotonically decreasing function of ξ at $E_* \geq \sqrt{2/3}$, and, at $E_* < \sqrt{2/3}$, first it increases, reaching a maximum at the depth $\xi = \xi_\nu$, and only then monotonically tends to zero. Dimensionless depths of extrema, ξ_λ and ξ_ν, are

$$\xi_{\{\lambda,\nu\}} = \frac{36E_*^3}{\exp(\sqrt{6}E_*)}\int_{\{1/3,1/2\}}^{(\sqrt{6}E_*)^{-1}}\eta'^2\exp(1/\eta')d\eta' \to \sqrt{2/3} + E_*, \quad (38)$$

which differ one from another only by the lower limits in the integral. The limit in the right side of (38) corresponds to $E_* \to 0$.

The following asymptotic formulas hold true at $\xi \to \infty$:

$$\left(\frac{e}{l_h^2N_h^2}\right)^{1/2} = \eta(\xi, E_*) \to \frac{\sqrt{6}E_*}{\ln(E_*\xi) + 4\ln[(\sqrt{6}E_*)^{-1}\ln(E_*\xi)]} \to 0, \quad (39)$$

$$l/l_h = \lambda(\xi, E_*) \to \frac{\xi[\ln(E_*\xi)]^4}{\sqrt{6}\{\ln(E_*\xi) + 4\ln[(\sqrt{6}E_*)^{-1}\ln(E_*\xi)]\}^3} \to \infty, \quad (40)$$

$$N/N_h = \nu(\xi, E_*) \to \frac{6E_*\{\ln(E_*\xi) + 4\ln[(\sqrt{6}E_*)^{-1}\ln(E_*\xi)]\}^2}{\xi[\ln(E_*\xi)]^4} \to 0. \quad (41)$$

Such a behavior of l and N in an infinitely deep fluid seems to be quite realistic (cf. high amplitude of internal waves, large thickness of turbulence spots, and very low density gradients in the deep ocean). As for the limit $e \to 0$ in (39), it immediately follows from the boundary condition (35).

Combining (26), (27) with (36), (39)–(41), we obtain

$$\frac{T - T_a}{T_s - T_a} = \frac{Q}{Q_h} \equiv \theta(\xi, E_*)$$

$$= \exp(\sqrt{6}E_* - \sqrt{6}E_*/\eta) \rightarrow \frac{36E_*^3}{\xi[\ln(E_*\xi)]^4}, \quad (42)$$

where the asymptotic expression corresponds to $\xi \rightarrow \infty$. According to (42) and (37), $(T - T_a) \propto Q$ is a monotonically decreasing function of ξ at any E_*. If $E_* < \sqrt{2/3}$, it has a bend at the point $\xi = \xi_\nu$. The function $\theta(\xi, E_*)$ has the following value at this point:

$$\theta_\nu = \exp(\sqrt{6}E_* - 2). \quad (43)$$

The expression for $\theta = 1 - \vartheta$ given by (42) and (37) and the third-order polynomial in (5a) may be quite similar at certain values of E_*.

6. Comparison with laboratory experiments

The vertical temperature profiles in thermocline were carefully examined in the Deardorff et al. (1980) laboratory experiments on penetrative convection. In those experiments, transition of temperature T from its value in the mixed layer to a certain value characteristic for the undisturbed layer (i.e., to T_a in the experiments with the initially two-layer fluid) is described by a monotonic function of z. As to the heat flux Q, in both cases of two-layer and linear background stratification, its profile is as follows: in the main part of the well-mixed layer, Q is a linearly decreasing function of z; it passes through zero and changes the sign at some level $z = h_-$; at the level $z = h > h_-$ it has the extremum, and then it monotonously drops practically to zero by the level $z = h_+ > h$. Certainly, it is reasonable to interpret the observed profiles of T and Q in terms of the solution (42) only in the layer $z > h$, which should just be called as the thermocline. So the values of l_h (characteristic amplitude of turbulent disturbances at the mixed layer–thermocline interface, which is proportional to the thickness of the turbulent entrainment layer in case of penetrative convection), T_s (the mixed-layer temperature), and T_a (the undisturbed-layer temperature) required for such an interpretation can be naturally determined by

$$l_h = h - h_-, \quad T_s = T(h), \quad T_a = T(h_+). \quad (44)$$

The empirical graph on Fig. 2, taken from Deardorff et al. (1980), presents the dependence of

$$\tilde{\theta} = [T(h_-) - T(z)]/[T(h_-) - T(h_+)]$$

$$\text{on} \quad \tilde{\xi} = (z - h_-)/(h_+ - h_-) \quad (45)$$

in the experiment E2 with two-layer fluid. The set of parameters measured in this experiment, and the estimates of the dimensionless entrainment rate E_* and our dimensionless constants C_e and C_H, are presented in Table 1. Deardorff et al. (1980) did not specify the moment of time corresponding to the published dimensionless temperature curve. That is why particular values of very approximate estimates of both constants were found for each count.

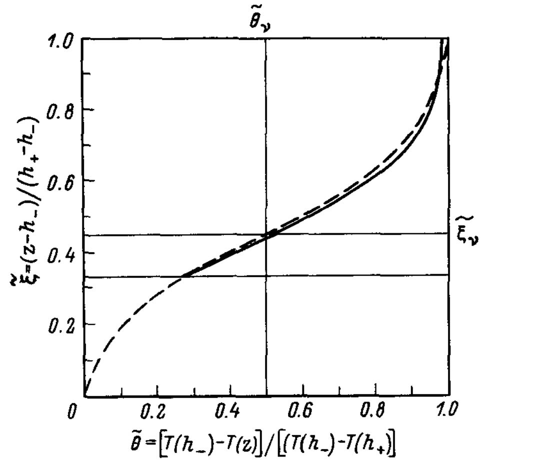

FIG. 2. Dimensionless temperature profiles in the penetrative convection experiment E2 (with a two-layer fluid) of Deardorff et al. (1980): dashed line represents the measured profile; solid line calculated from (37) and (42) at $C_e = 0.7, E_* = 0.66$. The thermocline is the region above the horizontal line $\tilde{\xi} = 0.33$; $\tilde{\xi}_\nu$ and $\tilde{\theta}_\nu$ are coordinates of the point of bend.

It was done as follows. First, $\tilde{\theta}$ can be expressed in terms of Eq. (42):

$$\tilde{\theta} = 1 - (T_s - T_a)/[T(h_-) - T_a]\theta. \quad (46)$$

Taking off the empirical value of $\tilde{\theta}$ at the point of bend (i.e., $\tilde{\theta}_\nu$) from the graph and using the measured $T(h_-)$, $T_s = T(h)$, and $T_a = T(h_+)$ values, we obtain the estimate of θ_ν from (46) and the estimate of E_* from (43). Next, knowing E_*, we can find the theoretical values of ξ_ν from (38). Then, since the formula holds true [according to definition (29a), (44a), and (45b)]:

$$[(h_+ - h_-)/(h - h_-)]\tilde{\xi} = 1 + C_e^{1/2}\xi. \quad (47)$$

Taking off the empirical value of $\tilde{\xi}$ at the point of bend (i.e., $\tilde{\xi}_\nu$) from the graph, substituting it and the theoretical value of ξ_ν found above into (47), and using the measured h_+, h, and h_- values, we obtain the estimate of C_e. Finally, substituting our estimates of E_* and C_e into (27) and (32) and using the measured values of β, $\dot{h}$, $T_s - T_a = T(h) - T(h_+)$, and $l_h = h - h_-$, we obtain the estimate of C_H. The data of the four counts show the great scatter. In addition to the above comment about approximate character of our estimates, it may reflect very low accuracy in determining the point of bend from the empirical curve. In fact,

TABLE 1. Measured values of parameters in experiment E2 of Deardorff et al. (1980), and estimates of dimensionless entrainment rate E_* and dimensionless constants C_e and C_H.

	Count			
	1	2	3	4
Data of measurements				
t, s	277	389	502	577
h_-, cm	21.8	23.1	24.8	26.2
h, cm	24.3	26.0	28.0	29.5
h_+, cm	29.4	31.7	34.7	36.8
$\dot{h}$, cm · c^{-1}	0.0166	0.0181	0.0230	0.0239
β, cm · s^{-1} · K^{-1}	0.234	0.238	0.243	0.247
$T(h_-)$, °C	22.50	23.19	23.98	24.22
$T(h)$, °C	23.15	23.70	24.17	24.45
$T(h_+)$, °C	24.82	25.00	24.98	25.10
Estimates				
E_*	0.668	0.669	0.662	0.657
C_e	0.713	0.596	0.756	0.919
C_H	0.0239	0.0237	0.0436	0.0579

new measurements are needed for convincing verification of the model. Anyway, on the average, the data mentioned above give the rough estimates

$$C_e = 0.7, \quad C_H = 0.04. \tag{48}$$

The solution given in section 5 is illustrated by theoretical profiles of λ, ν, η, and θ at $E_* = 0.66$ (corresponding to the experiments examined above) and $E_* = 0.4$, presented in Fig. 3.

Figure 4 shows the results of numerical simulation of the temperature profile evolution in the Deardorff and Willis (1982) laboratory experiments on mixed-layer and thermocline formation under the action of shear current developing in an annulus on the background of initially linear stratification. Unfortunately, there were no data in the work cited for immediate evaluation of l_h or e_h. That is why we had to use the ad hoc formula $l_h = 0.4h$, where 0.4 is the fitting coefficient. Notwithstanding the arbitrariness of the application of this formula, the observed and simulated behavior of temperature profiles revealed reasonable qualitative resemblance.

7. Conclusion

(i) Self-similarity formulation of the vertical temperature profile in the region of supercritical stability adjoining a well-mixed layer was suggested by Kitaigorodskii and Miropolsky (1970) as applied to the oceanic seasonal thermocline. It appears, however, that the formulation is confirmed much better by laboratory experiments (e.g., Linden 1975; Wyatt 1978) than by field measurements. Two different versions of dimensionless temperature profiles in thermocline were observed in natural water reservoirs in alternative cases of the mixed-layer deepening and collapsing (Mälkki and Tamsalu 1985).

(ii) Comparison of the experimental data on heat flux and vertical temperature gradient showed that the effective heat conductivity is much higher than the

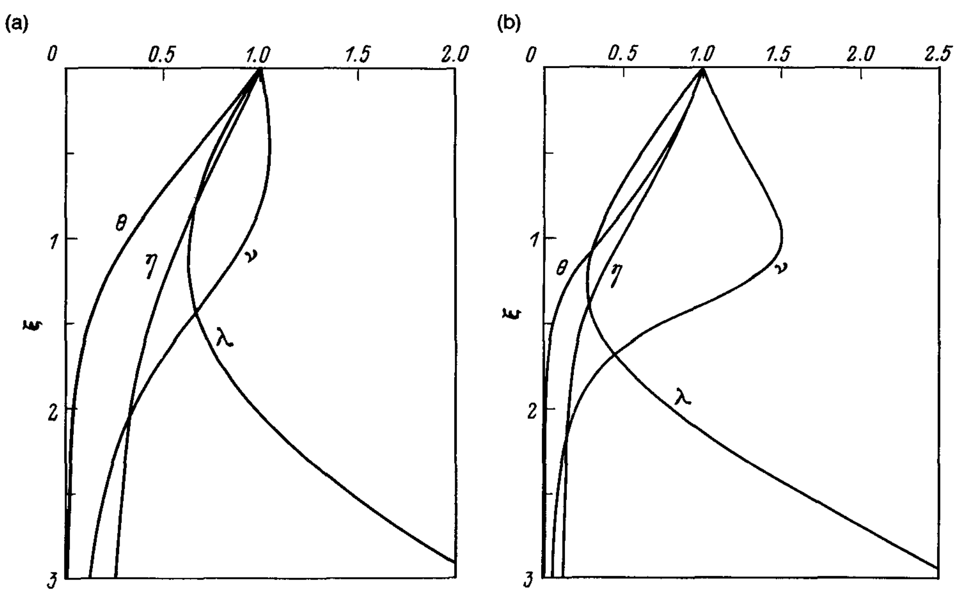

FIG. 3. Dimensionless vertical profiles of the turbulence length scale λ, buoyancy frequency ν, energetic parameter η, and temperature (or heat flux) θ, according to the solution given by (36), (37), (42): (a) $E_* = 0.66$, (b) $E_* = 0.4$.

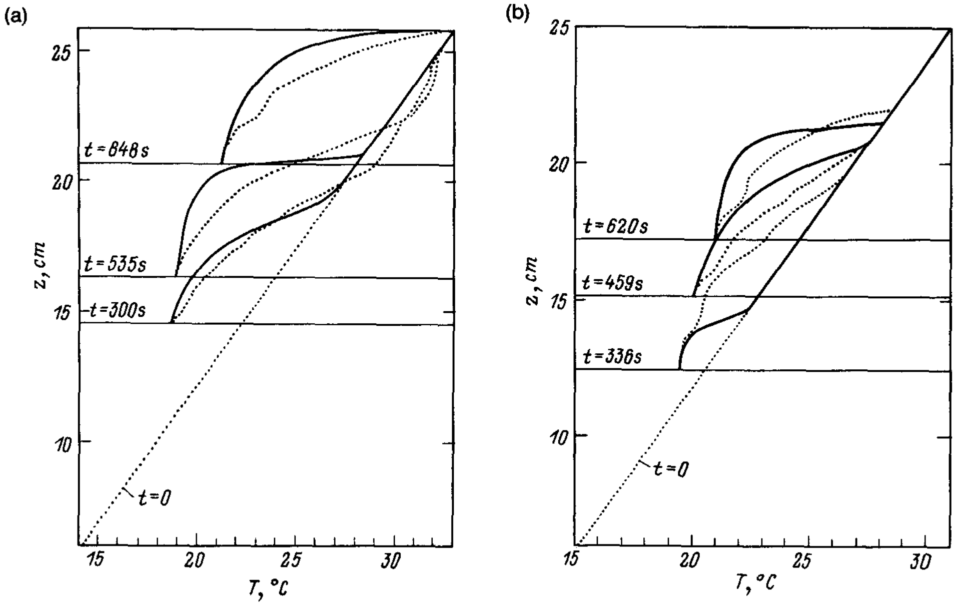

FIG. 4. Successive temperature profiles in Deardorff and Willis (1982) laboratory experiments: (a) experiment 1, (b) experiment 2. Dotted lines represent measured profiles; solid lines are plotted after the solution of the heat transfer equation (23) and the stationary version of the turbulent energy budget equation (24). The horizontal lines indicate the mixed-layer depths at different times (given in seconds).

molecular one and, rather unexpectedly, can increase with the increase of temperature gradient, in contrast with the usual behavior characteristic for weakly stable shear flows (Gill and Trefethen, unpublished).

(iii) Self-similarity of the temperature profile in a thermocline was explained, in the case of mixed-layer deepening, as being equivalent to the propagating-wave–type solution of the heat transfer equation (Turner 1978; Barenblatt 1978).

(iv) A simple theoretical model of the heat transfer due to intermittent turbulence produced by internal waves breaking and wave–turbulence interaction is suggested. The expression for the effective heat conductivity is derived from dimensional arguments using the following governing parameters: buoyancy parameter, temperature gradient, turbulence length scale. The model produces the shape of temperature profile in thermocline observed by Mälkki and Tamsalu in the case of mixed-layer deepening.

(v) A more advanced model is derived from consideration of turbulent energy budget and the aforementioned dimensional arguments. Analytical and numerical solutions are obtained simulating temperature profiles in thermocline in laboratory experiments on mixed-layer deepening (Deardorff et al. 1980; Deardorff and Willis 1982). The given energy balance model must be applicable to the problem of reconstruction of thermocline not only in the considered case of the mixed-layer deepening, but also in case of its steady state or collapse.

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