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Numerical models and intercomparisons of beach profile evolution

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Abstract

A modified non-linear cross-shore sediment transport relationship is developed based on equilibrium beach profile concepts and scaling relationships. This non-linear relationship provides a reasonable explanation for the significantly different time scales of beach evolution evident in various laboratory experiments. The proposed non-linear model called "CROSS" is calibrated and compared with the commonly employed linear transport relationship using laboratory data. A total of seven large scale wave tank experiments from three different facilities are examined. The results demonstrate that the non-linear transport model provides better overall predictions than the linear transport equations. The CROSS model and three other commonly used models are applied to predict beach erosion at Ocean City, Maryland during the November 11, 1991 and January 4, 1992 storms. Seven survey lines are available for comparison with the numerical simulations. Overall, CROSS, EDUNE and SBEACH (version 3.0) provide reasonable predictions for both dune erosion and the entire profiles. The sensitivity of CROSS to the transport coefficient, active water depth, storm surge levels and the storm wave heights are examined for the storm erosion at Ocean City. It appears that CROSS is quite insensitive to the transport coefficient. The sub-aqueous part of a profile is quite sensitive to the wave height and the subaerial part is less affected. The CROSS model provides better predictions with the ratio of active water depth to incoming wave height of 1 than with the ratio of 1.28, and the 20% increased storm surge yields a better simulation.

Keywords: Dune erosion; Numerical models; Storm effects; Beach erosion; Erosion prediction

1. Introduction

The increasing use of the coastal zone has made a working understanding of nearshore and beach processes an increasingly demanding goal in coastal studies.

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Accurate estimates of beach profile evolution in response to tides, storms and beach nourishment are required for a variety of regulatory and design purposes. Due to the complexities of beach profiles, sediment characteristics and concentrations, and wave and water level conditions, an analytical treatment is difficult and recourse to numerical modeling is required.

Sediment transport at a point in the nearshore zone can be considered in terms of cross-shore and long-shore components. It appears that under a number of coastal engineering scenarios of interest, the transport is dominated by either the cross-shore or long-shore component. The cross-shore component determines profile evolution primarily for beaches far away from structures and inlets and under cases of increased water levels, storms and beach nourishment. In contrast to longshore sediment transport modeling, which has been studied for about five decades, a focus on cross-shore sediment transport modeling is relatively recent (about 20 years) and uncertainty in predicting effects of all variables thus may be considerably greater.

Cross-shore sediment transport models can be broadly classified into two groups: "open loop" and "closed loop" models. An "open loop" model is not constrained a priori to the final profile and the sediment transport is determined by sediment concentrations and fluid velocities. "Closed loop" sediment transport models are based on equilibrium beach profile concepts (Bruun, 1954; Dean, 1977) and assume that a profile will eventually achieve equilibrium if exposed to the same conditions for a long time. Cross-shore transport is caused by deviations of a beach profile from the equilibrium. Several numerical models, including EDUNE (Kriebel, 1982; Kriebel and Dean, 1985; Kriebel, 1986), SBEACH (Larson et al., 1989; Larson and Kraus, 1989), and the model used in establishing a coastal hazard zone in Florida, termed the Coastal Construction Control Line (CCCL) (Chiu and Dean, 1984, 1986) have been developed according to equilibrium beach profile concepts.

Based on analysis of scaling relationships, a modified nonlinear cross-shore sediment transport model (CROSS) is proposed in this study. Three sets of large wave tank data are utilized, including three cases in the large German wave flume in Hannover (Dette and Uliczka, 1987), three cases of Saville's large wave tank experiments (Kraus, 1988), and one case in the SUPERTANK experiments at Oregon State University (Kraus and Smith, 1994) to calibrate and compare the proposed nonlinear equation with the linear. Additionally, the proposed CROSS model is compared with three existing models (EDUNE, SBEACH and CCCL) by beach erosion predictions at Ocean City, Maryland during the November 11, 1991 and January 4, 1992 storms (Stauble et al., 1993). Of these models, only SBEACH (both versions) and EDUNE include representation of dune overwash.

2. Equilibrium beach profile

According to the balance of destructive and constructive forces and assuming wave energy dissipation per unit water volume to be the dominant destructive force, Dean (1977) has proposed that a sediment of a specific size, d, will be stable in the presence of a particular level of wave energy dissipation per unit water volume, D_*, leading to the following for equilibrium beach profiles:

$$\frac{1}{h}\frac{\partial}{\partial y}(EC_G) = D_*(d) \tag{1}$$

in which h is the total water depth at a distance y offshore, E is the local wave energy density, and C_G is the local wave group velocity. With shallow water and spilling wave breaker assumptions, Eq. (1) can be integrated to:

$$h(y) = A(d)\,y^{2/3} \tag{2}$$

Approximately 500 profiles from the east and Gulf coast shorelines of the United States were examined and provided reasonable support for this equilibrium form (Dean, 1977). The so-called sediment scale parameter, $A(d)$, is determined from Eqs. (1) and (2) as

$$A(d) = \left[\frac{24}{5}\frac{D_*}{\rho g^{3/2}\kappa^2}\right]^{2/3} \tag{3}$$

where κ is the ratio of breaking wave height to water depth and $A(d)$ depends on sediment size, d, (Moore, 1982) or equivalently sediment fall velocity, w (Dean, 1987).

Two disadvantages of Eq. (2) are the infinite beach slope at the water line and the monotonic form of the profile. The first shortcoming is overcome by including gravity as a significant destructive force when a profile becomes steep. In this case, Eq. (2) is modified to include the beach face slope, m_0,

$$y = \frac{h}{m_0} + \left(\frac{h}{A}\right)^{3/2} \tag{4}$$

Larson (1988) developed an equation of similar form by considering the breaking wave model of Dally et al. (1985) and retaining the requirement of uniform wave energy dissipation per unit volume. Since the scale parameter, A, is only a function of sediment size, wave conditions are not included in Eq. (2).

Bodge (1992) and Komar and McDougal (1994) have proposed equilibrium beach profiles of exponential form which approach a finite depth at an infinite offshore distance. The formulations require fitting two parameters to the particular profile. Inman et al. (1993) developed an equilibrium relationship that treated the profile as two parts, the inner (bar-berm) and the outer (shorerise) portions. The two portions are matched at a breakpoint-bar and fit by two power curves. A total of seven parameters is required to represent a profile. These methods are diagnostic and generally useful for a beach for which measured data are available. In comparison, the method described earlier by Eq. (2) is prognostic requiring only quantification of the sediment size.

3. Scale analysis and discussion of transport relationships

A beach which is steeper than equilibrium has a smaller volume of water over which the energy from an incident wave is dissipated causing higher levels of turbulence and the actual energy dissipation per unit volume to be greater than the equilibrium value. As a result, the total destructive forces are greater than the constructive forces. The profile will respond to this imbalance of forces through redistribution of the sediment and thus adjustment of the profile toward equilibrium. Over time, sand will be carried from onshore to offshore and deposited near the breakpoint. Similar to this process, for a beach with a milder slope than equilibrium the sediment will be moved from offshore to onshore, thus approaching equilibrium.

Based on these concepts, Moore (1982), Kriebel (1982) and Kriebel and Dean (1985) proposed a proportionality between the seaward sediment transport rate per unit beach width, Q, and the deviation of local wave energy dissipation per unit volume from the equilibrium value at each location across the surf zone:

$$Q = K(D - D_*) \tag{5}$$

in which, D represents the local wave energy dissipation per unit water volume, and the transport coefficient, K, is assumed to be a dimensional constant. Based on linear wave theory,

$$D = \frac{5}{24}\rho g^{3/2}\kappa^2 \frac{\partial h^{3/2}}{\partial y} \tag{6}$$

The following scaling relationship is established from Eq. (5),

$$Q_r = \frac{(D - D_*)_{\text{model}}}{(D - D_*)_{\text{prototype}}} = (D - D_*)_r \tag{7}$$

For an undistorted model, according to the definition of D, the disequilibrium scale ratio can be expressed in terms of the length scale ratio, L_r, as

$$(D - D_*)_r = \sqrt{L_r} \tag{8}$$

On the other hand, the Froude relationship yields the time scale ratio, T_r,

$$T_r = \sqrt{L_r} \tag{9}$$

From the relationships presented, the cross-shore sediment transport, Q, should be scaled as:

$$Q_r = \frac{L_r^2}{T_r} = L_r^{3/2} \tag{10}$$

This equation, which was proposed independently by Vellinga (1982), provides a basis for evaluating transport models. Obviously, Eq. (7) in which $Q_r = L_r^{1/2}$ does not provide a valid scaling of the transport unless K varies with scale.

It is of interest to develop and test a transport model which can ensure convergence to the target (equilibrium) beach profile and also satisfy the scaling relationship given by Eq. (10). One approach is to consider the following form for the transport model

$$Q = K(D - D_*)|D - D_*|^{n-1} \tag{11}$$

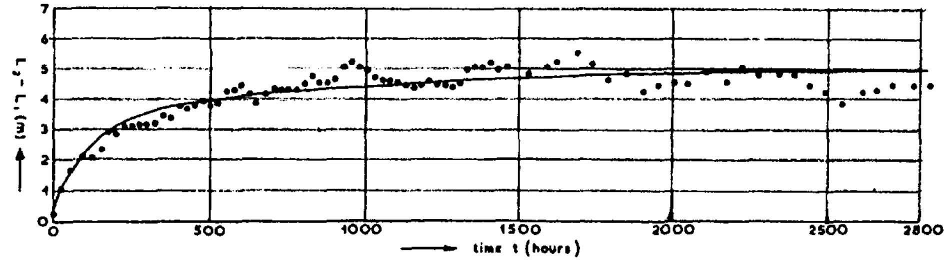

Fig. 1. Evolution of $(L_2 - L_1)$ versus time for Swart's Case B (from Swart, 1974).

which preserves the desired transport direction and results in the following scaling relationship

$$Q_r = K_r(D - D_*)_r |D - D_*|_r^{n-1} \tag{12}$$

Combining Eqs. (10) and (12), we have,

$$Q_r = K_r(D - D_*)|D - D_*|_r^{n-1} = K_r L_r^{n/2} = L_r^{3/2} \tag{13}$$

If K_r is independent of the length scale and equals unity, $n = 3$ is determined such that both the scaling relationship and convergence to the equilibrium profile are satisfied.

Sediment transport in the nearshore region is a complicated process. Under different conditions, some beaches approach equilibrium very fast, but others may vary slowly. The time scale of beach evolution may vary from hours to thousands of hours for different laboratory experiments. As shown in Figs. 1 and 2, experiments of Swart (1974) were conducted for durations exceeding 2800 hours with a monotonic approach to equilibrium still occurring whereas for the laboratory data of Dette and Uliczka (1987), equilibrium was nearly attained over a time scale of hours or tens of hours. In Fig. 1, $(L_2 - L_1)$ is the separation distance between two contours selected by Swart to represent the profile geometry. In an attempt to understand the causes of the different time scales, we examine the following equation

$$\frac{dx}{dt} = -k(x - x_*)|x - x_*|^{n-1} \tag{14}$$

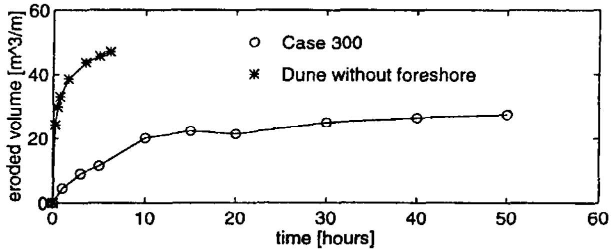

Fig. 2. Eroded volumes versus time for the German "dune without foreshore" case and Saville's Case 300.

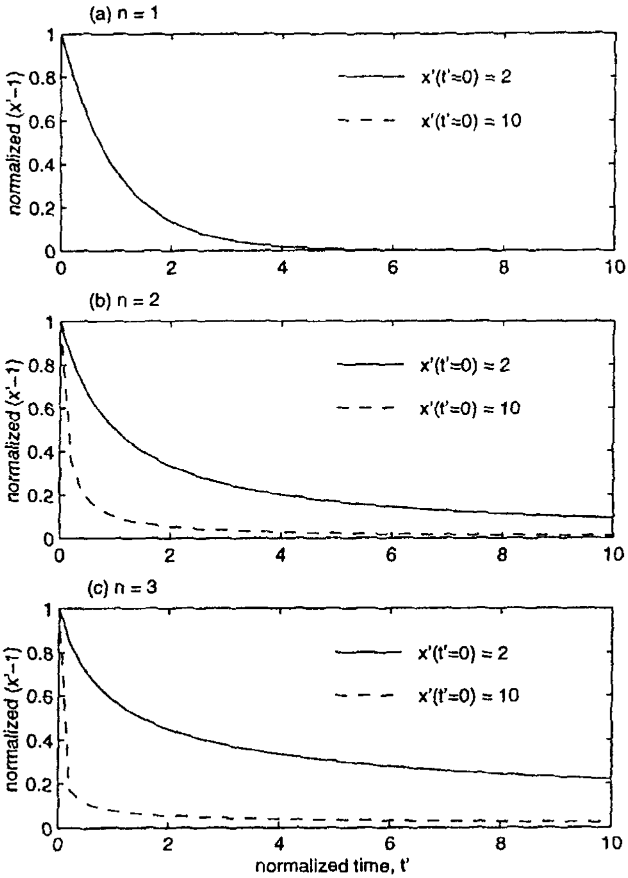

Fig. 3. Solution of Eq. (15) for three values of n.

which is reminiscent of the transport relationships discussed above. In Eq. (14), x_* is the equilibrium value of x. Nondimensionalizing with $x' = x/x_*$ and $t' = t/kx_*^{n-1}$, we have

$$\frac{\mathrm{d}x'}{\mathrm{d}t'} = -(x'-1)|x'-1|^{n-1} \tag{15}$$

With the initial condition $x'(t'=0) = x'_0$, the solution of Eq. (15) is given by

$$\frac{x'-1}{x'_0-1} = \mathrm{e}^{-t'} \quad \text{for } n = 1$$

and

$$\frac{x'-1}{x'_0-1} = \frac{1}{\left[(n-1)|x'_0-1|^{n-1}t'+1\right]^{1/(n-1)}} \quad \text{for } n \neq 1$$

Fig. 3 presents the solution in the form: $(x'-1)/(x'_0-1)$ versus t' for $n = 1$, 2 and 3, and $x'_0 = 2$ and 10 respectively. The time scale of the linear system ($n = 1$) is independent of the initial conditions and the two lines in Fig. 3a are coincident. However, for the non-linear systems, the initial conditions affect the time scale through the factor $|x'_0 - 1|^{n-1}$. As n increases, this factor becomes more and more significant.

As demonstrated above, a nonlinear transport equation provides an explanation for the range of time scales observed in laboratory experiments of beach profile evolution. In such nonlinear systems, a large deviation of the initial condition from the equilibrium corresponds to a smaller time scale of relative profile response. In the following study of laboratory data, the transport relationships, Eq. (11) with $n = 1$ and $n = 3$, are applied. The time dependent profile response is then determined by the numerical solution (Zheng, 1996) of the transport and continuity equations, the latter being

$$\frac{\partial y}{\partial t} = -\frac{\partial Q}{\partial h} \tag{16}$$

where y is the offshore position of a particular contour, h, from a reference baseline.

4. Calibration of cross model with laboratory experiments

The proposed non-linear transport model with $n = 3$ is compared with the linear transport relationship, $n = 1$, for seven large wave tank experiments from three different facilities. Among these experiments, five were carried out with monochromatic wave conditions, and the other two were conducted with random waves. Calibrations of the transport relationships are accomplished by a series of simulations in which trial K values are used to simulate profile evolution for each experiment. The errors between the predicted and observed eroded volumes are obtained at various times. The best-fit K value is determined as the value yielding the overall least squares error. The active profile considered in the numerical models is from the wave run-up limit to the wave breaking point, where the breaking water depth is 1.28 times the breaking wave height according to the McCowan (1894) theory. Outside this active region, the net sediment transport is set to zero. At each time step, the water level is determined by the sum of storm surge, tide and wave set-up.

The wave set-up in the surf zone is calculated according to the balance of pressure and radiation stress gradients. Based on linear wave theory, the set-up, $\bar{\eta}$, is given (Bowen et al., 1968) as

$$\bar{\eta}(h) = \bar{\eta}_b + \frac{3\kappa^2/8}{1 + 3\kappa^2/8}(h_b - h) \tag{17}$$

where κ is the ratio of breaking wave height to breaking water depth, and $\bar{\eta}_b$ and h_b are the set-down and the water depth at the breaking point, respectively. For shallow water, $\bar{\eta}_b$ is given by Longuet-Higgins and Stewart (1964) as $-0.0625\ \kappa^2 h_b$. The formula of Hunt (1958) is applied to calculate the wave run-up

$$R = F_R H_b \frac{m}{\sqrt{H_b/L_0}} \tag{18}$$

Table 1
Wave height, period, and water depth in horizontal section of the tank in Saville's experiments

Case No.	Wave height (m)	Wave period (s)	Water depth (m)	H/wT
300	1.68	11.33	4.27	4.94
400	1.62	5.60	4.42	9.64
500	1.52	3.75	4.57	13.51

where R is run-up height measured vertically upward from the still water level, F_R is a run-up coefficient, set to 1, m is the effective beach slope averaged from the run-up limit to the wave breaking point, and L_0 is the deep water wave length.

Three characteristic profile slopes (dune, shoreline and offshore slopes) must be specified in the numerical model. The dune slope, which is defined as the averaged dune scarp slope after erosion, is the maximum slope that the profile is allowed to achieve. The shoreline slope is the anticipated profile slope between the shoreline and the run-up limit. After each time step, if the profile is steeper than the given shoreline slope at a point, the profile is smoothed toward the shoreline slope with an exponential "folding time" of 0.1 hours. Sand conservation across the profile is ensured by the continuity equation, so that the volume eroded from the beach face must be deposited offshore. The offshore slope is introduced to control the slope at the seaward end of deposited volume.

4.1. Saville's experiments

Saville's experiments (Kraus, 1988) for Cases 300, 400, and 500 are investigated here. These three cases had the same initial beach slope of 1 : 15 and a mean sand diameter of 0.22 mm. The corresponding fall velocity for the sand at a temperature of 20°C is 3 cm/s and the profile scale parameter, A, is 0.106 $m^{1/3}$. Regular waves were run for all cases. The wave and water level conditions for the three cases are presented in Table 1. The initial and final profiles for each of these experiments are presented in Fig. 4 in order of increasing fall velocity parameter, H/wT. As the fall velocity parameter increases the offshore bar becomes more and more significant. In Case 500, the height of the offshore bar was about 1.5 m. According to observations recorded during the experiment, a second wave breaking position occurred in this case.

In each case, the dune, shoreline and offshore slopes used in the numerical model are determined according to the observed final profiles. The slope values are shown in Table 2. The root mean square error of eroded volume is defined as

$$\mathrm{Err} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\mathrm{Vol}_p(t_i) - \mathrm{Vol}_m(t_i)\right]^2} \tag{19}$$

where Vol_p and Vol_m are the predicted and measured eroded volumes, respectively, and t_i denotes the time at which the measurements are available. The best-fit K values and the corresponding errors of eroded volume for the three cases are presented in Table 2. The best-fit K values vary from 5.77×10^{-10} to 8.55×10^{-10}, a factor of 1.48 for $n = 3$. The corresponding factor for the best-fit K values for $n = 1$ is 2.36. Comparisons

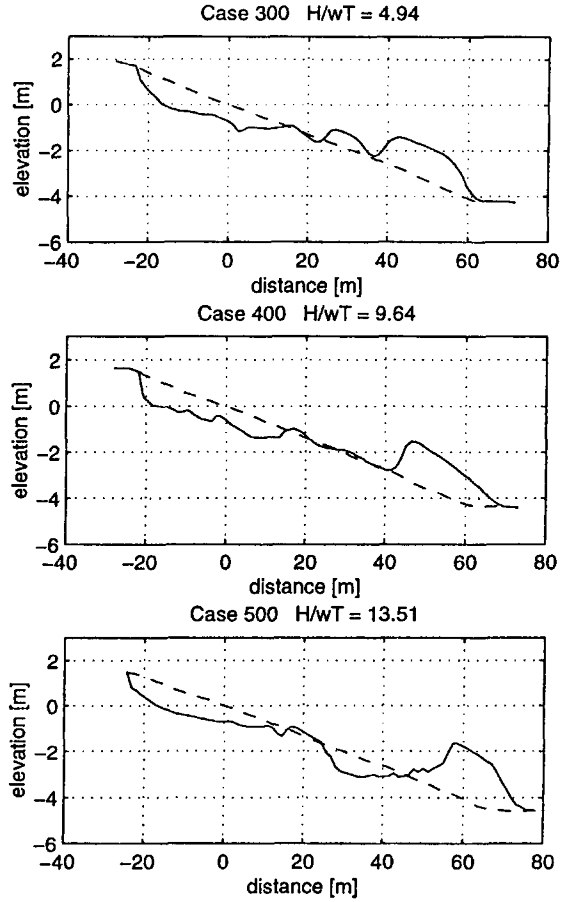

Fig. 4. The initial (dashed) and final (solid) profiles of Cases 300, 400, and 500 in Saville's experiments.

of predicted and measured eroded volumes are shown in Fig. 5 for the predictions of both the linear ($n = 1$) and non-linear ($n = 3$) sediment transport relationships. It appears that the non-linear transport relationship provides a better fit for Cases 300 and 400, while the linear transport relationship presents better results for Case 500. Since Case 500 had a very significant offshore bar formed during beach erosion (Fig. 4), thus it may not be appropriate to calculate wave energy dissipation per unit volume according to the local depth. The profile evolutions are compared in Fig. 6 for Case 400. It is noticed that the numerical model predicts a smooth, monotonic profile form and cannot represent bar formation.

4.2. Large German Wave Flume

Three series of experiments were carried out in the German Large Wave Flume (Dette and Uliczka, 1987; Dette et al., 1992). Two of these experiments had the same

Table 2
The numerical model inputs and best-fit results for laboratory experiments

Case			Slope			Best-fit K		Error [a] $[m^2]$	
			dune	shoreline	off-shore	$n=3$ $[m^8 s^2/N^3]$	$n=1$ $[m^4/N]$	$n=3$	$n=1$
Saville's experiments	Case 300		0.50	0.15	0.20	8.55×10^{-10}	3.45×10^{-6}	2.08	3.37
	Case 400		1.00	0.17	0.15	7.97×10^{-10}	3.71×10^{-6}	1.78	2.40
	Case 500		0.50	0.13	0.30	5.77×10^{-10}	1.57×10^{-6}	3.99	1.88
German Large Wave Flume	Regular waves	dune w/o foreshore	3.00	0.20	0.20	7.64×10^{-10}	2.03×10^{-5}	0.74	8.08
	Regular waves	dune with foreshore	3.00	0.20	0.162	1.03×10^{-9}	8.13×10^{-6}	0.46	1.11
	Irregular waves	dune w/o foreshore	3.00	0.18	0.50	4.51×10^{-10}	2.11×10^{-5}	1.58	2.48
SUPERTANK ST-10			0.6	0.22	0.16	5.26×10^{-10}	3.03×10^{-6}	0.55	0.37

[a] Error represents the root mean square between predicted and observed eroded volume as shown in Eq. (19).

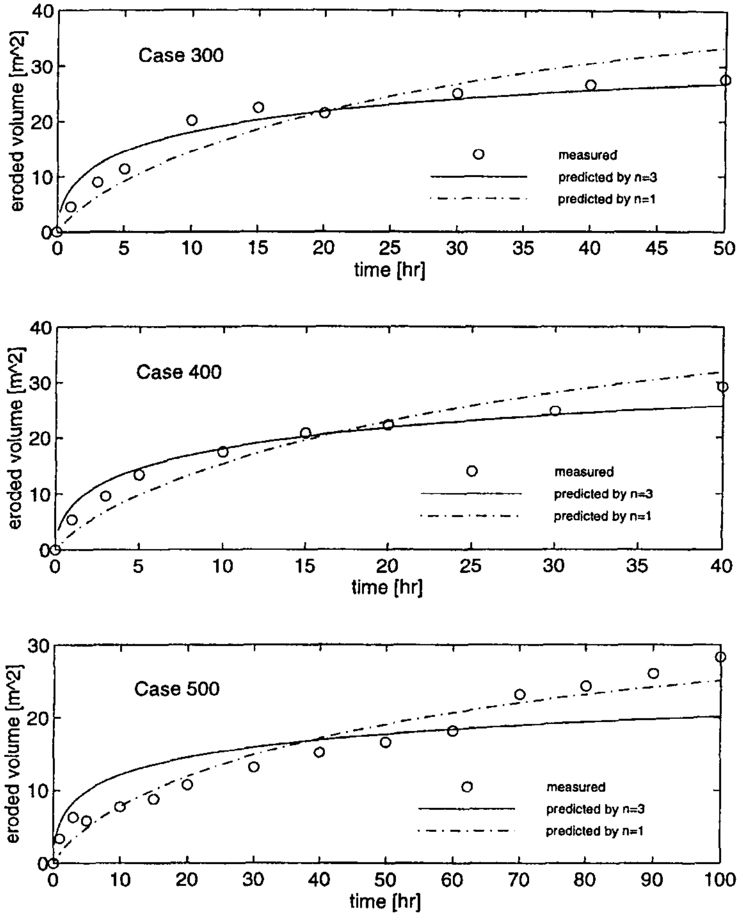

Fig. 5. Comparisons of predicted (with best-fit K values) to observed eroded volumes for Cases 300, 400 and 500 in Saville's experiments.

constant wave conditions and different initial profiles. Regular waves with a wave height of 1.5 meters and a period of 6 seconds were generated in a water depth of 5 meters. The sand used for both experiments had a mean diameter of 0.33 mm, which corresponds to a sediment scale parameter of 0.13 $m^{1/3}$. Two initial profiles were termed "dune without foreshore" and "dune with foreshore". The "dune without foreshore" had a dune crest of 2 meters above still water level (SWL) and a seaward slope of 1 : 4 down to the channel floor. The "dune with foreshore" had a slope of 1 : 4 from the dune crest of 2 meters above SWL to 1 meter below SWL followed by a slope of 1 : 20 down to the channel floor. The third experiment, a "dune without foreshore" was conducted with the same sediment as the other two but irregular waves with a significant wave height of 1.5 m, peak spectral wave period of 6 s and a water depth of 5 m. A

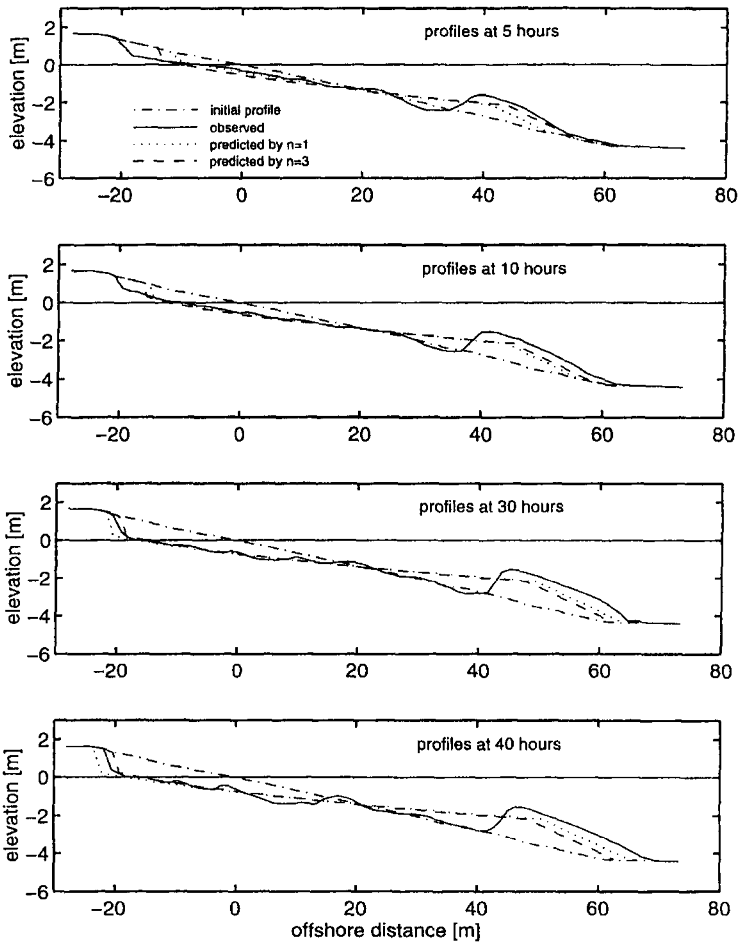

Fig. 6. Case 400 from Saville's experiments. Comparisons of predicted to observed profiles at different times for the best-fit K values for each transport relationship.

Jonswap-spectrum was applied to generate irregular waves with a repetition interval of 256 seconds.

During numerical simulation, the active water depth at each time step is determined as 1.28 times the incident wave height. The input of the dune, shoreline and offshore slopes presented in Table 2 are determined based on the observed final profiles in each case. In the irregular wave cases, the time series of measured wave heights and periods are applied. The results of the best-fit K values and the corresponding root mean square errors of eroded volumes are presented in Table 2 for both nonlinear ($n = 3$) and linear ($n = 1$) transport relationships. Fig. 7 presents comparisons of predicted to measured eroded volumes for the three experiments. The predicted and observed profiles are compared for different times in Fig. 8 for the case "dune without foreshore" with

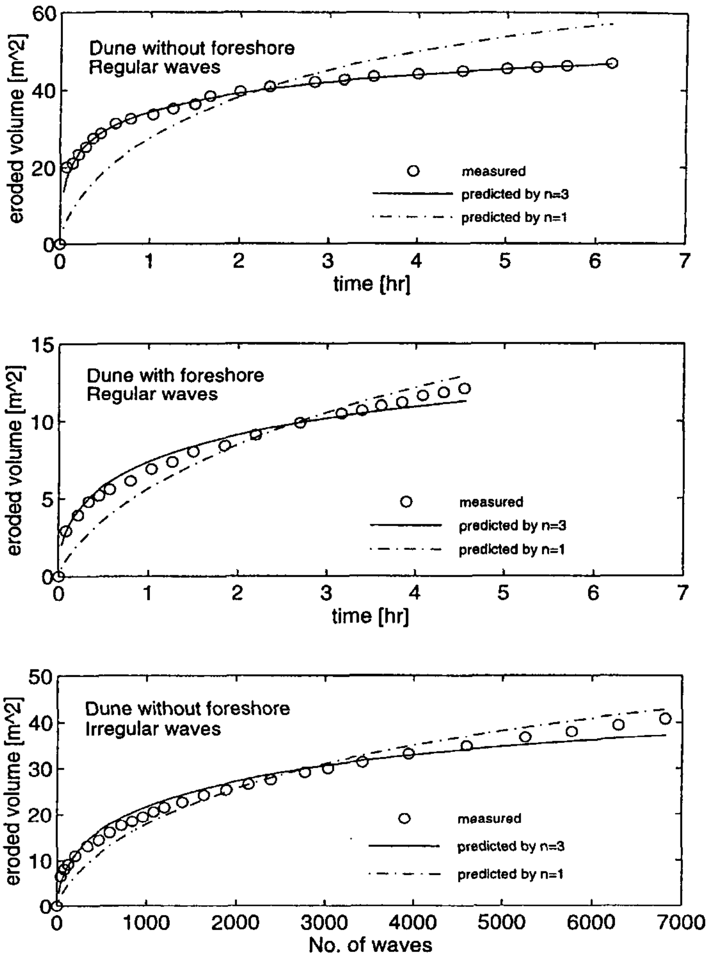

Fig. 7. Comparisons of predicted (with best-fit K values) to observed eroded volumes for the two tests with constant waves in the German Large Wave Flume.

random waves. Overall $n = 3$ provides better predictions for eroded volume than $n = 1$ for the three cases. The results confirm that the non-linear model has capabilities to handle different initial condition, whereas the linear transport relationship cannot represent well the difference in time scale of profile evolution caused by the different initial conditions. Fig. 7 clearly shows that the linear transport relationship has difficulty in simulating the rapid response in the case of "dune without foreshore" with regular waves, although it provides acceptable predictions for the other two cases.

4.3. SUPERTANK experiments

The SUPERTANK project was conducted in the large wave tank at the Otto Hinsdale Wave Research Laboratory, Oregon State University (Kraus and Smith, 1994, Smith and

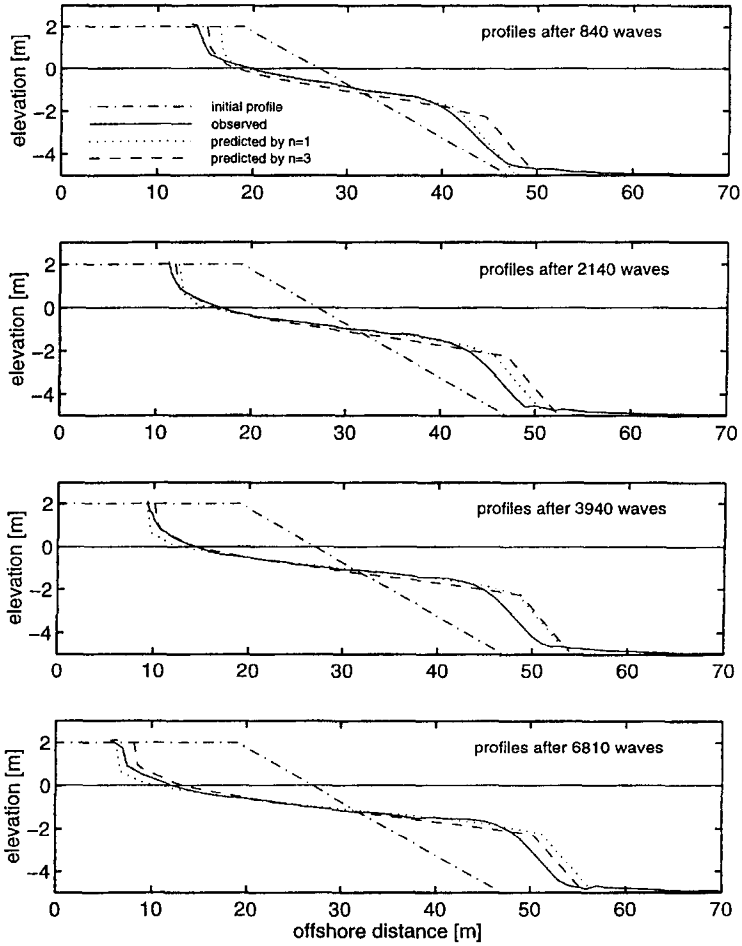

Fig. 8. Test of "dune without foreshore" with irregular waves. Comparisons of predicted to observed profiles at four different times for the best-fit K values for each transport relationship.

Kraus, 1995). Among the SUPERTANK experiments, ST-10 was the longest test (21 hours of wave action) and conducted to observe beach response to erosive waves. The test was run with combinations of random and monochromatic waves with random waves used for most of the experiment duration. The time histories of significant wave height and peak spectral wave period at a water depth of about 2.6 meters are shown in Fig. 9, where "mon" represents the intervals with monochromatic wave conditions. The beach sediment consisted primarily of very uniform sand with median grain-size of 0.22 mm. Under random wave conditions, the joint distribution of wave periods and heights was applied to generate wave height and period according to the input of significant wave heights and peak spectral periods shown in Fig. 9. The input dune, shoreline and offshore slopes determined from the measured final beach profile are presented in Table

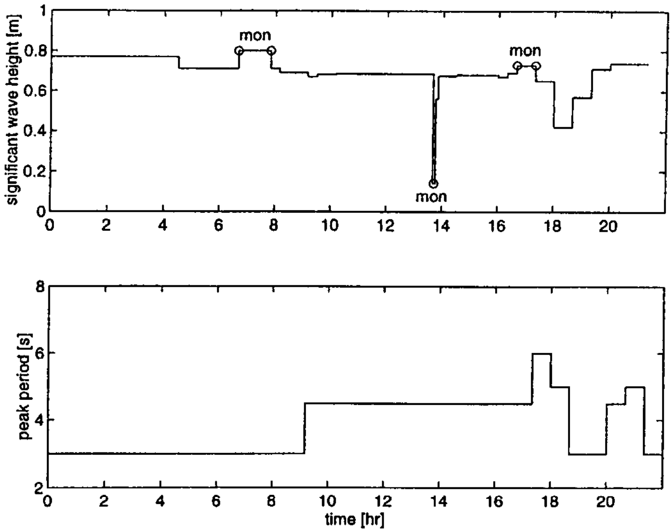

Fig. 9. Significant wave height and peak spectral wave period time history of ST-10 test in SUPERTANK. "Mon" denotes monochromatic waves.

2 with the best-fit K values and the corresponding errors of eroded volume together. The time histories of predicted eroded volumes are compared with the observations in Fig. 10. The linear transport model presents a somewhat smaller error for the beach erosion process.

In summary of the calibration, a total of seven experiments have been employed for the calibration and comparison of the proposed non-linear versus the linear transport relationships. Five of these were conducted with regular wave conditions and the other two were carried out with random waves. Different sediment sizes, initial profiles, and wave conditions characterized the experiments. It appears that the fine sediment generally has relative mild dune, shoreline and offshore slopes and the coarse sand corresponds to somewhat steeper slopes, consistent with early findings by Bascom

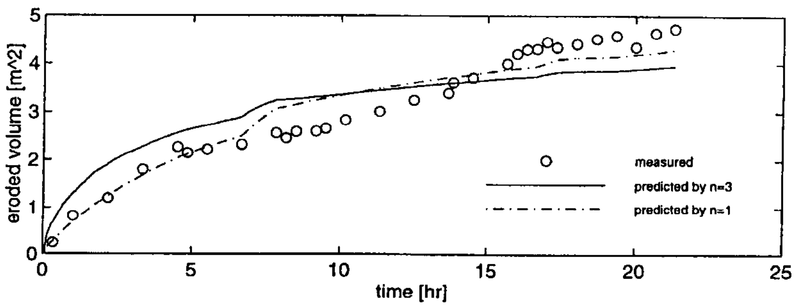

Fig. 10. Comparisons of predicted with best-fit K values to observed eroded volumes for the test ST-10 in SUPERTANK.

(1951). The best-fit transport coefficient, K, in the non-linear ($n = 3$) relationship has a much narrower range than that for the linear relationship. The average best-fit K value for the non-linear relationship is 7.14×10^{-10} m^8 s^2/N^3 with a variation from -37% to $+44\%$, whereas the best-fit K values of the linear relationship range between -74% to $+248\%$ of the average value of 6.07×10^{-6} m^4/N. On average, the non-linear relationship yields an error of 1.58 m^2 for the eroded volumes and the linear relationship yields 2.81 m^2. Based on these results, it appears that the non-linear transport relationship with n = 3 provides a more appropriate cross-shore sediment transport model. Thus, the transport coefficient of 7.14×10^{-10} m^8 s^2/N^3 is adopted in CROSS according to the average of the best-fit K values in the seven experiments. The input dune slope is usually quite steep and may be considered as 1 for most cases, although it can be much steeper for a laboratory experiment with coarse sediment. The offshore slope may be taken as 0.15. In laboratory experiments, this offshore slope may be steeper than the value of 0.15 for a steep initial profile. The shoreline slope is usually steeper for laboratory experiments than for field conditions. The value of 0.18 may be used as laboratory shoreline slopes. Under field conditions, it is recommended that the measured pre-storm shoreline slope be applied as the shoreline slope input.

5. Storm and beach profile characteristics at Ocean City, Maryland

The Ocean City, Maryland beach was nourished by the State of Maryland and Federal Government in 1988, 1990 and 1991 for storm protection purposes. The entire project was finished in August 1991. After the project, several storms occurred in late 1991 and early 1992. Among the 1991–1992 winter storms, the January 4, 1992 storm was most severe with a peak surge of 2 meters (Kraus and Wise, 1993). Beach profiles were surveyed on November 2, 1991 and January 11, 1992 before and after the storm respectively (Stauble et al., 1993). During this period, an additional storm occurred on November 11, 1991. The profile responses using SBEACH have previously been simulated by Kraus and Wise (1993) for the changes occurring between November 2, 1991 and January 11, 1992, however only the large January 4, 1992 storm was represented in their simulations. Both storms will be included in the numerical simulations presented herein.

In the following study, the water depths and profile elevations are referenced to NGVD (National Geodetic Vertical Datum), which lies 0.02 meters below mean water level for Ocean City. The wave and storm surge characteristics applied in the numerical simulations were measured during the storms by two gages located just offshore of Ocean City in a water depth of approximately 10 meters. These gages were installed by the Coastal Engineering Research Center, U.S. Army Corps of Engineers Waterways Experiment Station. The results are shown in Figs. 11 and 12 for the November 11, 1991 and January 4, 1992 storms, respectively.

Seven survey lines located from 37th Street (south) to 124th Street (north) are available for this effort. In this study area, the sand size varies quite significantly from dune to offshore. Sediment samples along six profiles located at 37th, 56th, 66th, 81st, and 92nd and 103rd Streets were collected and analyzed (Stauble et al., 1993). Since the

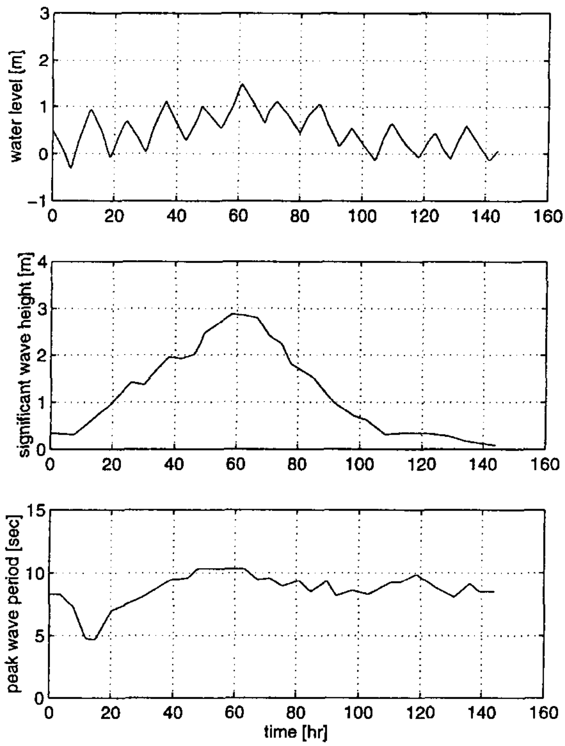

Fig. 11. Water level, significant wave height and peak spectral wave period time history for the November 11, 1991 storm at Ocean City, MD (from Stauble et al., 1993). Note: zero time corresponds to 0:00 on November 8, 1991.

four survey lines located at 45th, 63rd, 74th, and 124th Streets have no sediment data available, the grain size of the profile at 103rd Street is used for the profile at 124th Street, and the grain sizes at the other three profiles are determined by interpolation. The mean grain sizes at 11 morphologic zones are shown in Table 3 for the seven survey lines.

The measured pre- and post-storm profiles are shown in Fig. 13. It appears that the storm-caused erosion is quite different for profiles at different locations. Based on the measured profiles, the total volumes gained or lost from the pre-storm profile to the post-storm profile are calculated for each profile and presented in Table 4. It is clear that net volume changes are quite different from zero for the profiles. To remove this effect, which is considered to be due to gradients in longshore sediment transport, each post-storm profile is adjusted by shifting the profile horizontally a distance Δy to yield zero net volume change. The value of Δy can be calculated by

$$\Delta y = \frac{1}{h_{\text{total}}} \int_{y_0}^{y_\infty} (h_{\text{mb}} - h_{\text{ma}}) \mathrm{d} y \tag{20}$$

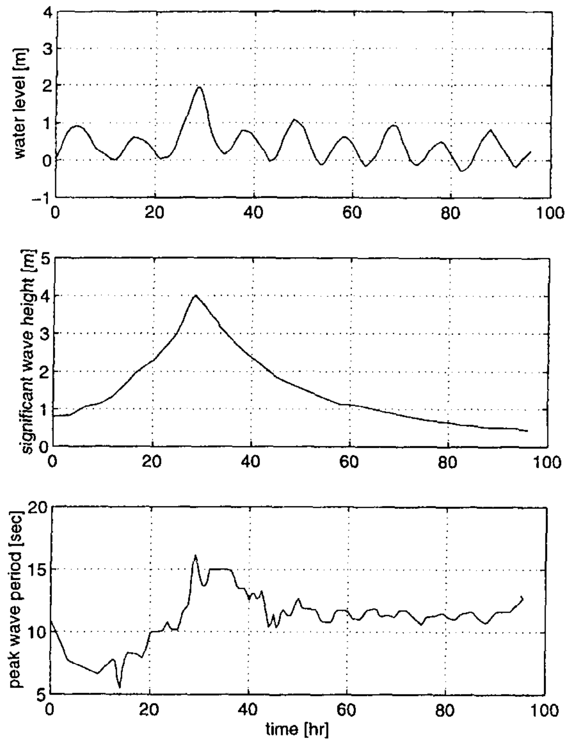

Fig. 12. Water level, significant wave height and peak spectral wave period time history for the January 4, 1992 storm at Ocean City, MD (from Stauble et al., 1993). Note: zero time corresponds to 0:00 on January 3, 1992.

Table 3
Average mean grain size in millimeters

Sample location	37th St.	45th St.	56th St.	63rd St.	74th St.	103rd St. [a]
Dune base	0.35	0.34	0.33	0.39	0.39	0.44
Berm crest	0.29	0.31	0.34	0.35	0.35	0.38
Mean-tide line	0.32	0.30	0.27	0.27	0.29	0.38
Swash zone	0.34	0.38	0.44	0.47	0.48	0.39
Nearshore trough	0.31	0.33	0.36	0.45	N/A	N/A
Nearshore bar	0.29	0.36	0.46	0.44	N/A	N/A
−1.52 m contour	0.29	0.26	0.21	0.26	0.32	0.23
−3.05 m contour	0.21	0.21	0.22	0.22	0.21	0.21
−4.57 m contour	0.20	0.22	0.24	0.21	0.22	0.21
−6.10 m contour	0.29	0.23	0.14	0.15	0.28	0.17
−7.62 m contour	0.34	0.25	0.13	0.12	0.17	0.16

[a] The grain size in this column is also used as the grain size of the profile at 124th St.

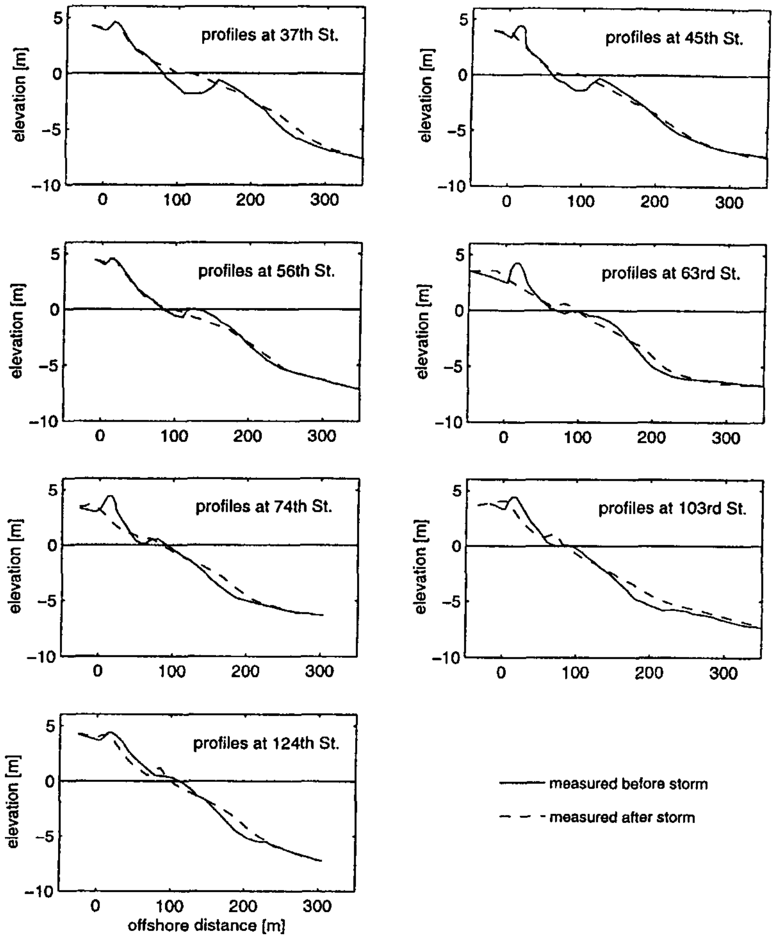

Fig. 13. Measured pre-storm (solid line) and post-storm (dashed line) profiles.

Table 4
Measured volume change during the storm and adjustment Δy

Street	Net volume change (m^3/m)	Adjustment Δy (m)	h_{total} (m)
37th	137.99	−11.29	12.22
45th	31.59	−2.83	11.16
56th	−20.25	1.97	10.28
63rd	−1.65	0.16	10.31
74th	20.92	−2.19	9.55
103rd	79.73	−7.09	11.25
124th	19.64	−1.80	10.91

Table 5
Measured eroded volumes and beach retreat at the 3 m contour

Street	Eroded volume (m^3/m)		Retreat at 3 m contour (m)	
	without adjust.	with adjust.	without adjust.	with adjust.
37th	6.75	48.50	2.10	13.39
45th	20.36	29.42	8.30	11.13
56th	9.12	2.72	2.30	0.33
63rd	44.76	44.46	29.78	29.62
74th	50.05	54.66	21.65	23.84
103rd	42.40	62.51	13.59	20.68
124th	40.08	46.17	10.11	11.91

where subscripts mb and ma denotes profile elevation measured before and after storms, respectively, y_0 and y_∞ are the offshore distance coordinates at the baseline and offshore profile change limit, respectively, and h_{total} is the total elevation of the active post-storm profile and is based on inspection of each individual profile. The sign of Δy is positive for a seaward translation. The profile retreat at the 3 meter contour and the eroded volumes with and without the shifting adjustments are shown in Table 5.

6. Comparisons of four erosion models

The predictions of the proposed CROSS model and three existing models: EDUNE (Kriebel, 1982, 1986; Kriebel and Dean, 1985), two versions of SBEACH (Larson et al., 1989, Larson and Kraus, 1989) and CCCL (Chiu and Dean, 1984, 1986), are compared with the storm erosion measured at Ocean City, Maryland. The input parameters for each model in this study are selected to represent the conditions for which each model was calibrated. The run conditions for each model are described briefly as follows:

6.1. CROSS

The dune and offshore slopes are set equal to 1 and 0.15, respectively, as default conditions, and the shoreline slope is set to the average shoreline slope value of the measured pre-storm profiles (0.05). Two different sets of profile parameters are compared in this model. First, along each measured profile the variable profile parameter, A, is determined according to the variable sediment size listed in Table 3. Second, as a basis for comparison with the other three models, a constant grain size of 0.35 mm, which was recommended by Kraus and Wise (1993) is used. During each storm, a random wave series is generated according to the time history of the measured significant wave height and peak spectral wave period. Both wave set-up and run-up are included. The active water depth is 1.28 times the instantaneous incoming individual wave height.

6.2. CCCL

This model has been used in establishment of the Coastal Construction Control Line (CCCL) in Florida. The CCCL is a line which depicts the landward limit of impact of a 100 year return period storm event. The default input dune slope is set to 1. Since this model cannot handle variable sediment size along a profile, a uniform grain size of 0.35 mm is applied. The set-up at the shoreline is calculated by Eq. (17) according to the significant wave heights. No wave run-up is included in the model. The input water level during a storm is given by adding the set-up at the shoreline to the measured storm surge. After running two storms, consistent with model application, a factor of 2.5 is applied to those contours which receded. This model does not incorporate a transport equation, but rather considers the profile to approach equilibrium with an exponential folding time scale of 13 hours.

6.3. EDUNE

The default input dune slope is set to 1 and the input shoreline is taken as 0.05 which is the average shoreline slope of the measured pre-storm profiles. For the same reason as CCCL, a sediment size of 0.35 mm is used. The input significant wave height is applied as a regular wave height for each time step. In EDUNE, the wave run-up is a constant value throughout the erosion simulation and is used to control the location of the dune scarp above the peak still water flood level. According to the EDUNE Users Manual (Kriebel, 1989), the value of run-up should be based on field data or experience from a particular location and does not necessarily simulate a realistic wave run-up limit at each time step. Calibrated to match the dune erosion level of post-storm profiles at Ocean City, the run-up calculated by Hunt's formula for the maximum significant wave height is used in the entire simulation time for each storm, which is 0.91 and 1.52 m for the November 1991 and January 1992 storms, respectively. No set-up is included in the simulations. The transport coefficient for this application is the program default value of 8.73×10^{-6} m^4/N.

6.4. SBEACH

Although source codes for this model are not available, both versions (2.0 and 3.0) of SBEACH model are believed to incorporate wave run-up and set-up. The maximum slope that a predicted profile is allowed to achieve is required and is set to 17.5° as a default condition; this corresponds to a slope of 0.32. A uniform sediment size of 0.35 mm is applied. The wave conditions used in the SBEACH model simulations are the same as the measurements shown in Figs. 11 and 12. Both versions of the SBEACH model provide the choice of wave type (monochromatic or irregular). The option of irregular waves is chosen for this study. The version 3.0 of SBEACH, which became available in September 1994, is not documented to the same degree as the earlier version.

7. Measures of model performance

The numerical results from the four models are quantified in terms of several parameters. A comparison of measured and predicted profile changes is provided by the residual parameter, Res, defined in non-dimensional form as:

$$\mathrm{Res} = \frac{\sum_{i=1}^{n} \left(h_{\mathrm{p}i} - h_{\mathrm{ma}i} \right)^2}{\sum_{i=1}^{n} \left(h_{\mathrm{mb}i} - h_{\mathrm{ma}i} \right)^2} \tag{21}$$

where h is the elevation from mean water line, the subscripts p and m denote predicted and measured, respectively, b and a indicate before and after storm conditions, respectively, i represents the ith location on the profile and the sums extend across the entire active profile. The minimum possible value of Res is zero, which would correspond to a perfect simulation. The agreement between calculated and measured dune erosion is quantified by the eroded volume and the beach retreat at the 3 meter contour. To provide a measure of erosion and retreat, two different errors are presented: the root mean square error, $\mathrm{ERR}_{\mathrm{rms}}$, and the algebraic average error, $\mathrm{ERR}_{\mathrm{ave}}$. These are expressed as:

$$\mathrm{ERR}_{\mathrm{rms}} = \frac{\sum_{j=1}^{n} \left(S_{\mathrm{p}j} - S_{\mathrm{m}j} \right)^2}{\sum_{j=1}^{n} S_{\mathrm{m}j}^2}$$

$$\mathrm{ERR}_{\mathrm{ave}} = \frac{\sum_{j=1}^{n} \left(S_{\mathrm{p}j} - S_{\mathrm{m}j} \right)}{\sum_{j=1}^{n} S_{\mathrm{m}j}} \tag{22}$$

where S is an eroded volume or beach retreat, and j denotes the jth beach profile line. $\mathrm{ERR}_{\mathrm{rms}}$ represents a factor of simulation accuracy and $\mathrm{ERR}_{\mathrm{ave}}$ provides a measure of over or under-prediction of erosion. The measure, Res, is based on local differences across the active profile whereas $\mathrm{ERR}_{\mathrm{rms}}$ and $\mathrm{ERR}_{\mathrm{ave}}$ are based on total differences or differences at a particular elevation.

The residuals, predicted eroded volumes and beach retreat at the 3 meter contour are presented in Table 6, in which "without adjustment" means data given by the original measured post-storm profiles, and "with adjustment" means data given by horizontally shifted post-storm survey profiles to satisfy no net volume change. The onshore limit of predicted eroded volume for each profile is determined by the onshore cross over of the predicted post-storm and the measured pre-storm profiles, while the offshore limit is determined by the offshore cross over of the predicted post-storm and measured pre-storm profiles or the shoreline of the predicted profile depending on which is closer. It is noticed that all four models yield large residuals for the 56th Street profile because

Table 6
The predicted residuals, and measured and predicted eroded volumes and beach retreat at the 3-meter contour

Profile	Model		Residual w/o adjust.	Residual with adjust.	Eroded vol. (m^2)	Retreat at 3-m contour (m)
37th St.	Measurements	w/o adjust.	–	–	6.75	2.10
		with adjust.	–	–	48.50	13.39
	CROSS (var. sand size)		0.614	0.454	22.04	8.04
	CROSS (fixed sand size)		0.508	0.339	19.37	6.75
	CCCL		1.286	0.443	83.22	18.34
	EDUNE		0.588	0.512	9.15	4.80
	SBEACH (ver. 2.0)		1.074	1.075	16.42	3.55
	SBEACH (ver. 3.0)		0.664	0.613	30.01	4.84
45th St.	Measurements	w/o adjust.	–	–	20.36	8.30
		with adjust.	–	–	29.42	11.13
	CROSS (var. sand size)		0.525	0.496	20.13	8.71
	CROSS (fixed sand size)		0.287	0.280	19.39	8.36
	CCCL		1.067	0.851	60.56	18.34
	EDUNE		0.624	0.604	18.30	6.51
	SBEACH (ver. 2.0)		1.018	0.971	15.21	2.96
	SBEACH (ver. 3.0)		0.736	0.713	23.34	6.79
56th St.	Measurements	w/o adjust.	–	–	9.12	2.30
		with adjust.	–	–	2.72	0.33
	CROSS (var. sand size)		1.539	1.584	12.36	5.74
	CROSS (fixed sand size)		0.765	0.785	11.48	5.28
	CCCL		1.506	1.761	31.62	11.86
	EDUNE		1.812	1.725	4.68	3.73
	SBEACH (ver. 2.0)		2.971	2.971	4.76	2.28
	SBEACH (ver. 3.0)		1.588	1.526	5.56	3.32
63rd St.	Measurements	w/o adjust.	–	–	44.76	29.78
		with adjust.	–	–	44.46	29.62
	CROSS (var. sand size)		0.854	0.854	17.71	9.27
	CROSS (fixed sand size)		0.596	0.595	17.12	9.07
	CCCL		1.960	1.966	53.68	20.08
	EDUNE		0.797	0.795	19.88	8.93
	SBEACH (ver. 2.0)		0.803	0.802	14.76	4.59
	SBEACH (ver. 3.0)		0.646	0.645	25.68	14.65
74th St.	Measurements	w/o adjust.	–	–	50.05	21.65
		with adjust.	–	–	54.66	23.84
	CROSS (var. sand size)		0.511	0.524	25.41	11.69
	CROSS (fixed sand size)		0.450	0.463	21.33	9.71
	CCCL		0.610	0.515	86.72	30.99
	EDUNE		0.534	0.530	39.20	16.18
	SBEACH (ver. 2.0)		0.936	0.935	16.72	4.11
	SBEACH (ver. 3.0).		0.526	0.527	29.73	12.68
103rd St.	Measurements	w/o adjust.	–	–	42.40	13.59
		with adjust.	–	–	62.51	20.68
	CROSS (var. sand size)		0.329	0.274	35.18	15.23
	CROSS (fixed sand size)		0.261	0.251	28.88	13.41
	CCCL		2.233	1.499	123.05	28.92

Table 6 (continued)

Profile	Model		Residual		Eroded vol. (m^2)	Retreat at 3-m contour (m)
			w/o adjust.	with adjust.		
	EDUNE		0.548	0.413	54.66	20.04
	SBEACH (ver. 2.0)		0.596	0.590	19.54	7.03
	SBEACH (ver. 3.0)		0.510	0.433	35.20	10.61
124th St.	Measurements	w/o adjust.	–	–	40.08	10.11
		with adjust.	–	–	46.17	11.91
	CROSS (var. sand size)		0.931	0.895	40.08	12.45
	CROSS (fixed sand size)		0.503	0.493	23.36	9.08
	CCCL		3.131	2.821	93.83	31.70
	EDUNE		1.000	0.968	48.02	14.79
	SBEACH (ver. 2.0)		1.112	1.136	10.08	3.31
	SBEACH (ver. 3.0)		0.802	0.805	23.88	4.87

this profile had very little change after two storms and the denominator of Eq. (21) becomes very small.

The residuals averaged over seven profiles and the two kinds of error defined above for eroded volume and beach retreat at the 3 meter contour with respect to the original measured profiles and the horizontal shifted profiles are presented in Tables 7 and 8.

Table 7
The average residuals and errors of eroded volume and beach retreat at the 3 meter contour with respect to the original measured profiles

Model	Average residual	Error of eroded volume		Error of retreat	
		ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
CROSS (variable sand size)	0.758	0.193	−0.190	0.334	−0.190
CROSS (fixed sand size)	0.481	0.262	−0.340	0.350	−0.298
CCCL	1.685	2.222	1.495	1.072	0.971
EDUNE	0.843	0.116	−0.092	0.314	−0.146
SBEACH (ver. 2.0)	1.216	0.423	−0.544	0.617	−0.683
SBEACH (ver. 3.0)	0.782	0.196	−0.188	0.207	−0.342

Table 8
The average residuals and errors of eroded volume and beach retreat at the 3 meter contour with respect to the horizontal shifted measured profiles

Model	Average residual	Error of eroded volume		Error of retreat	
		ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
CROSS (variable sand size)	0.725	0.227	−0.401	0.283	−0.359
CROSS (fixed sand size)	0.458	0.319	−0.511	0.327	−0.444
CCCL	1.331	0.707	0.847	0.480	0.561
EDUNE	0.792	0.182	−0.328	0.260	−0.324
SBEACH (ver. 2.0)	1.211	0.470	−0.663	0.623	−0.749
SBEACH (ver. 3.0)	0.752	0.183	−0.399	0.259	−0.479

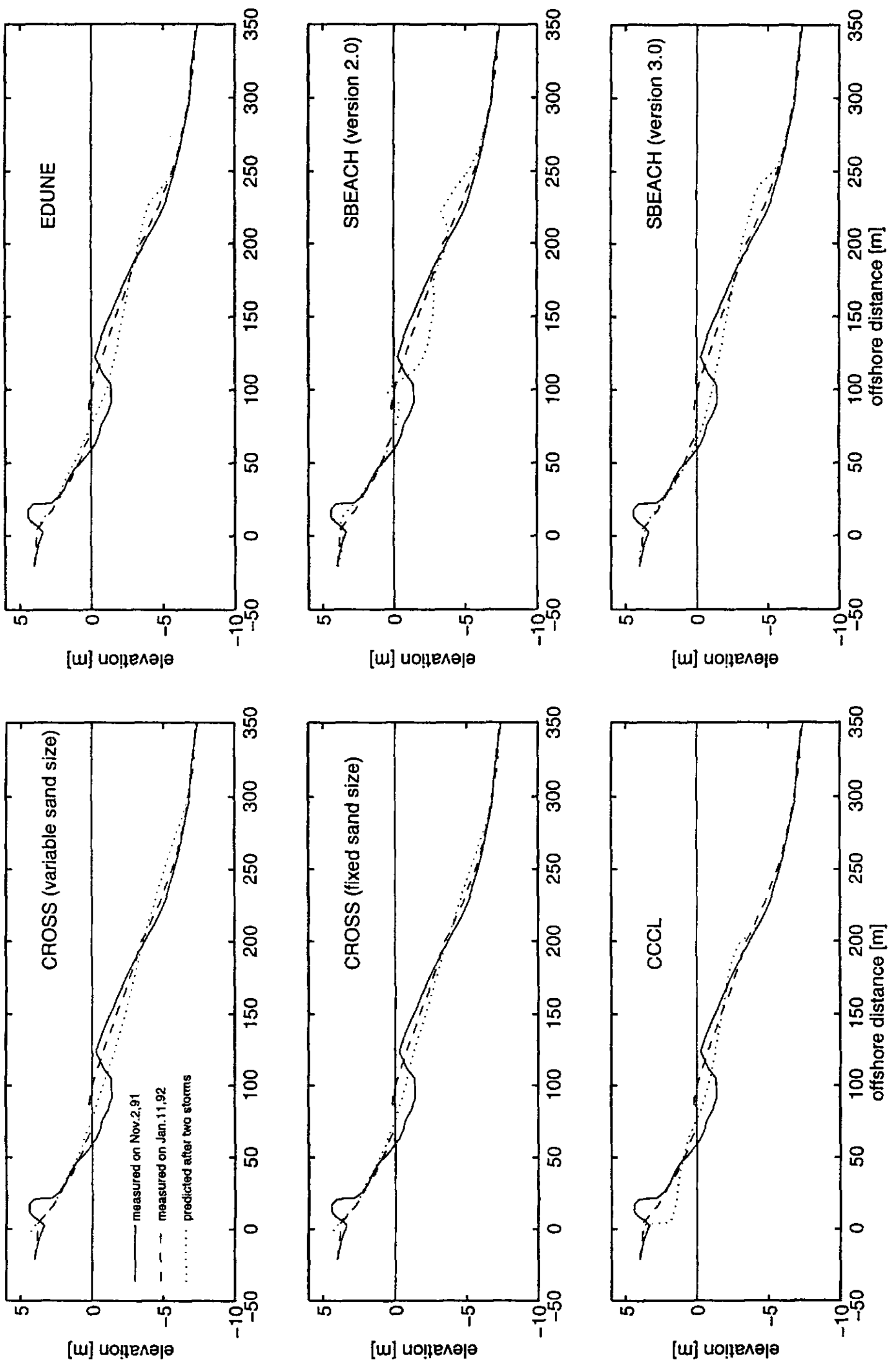

Fig. 14. Comparisons of predicted and measured profiles at 45th Street.

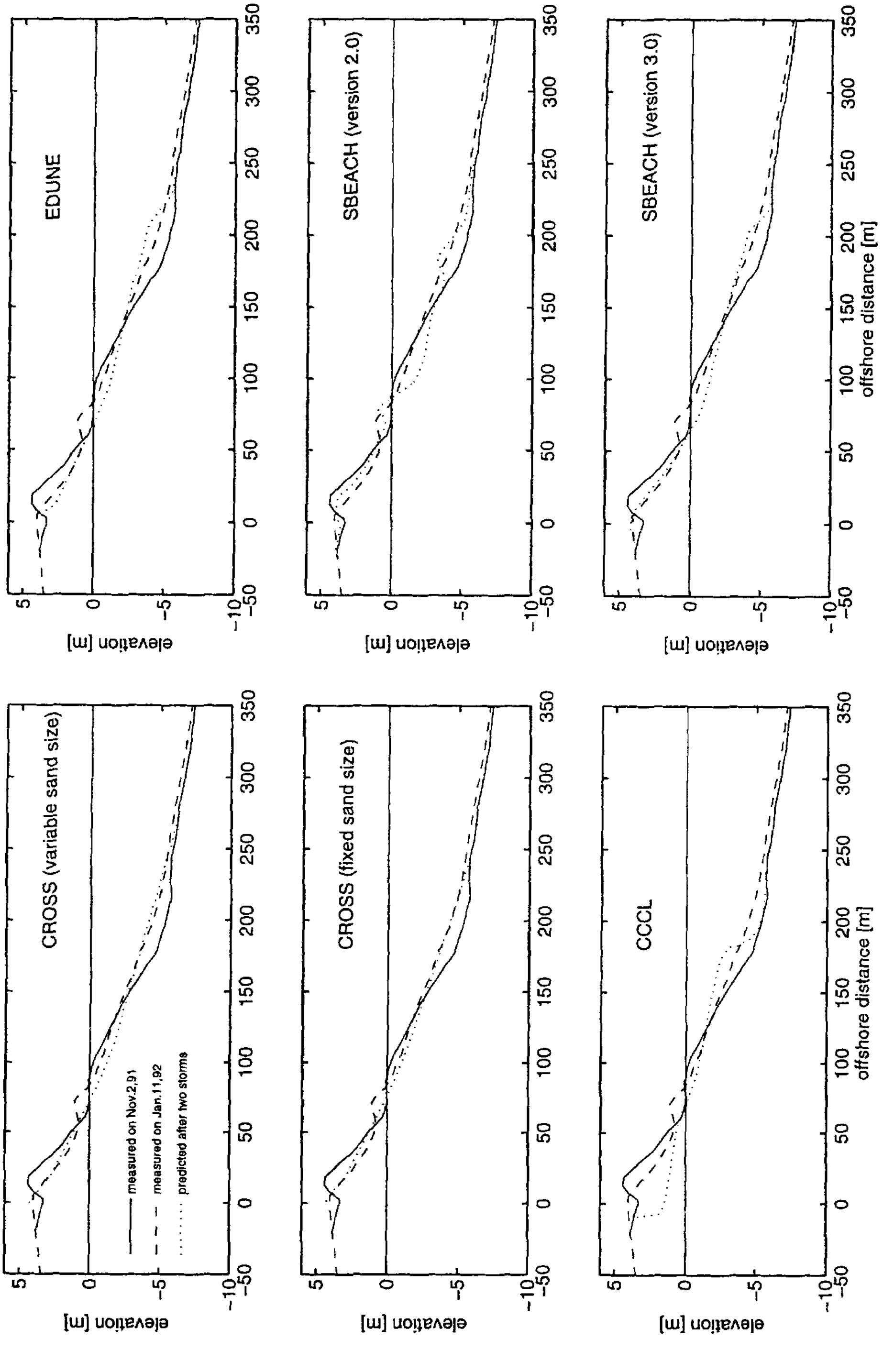

Fig. 15. Comparisons of predicted and measured profiles at 103rd Street.

Since all five models discussed here only include cross-shore sediment transport, the results presented "with adjustment" are considered more appropriate. Overall, CCCL overpredicts the dune erosion during the two storms, whereas the other three models yield underpredictions. CROSS with fixed sand size yields the smallest residuals. Judged by eroded volume and beach retreat at the 3 meter contour, CROSS with variable sand size, EDUNE and SBEACH version 3.0 present comparable predictions. Among the four models, only EDUNE and SBEACH incorporate dune overwash processes. In the CROSS model, the profile shoreward of the dune crest is treated as a horizontal surface with the same elevation as the dune crest. Therefore, the numerical simulations of CROSS do not include the effect of overwash. It is expected that the underpredictions of CROSS could be improved by incorporating dune overwash processes in the model. Comparisons between predicted and measured profiles for two of the seven survey lines are shown in Figs. 14 and 15 for the beach at 45th and 103rd Streets, respectively.

8. Sensitivity study of cross model

The transport coefficient and the active water depth are two important factors in the CROSS model. It is interesting to investigate the sensitivity of the model to these two factors. Generally in applications of field storm erosion, some uncertainties always exist in measured wave conditions during a storm. To evaluate the effects of these uncertainties, the sensitivities of the CROSS model to the wave height and storm surge are discussed. To better represent beach sediment characteristics, the variable sand size distributions across the profiles as listed on Table 3 are applied for the studies.

8.1. Transport coefficient

According to the calibrations of seven experiments, a standard average transport coefficient, K, of 7.14×10^{-10} m^8 s^2/N^3 is used in CROSS for the numerical simulations of the erosion due to the two storms. For sensitivity analysis, transport coefficients of 8.57×10^{-10} and 5.71×10^{-10} m^8 s^2/N^3, corresponding to $\pm 20\%$ changes, are applied. The average residual and the root mean square and the algebraic average errors of eroded volume and beach retreat at the 3 meter contour over the seven measured profiles are shown in Table 9. With a 20% change in K value, the variations of the root mean square and the algebraic average errors for dune erosion are less than 3% and 7%, respectively. It appears that CROSS is quite insensitive to the transport coefficient for the beach erosion during the two storms at Ocean City. This is due, in part, to the non-linear transport relationship in CROSS resulting in changes occurring relatively slowly when the conditions are close to equilibrium.

8.2. Active water depth

In the CROSS model, the sediment transport rate is related to the wave energy dissipation caused by wave breaking. The active water depth is defined as the breaking water depth which is 1.28 times the instantaneous breaking wave height. Depending on

Table 9
Sensitivity study of CROSS. The average residuals and errors of eroded volume and beach retreat at the 3-meter contour predicted by the $\pm 20\%$ changes in K values

Adjustment	K	Average residual	Error of eroded vol.		Error of retreat	
			ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
Without adjustment	Standard	0.758	0.193	−0.190	0.334	−0.190
	Decreased	0.665	0.205	−0.228	0.347	−0.216
	Increased	0.782	0.176	−0.110	0.322	−0.113
With adjustment	Standard	0.725	0.227	−0.401	0.283	−0.359
	Decreased	0.636	0.250	−0.428	0.306	−0.379
	Increased	0.746	0.191	−0.341	0.256	−0.297

different wave conditions and beach slope, the ratio of the breaking water depth to the breaking wave height may differ from 1.28. In this sensitivity study, an active water depth equal to the incoming wave height is investigated. The average residuals and errors of eroded volume and beach retreat at the 3 m contour are compared in Table 10. The profile residuals and errors of dune erosion decrease considerably with a change in the ratio of active water depth to incoming wave height from 1.28 to 1. It appears that the active water depth of 1.28 times the wave height, which is based on laboratory calibrations of CROSS, may be too large for the Ocean City storm erosion, Recall that five of the seven laboratory experiments used in the calibration of CROSS were carried out with regular waves, it appears that there are differences in the active water depth between monochromatic and random waves. The ratio of 1.28 provides reasonable predictions for profiles under constant wave conditions, while for a profile under irregular wave conditions, the active water depth equal to the instantaneous wave height presents better results. This phenomenon was also noticed by Vellinga (1982), who found in his laboratory studies with irregular waves that the active water depth was equal to approximately 0.75 times the significant wave height.

8.3. Storm surge

During a storm, beach erosion is related to the augmented water levels of the storm surge. A higher surge level and a longer surge duration will cause more severe dune

Table 10
Sensitivity study of CROSS. The average residuals and errors of eroded volume and beach retreat at the 3-meter contour predicted by active water depth as 1.28 times and equal to the instantaneous wave heights

Adjustment	Ratio [a]	Average residual	Error of eroded vol.		Error of retreat	
			ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
Without adjustment	1.28	0.758	0.193	−0.190	0.334	−0.190
	1.00	0.587	0.194	−0.117	0.322	−0.099
With adjustment	1.28	0.725	0.227	−0.401	0.283	−0.359
	1.00	0.554	0.189	−0.346	0.255	−0.287

[a] This is the ratio of active water depth to incoming wave height.

Table 11
Sensitivity study of CROSS. The average residuals and errors of eroded volume and beach retreat at the 3-meter contour predicted by the ±20% change in storm surges

Adjustment	Storm surge	Average residual	Error of eroded vol.		Error of retreat	
			ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
Without adjustment	Standard	0.758	0.193	−0.190	0.334	−0.190
	Decreased	0.791	0.217	−0.296	0.369	−0.301
	Increased	0.673	0.188	−0.064	0.305	−0.043
With adjustment	Standard	0.725	0.227	−0.401	0.283	−0.359
	Decreased	0.760	0.284	−0.479	0.348	−0.446
	Increased	0.635	0.178	−0.307	0.229	−0.242

erosion. Generally, it is difficult to reconstruct, from limited measurements, the exact storm surge conditions for numerical simulation purposes. To investigate the sensitivity of the CROSS model to storm surge, the water levels measured in the two storms (Figs. 11 and 12) were increased and decreased 20%, respectively. The averaged residual and errors of eroded volume and beach retreat at the 3 meter contour as predicted by different surge levels are compared in Table 11. With a 20% variation in storm surge, the changes of dune erosion and the profile residual are less than 15% and 10%, respectively. Overall, it appears that the 20% increased storm surge provides a better simulation for the beach erosion.

8.4. Wave height

Since the exact wave parameters during a storm are generally not available, the effects of wave heights on a numerical model are one of the most difficult problems in coastal engineering applications. For the two storms which occurred at Ocean City, the wave heights presented in Figs. 11 and 12 are increased and decreased by 20% to test the sensitivity of the CROSS model. It appears that the subaqueous part of a profile is quite sensitive to the change of wave heights and the subaerial part is affected less. As a

Table 12
Sensitivity study of CROSS. The average residuals and errors of eroded volume and beach retreat at the 3-meter contour predicted by the ±20% change in wave heights

Adjustment	Wave height	Average residual	Error of eroded vol.		Error of retreat	
			ERR_{rms}	ERR_{ave}	ERR_{rms}	ERR_{ave}
Without adjustment	Standard	0.758	0.193	−0.190	0.334	−0.190
	Decreased	0.411	0.344	−0.416	0.408	−0.386
	Increased	1.146	0.204	0.093	0.301	0.082
With adjustment	Standard	0.725	0.227	−0.401	0.283	−0.359
	Decreased	0.386	0.389	−0.567	0.400	−0.513
	Increased	1.095	0.147	−0.191	0.202	−0.143

result, the eroded volume and beach retreat are less affected by the change of wave heights, while the variations of the profile residual are much more significant. The average residuals and errors of eroded volume and beach retreat at the 3 meter contour predicted by different wave heights are presented in Table 12. Due to the variations in wave heights, the changes of average residuals are higher than 30% and the changes in the errors of eroded volume and beach retreat at the 3 meter contour are about equal to or less than 20%.

9. Conclusions and discussion

Based on equilibrium beach profile concepts, sediment transport is caused by deviations of a beach from its equilibrium form. According to scaling analysis, the linear transport relationship is modified and a non-linear transport equation is developed, in which the transport rate is proportional to the cube of the difference of the local wave energy dissipation per unit volume from equilibrium. This non-linear transport relationship can explain the significantly different time scales of profile evolution between various experiments. An analytical examination of a similar process demonstrates that the initial condition causes considerable differences in the response time scale for a nonlinear system but does not affect that of a linear system.

The proposed non-linear transport relationship is calibrated and compared with the linear transport relationship based on laboratory data. A total of seven large scale wave tank experiments from three different facilities are included. It is found that the transport coefficient K in the non-linear transport relationship has a much narrower range than that of the linear. The average best-fit K values of the non-linear model is 7.14×10^{-10} $m^8\ s^2/N^3$ with a variation from -37% to 44%, whereas the best-fit K values of the linear model range from -74% to 248% of the average value of 6.07×10^{-6} m^4/N. On average, the non-linear model yields much less error for the eroded volumes than the linear model. Thus, it appears that the non-linear transport relationship is more appropriate to represent cross-shore sediment transport. The model ''CROSS'' is developed based on the non-linear relationship and the average best-fit K value of 7.14×10^{-10} $m^8\ s^2/N^3$ is recommended for applications.

The CROSS model is compared with CCCL, EDUNE and SBEACH for the November 1991 and January 1992 storms at Ocean City, Maryland. Among the four models, CCCL is the only one overpredicting average dune erosion, and the other three result in underprediction. The overprediction by the CCCL model is consistent with its calibration which incorporates natural longshore variability of dune erosion. The run-up used in EDUNE is constant over each of the storm simulation times and is determined for a particular storm by using the maximum significant wave height in Eq. (18) and calibrating to match maximum dune erosion elevation of the post-storm profiles. Considering profile residuals and errors of eroded volume and beach retreat together, CROSS, EDUNE and SBEACH (version 3.0) present reasonable predictions for both dune erosion and entire profiles.

Sensitivity studies demonstrate CROSS to be quite insensitive to the transport coefficient for beach erosion during the two storms at Ocean City. With a decrease in the ratio of active water depth to incoming wave height from 1.28 to 1, the predictions of CROSS are improved. Compared with the results of laboratory experiments, it appears the ratio of 1.28 provides better predictions for profiles under monochromatic waves and the ratio of 1.0 presents better results for a profile under irregular waves. A $\pm 20\%$ variation in storm surge caused changes of average profile residuals and dune erosion errors as predicted by CROSS of less than 10% to 15%. On average, the 20% increased storm surge provides better agreement for both profiles and dune erosion. In the CROSS model, the subaqueous part of a profile is quite sensitive to the wave heights and the subaerial part is affected less. As a result, the variations in eroded volume and beach retreat are generally less than 20% with a $\pm 20\%$ change of wave heights. However, the variations of the profile residual are much more significant and the changes of average residuals are higher than 30%.

To better represent a profile change between the set-up limit to the run-up limit, swash mechanisms should be represented. Under monochromatic wave conditions, an offshore bar usually forms near the break point. Seaward of the offshore bar, there is no significant observable profile change in experiments. Therefore, the active water depth is defined as the breaking water depth in CROSS which is 1.28 times the breaking wave height. However, in applications of CROSS it appears that the active water depth of 1.28 times wave height is too large for Ocean City storm erosion. Decreasing active water depth to the instantaneous wave height provides improved predictions. It is also seen in the experiment of ''dune without foreshore'' with random waves in the German large wave flume that the limiting depth of beach profile change is about the same as the incoming significant wave height. It appears that the mechanism of beach erosion under regular and irregular waves is different. More research should be conducted in future studies to evaluate the cause.

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