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Bottom friction effects in the combined flow field of random waves and currents

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ABSTRACT

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Bottom friction effects are quantified for the combined flow field comprising random waves propagating in the presence of currents. From the available methods of analysis for the monochromatic wave case a suitable theoretical model is first chosen using as criteria ease of implementation, rigorous derivation, and satisfactory description of the physical mechanism of the interaction. This model is then extended in order to deal with random waves and currents. Suitable formulae for the bed shear stress are first developed for the extreme cases when the wave field is much stronger than the current and vice versa, and then a derivation procedure for the general case is described, including a suitable parameterization which significantly reduces the required computational load. The parameterisation for the general case is compared with results from the expressions for the two extreme cases and very good agreement is obtained. Comparisons are also made with results using simple representations of a random wave field in terms of average quantities like the significant or root mean square waveheight and it is shown that such representations which are based on monochromatic wave ideas overpredict the corresponding coefficients. Finally results from the full parameterized solution for the general case of random waves propagating in the presence of currents are compared with two sets of full scale data and good agreement is obtained. It is shown that the calculation formulae developed and presented in this work offer reliable means of treating the complex interaction taking place at the sea bed between random waves and currents.

INTRODUCTION

Various models for the bottom friction in combined monochromatic wave and current flows are available. The modelling of the combined flow field closely follows the analysis used for the pure current case by adopting one of three general approaches, based either on eddy viscosity considerations, or ideas from mixing length theory or finally on dimensional analysis. In the first case, a linear variation of the eddy viscosity at the sea bed is used, although

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the proportionality coefficients vary from model to model and, depending on the flow regime, even within each particular model. Approaches based on mixing length ideas explicitly assume that the same mixing length may be used for the current only, wave only and the combined flow field while dimensional analysis based procedures directly apply the methodology used for the current only case to the combined flow field.

Several important characteristics of the combined flow field have long been recognized and are well documented. It is generally established that the turbulence inside the wave boundary layer is enhanced thus causing more drag to the current. However, the effect of this interaction on the waves is considered to be small owing to the high gradient of the wave orbital velocity near the sea bed. The increased drag felt by the current results in a change of the current velocity distribution, with the current velocity magnitude generally being reduced. Outside the wave boundary layer, the two flows can be solved for independently and linearly superimposed, given that the wave is considered as a potential flow. However, all currently available models assume that waves are monochromatic and unidirectional, and the correctness of this assumption needs to be seriously questioned.

This paper sets out to develop a bottom friction calculation procedure for the combined random wave and current flow field. A suitable model for the combination of monochromatic, regular waves and a current is chosen first, the requirements being that it be rigorous, relatively straightforward to implement, and show good agreement with experimental data. This model is then extended so as to take into account the effects of the randomness of the wave motion on the combined flow field. The cases of a weak or strong current relative to the waves are treated first, owing to the scope for simplification of the governing equations. Subsequently the general case is treated and parameterised so as to render the governing equations more amenable to numerical treatment. The parameterisation for the general case is shown to agree well with results from the expressions for the two extreme cases previously mentioned.

It is often the case that a target sea state with a given spectrum is approximately described by average quantities like the significant wave height H_s or the root-mean-square waveheight H_{rms}, and the period T_p corresponding to the spectral peak or the average zero-crossing period T_z. This approach is attractive owing to its simplicity but its main failing is that it is basically a monochromatic wave approach which ignores contributions from individual components of the spectrum. Results derived from the expressions developed in this work are compared with results based on monochromatic representations of the sea spectral density and it is shown that such approaches overestimate bottom friction effects.

Finally results from the formulae describing the bottom friction effects in the combined flow field when random waves propagate in the presence of currents are compared with available full scale data and good agreement is obtained. It is shown that the developed model is capable of describing bottom friction effects under a wide variety of conditions.

FUNDAMENTAL CONSIDERATIONS FOR THE MONOCHROMATIC WAVE CASE

It is not the aim of this paper to review the many available models of bottom friction effects for the combined flow field involving monochromatic regular waves propagating in the presence of a current. In this section a suitable theoretical vehicle is chosen for further treatment, with a view to incorporating into it the randomness of the wave motion.

The available bottom friction models for the combined regular wave and current case range from the very simple (Ebersole et al., 1980) to the more complex (Lundgren, 1972; Grant and Madsen, 1979; Fredsoe, 1984) which aim at providing more insight into the physics of the interaction at the expense of computational efficiency. For practical applications, the problem of obtaining acceptable accuracy with modest computational requirements remains an acute one. In this context the contribution of Bijker (1966) is an important one, because his model for the calculation of the mean bottom friction experienced by the current in the combined flow field is both easy to implement and reasonably rigorous with regard to derivation procedure. The fundamental assumption of Bijker's analysis is that the instantaneous velocity profile for any flow is logarithmic and that the thickness of the constant stress layer is known. The total velocity at every point over the vertical and at every time instant is taken equal to the sum of a reduced wave velocity and the current velocity. Moreover, the gradient of the velocity at a particular point, hereafter referred to as Bijker's point (Fig. 1), can be simply replaced by a simple ratio for any flow. The mean bottom stress can therefore be written as

$$\tau=\left\langle \rho l^2\left|\frac{\partial u}{\partial y}\right|u\frac{\partial u}{\partial y}\right\rangle \tag{1}$$

where u is the instantaneous velocity of the flow concerned.

An attempt was made by Swart (1974) to improve the evaluation of the velocity at Bijker's point by relating the component velocities at that point to the free stream wave velocity and the mean velocity of the current. However comparisons with experimental data produced no conclusive proof of the superiority of Swart's analysis.

In another attempt to improve upon Bijker's method, O'Connor and Yoo (1988) related the energy dissipation rate to the current velocity reduction, a trend present in the results of a number of experiments. In the authors' opinion, O'Connor's and Yoo's model is based on up to date understanding of the

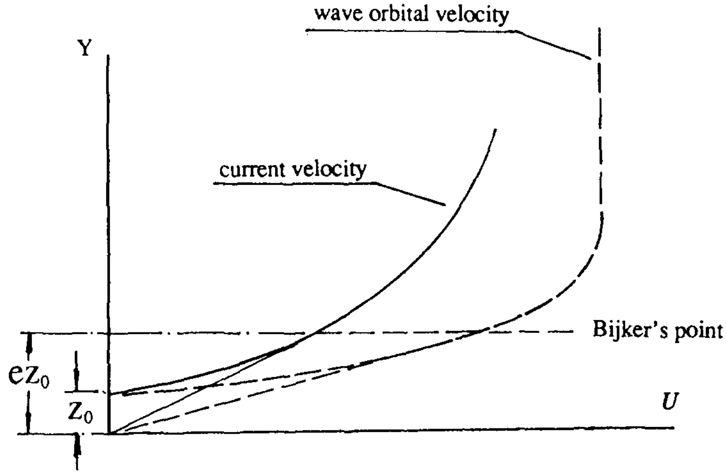

Fig. 1. Schematic diagram of wave and current velocity distributions considered by Bijker (1966).

physics of wave and current boundary layers, it shows good agreement with experimental data and, furthermore, it has the advantage of being relatively simple to implement. Therefore, this model is chosen as the basis of the analysis to be presented in the following sections. For more details on the features of O'Connor's and Yoo's model the reader is referred to the original paper (O'Connor and Yoo, 1988) and only the final equations are given here for the sake of completeness.

If Chezy's law is applied to the current and wave flows, respectively, then the following equations apply

$$\tau_c = \rho C_c U_c^2 \tag{2}$$

$$\tau_w = \rho C_w U_{wm}^2 \tag{3}$$

where τ_c and τ_w are the shear stresses corresponding to the current only and wave only flows, respectively, C_c and C_w are the bed friction coefficients, and U_c, U_{wm} are the corresponding depth mean current velocity and free stream maximum wave orbital velocity, respectively. The shear stress at Bijker's point has a component τ_p in line with the current, a component τ_n perpendicular to the current, and

$$\tau_p = \beta \tau_c \tag{4}$$

$$\tau_n = \gamma \tau_c \tag{5}$$

where β and γ are given below. The total velocity at Bijker's point is

$$u_t = (\alpha^2 C_c U_c^2 + C_w U_w^2 + 2\alpha C_c^{1/2} C_w^{1/2} U_c U_w \sin\theta)^{1/2} \tag{6}$$

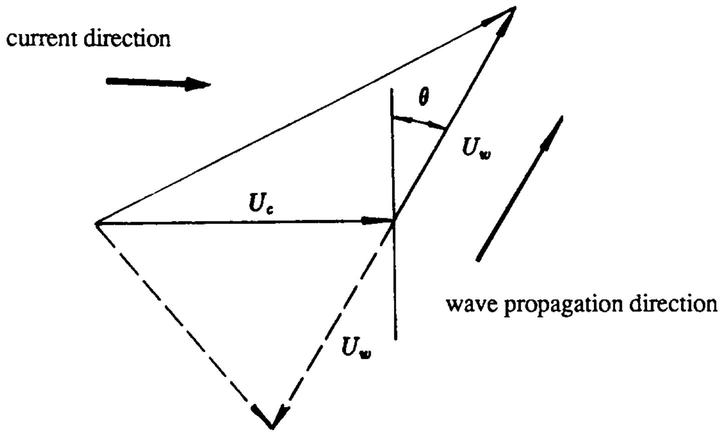

Fig. 2. Wave and current propagation directions and vector summation of velocities.

and the full expressions for all quantities are as follows, the directions of wave and current propagation are schematically shown in Fig. 2.

$$\beta=\alpha^2\langle (1+\mu_\alpha\sin\theta\sin\lambda)(1.0+\mu_\alpha^2\sin^2\lambda+2\mu_\alpha\sin\theta\sin\lambda)^{1/2}\rangle \tag{7}$$

$$\gamma=\alpha^2\langle \mu_\alpha\cos\theta\sin\lambda(1.0+\mu_\alpha^2\sin^2\lambda+2\mu_\alpha\sin\theta\sin\lambda)^{1/2}\rangle \tag{8}$$

$$\delta=\langle (|\sin\lambda(\sin\lambda+\mu_\alpha^{-1}\sin\theta)|(\mu_\alpha^{-2}+\sin\lambda+2\mu_\alpha^{-1}\sin\theta\sin\lambda)^{1/2}\rangle \tag{9}$$

$$\frac{1}{\alpha}=\left(\beta+DD\cdot\delta\frac{C_w U_{wm}^3}{C_c U_c^3}\right)^{1/3} \tag{10}$$

where

$$\mu_\alpha=\frac{1}{\alpha}\cdot\frac{C_w^{1/2}U_{wm}}{C_c^{1/2}U_c} \tag{11}$$

$$DD=\frac{0.6|\sin\theta|+0.2|\cos\theta|}{0.6} \tag{12}$$

where α is the ratio between the current velocity in the combined flow field and the current velocity for the current only flow at Bijker's point, θ is the angle between the wave propagation direction and the direction perpendicular to the current, δ is the energy dissipation ratio of the combined flow to the current only flow, and λ is the phase of the wave.

As it has already been mentioned the model results agree fairly well with the available experimental data. However, it should be noted that the best agreement is obtained when μ_α is approximately equal to unity, which implies that the current and the waves have approximately the same strength.

BOTTOM FRICTION EFFECTS IN COMBINED RANDOM WAVE AND CURRENT FLOWS

In order to evaluate the effects of the randomness of the wave motion, the analysis briefly discussed in the previous section is carried out in the probability domain. The fundamental difference between the treatments afforded to the monochromatic and random wave cases is that, instead of taking time averages over one wave period in order to evaluate the various expressions, statistical expectations are used. If, for example, y is a function of the variable x, $y=f(x)$, and the p.d.f. (probability density function) of x is $p(x)$, then the expected value of y, $E(y)$ is

$$E(y)=\int f(x)p(x)\mathrm{d}x \tag{13}$$

In order for the formulae presented in the previous section to be applicable to random waves propagating in the presence of currents, each individual formula must be re-evaluated with the time average being replaced by an expected value.

In the following derivations the linear random model for the sea is used, according to which the sea state is the result of the linear summation of a large number of first order sinusoidal components, and the free surface displacement, water particle kinematics and the dynamic pressure are all Gaussian random variables. This model is especially suitable for describing the flow field in the vicinity of the sea bed not only because of the fact that for that region no regular wave theory has been proven superior to others under all test conditions(see, e.g., Kirkgoz, 1986), but also because the effects of nonlinear terms are small near the sea bed. The variation of the free surface displacement is therefore given by

$$\eta=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{ij}\cos(k_jx\cos\theta_i+k_jy\sin\theta_i-\omega_jt+\epsilon_{ij}) \tag{14}$$

where $a_{ij}=(2.0\cdot S(\omega,\theta)\Delta\omega\Delta\theta)^{1/2}$, k_j is the wave number of the jth spectral component, θ_i is the wave directional angle, ϵ_{ij} is a random phase uniformly distributed over the interval $[0,2\pi]$, and $S(\omega,\theta)$ is the spectral density function of η.

If it also assumed that the spectral density function of the free surface displacement is narrow-banded, then the wave amplitude is distributed according to the Rayleigh distribution function

$$p(a)=\frac{a}{m_0}\exp\left(-\frac{a^2}{2m_0}\right) \tag{15}$$

where

$$m_i = \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \omega^i S(\omega,\theta) \mathrm{d}\omega \mathrm{d}\theta$$

The standard deviations of the orbital displacement, σ, and the horizontal velocity, σ_u, at sea bottom which will be used later on are given by

$$\sigma^2 = \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \frac{1}{\sinh^2(kh)} S(\omega,\theta) \mathrm{d}\omega \mathrm{d}\theta \tag{16}$$

$$\sigma_u^2 = \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \frac{\omega^2}{\sinh^2(kh)} S(\omega,\theta) \mathrm{d}\omega \mathrm{d}\theta \tag{17}$$

Having established the statistical and spectral properties necessary for the subsequent derivations, we now proceed to evaluate the bottom friction effects for the combined flow field when random waves propagate in the presence of currents. In this context it should be noted that the bottom friction equations cannot be derived analytically since the whole analysis is based on a quadratic law for the shear stress. However, when the current is either weak or strong relative to the waves, the derivations are relatively straightforward. These two cases are examined first, followed by the general case for which a parametric approach will be adopted.

Wave dominant case

For the wave dominant case u_t is given by Eq. (6), and this quantity can be further approximated by the following equation which is similar to that derived by Liu and Darlymple (1978) for regular waves

$$u_t^2 = \left(C_w^{1/2} |U_w| + \frac{\alpha C_c^{1/2} U_w \sin(\theta+\alpha_1)}{|U_w|} \right)^2 + \alpha^2 C_c U_c^2 \cos^2(\theta+\alpha_1) \tag{18}$$

The last term is small for the case when the wave field is dominant and it is, therefore, neglected. Thus

$$u_t = C_w^{1/2} |U_w| + \alpha \frac{C_c^{1/2} U_c U_w \sin(\theta+\alpha_1)}{|U_w|} \tag{19}$$

where α_1 expresses the wave directional spreading from a principal direction of wave propagation.

The formula for β (Eq. 7) becomes

$$\beta C_c U_c^2 = (\alpha C_c^{1/2} U_c + C_w^{1/2} U_w \sin(\theta+\alpha_1)) u_t \tag{20}$$

Taking the simplified expression for u_t into account, the above formula for β can be written as

$$\beta C_c U_c^2 = E\Bigg[\alpha C_c^{1/2} C_w^{1/2} U_c |U_w| + C_w U_w |U_w| \sin(\theta+\alpha_1) + \frac{\alpha^2 C_c U_c^2 U_w \sin(\theta+\alpha_1)}{|U_w|} + \frac{\alpha C_w^{1/2} C_c^{1/2} U_c U_w^2 \sin^2(\theta+\alpha_1)}{|U_w|}\Bigg] \quad (21)$$

The expected values of the second and third terms are zero due to the linear model used to describe all random variables of interest. Therefore

$$\beta = \alpha C_c^{-1/2} E[C_w^{1/2} |U_w|] (1 + E[\sin^2(\theta+\alpha_1)]) \frac{1}{U_c} \quad (22)$$

and the formulae for γ and δ become

$$\gamma = \frac{\alpha E[C_w^{1/2} |U_w|]}{C_c^{1/2} U_c} E[\sin(\theta+\alpha_1)\cos(\theta+\alpha_1)] \quad (23)$$

$$\delta = \frac{E[C_w U_w^2 |U_w|] E[K_1] + \alpha^2 E[|U_w|] C_c U_c^2 E[K_2]}{C_c U_c^3} \quad (24)$$

where

$$K_1 = |\sin(\theta+\alpha_1)| + \frac{1}{3} |\cos(\theta+\alpha_1)| \quad (25)$$

$$K_2 = \sin^2(\theta+\alpha_1)\left(|\sin(\theta+\alpha_1)| + \frac{1}{3} |\sin(\theta+\alpha_1)| \right) \quad (26)$$

Further manipulation of Eqs. (22)–(26) yields the following formulae which may be used for practical calculation purposes

$$\frac{1}{\alpha} = (\beta+\delta)^{1/3} \quad (27)$$

$$\beta = \left(\frac{2}{\pi}\right)^{1/2} \frac{\alpha \sigma_u}{C_c^{1/2} U_c} E[C_w^{1/2}] (1 + K_{s2}) \quad (28)$$

$$\gamma = \frac{1}{2}\left(\frac{2}{\pi}\right)^{1/2} \frac{\alpha \sigma_u}{C_c^{1/2} U_c} E[\sin 2(\theta+\alpha_1)] E[C_w^{1/2}] \quad (29)$$

$$\delta = \left(\frac{2}{\pi}\right)^{1/2} \frac{\sigma_u}{C_c U_c^3} \left[2E[C_w[\sigma_u^2\left(K_{s1} + \frac{1}{3}K_{c1}\right) + \alpha^2 C_c U_c^2 \left(K_{s3} + \frac{1}{3}K_{c3}\right)\right] \quad (30)$$

where

$$K_{si} = E[\,|\sin^{i}(\theta+\alpha_1)|\,] \tag{31}$$

$$K_{ci} = E[\,|\sin^{i-1}(\theta+\alpha_1)\cos(\theta+\alpha_1)|\,] \tag{32}$$

where $i = 1,2,3$. Appropriate expressions for $E[C_w]$ and $E[C_w^{1/2}]$ are given in the following section dealing with the current dominant case.

It should be noted that, if we consider a long random wave record derived from a given target spectrum of free surface displacement, it is reasonable to argue that not every wave from the record, irrespective of how it is defined, represents a flow field which is "strong" compared with that of the current. The wave height and orbital velocity are distributed over the range $(0.0, +\infty)$ and $(-\infty, +\infty)$, respectively. It is, therefore, clear that some individual waves are not necessarily strong, which is the basis of the analysis for the present case. However, the "strong" case can be understood as implying that for most of the individual waves from a record, the flow conditions can be considered strong relative to the current and, therefore, the part of the total record which is neglected will be small compared with the overall length of the record. Moreover, it should be noted that this assumption also holds true for monochromatic regular waves, because the wave orbital velocity varies sinusoidally and its absolute value is not always larger than the current velocity, except in the case where the velocity of the current is zero.

It should also be noted that wave propagation direction and wave amplitude are assumed to be independent of each other.

Current dominant case

In a similar way as the case of a weak current has been tackled, the quadratic function used in the calculation of the total velocity may be simplified by a perturbation analysis, whereby the quantities β, D_w and δ may be evaluated, via a Taylor series expansion, to second or higher orders. Using such an expansion the quantity u_t can be written as

$$u_t = U(\alpha^2 + 2\alpha D\sin\theta + D^2)^{1/2}$$
$$= U\left(\alpha + \left.\frac{\partial u_t}{\partial D}\right|_{D=0} D + \frac{1}{2}\left.\frac{\partial u_t^2}{\partial D^2}\right|_{D=0} D^2 + \dots\right) \tag{33}$$

assuming, of course, that $D = u_w/U_c \ll 1$. To the specified order

$$u_t = \alpha C_c^{1/2} U_c + C_w^{1/2} U_w \sin(\theta+\alpha_1)$$
$$+ \frac{1}{2}\alpha^{-1} C_c^{-1/2} U_c^{-1} C_w U_w^2 \cos^2(\theta+\alpha_1) \tag{34}$$

which leads to the following expressions for β, γ and δ

$$\beta = \alpha^2 + \frac{1}{2}\frac{E[C_w]E[U_w^2]}{C_c U_c^2} + \frac{1}{2}\frac{E[C_w]E[U_w^2]E[\sin^2(\theta+\alpha_1)]}{C_c U_c^2} \tag{35}$$

$$\gamma = \frac{E[C_w]E[U_w^2]E[\sin(\theta+\alpha_1)\cos(\theta+\alpha_1)]}{C_c U_c^2} \tag{36}$$

$$\delta = \frac{E[\,|D^*\sin(\theta+\alpha_1)|\,] + \frac{1}{3}E[\,|D^*\cos(\theta+\alpha_1)|\,]}{C_c U_c^3} \tag{37}$$

where

$$D^* = |U_w|\alpha^2 C_c U_c^2 \sin\theta \tag{38}$$

The expected values of all quantities involve the Chezy coefficient C_w, which is a function of the wave properties. This coefficient, which is related to a wide range of parameters characterising the flow, will be parameterized based on the following considerations.

– C_w itself is a period average parameter and it is physically related to the turbulence characteristics, which are not constant over one wave period. In regular waves, a constant C_w is taken to represent a whole period during which the velocity varies sinusoidally. Therefore, a constant C_w may also be used for the random wave case.

– C_w is relatively insensitive to the quantity a/k_s, where a is the wave amplitude, and k_s is the bed roughness.

Using the same expression as O'Connor and Yoo (1988) for C_w, an expression for the coefficient $C_w^{1/2}$ may be derived as shown below

$$C_w = 0.12 \text{ for } a/k_s \leq 2.0 \tag{39}$$

$$C_w = \exp[5.213(a/k_s)^{-0.194} - 6.67] \text{ for } a/k_s > 2.0 \tag{40}$$

Therefore,

$$E[C_w^{1/2}] = \int_0^{+\infty} C_w^{1/2} p(a)\mathrm{d}a$$

$$= \int_0^{2k_s} (0.12)^{1/2} p(a)\mathrm{d}a$$

$$+ \int_{2k_s}^{+\infty} [\exp(5.213(a/k_s)^{-0.196} - 6.67)]^{1/2} p(a)\mathrm{d}a \tag{41}$$

where $p(a)$ is the p.d.f. of the wave amplitude.

After carrying out the numerical integration, the above expression can be further simplified as

$$\log_{10}(C_w^{1/2}) = -0.485 - 0.234\log_{10}(\sigma/k_s) \tag{42}$$

If an expression for C_w is sought, rather than $C_w^{1/2}$, it can easily be shown that

$$\log_{10}(C_w) = -0.963 - 0.467\log_{10}(\sigma/k_s) \tag{43}$$

Summarising the results for the case when the current is dominant, the following formulae may be used for practical calculation purposes

$$\frac{1}{\alpha} = (\beta+\delta)^{1/3} \tag{44}$$

$$\beta = \alpha^2 + \frac{1}{2}\frac{E[C_w]\sigma_u^2}{C_c U_c^2}(1+K_{s2}) \tag{45}$$

$$\gamma = \frac{1}{2}\frac{E[C_w]\sigma_u^2}{C_c U_c^2}E[\sin 2(\theta+\alpha_1)] \tag{46}$$

$$\delta = \left(\frac{2}{\pi}\right)^{1/2}\frac{\alpha^2\sigma_u}{U_c}\left(K_{s2} + \frac{1}{3}K_{c2}\right) \tag{47}$$

where K_{si} and K_{ci} are as given by Eqs. (31) and (32).

The derivation for the general case

For the general case, when the strengths of the waves and the current are comparable, the expressions for the quantities of interest involve triple integrals, which makes the evaluation rather tedious. Therefore some parameterizations are introduced with a view to deducing expressions for β and δ which are simpler to evaluate.

The general formulae for β and δ are as follows

$$\beta = \int\int\int \alpha\left[1.0 + \frac{C_w^{1/2} U_w \sin(\theta+\alpha_1)}{\alpha C_c^{1/2} U_c}\right] u_t f(a,u,\alpha_1)\, d\alpha_1\, du\, da \tag{48}$$

$$\delta = \int\int\int \frac{D_w^* |\sin(\theta+\alpha_1)| + 0.333 D_w^* |\cos(\theta+\alpha_1)|}{C_c U_c^3} u_t f(a,u,\alpha_1)\, d\alpha_1\, du\, da \tag{49}$$

where

$$u_t = [\alpha^2 C_c Uc^2 + C_w U_w^2 + 2\alpha C_c^{1/2} U_c C_w^{1/2} U_w^2 \sin(\theta+\alpha_1)]^{1/2} \tag{50}$$

$$D^* = |U_w(C_w^{1/2} U_w + C_c^{1/2} U_c \sin(\theta+\alpha_1))|\, u_t \tag{51}$$

$$f(a,u,\alpha_1)=p(\alpha_1)\frac{a}{\pi\sigma_u^2}\exp\left(-\frac{a^2}{2\sigma_u^2}\right)\cdot\frac{1}{(Ka^2-u^2)^{1/2}} \tag{52}$$

The integration can be carried out numerically. In the first step the integration is carried out with respect to u using the results of Yoo (1989) who considered the regular monochromatic wave case only. The resulting double integral may then be evaluated using any one of a number of available numerical integration methods. In the present work Simpson's integration method is used, to a pre-specified degree of accuracy, and then the numerical results are regressed using the Chi-square regression method. The final expressions are

$$\frac{1}{\alpha}=(\beta+\delta)^{1/3} \tag{53}$$

$$\beta=\alpha^2(D_{\beta1}+D_{\beta2}B) \tag{54}$$

$$\gamma=\alpha^2(D_{\gamma1}+D\gamma_2B_1) \tag{55}$$

$$\delta=\frac{\alpha^3C_cU_c^{1/2}}{E[C_w^{1/2}]}(D_{\delta1}+D_{\delta2}B) \tag{56}$$

where

$$D_{\beta1}=0.85+0.83\sigma_1^{1.147} \tag{57}$$

$$D_{\beta2}=-0.364\sigma_1^{1.051} \tag{58}$$

$$D_{\delta1}=0.0543\sigma_1+0.029\sigma_1^2+1.594\sigma_1^3 \tag{59}$$

$$D_{\delta2}=-0.255\sigma_1-0.046\sigma_1^2 \tag{60}$$

$$D_{\gamma1}=0.393\sigma_1^{1.02} \tag{61}$$

$$D_{\gamma1}=-0.393\sigma_1^{1.02} \tag{62}$$

$$B=\int\cos2(\theta+\alpha_1)p(\alpha_1)\mathrm{d}\alpha_1 \tag{63}$$

$$B_1=\int|\cos2(\theta+\alpha_1)|p(\alpha_1)\mathrm{d}\alpha_1 \tag{64}$$

$$\sigma_1=\frac{C_w^{1/2}\sigma_u}{\alpha C_c^{1/2}U_c} \tag{65}$$

In order to verify whether the above formulae, corresponding to the general case where the strengths of the wave and current fields are comparable, have been satisfactorily parameterised, the repsentations for β_1 and δ_1 are compared with the simpler expressions for the two extreme cases using, of course, the appropriate ranges of the independent variable. The comparisons are made for the situation when the waves propagate in the same direction as the current with B being equal to -1 in this case.

From the weak current model

$$\beta_1 = \frac{\beta}{\alpha^2} = \left(\frac{8}{\pi}\right)^{1/2} \sigma_1 \qquad (66)$$

$$\delta_1 = \frac{\delta C_w^{1/2}}{\alpha^3 C_c^{1/2}} = \left(\frac{2}{\pi}\right)^{1/2} (2\sigma_1^3 + \sigma_1) \qquad (67)$$

where σ_1 was given in Eq. (65).

The above formulae for β_1 and δ_1 are equivalent to Eqs. (28) and (30) for the case of waves propagating in the same direction as the current. For the strong current case the corresponding expressions are

$$\beta_1 = 1 + \sigma_1^2 \qquad (68)$$

$$\delta_1 = \left(\frac{2}{\pi}\right)^{1/2} \sigma_1 \qquad (69)$$

while for the general case

$$\beta_1 = D_{\beta 1} - D_{\beta 2} \qquad (70)$$

$$\delta_1 = D_{\delta 1} - D_{\delta 2} \qquad (71)$$

The quantities β_1 and δ_1 are plotted in Fig. 3 and Fig. 4, respectively. It can be seen that the representations of the two terms for the general case are rather good at the extremes of the range, when compared with the formulae applicable to the two extreme situations. The agreement is especially good for β_1.

Suitability of simple representations of the random wave field

It is a common practice in coastal enginnering to deploy average quantities, characterising the entire random wave field, and then incorporate these quantities in formulae which are strictly applicable to the regular and monochromatic wave case only, the aim being to make the calculation procedure simpler. Popular choices are H_s, the significant wave height, and H_{rms}, the root mean square of the wave height. In this section the weak current model is used to investigate the effect of using H_s or H_{rms} rather than the formulae which were derived in the previous sections based on a given waveheight distribution. It is assumed that the waves propagate in the same direction with the current and, furthermore, that the values of the current reduction factor α appearing in Eqs. (72) and (74) are equal. The expression for u_t as appropriate to the regular wave case may be therefore simplified as previously shown. When the full random wave spectrum is taken into account

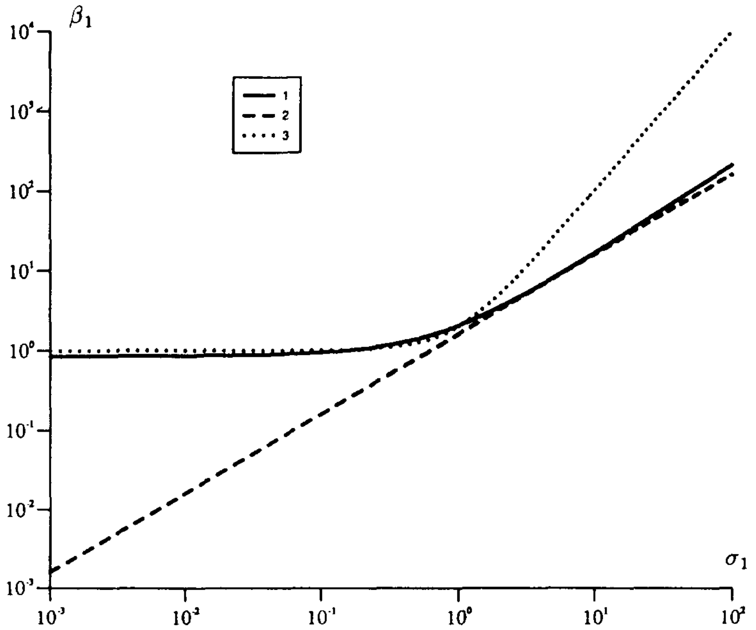

Fig. 3. Evaluation of β for (1) the general case (Eq. 70); (2) the wave dominant case (Eq. 66); (3) the current dominant case (Eq. 68).

$$\beta = \alpha^2 \left(\frac{8}{\pi}\right)^{1/2} \sigma_1 \tag{72}$$

$$\delta = \frac{\alpha^3 C_c^{1/2}}{C_w^{1/2}} \left(\frac{2}{\pi}\right)^{1/2} (2\sigma_1^3 + \sigma_1) \tag{73}$$

while for a regular wave

$$\beta = \left\langle \left(\alpha + \frac{C_w^{1/2} U_w}{C_c^{1/2} U_c}\right) \left[\frac{C_w^{1/2} |U_w| + \dfrac{\alpha C_c^{1/2} U_c U_w}{|U_w|}}{C_c^{1/2} U_c}\right] \right\rangle = \frac{4}{\pi} \cdot \frac{\alpha C_w^{1/2} U_{wm}}{C_c^{1/2} U_c} \tag{74}$$

$$\delta = \frac{2}{\pi} \frac{\alpha^2 U_{wm}}{U_c} + \frac{7}{6\pi} \cdot \frac{C_w U_{wm}^3}{C_c U_c^3} \tag{75}$$

The quantity U_{wm} above can be calculated using the significant height H_s in the appropriate formula from linear wave theory for regular, monochromatic waves. In this case

$$U_{sig} = 2.0\sigma_u \tag{76}$$

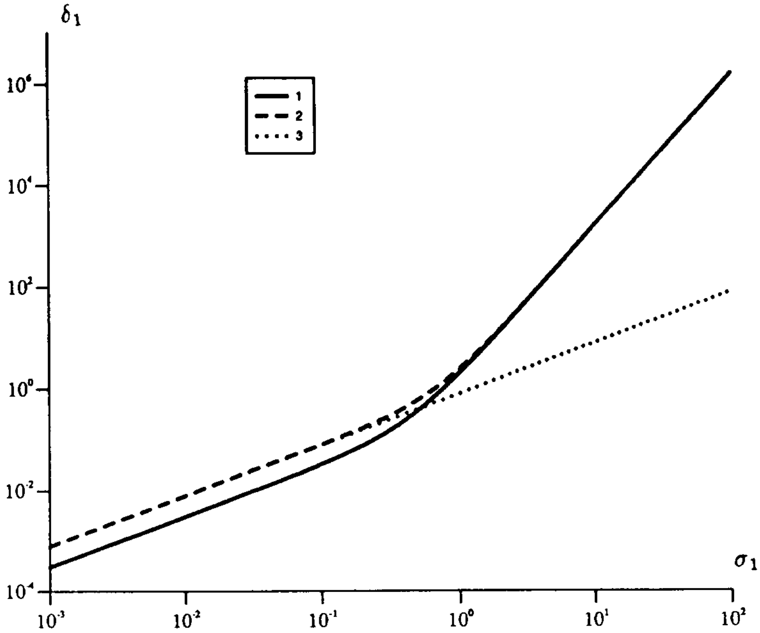

Fig. 4. Evaluation of δ for (1) the general case (Eq. 71); (2) the wave dominant case (Eq. 67); (3) the current dominant case (Eq. 69).

$$\beta_{sig}=\alpha^2\frac{8}{\pi}\sigma_1 \tag{77}$$

$$\frac{\beta_{sig}}{\beta}=1.60 \tag{78}$$

If the root mean square height H_{rms} is used

$$U_{rms}=\sqrt{2.0}\sigma_u \tag{79}$$

$$\beta_{rms}=\alpha^2\frac{4\sqrt{2}}{\pi}\sigma_1 \tag{80}$$

$$\frac{\beta_{rms}}{\beta}=1.13 \tag{81}$$

As a direct conclusion from this simple test it can, therefore, be said that both β_{sig} and β_{rms} overestimate the actual values. However, since the same value for α was used in all three cases, it cannot be said that the above test offers conclusive evidence as to the suitability of the chosen simple representations of the random wave field. Further tests are made in order to establish the range of validity of the simple representations as given above using, again, the stong wave case formulae. In the calculations carried out in order to produce the results given below, the velocity reduction coefficients α have been calculated from the respective formulae.– Using the full formulae for random waves, with $T_z=5.0$ s, $\sigma_u=0.1$ m/s, $U_c=0.1$ m/s, $k_s=0.1$ m and $D=2.0$ m, then $\beta=2.26$.– Using H_s in Eqs. (74) and (75), with $T=5.0$ s, $U_{sig}=0.2$ m/s, $U_c=0.1$ m/s, $k_s=0.1$ m and $D=2.0$ m, then $\beta=2.94$.– Using H_{rms} in the same equations with $T=5.0$ s, $U_{rms}=0.14$ m/s, $U_c=0.1$ m/s, $k_s=0.1$ m and $D=2.0$ m, then $\beta=2.88$.

In the above k_s is the bottom roughness, D the water depth, and the known ratio $H_s/H_{rms}=\sqrt{2}$ has also been used for the quantities U_{sig} and U_{rms}.

Owing to the fact that the velocity reduction coefficients α for each of the three calculations do not have the same value, the values of the ratios of β are not the same as those derived from the previous. It is still clear, however, that the approximate representations of wave height tend to over-estimate the bottom friction coefficients.

The quantities B and B_1 which appear in the general calculation formulae may be calculated from the directional spreading function of the spectrum of variance of free surface displacement. Due to our lack of knowledge on the p.d.f. of wave propagation direction and in order to take this spreading into full account numerical integration is required over the directional range. However, for some simple directional spreading functions, evaluation of the integrals may be achieved analytically. For example, using the directional spreading function proposed by Arthur (1949)

$$G(\alpha)=\begin{cases} 2/\pi\cos^2\alpha & |\alpha|<\pi/2 \\ 0 & |\alpha|>\pi/2 \end{cases} \tag{82}$$

yields

TABLE 1

Effect of directional spreading on β

angle (°)	no spreading ($B=\cos 2\theta$)	including spreading ($B=0.5\cos 2\theta$)
90	2.90	2.56
80	2.86	2.53
70	2.74	2.48
60	2.56	2.38
50	2.33	2.27
40	2.08	2.15
30	1.86	2.03
20	1.67	1.94
10	1.55	1.88
0	1.50	1.86

$B = 0.5\cos 2\theta$

Taking $D = 1.0$ m, $U_c = 0.1$ m/s, $k_s = 0.05$ m, $\sigma_u = 0.09$ m/s and $T_z = 5.0$ s, the results given in Table 1 are obtained.

As expected, directional spreading reduces β when the main direction of wave propagation is aligned with the current, while it increases β when the main direction of wave propagation is perpendicular to the current direction.

COMPARISONS WITH FIELD DATA

It is only comparatively recently that systematic experiments on the turbulence characteristics of the waves (Sleath, 1987) and the combined effect of waves and current in the bottom boundary layer have been carried out (Bakker and Van Doorn, 1978; Kemp and Simons, 1982). However, the combined properties of random waves and currents for all combinations of the governing parameters are still not available. This state of affairs is partly due to the time scales involved for averaging purposes which are difficult to define.

Field data are chosen for verification purposes. It should be noted, however, that the presented comparisons cannot be claimed to offer conclusive proof of the suitability or otherwise of the formulae derived herein since the full scale conditions are so complex that additional mechanisms may be involved which are not accounted for in the theoretical model.

There are two sets of full scale data available, and it should be made clear that the wave spectrum information for either set is incomplete. The experimental data are compared with results from the full parameterized model using Eqs. (53)–(56).

Case 1. The data of Cacchine and Drake (1982)

Records over eighty days were obtained and data were sampled at one sample per second for 60 s in each hour. The bottom roughness was estimated by Cacchine and Drake as $k_s = 0.3$ m and the mean depth was taken as 18 m. Although the value of k_s is rather large, Cacchine and Drake suggested that flow stratification might be responsible for such a high value and in this context the present authors have elected not to use a different value for this parameter. Water particle velocities were only measured near the bottom (0.2, 0.5, 0.7 and 1.0 m above the bottom, denoted by S_{20}, S_{50}, S_{70} and S_{100}, respectively, in Table 2), and the depth averaged current velocity was approximately 0.3 m/s. The main parameters describing the experimental data are summarised in Table 2.

For comparison purposes linear wave theory is first used to calculate the wave orbital velocity at the sea bed according to the given wave height. This

TABLE 2

The experiment results of Cacchine and Drake (1982)

Parameter	S_{100} (m/s)	S_{70} (m/s)	S_{50} (m/s)	S_{20} (m/s)	$(S_w)_{max}$ (m/s)	T (s)	H (m)	U^* (m/s)
Mean	0.289	0.272	0.246	0.136	0.269	8.9	1.3	0.04

TABLE 3

Input parameters for the random wave model

Parameter	U_c (m/s)	D (m)	k_s (m)	$U_{w,max}$ (average) (m/s)
Value	0.3	18.0	0.3	0.269

results in a maximum value for the wave orbital velocity at the bottom of 0.332 m/s, which is larger than the measured value of 0.3 m/s. Since it is considered that the measured value at 1m above the sea floor is more reliable, this value has been used instead in the theoretical formulae presented herein. The wave propagation angle is chosen to coincide with that of the mean current. The input values for the calculation are given in Table 3.

The calculated result yields $\beta=2.031$ while from the experimetal data $\beta=2.356$. If Grant and Madsen's (1979) model is used, which assumes that the waves are regular, then $\beta=1.825$. It is clear that there is close agreement between the results of the present theoretical model and the experimental data.

Case 2. The data of Grant et al. (1984)

The second set of full scale data used for verification purposes was obtained by Grant et al. (1984). The depth of water was 90 m, the roughness, as estimated by the authors, was 0.002 m, and water particle velocities were measured at approximately 0.3, 0.55, 1.05 and 2.05 m above the sea bottom. It should be noted that this particular data set is incomplete for the purpose of comparison with the present model, owing to the fact that no information is available on the wave height probability distribution and the depth mean current velocity. Bearing this in mind, values appropriate to the data collection site are chosen for quantities which were not measured. Furthermore, the original data were given by Grant et al. in a form suitable for use in their model, and in this work some values are modified to correspond to the definitions used herein. Although some arguments against the particular value of k_s used by Grant et al. have been advanced (Huntley, 1985) and an ad hoc manner may be used to adjust this value, it was decided not to alter the sug-

TABLE 4

Comparison between the models and the experimental results of Grant et al. (1984)

No.	U_c (10^{-2} m/s)	σ_u (10^{-2} m/s)	θ (°)	β	β_{meas}	β_{GM}
1	9.82	2.60	60	1.51	2.21	1.66
2	10.3	2.92	60	1.46	1.12	1.65
3	7.94	2.72	60	1.53	2.62	1.90
4	7.27	3.21	60	1.58	1.54	1.93
5	6.72	2.36	60	1.61	1.59	1.74
6	6.50	2.75	20	1.12	1.83	1.81
7	6.64	2.46	20	1.13	1.37	1.73
8	6.52	2.29	20	1.12	2.58	1.75
9	6.84	2.04	20	1.13	2.37	1.60
10	6.74	3.54	20	1.08	1.66	1.88
11	7.07	2.74	20	1.09	1.39	1.76
12	9.31	2.36	20	1.13	2.33	1.55
13	7.09	3.11	60	1.60	1.74	1.90
14	4.13	2.71	60	2.17	1.82	2.06
15	4.52	3.09	60	2.05	2.09	2.19

gested k_s value. In the table below measured and predicted values for β are shown, for a water depth of 90 m and a k_s value of 0.06m, which is equal to the value 0.002 multiplied by 30 as defined by Bijker (1966). The data in the column headed β are derived from the present model, and those headed β_{meas} and β_{GM} correspond to the measured values and those calculated by Grant et al. (1984) using Grant and Madsen's model (1979), respectively (see Table 4).

Inspection of the presented results produces no conclusive evidence as to the superiority of the calculation method proposed in this work. However, an interesting trend shows up in the results, whereby for the cases where the angle between the waves and the current was large, predictions by the present model are lower than measured values. It is suggested that the reason for this trend relates to the fact that during the field data collection program the total turbulence induced stresses were measured while the calculated results using the method proposed in this work relate not to the total stress but to the stress felt by the current.

CONCLUSIONS

A general method has been presented for the evaluation of bottom friction effects in the combined flow field of random waves propagating in the presence of currents, the derivation being based on the regular wave model of O'Connor and Yoo (1988). The derived formulae for the sea bed friction effects include information about the distribution of variance of free surface displacement with frequency, as well as about the spreading of variance with direction.

Appropriate formulae were first derived for the two simple cases when the current is either strong or weak relative to the wave field. Suitable formulae were also derived for the general case when no simplifying assumptions can be made. For the general case the relevant expressions are quite complex but it has been shown that they may be parameterised, which produces much simpler forms. It has also been shown that the parameterised formulae for the general case agree well with the two sets of expressions for the two extreme cases using the appropriate range of values for the independent variables.

For the regular wave case it is well established that the combined wave and current flow field causes an increase of the shear felt by the current, which might be considerable. A similar trend has been verified for the random wave case. However, taking into account the randomness and directionality of wave motion produces a smoothing effect and in this context a representative regular wave based approach is likely to over-estimate the bottom friction, which would logically lead to underestimating the nearshore wave-induced current velocities. Two simple representations of a random wave field in terms of H_s and H_{rms} have been tested in this work, and it has indeed been verified that they tend to overestimate the bottom friction effects.

Finally comparisons were made between results from the formulae derived for the purposes of this work and two sets of full scale data, and it has been shown that the present method offers a good description of measured mean bottom friction characteristics in combined wave and current flow fields. However, owing to the uncertainties associated with the choice of appropriate values for parameters related to the full scale measurements, it cannot be claimed that the presented comparisons are firmly conclusive as to the superiority of the method described in this work.

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