# Modeling of the Turbulence in the Water Column under Breaking Wind Waves

## HONG ZHANG\* and ENG-SOON CHAN

Tropical Marine Science Institute, National University of Singapore, Singapore 119223

(Received 12 November 2001; in revised form 2 December 2002; accepted 3 December 2002)

Past studies have shown that there is a wave-enhanced, near-surface mixed-layer in which the dissipation rate is greater than that derived from the "law of the wall". In this study, turbulence in water columns under wind breaking waves is investigated numerically and analytically. Improved estimations of dissipation rate are parameterized as surface source of turbulent kinetic energy (TKE) for a more accurate modelling of vertical profile of velocity and TKE in the water column. The simulation results have been compared with the experimental results obtained by Cheung and Street (1988) and Kitaigorodskii et al. (1983), with good agreement. The results show that the numerical full model can well simulate the near-surface wave-enhanced layer and suggest that the vertical diffusive coefficients are highly empirical and related to the TKE diffusion, the shear production and the dissipation. Analytical solutions of TKE are also derived for near surface layer and in deep water respectively. Near the surface layer, the dissipation rate is assumed to be balanced by the TKE diffusion to obtain the analytical solution; however, the balance between the dissipation and the shear production is applied at the deep layer. The analytical results in various layers are compared with that of the full numerical model, which confirms that the wave-enhanced layer near the surface is a diffusion-dominated region. The influence of the wave energy factor is also examined, which increases the surface TKE flux with the wave development. Under this region, the water behavior transits to satisfy the classic law of the wall. Below the transition depth, the shear production dominantly balances the dissipation.

## 1. Introduction

The surface layer of the ocean is a mixed-layer controlled mainly by the thermal and momentum fluxes. A better understanding of the turbulence under breaking waves is essential in determining the mechanisms of heat and momentum exchanges between atmosphere and ocean. However, it is typically difficult to measure the velocity and dissipation rates of the turbulence near the ocean surface, especially when there are extreme waveinduced motions. The absence of stable observation platforms and the complexity of extracting the essential components describing mixing also make measurements difficult.

In the past a great deal of effort has been directed towards determining the near-surface vertical distribution of the velocity. By tracking drifters and drogues, using acoustic travel time and compass sighting techniques, it has been found that velocity profiles were logarithmic at depths up to the order of 1 m below the surface water (Churchill and Csanady, 1983). In laboratory studies, it has been noted that waves affected the mean flow, and velocity profiles remained essentially logarithmic with depth (Cheung and Street, 1988). Using linear statistical techniques, the root-mean-square (rms) turbulent velocities obtained through field measurement exhibit a strong dependence on the wave energy (Kitaigorodskii *et al.*, 1983). They found that a dissipation rate in the upper layer is two orders of magnitude larger than the expected value for a constant stress layer, and this intense turbulence is generated by waves.

Keywords: • Turbulent kinetic

energy,

layer.

· dissipation,

· wave breaking,

· wave-enhanced

· surface roughness.

When there are breaking waves, wave-turbulence interaction could be responsible for the enhanced nearsurface dissipation of the turbulent kinetic energy (TKE) (Anis and Moum, 1995). The dissipation rate is closely related to wind stress production and exhibits an exponential decay with depth. Derivation of vertical diffusive

<sup>\*</sup> Corresponding author. E-mail: tmszh@nus.edu.sg

Copyright © The Oceanographic Society of Japan.

coefficients through acoustic measurements of bubbles has been attempted using an upward-directed, high-frequency sonar (Thorpe, 1984). However, the data were too scattered and only the trend of vertical diffusion coefficient of TKE near the surface could be derived. The experiment can only be used to demonstrate the technique. The observation data were also inadequate to yield the diffusion coefficient as a function of water depth and wind speed. To date, empirical parameters are still needed to predict the diffusion due to turbulent mixing.

In the earlier studies, the surface layer dissipation rates were derived based on the classical law of the wall. In measurements, however, much higher dissipation rates were found near the ocean surface (Kitaigorodskii et al., 1983). The upper layer, up to a depth of  $10\zeta_{rms}$  ( $z_{rms}$  is the rms wave amplitude), is a region of intense turbulence generated by wave breaking. Similarly, dissipation rates near the surface of a large lake were found to be one or two orders of magnitude greater than those estimated through the law of the wall (Terray et al., 1996). The scaling of the dissipation rate as a function of wind speed, wave characteristics and depth has been proposed by Terray et al. (1996). Near the surface, within one wave height, the dissipation rate is great and remains constant. Beyond this region it decays with depth as  $z^{-2}$ . In the deepest layer, the dissipation rate is controlled by the law of the wall. These estimations can be applied to new models to simulate many physical, chemical and biological processes related to the turbulent intensity of mixing in the very near surface layer.

In more recent years, numerical modeling has been extensively applied to analyze the dynamics of turbulent mixing. Typical turbulent mixed layer models may be classified into two basic types according to their formulation: differential and bulk models. Models such as those developed by Mellor and Yamada (1974, 1982), and Lemos (1991) are models in the sense that the equations for momentum, heat, salt and TKE are used in their primitive form and are not integrated over the mixed layer. The mixed layer in these models is defined as a region where the local TKE is sufficient to provide a certain level of vertical mixing. Craig and Banner (1994) and Craig (1996) employed an improved "level 2 1/2" closure model to predict near-surface turbulence, in which the dissipation rate decays as  $\varepsilon = q^3/Bl$ , where B is a constant of proportionality, q is turbulent velocity and l is mixing length. The influence of wave breaking was modeled by including turbulent energy input at the surface. The waveenhanced layer was found. In the latter paper, wind effects are considered as a boundary input. In bulk models, e.g. those of Garwood (1977) and Niiler (1975), the mixed layer was assumed to be a well mixed layer, which is uniform in temperature and salinity. The governing equations for the bulk models were obtained by integrating the primitive equations over the depth of the mixed layer. Martin (1985) tested the ability of the mixed layer models to simulate the seasonal evolution of the mixed layer at weather-ship stations November and Papa, in the eastern North Pacific. The models were sensitive to "external" parameters such as surface heat flux, sea-water turbidity, and ambient diffusivity below the mixed layer.

Most recently, Burchard (2001) simulated dynamics in the wave-enhanced layer by using a  $k-\varepsilon$  model, and showed that the measured near-surface dissipation rate under-breaking waves can be simulated by considering a shear-dependent closure for the second moments.

In our study, we have adopted an enhanced estimate of the dissipation rates based on the proposal by Terray et al. (1996). Recent advanced field measurement techniques have been successfully applied to the sea surface layer measurement. Based on extensive field data, Terray et al. (1996) proposed the wave-dependent scaling of the dissipation rate which was controlled by various characteristics of the wave field, leading to a better description of the vertical turbulent structure near the ocean surface. The proposed scaling included the turbulent kinetic energy. With the improved estimation of dissipation rate as TKE source term to describe the TKE fluxes, the velocity and TKE were modeled and verified with measurement data from Cheung and Street (1988) and Kitaigorodskii et al. (1983). The effects of diffusion and shear production were analyzed analytically in different depth zones, respectively. The results showed that the diffusion was more important at the surface, but that shear production was dominant in deep water.

#### 2. Description of the Model

#### 2.1 Governing equations

In this study, we have developed a one-dimensional, steady-state model of wave-enhanced turbulence in the ocean surface, similar to the model described by Craig and Banner (1994) and Craig (1996), but with a more accurate and realistic formula of the dissipation rate. As illustrated in Fig. 1, the sea-bed is located at z = -H and the surface is located at z = 0. Wave breaking is assumed to be dominant and therefore to play an important role in enhancing turbulence in the upper ocean layer. Neglecting the stratification, the governing equations for the model include momentum equations and a TKE equation as follows:

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho_0} \frac{\partial P}{\partial x} + \frac{\partial}{\partial z} \left( K_m \frac{\partial u}{\partial z} \right) + F_x \tag{1}$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho_0} \frac{\partial P}{\partial y} + \frac{\partial}{\partial z} \left( K_m \frac{\partial v}{\partial z} \right) + F_y$$
(2)





Fig. 1. Structure of the water layers,  $Z_b = 0.6h_s$ ,  $Z_t = 0.3 \kappa c h_s/u_*$ .

$$\frac{D\frac{q^2}{2}}{Dt} = \frac{\partial}{\partial z} \left( K_q \frac{\partial \frac{q^2}{2}}{\partial z} \right) + K_m \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right) + F_q - \varepsilon.$$
(3)

In (1) and (2),  $\rho_0$  and  $\rho$  are reference and in-situ densities, z is the vertical coordinate, f is the Coriolis parameter,  $K_m$  is the eddy viscosity, and  $F_x$ ,  $F_y$  are horizontal diffusion terms. In (3),  $K_q$  is the vertical diffusion coefficient for TKE;  $\varepsilon$  is the dissipation due to turbulent motion; q is the turbulent velocity scale, defined as the square root of twice the TKE ( $q^2/2$ ).

Neglecting advection, horizontal diffusion and Coriolis force, the momentum and TKE equations in the steady case are given by

$$\frac{\partial}{\partial z} \left( K_m \frac{\partial u}{\partial z} \right) = 0 \tag{4}$$

$$\frac{\partial}{\partial z} \left( K_m \frac{\partial v}{\partial z} \right) = 0 \tag{5}$$

$$\frac{\partial}{\partial z} \left( K_q \frac{\partial \frac{q^2}{2}}{\partial z} \right) + K_m \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right) - \varepsilon = 0.$$
 (6)

The first term in (6) represents the vertical diffusion of TKE and the second term is the energy generated by ve-

locity shear. Following Mellor and Yamada (1982), the eddy viscosity  $K_m$  and the vertical TKE diffusive coefficient  $K_a$  can be expressed as:

$$K_m = S_m lq \tag{7}$$

$$K_q = S_q lq, \tag{8}$$

where l is the mixing length (turbulent length scale);  $S_m$  and  $S_q$  are empirical momentum and turbulent coefficients, respectively.

## 2.2 Mixing length and surface roughness

The mixing length l can be modeled simply as the mixed layer depth (Skyllingstad and Denbo, 1995), or in a more complex form using a differential equation similar to (6). In the present study, l is assumed to follow "the law of the wall" near the water surface, i.e.,  $l = \kappa(z_0 - z)$ , in which  $\kappa$  is the von Karman constant, assumed to be 0.4;  $z_0$  is the roughness length for the surface. This reflects the loss of momentum to sea surface. Here, an empirical model of Donelan *et al.* (1993) is applied to compute the roughness length according to the wind speed and the wave status,

$$z_0 = 3.7 \times 10^{-5} \frac{U_{10}^2}{g} \left(\frac{U_{10}}{C_p}\right)^{0.9}$$
(9)

in which  $U_{10}$  is the wind velocity at 10 meters height above the water surface and  $C_p$  is wave velocity corresponding to the spectral peak frequency. The wind velocity profile has a logarithmic height dependence

$$U_{10} = \frac{u_{*a}}{\kappa} \ln \frac{10}{z_0}.$$
 (10)

The friction velocity of air  $u_{*a}$  and water  $u_{*w}$  can be written as:

$$\tau_s = \rho_a C_{10} U_{10}^2, \tag{11}$$

$$u_{*a} = \sqrt{\tau_s / \rho_a} \tag{12}$$

$$u_{*w} = \sqrt{\tau_s / \rho_w} \tag{13}$$

where  $\tau_s$  is surface stress;  $\rho_a$  and  $\rho_w$  are the density of air and water respectively; and  $C_{10}$  is the drag coefficient,  $C_{10} = (0.8 + 0.065 U_{10}) 10^{-3}$  (Wu, 1982).

The wave energy factor  $\alpha$  is described as in Terray *et al.* (1996):

$$\alpha = \frac{c}{u_*}, \quad \begin{cases} c \sim 0.5c_p, & \text{for } \frac{c_p}{u_*} < 300\\ c \sim 150u_*, & \text{for } \frac{c_p}{u_*} > 300 \end{cases}$$
(14)

where  $c_p$  is the wave phase velocity and  $c_p/u_*$  is defined as the wave age. Equation (9) indicates that the surface roughness is inversely associated with wave age, since  $U_{10}/C_p \sim u_{*w}/C_p \sim 1/\alpha$  which represents the wave age and characterizes the wind stress and the wave development stages. The surface roughness would influence the dissipation under wind wave breaking.

#### 2.3 Dissipation rate

In general, the surface layer dissipation estimates agree satisfactorily with the structure of a classical law of the wall. This has been widely applied in many TKE modeling exercises, such as Noh and Kim (1999), Craig and Banner (1994) and Mellor and Yamada (1982). The dissipation due to turbulent motion is scaled as  $\varepsilon = q^3/Bl$ (Batchelor, 1953), which does not take account of the influence of wind and wave. However, for wind-driven waves, field measurements in the upper layer of the ocean showed that dissipation rate  $\varepsilon(z)$  under wind wave breaking is much higher than predicted by the scaling of  $u_*^{3}/\kappa_z$ (Soloviev et al., 1988). Kitaigorodskii et al. (1983) found that  $\varepsilon(z)$  is two order greater than the value given by the law of the wall. Melville (1982) suggested that dissipation decayed as  $z^{-n}$ , with *n* ranging 3.0–4.6. Terray *et al.* (1996) also proposed a scale for the dissipation rate based on wind and wave parameters and the water depth, as follows:

$$\varepsilon = \frac{0.3u_*^2 ch_s}{z_b^2}, \qquad -z_b \le z \le 0, \quad z_b = 0.6h_s$$
(15)

$$\varepsilon = \frac{0.3u_*^2 ch_s}{z^2}, \ -z_t \le z \le -z_b, \quad z_t = \frac{0.3\kappa ch_s}{u_*}$$
(16)

$$\varepsilon = \frac{u_*^3}{\kappa(-z)}, \qquad -H \le z \le -z_t, \tag{17}$$

where c is the effective phase velocity. This estimate is based on the assumption that the water is deep and the region is directly stirred by the wave breaking. This threelayer structure is illustrated in Fig. 1. Near the surface, within one wave height depth, the dissipation rate is high and is maintained constant. At sufficient depth the dissipation rate declines asymptotically to values given by traditional wall law. A comparison between the dissipation rate obtained from the improved prediction using (15)–(17) and the wall law is shown in Fig. 6, which illustrates that Eqs. (15)– (17) can describe a dissipation rate closer to the measurements reported by Kitaigorodskii *et al.* (1983). We have therefore applied them to our numerical and analytical model for analyzing the velocity and TKE in the water column under wind-wave breaking.

#### 2.4 Boundary conditions

The shear flow near the air/water interface is analogous to the turbulent flow over a rough surface. For this sort of flow it is often assumed that there is a region adjacent to the surface where the stress can be considered constant. Churchill and Csanady (1983) stated that a sublayer exists with linear velocity, very close to the surface. Assuming that the wind direction is the same as that of horizontal velocity u, the following conditions (Craig and Banner, 1994) are satisfied at the surface layer (z = 0),

$$K_m \frac{\partial u}{\partial z} = u_*^2 \tag{18}$$

$$K_m \frac{\partial v}{\partial z} = 0 \tag{19}$$

$$K_q \frac{\partial \frac{q^2}{2}}{\partial z} = \alpha u_*^3.$$
 (20)

If the momentum equation (4) and boundary condition (18) are satisfied, one can conclude that boundary condition (18) is valid for the water column in the entire range -H < z < 0.

In the work reported by Ly (1986), the TKE at z = 0 is assumed to satisfy the boundary condition,  $q^2/2 = 4.66u_*^2$ , which is a function of wind friction velocity only and is not related to the wave parameters. However, in Craig (1996)'s analytical analysis, TKE is derived as a function of wave age and the diffusive coefficients at the surface,

$$q^3 = \beta u_*^3, \tag{21}$$

$$\beta = \left(\frac{B}{S_m}\right)^{3/4} + \alpha \left(\frac{3B}{S_q}\right)^{1/2}, \qquad (22)$$

which is applied in the present analysis.

At the sea-bed z = -H, a zero-flux TKE is assumed, i.e.

$$\frac{\partial \frac{q^2}{2}}{\partial z} = 0,$$
 (23)

and the no-slip condition is imposed

$$u = 0, v = 0.$$
 (24)

## 2.5 Numerical method

In order to solve this problem, we must resort to a numerical computation scheme, as follows. Neglecting the Coriolis force, the right-hand sides of (4) and (5) equal zero. Substituting the following two equations into (4) and (5),

$$\frac{\partial u}{\partial z} = \frac{u_*^2}{lqS_m} \tag{25}$$

$$\frac{\partial v}{\partial z} = 0, \tag{26}$$

the problem then reduces to solving (25), and rewriting (6) as

$$\kappa^2 s_q \left( \frac{\partial Q}{\partial l} + l \frac{\partial^2 Q}{\partial l^2} \right) + \frac{u_*^4}{s_m l (3Q)^{1/3}} - \varepsilon = 0$$
(27)

$$Q = \frac{q^3}{3},\tag{28}$$

which can be viewed as the discretization of points along the water column. The vertical domain is discretized as  $0 = z[0] > z[1] \dots > z[i] \dots > z[N+1] = -H$ . Equations (25) and (27) can be discretize as follows:

$$\kappa^{2} s_{q} \left( \frac{Q_{i-1} - Q_{i}}{l_{i-1} - l_{i}} + l_{i-1} \frac{Q_{i-1} - 2Q_{i} + Q_{i+1}}{(l_{i-1} - l_{i})(l_{i} - l_{i+1})} \right) + \frac{u_{*}^{4}}{s_{m} l_{i} (3Q_{i})^{1/3}} - \varepsilon_{i} = 0$$
(29)

$$\frac{u_{i-1} - u_i}{l_{i-1} - l_i} = -\frac{u_*^2}{\kappa s_m l_i (3Q_i)^{1/3}},$$
(30)

which can be solved by applying Newton's method. An initial guess is made for the unknown values of the  $(Q_1, Q_2, ..., Q_i, ..., Q_N)$ . All the other dependent variables are computed on the basis of this guess. They are then even-

tually updated by the Newtonian Iteration Scheme until they converge with an absolute error of less than  $10^{-8}$  at all grids. The boundary point of  $Q_0$  can be calculated by (21).  $Q_{N+1}$  will be discussed in the next section. The numerical results are shown in Section 4.

### 3. Analytical Analysis

### 3.1 Shear production balancing dissipation in deep layer

To analyze the effects of the shear generation of TKE, we simplify (6) using only terms describing the balance between the shear production and dissipation of the turbulent kinetic energy in deep layer, which can be written as

$$\kappa lqs_m\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right) = \varepsilon.$$
 (31)

Substituting (25) and (26) into (31) leads to

$$q = \frac{u_*^2 z_b^2}{0.3\kappa S_m h_s cl}, \qquad -z_b \le z \le 0$$
(32)

$$q = \frac{u_*^2 z^2}{0.3 \kappa S_m h_s cl}, \qquad -z_t \le z \le -z_b$$
(33)

$$q = \frac{u_*}{S_m}, \qquad -H \le z \le -z_t. \tag{34}$$

Substituting (32)–(34) into (25) and integrating it with the boundary condition of u = 0 at z = H yields

$$u = -\frac{0.3h_sc}{z_b^2} z + \frac{u_*}{\kappa} \ln \frac{z_t + z_0}{H + z_0} - \frac{0.6h_sc}{z_b} + \frac{0.3h_sc}{z_t}, -z_b \le z \le 0$$
(35)

$$u = \frac{u_*}{\kappa} \ln \frac{z_t + z_0}{H + z_0} + \frac{0.3h_s c}{z} + \frac{0.3h_s c}{z_t}, \quad -z_t \le z \le -z_b \quad (36)$$

$$u = \frac{u_*}{\kappa} \ln \frac{z_0 - z}{H + z_0}, \qquad -H \le z \le -z_t. \quad (37)$$

In the deeper depth  $(-H \le z \le -z_t)$ , *q* is strictly a constant and *u* varies logarithmically. This layer is defined as "the shear layer". The distribution of *q* obtained using the above analytical method is shown by dashed lines in Fig. 2. When the results are compared with the full numerical model, it is found that they are in good agreements in



Fig. 2. Comparison of profiles  $q/u_*$  (Cheung and Street, 1988), obtained by numerical full model and the analytical solutions, of case 4 in Table 1.

deeper water, since the shear generation of TKE is balanced dominantly by dissipation. This shear layer is unaffected by the presence of the wave-enhanced dissipation near the ocean surface. Therefore, at the sea bottom,  $q_{N+1} = u_*/S_m$  is used as a boundary condition. However, the analytical solutions in this section for upper layers are meaningless and can be ignored, since in the near surface upper layers the influence of diffusion is significant.

# 3.2 Diffusion balancing dissipation at near surface layer

The effect of diffusion is more significant in TKE production at the surface. To eliminate the shear production, the TKE equation (6) can be rewritten as a balance of the downward TKE diffusion and its dissipation,

$$\frac{\partial}{\partial z} \left( K_q \frac{\partial \frac{q^2}{2}}{\partial z} \right) = \varepsilon.$$
(38)

By integrating (38) with the boundary condition (21) at z = 0, q can be solved for analytically as follows:

$$q^{3} = \beta u_{*}^{3} - \frac{3\alpha u_{*}^{3}}{\kappa s_{q}} \ln \frac{z_{0} - z}{z_{0}} + \frac{0.9ch_{s}u_{*}^{2}}{\kappa s_{q}z_{b}^{2}} \left( -z - z_{0} \ln \frac{z_{0} - z}{z_{0}} \right),$$
$$-z_{b} \le z \le 0 \qquad (39)$$

$$q^{3} = \frac{0.9ch_{s}u_{*}^{2}}{\kappa s_{q}}$$

$$\left(\frac{1}{z_{0}-z} + \frac{2}{z_{b}}\ln\frac{z_{0}-z_{b}}{z_{0}-z} + \frac{z_{b}}{z_{b}^{2}}\ln\frac{z_{0}}{z_{0}-z} - \frac{1}{z_{b}} - \frac{1}{z_{0}-z_{b}}\right)$$

$$+\beta u_{*}^{3} - \frac{3\alpha u_{*}^{3}}{\kappa s_{q}}\ln\frac{z_{0}-z}{z_{0}},$$

$$-z_{t} \leq z \leq -z_{b}. \quad (40)$$

Close to the surface, Eqs. (39) and (40) hold. But we did not derive the analytical solution through (38) in deeper water, since Eq. (38) could not be valid there. No analytic solution of u has so far been derived due to its complexity. Figure 2 shows a comparison of the profile of q obtained by analytical solutions and the full numerical model. The solid line is the result given by the analytical solution from the balancing between the diffusion and the dissipation, which is in good agreement with that of the numerical solutions (dashed-dotted line, in Fig. 2) near the surface. This very near surface layer ( $-z_b \le z \le 0$ ) is defined as the "wave-enhanced layer" following Craig and Banner (1994). In this wave-enhanced layer, the TKE from wave breaking is the main source to balance the dissipation.

A comparison of the results from the full model with the analytical solutions based on the balance between the shear production and dissipation is given by a plot of the TKE results obtained by numerical and analytical methods given in Fig. 2, which clearly illustrates TKE transition from the wave-enhanced layer to the shear layers. In the diffusive near-surface layer, a balance between the surface flux of TKE and dissipation is the dominant effect while further beneath this layer, dissipation would satisfy the classic law of the wall, which shows that the shear generation of TKE is balanced by dissipation

#### 4. Comparison with Experimental Data

# 4.1 Calculation of $S_m$ and $S_q$

The Level 2 1/2 model described by Mellor and Yamada (1982) gives:

$$G_m = \frac{l^2}{q^2} \left(\frac{\partial u}{\partial z}\right)^2 \tag{41}$$

$$G_h = -\frac{l^2}{q^2} \beta_1 g \frac{\partial \theta}{\partial z}$$
(42)

$$\frac{P_s + P_b}{\varepsilon} = B1 \left( S_m G_m + S_h G_h \right), \tag{43}$$

Table 1. Characteristic parameters for different cases of wind-generated waves.

	$u_{\infty}$ (m/s)	$u_s (\mathrm{mm/s})$	$\Delta$ (mm)	$u_* (\text{mm/s})$	$\overline{\tilde{u}}_o \text{ (mm/s)}$	$z_{\eta}$ (mm)	$f_D$ (Hz)
case 1	4.7	93	264	7.20	76.67	19.9	3.5
case 2	6.7	137	249	11.30	108.34	32.6	2.7
case 3	9.9	204	354	17.50	139.84	45.1	2.4
case 4	13.1	270	298	27.50	194.06	58.2	2.0

The parameters are denoted as:

 $u_{\infty}$  - the wind speed,

 $u_S$  - the Eulerian surface drift,

 $\tilde{u}_{o}$  - the rms surface orbital velocity,

 $\Delta$  - the depth at which the turbulent shear stress vanishes,

 $z_{\eta}$  - the wave-decay depth,

 $f_D$  - the dominant frequency of the wave.

where  $\theta$  is temperature or salinity,  $P_s$  is the shear production of turbulent energy, and  $P_b$  is the buoyant production. For the experiment of wind generated wave breaking (Cheung and Street, 1988), it was assumed that there is no stratification, the  $\theta$  flux at the surface boundary layer is zero, i.e.  $\partial \theta / \partial z = 0$ , which leads to  $G_h = 0$ . Then  $S_m$  and  $S_h$  can be written as,

$$S_h = A_2 \left( 1 - \frac{6A_1}{B_1} \frac{P_s + P_b}{\varepsilon} \right) \tag{44}$$

$$S_m = A_1 \left( 1 - 3C_1 - \frac{6A_1}{B_1} \frac{P_s + P_b}{\varepsilon} \right),$$
(45)

where  $(A_1, B_1, A_2, B_2, C_1) = (0.92, 16.6, 0.74, 10.1, 0.08)$ . In the following analysis,  $s_q$  is set equal to  $s_h$ . Their experiment ignores the buoyancy effects. At the surface layer, the diffusion term is dominant, while the shear production is relatively small compared to dissipation. As  $P_s/\varepsilon << 1$ ,  $S_m$  and  $S_h$  equal 0.7 and 0.74, respectively, as applied to the analytical solution in Section 3. However, in the numerical solution discussed in Subsection 2.4,  $S_m$  and  $S_h$  are solved numerically using (44) and (45), which are depth dependent.

# 4.2 Experiments of Cheung and Street (1988) and Kitaigorodskii et al. (1983)

In our study, we compared the numerical solutions with the experimental results of Cheung and Street (1988) and Kitaigorodskii *et al.* (1983). Cheung and Street (1988) conducted a series of experiments in a laboratory channel to obtain mean and turbulent velocity data in the water layer beneath wind-generated water waves. Wind-generated wave experiments were run at seven different wind speeds from  $1.5-13.1 \text{ ms}^{-1}$ . When the wind speed was

greater than  $3.5 \text{ ms}^{-1}$  (Thorpe, 1992), it was sufficient to cause breaking waves. Therefore, the lowest three speeds did not create fully turbulent boundary layers, and hence only the runs with speeds of  $4.7-13.1 \text{ ms}^{-1}$  are considered in our comparison. Table 1 lists the characteristic parameters of the runs.

In comparison between the numerical and experimental data, u and z have been converted from the nondimensional parameters  $u^+$  and  $z^+$ ,

$$u = u_S - u^+ u_* \tag{46}$$

$$z = \frac{z^+ u_*}{v_w} \tag{47}$$

where  $v_w$  is the kinematic viscosity of water. The phase velocity  $c_p$  can be estimated as  $z_\eta f_D$  from the data in Table 1. The wave energy factor  $\alpha$  and bottom roughness  $z_0$ can be calculated by (14) and (9) respectively. The significant wave height  $h_s$  can be calculated by using the given rms surface orbital velocity as follows,

$$h_s = \frac{2\sqrt{2\tilde{u}_o}}{\pi f_D}.$$
(48)

Table 2 shows the wave-age-dependent  $c_p$  and  $z_0$ , which are used in the analytic and numerical analysis.

We used the full numerical model to calculate q and u, and compared the values with the experimental data. The results are shown in Figs. 3 and 4, which show that the measurements of q and u are in good agreement with both the analytical and numerical solutions.

The measurement results of turbulent velocity components beneath nature, wind-generated waves on Lake



Table 2. The wave age dependent  $c_p$  and  $z_0$  for different cases of wind-generated waves.

Fig. 3. Profile comparison of measured  $q/u_*$  (Cheung and Street, 1988) with numerical and analytical solutions of cases 1–4 in Table 1.

Ontario published by Kitaigorodskii *et al.* (1983) are also compared with the numerical solution. The wave parameters are listed in Table 3. Table 4 lists the measured q and  $\varepsilon$  at different depths. First, the dissipation rate calculated by using  $\varepsilon = q^3/Bl$  and (15)–(17) are compared and shown in Fig. 5. As the depths of the measured data are out of the wave-enhanced layer, the analytical solution is not valid for the comparison of q. Therefore, it is only compared with the full numerical model. A satisfactory agreement is also found, as shown in Fig. 6.

#### 4.3 The effects of wave energy factor $\alpha$

For wind generated waves, the wave energy factor  $\alpha$  is a good measure of the sea state. The experimental values of  $\alpha$  found by Cheung and Street (1988) are listed in Table 2. If the wave energy factor increases, it means that the surface flux of the TKE is increasing. From (9) and (21), the following relationships exist,



Table 3. Characteristic parameters for different cases of field experiments.

Fig. 4. Profile comparison of measured u (Cheung and Street, 1988) with numerical and analytical solutions of cases 1-4 in Table 1.

$$z_0 \sim \alpha^{-0.9} \tag{49}$$

$$\frac{q}{u_*} \sim \alpha^{1/3}.$$
 (50)

Figure 7 shows the profile of  $q/u_*$  for different values of  $\alpha$ . It is found that in the wave-enhanced layer the wave energy factor  $\alpha$  has a significant influence on the model results; however, deep in the water, its effect on q and u vanishes.

### 5. Conclusions

The classic wall law is widely used in estimating the dissipation and modeling of TKE. However, recent experimental results show that a wave enhanced turbulent layer exists in a wind-force aquatic surface in which the dissipation rate is higher than the prediction given by the law of the wall. In the present model, the dissipation is

Table 4. The measured q and  $\varepsilon$  in Kitaigorodskii et al. (1983).

	Depth (cm)	q (cm/s)	$\varepsilon$ (cm <sup>2</sup> /s <sup>3</sup> )
case 5	44	9.6	3.7
	62	8.1	1.7
	89	5.8	0.7
case 6	67	7.5	2.3
	117	6.1	1.2



Fig. 5. Comparison of measured dissipation rate (Kitaigorodskii *et al.*, 1983) with results by  $\varepsilon = q^3/Bl$  and the estimation of Terray *et al.* (1996) which is used to modify our model.

described by a three-layer structure, and scaling with both wind forcing and wave parameters (Terray *et al.*, 1996), which can predict the dissipation rate more accurately and realistically. The modified model results of velocity and TKE are in reasonably good agreement with those of laboratory (Cheung and Street, 1988) and field (Kitaigorodskii *et al.*, 1983) measurements.

Near the surface the model is simplified by balancing the diffusion of TKE with the dissipation rate. The analytical solution can be derived for the near-surface layer. In deep water, the shear production is the only term to balance the dissipation rate. The analytical solution can also be found for both u and q. The good agreements obtained in various layers confirm that the wave-enhanced layer near the surface is a diffusion-dominated region



Fig. 6. Comparison of measured  $q/u_*$  (Kitaigorodskii *et al.*, 1983) with the results predicted by the numerical full model of cases 5 and 6 in Table 3.



Fig. 7. The profiles of  $q/u_*$  and u of case 4 in Table 1 for different  $\alpha$ .

(Craig, 1996). Under this region, the water behavior transits to satisfy the classical law of the wall. Below the transition depth, the shear production predominantly balances the dissipation. Therefore, comparison among measurements, analytical solutions and full model solutions illustrates that the present model appears to work well in reproducing our present knowledge, simulating turbulence in the ocean surface layer under wind-wave breaking, in which the wind-wave effects have been taken into account in TKE equation.

It has also been found that the wave energy factor has significant influence on the surface layer. During the development phase of the wave field, with increasing wave age, the surface roughness reduces and more wind energy might go into the wave field and is lost by wave breaking, this does not affect the mixing in deeper water.

This model has produced predictions that can be applied to benchmark further experimental results. It can also be extended in the future to model the transport of chemical and physical process under wind-wave breaking. When applying this wave-enhanced mixing layer scheme to the general circulation model, however, the temporal and horizontal spatial effects are neglected. The influence of variable time and geometry on the wave-enhanced layer needs to be investigated and evaluated in future studies.

# Notation

- $C_{10}$  drag coefficient for  $U_{10}$
- c effective wave phase velocity  $[LT^{-1}]$
- $c_p$  wave phase velocity [LT<sup>-1</sup>]
- $f_D$  dominant wave frequency [T<sup>-1</sup>]
- *H* water depth [L]
- $h_s$  significant wave height [L]
- $K_m$  eddy viscosity [L<sup>2</sup>T<sup>-1</sup>]
- $K_q$  vertical TKE diffusive coefficient [L<sup>2</sup>T<sup>-1</sup>]
- *l* mixing length [L]
- q turbulent velocity scale  $[LT^{-1}]$
- $q^+$  nondimensionized turbulent velocity scale
- $S_m$  empirical momentum coefficient
- $S_q$  empirical TKE coefficient
- t time [T]
- $U_{10}$  wind velocity at 10 m height above the sea surface [LT<sup>-1</sup>]
- *u*, *v*horizontal velocity components [LT<sup>-1</sup>]
- $u_S$  Eulerian surface drift [LT<sup>-1</sup>]
- $\overline{\tilde{u}}_{o}$  rms surface orbital velocity [LT<sup>-1</sup>]
- $u_{\infty}$  wind velocity in experiment [LT<sup>-1</sup>]
- $u_*$  friction velocity at the surface [LT<sup>-1</sup>]
- $u_{*a}$  friction velocity of air [LT<sup>-1</sup>]
- $u_{*b}$  friction velocity of sea bed [LT<sup>-1</sup>]
- $u^+$  nondimensionized horizontal velocity
- z vertical coordinate [L]
- $z_0$  surface roughness length [L]
- $z_{0b}$  bottom roughness length [L]
- $z_{\eta}$  wave decay depth [L]
- $z^+$  nondimensinized vertical coordinate

- $\alpha$  wave energy factor
- $\varepsilon$  TKE dissipation rate [L<sup>2</sup>T<sup>-3</sup>]
- $\kappa$  von Karman constant
- $\delta$  viscous sub layer thickness [L]
- $\Delta$  depth at which the turbulent shear stress vanishes [L]
- $v_w$  dynamic viscosity of water [L<sup>2</sup>T<sup>-1</sup>]
- $\rho_a$  density of air [ML<sup>-3</sup>]
- $\tau_s$  surface stress [ML<sup>-1</sup>T<sup>-2</sup>]

#### References

- Anis, A. and J. N. Moum (1995): Surface wave-turbulence interaction: Scaling  $\varepsilon(z)$  near the sea surface. J. Phys. Oceanogr., 25, 2025–2045.
- Batchelor, G. K. (1953): *The Theory of Homogenous Turbulence*. Cambridge University Press.
- Burchard, H. (2001): Simulation the wave-enhanced layer under breaking surface waves with two-equation turbulence models. J. Phys. Oceanogr., 31, 3133–3145.
- Cheung, T. K. and R. L. Street (1988): The turbulent layer in water at an air-water interface. J. Fluid Mech., **194**, 133–151.
- Churchill, J. H. and G. T. Csanady (1983): Near-surface measurements of Quasi-Lagrangian velocities in open water. J. Phys. Oceanogr., 13, 1669–1680.
- Craig, P. D. (1996): Velocity profiles and surface roughness under breaking waves. J. Geophys. Res., **101**, 1265–1277.
- Craig, P. D. and M. L. Banner (1994): Modeling wave-enhanced turbulence in the ocean surface layer. J. Phys. Oceanogr., 24, 2546–2559.
- Donelan, W. A., F. W. Dobson and S. D. Smith (1993): On the depandence of sea surface rougness on wave development. *J. Phys. Oceanogr.*, 23, 2143–2145.
- Garwood, R. W. (1977): An oceanic mixed-layer model capable of simulating cyclic sates. J. Phys. Oceanogr., 7, 455–471.
- Kitaigorodskii, S. A., M. A. Donelan, J. L. Lumley and E. A. Terray (1983): Wave-turbulence interactions in upper ocean. Part II: Statistical characteristics of wave and turbulent components of the random velocity field in the marine surface layer. J. Phys. Oceanogr., 13, 1988–1999.
- Lemos, C. M. (1991): Wave Breaking. Springer-Verlag.
- Ly, L. N. (1986): Modeling the interaction between the atmospheric and oceanic boundary layers, Including a surface wave layer. J. Phys. Oceanogr., **16**, 1430–1443.
- Martin, P. J. (1985): Simulation of the mixed layer at OWS November and Papa with several models. J. Phys. Oceanogr., **90**, 903–916.
- Mellor, G. L. and T. Yamada (1974): A hierarchy of turbulence closure models for planetary boundary layers. J. Atmos. Sci., 31, 1791–1806.
- Mellor, G. L. and T. Yamada (1982): Development of a turbulence closure model for geophysical fluid problems. *Rev. Geophys.*, 20, 851–875.
- Melville, W. K. (1996): The role of surface-wave breaking in air-sea interaction. Ann. Rev. Fluid Mech., 28, 279–321.
- Niiler, P. P. (1975): Deepening of the wind mixed layer. J. Mar. Res., **33**, 405–422.

- Noh, Y. and H. J. Kim (1999): Simulations of temperature and turbulence structure of the oceanic boundary layer with the improved near-surface process. J. Geophys. Res., 104, 15621–15634.
- Skyllingstad, E. D. and D. W. Denbo (1995): An ocean largeeddy simulation of Langmuir circulations and convection in the surface mixed layer. J. Geophys. Res., 100, 8501– 8522.
- Soloviev, A. V., N. V. Vershinsky and V. A. Bezverchnii (1988): Small-scale turbulence measurements in the thin surface layer of the ocean. *Deep-Sea Res.*, **35**, 1859–1874.
- Terray, E. A., M. A. Donelan, Y. C. Agrawal, W. M. Drennan,

K. K. Kahma, A. J. Williams, III, P. A. Hwang and S. A. Kitaigorodskii (1996): Estimates of kinetic energy dissipation under breaking waves. *J. Phys. Oceanogr.*, **26**, 792–807.

- Thorpe, S. A. (1984): On the determination of  $k_v$  in the nearsurface ocean from acoustic measurements of bubbles. J. *Phys. Oceanogr.*, **14**, 855–863.
- Thorpe, S. A. (1992): Bubble clouds and the dynamics of the upper ocean. *Quarterly Journal od the Meteorological*, **118**, 1–22.
- Wu, J. (1982): Wind-stress coefficients over sea surface from breeze to hurricane. J. Geophys. Res., C87, 9704–9706.