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OCEAN ACOUSTICS AND UNDERWATER SOUND

# The Effect of Anisotropy of a Rough Sea Surface on the Generation of Acoustic Radiation

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**Abstract**—A model that relates the frequency–angular characteristics of wind waves on the sea surface to the frequency spectrum of acoustic radiation generated by them is constructed. Based on empirical wave energy distribution functions, it is shown that, in the vicinity of the peak of the wind wave spectrum, the intensity of acoustic radiation strongly depends on the model chosen for the angular distribution function $\theta(\alpha)$ and on its parameters. At high frequencies, four to five times higher than the dominant wave frequency, it is possible to assume that, to a first approximation, $\theta(\alpha) = \text{const}$. In the intermediate frequency range, between the dominant frequency and the high frequencies, the predominant contribution to the sound radiation is made by the waves that travel in the direction close to orthogonal with respect to the wind.

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## INTRODUCTION

Acoustic radiation produced by wave motion in the ocean is one of the main sources of underwater noise [1, 2]. Physical mechanisms underlying the generation of sound by wind waves on the sea surface have long been known [3, 4]. However, the problem of estimating the spectrum of acoustic noise produced by these waves is far from being solved. This is primarily because of the lack of information on the properties of the wind waves themselves.

The sound is produced by counterpropagating waves of the same frequency. As a rule, interaction is considered between the wind-generated waves and the waves traveling in opposition to the wind [5]. It has been shown [6] that the interaction between these waves results in the energy transfer to the waves that travel in opposition to the wind. As follows from the analysis of weakly turbulent spatial spectra, this effect, though less pronounced, can be caused by the nonresonant interaction [7]. The presence of spectral components of the wave field that travel in opposition to the wind was revealed experimentally by Doppler radar observations of the sea surface [8]. An analysis of the physical mechanisms that produce counterpropagating waves in the gravity-capillary range is given in [5].

In addition to the wave components that propagate in the wind direction, acoustic radiation can be produced by the components that propagate at an angle to the principal wind direction. Since, at frequencies several times higher than the dominant wave frequency, the angular distribution is no longer directional [9], the contribution of these components can be substantial.

This paper presents a model that relates frequency–angular characteristics of wind waves to the spectrum of acoustic radiation generated by them. Based on empirical angular energy distribution functions of waves on the sea surface in the gravity-wave range, the acoustic radiation is analyzed at frequencies on the order of the frequency of the dominant component of the wave field.

## DEPENDENCE OF THE ACOUSTIC PRESSURE SPECTRUM ON THE ANGULAR ENERGY DISTRIBUTION OF SPECTRAL COMPONENTS OF THE WIND WAVE FIELD

The expression that relates the mean-square value of the acoustic pressure produced by wind waves to the two-dimensional spectrum $\Xi_\xi(\mathbf{k})$ of wind waves was obtained in [4]:

$$\overline{p^2} = \frac{\pi\rho^2}{4C_S^2}\int \omega^2(5\omega^2 - kg)^2 m(\mathbf{k})\Xi_\xi^2(\mathbf{k})d\mathbf{k}\bigg|_{k = k(\omega)}, \quad (1)$$

where $\mathbf{k}$ is the wave vector; $\rho$ is the density; $C_s$ is the velocity of sound; and the dimensionless coefficient $m(\mathbf{k})$ determines the level of standing waves, $m = 1$ and 0 corresponding to purely traveling and purely standing waves, respectively. Expression (1) is valid for waves that satisfy the dispersion relation

$$\omega^2 = gk + \gamma k^3, \quad (2)$$

where $\omega$ is the circular frequency, $g$ is the acceleration of gravity, $k$ the wave number, and $\gamma$ is the surface tension coefficient.

Let us change to the polar coordinate system in Eq. (1), from components $k_x$ and $k_y$ of the wave vector to its absolute value $k$ and direction $\alpha$:

$$\Psi_\xi(k, \alpha) = \frac{\partial(k_x, k_y)}{\partial(k, \alpha)} \Xi_\xi(k_x, k_y),$$

where the Jacobian is $\frac{\partial(k_x, k_y)}{\partial(k, \alpha)} = k$.

Let us represent the spectrum of wave numbers and directions in the form

$$\Psi_\xi(k, \alpha) = \theta(\alpha, k)\Phi_\xi(k). \tag{3}$$

The procedure of changing the variables in the wind wave spectra relies on the normalization condition and dispersion relation. According to the normalization condition, the integral of any wave spectrum with respect to all the variables is equal to the variance of the surface roughness height.

Dispersion relation (2) is valid for a wide range of waves. For gravity waves, it can be simplified by dropping the second term. Using the dispersion relation in the form

$$\omega^2 = gk, \tag{4}$$

we obtain the expression that relates the wave number spectrum to the frequency spectrum for gravity waves in a deep sea:

$$S(\omega) = \frac{dk}{d\omega}\Phi(k),$$

where $dk/d\omega = 2\omega/g$. Let us express the coefficient $m$ in terms of the angular energy distribution function $\theta(\alpha)$ of waves. Taking into account that $m(\alpha) = m(\alpha + \pi)$, we obtain

$$m(\alpha) = \begin{cases} 0, \text{ for } \theta(\alpha) = \theta(\alpha+\pi) = 0 \\ \theta(\alpha)/\theta(\alpha+\pi), \text{ for } \theta(\alpha) < \theta(\alpha+\pi) \\ \theta(\alpha+\pi)/\theta(\alpha), \text{ for } \theta(\alpha) > \theta(\alpha+\pi) \\ 1, \text{ for } \theta(\alpha) = \theta(\alpha+\pi) \neq 0. \end{cases}$$

For the waves that satisfy condition (4), expression (1) can be rewritten as

$$\overline{p^2} = \iint \frac{2\pi\rho^2 g^2}{C_S^2}\omega^3 m(\alpha)\{\theta(\alpha)S_\xi(\omega)\}^2 d\omega d\alpha.$$

Therefore, the frequency spectrum of the squared acoustic pressure is related to the spectrum of wind waves as

$$S_p(\omega) = \frac{2\pi\rho^2 g^2}{C_S^2}\omega^3 S_\xi^2(\omega)\int_0^\pi m(\alpha)\theta^2(\alpha)d\alpha. \tag{5}$$

The integral on the right-hand side of Eq. (5) describes the dependence of the sound intensity generated by gravity waves on their spatial energy distribution. Since this integral has not been analyzed earlier, let us consider it in more detail.

## FREQUENCY–ANGULAR CHARACTERISTICS OF GRAVITY WAVES ON THE SEA SURFACE

Two models of the angular distribution function are used most frequently [10, 11]:

$$\theta_1(\alpha) = N_1\cos^{2s}\left(\frac{(\alpha-\alpha_0)}{2}\right), \tag{6}$$

$$\theta_2(\alpha) = N_2\mathrm{sech}^2[\beta(\alpha-\alpha_0)], \tag{7}$$

where

$$N_i = 1/\int_0^{2\pi}\theta_i(\omega, \alpha)d\alpha$$

are the normalizing coefficients, $i = 1, 2$; $\alpha_0$ is the principal wind direction; and $s$ and $\beta$ are the dimensionless parameters.

The narrowest angular distribution is observed at the frequency of the peak of the wave spectrum $S_\xi(\omega)$. Let us denote this frequency as $\omega_0$. At present, there is no general opinion regarding the parametrization of the angular distribution. According to [11], it can be described in terms of the dimensionless frequency $\Omega = \omega/\omega_0$ alone, and the parameter $\beta$ in Eq. (7) can be approximated by the expression

$$\beta = \begin{cases} 2.44\left(\frac{\Omega}{0.95}\right) \text{ for } \Omega \le 1.6 \\ 1.24 \text{ for } \Omega > 1.6. \end{cases} \tag{8}$$

Approximation (8) was constructed with the use of experimental data only for $\Omega \le 1.6$. For higher frequencies, experimental data were absent, and the parameter $\beta$ was assumed to be constant. In [12], this assumption was verified using stereoscopic photographs of the rough sea surface. It was found that, in the region $\Omega > 1.6$, the behavior of the parameter $\beta$ can be described by the expression

$$\beta = 10^{-0.4 + 0.8393\exp(-0.567\ln(\Omega^2))}.$$

Somewhat different results were obtained in the JONSWAP experiment [10]. The widening rate of the angular distribution, which is determined by the parameter $s$ in model (6), was found to depend on the age of the wave. At frequencies above the frequency of the spectral peak, this parameter is

$$s = 9.77\Omega^{-2.33 - 1.45(\zeta - 1.17)},$$

where $\zeta = U_{10}/C_0$ is the inverse of the wave's dimensionless age, $U_{10}$ is the wind speed at a height of 10 m, and $C_0$ is the phase velocity of dominant waves.

Consider three scenarios. The first and second scenarios assume that the angular distribution is described by model (6) at a fully developed ($\zeta = 1.3$) and developing ($\zeta = 3$) surface roughness, respectively. The third scenario refers to model (7). The angular distribution functions obtained for these scenarios are shown in Fig. 1.

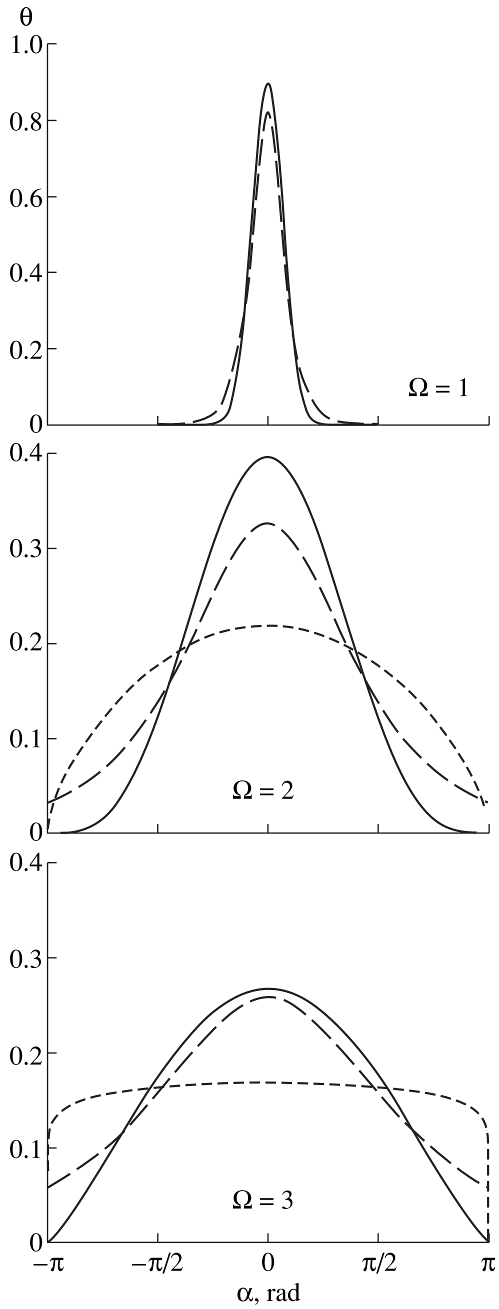


**Fig. 1.** Angular energy distribution function of waves $\theta(\alpha)$: model (6) with $\zeta = 1.3$ and 3 (the solid and short-dashed lines, respectively); model (7) (the long-dashed line).

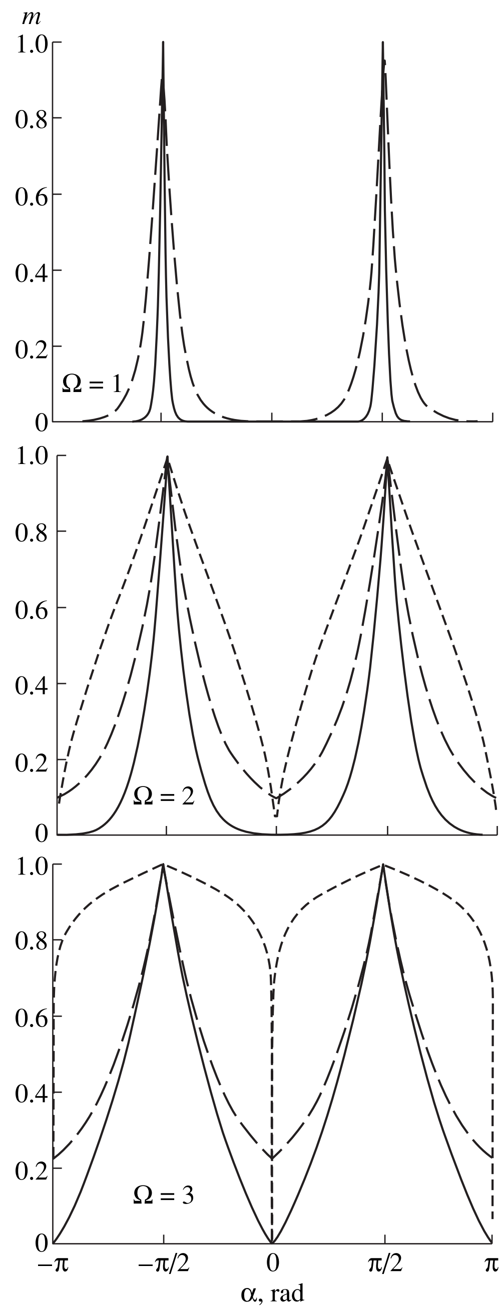


**Fig. 2.** Angular dependence of the coefficient $m$: model (6) with $\zeta = 1.3$ and 3 (the solid and short-dashed lines, respectively); model (7) (the long-dashed line).

At the frequency of the spectral peak, $\Omega = 1$, all the three functions $\theta(\alpha)$ are narrow. The slowest variation of the width of the angular distribution with frequency is observed for the fully developed surface roughness, while the fastest variation occurs for the developing roughness. In the latter case, at $\Omega = 3$, the angular distribution is almost isotropic. Figure 2 shows the parameter $m$ calculated as a function of azimuth angle $\alpha$ for angular distributions illustrated in Fig. 1. In the limiting case of an isotropic roughness, $\theta = \text{const} = (2\pi)^{-1}$ and, accordingly, $m \equiv 1$.

The integral

$$I = \int_0^{\pi} m(\alpha)\theta^2(\alpha)d\alpha \tag{9}$$

varies in a wide range. Its frequency behavior is shown in Fig. 3. The maximum growth rate of $I$ is observed in

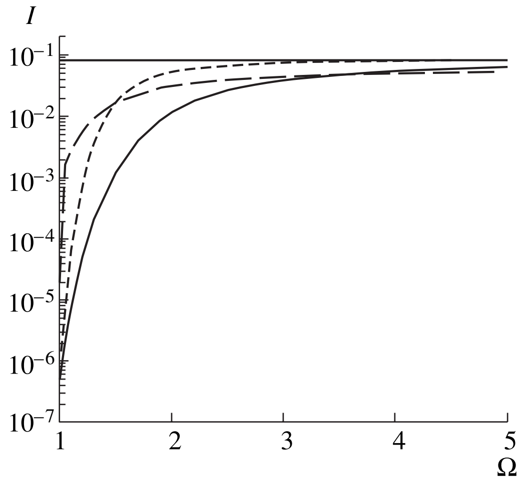


**Fig. 3.** Integral $I$ characterizing the effect of the angular energy distribution of waves versus the dimensionless frequency $\Omega$: model (6) with $\zeta = 1.3$ and 3 (the solid and short-dashed lines, respectively); model (7) (the long-dashed line).

the vicinity of $\Omega = 1$. At higher frequencies, as the roughness approaches the isotropic state, the integral $I$ should tend to its maximum value of $I = (4\pi)^{-1}$. For model (7), this condition is not satisfied. Model (7) is part of a more general model of the frequency–angular spectrum, which is constructed for the wave scales $\Omega \leq 3.5$ [11]; presumably, it is not correct to extend it to higher values of $\Omega$.

The results presented in Fig. 3 show that the angular energy distribution of wind waves in the vicinity of the spectral peak, i.e., on the scales where the major part of the wind wave energy is concentrated, should be studied in more detail. Depending on the model chosen for the function $\theta(\alpha)$, the integral $I$ may vary over more than one order of magnitude. Accordingly, as follows from Eq. (5), the pressure spectral density will also be different for different models.

At high frequencies, where the angular distribution is wider, the choice of the model and its parameters becomes less critical. In this region, to a first approximation, one can assume that $\theta(\alpha) = \text{const}$.

Now, let us determine the wave directions that make the greatest contribution to the sound generation. The contributions of individual components of the wave field are determined by the integrand in Eq. (9). Depending on the choice of the model of angular distribution, the values of the function $F(\alpha) = m(\alpha)\theta^2(\alpha)$ at the frequency of the dominant waves may differ by

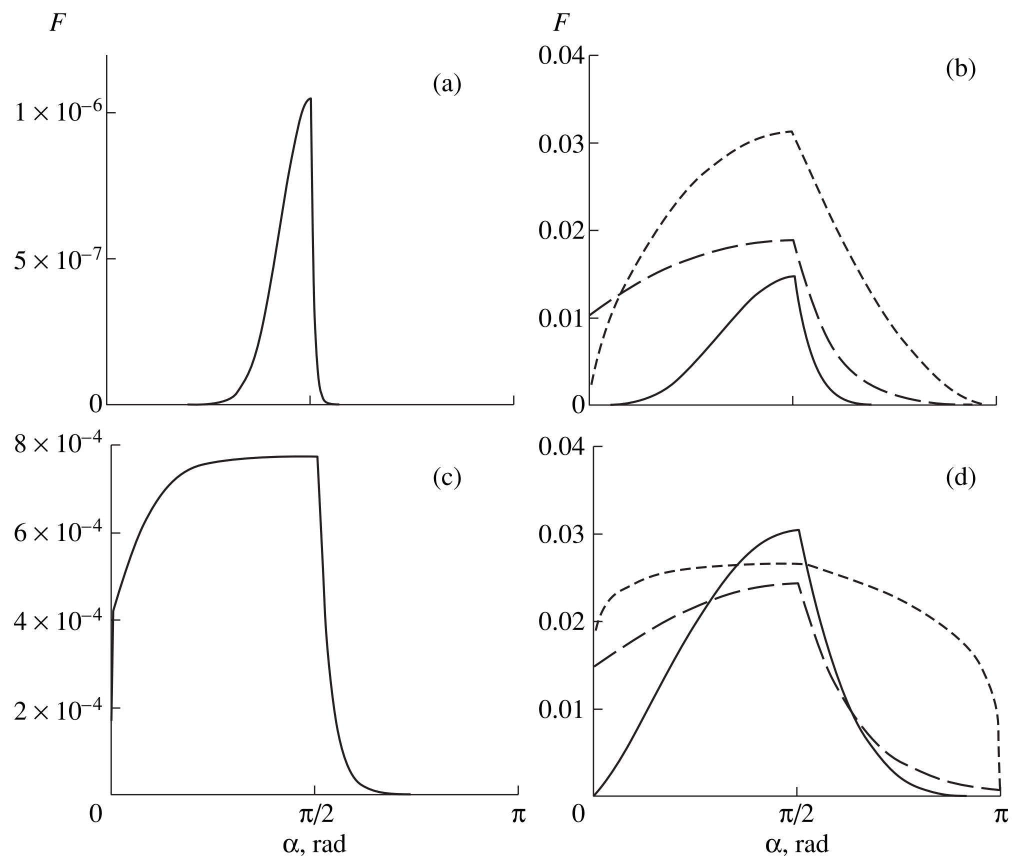


**Fig. 4.** Function $F(\alpha)$ at $\Omega = 1$ for (a) model (6) and (c) model (7) and at $\Omega$ = (b) 2 and (d) 3: model (6) with $\zeta = 1.3$ and 3 (the solid and short-dashed lines, respectively); model (7) (the long-dashed line).

almost two orders of magnitude (Fig. 4). At higher frequencies, the difference decreases. In the intermediate frequency range, between the dominant frequency, at which the angular distribution is narrow, and the high frequencies, at which the angular distribution is almost isotropic, the greatest contribution is made by the waves that travel in the direction close to that orthogonal to the wind.

Note that the above analysis refers to gravity waves. A relationship similar to Eq. (5) can also be obtained for gravity-capillary waves. Using the expression for the pressure spectrum reported in [4] and representing the frequency–angular spectrum of the wind waves in form (3), we obtain

$$S_p(\omega) = \frac{\pi\rho^2}{4C_S^2}\omega^2(5\omega^2 - kg)^2 S_\xi^2(\omega)\left(k\frac{dk}{d\omega}\right)I\Bigg|_{\omega^2 = gk + \gamma k^3}.$$

## CONCLUSIONS

The model proposed by L.M. Brekhovskikh [4] for the acoustic radiation generated by wind waves has undergone a further development. Formulas that directly relate the spectrum of acoustic radiation to the angular energy distribution function of wind waves are constructed.

Based on empirical wave energy distributions, it is shown that, in the region of the waves that carry the major portion of energy, the acoustic radiation intensity strongly depends on the choice of the model of the angular distribution function $\theta(\alpha)$ and on its parameters. At high frequencies, four to five times higher than the frequency of the dominant waves, one can assume that, to a first approximation, $\theta(\alpha) = \text{const}$.

In the frequency range between the dominant frequency, at which the angular distribution is narrow, and high frequencies, at which the angular distribution is almost isotropic, the predominant contribution to sound radiation is made by the waves that travel in the direction close to that orthogonal to the wind.

## REFERENCES

1. M. Garces, J. Aucan, D. Fee, et al., Geophys. Res. Lett. **33**, 05611 (2006).
2. A. N. Serebryanyĭ, A. V. Furduev, A. A. Aredov, and N. N. Okhrimenko, Dokl. Akad. Nauk **402**, 543 (2005) [Dokl. Phys. **52**, (2005)].
3. M. S. Longuet-Higgins, Philos. Trans. R. Soc. London, Ser. A **243**, 1 (1950).
4. L. M. Brekhovskikh, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **2**, 970 (1966).
5. K. A. Naugol'nykh and S. A. O. Rybak, Akust. Zh. **49**, 100 (2003) [Acoust. Phys. **49**, 88 (2003)].
6. M. L. Banner and I. R. Young, J. Phys. Oceanogr. **24**, 1550 (1994).
7. M. M. Zaslavskiĭ and E. G. Panchenko, Izv. Akad. Nauk, Fiz. Atmos. Okeana **31**, 282 (1995).
8. D. D. Crombie, K. Hasselmann, and W. Sell, Boundary-Layer Meteorol., No. 13, 45 (1978).
9. A. V. Babanin and Yu. P. Soloviev, Mar. Freshwater Res. **49**, 89 (1998).
10. D. E. Hasselman, M. Dunckel, and J. A. Ewing, J. Phys. Oceanogr. **10**, 56 (1980).
11. M. A. Donelan, J. Hamilton, and W. H. Hui, Philos. Trans. R. Soc. London, Ser. A **315**, 509 (1985).
12. M. L. Banner, J. Phys. Oceanogr. **20**, 966 (1990).

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