

Wave reflection in 3D conditions

Barbara Zanuttigh^{a,*}, Thomas Lykke Andersen^b

^a University of Bologna, DISTART, Viale Risorgimento 2, 40136 Bologna, Italy

^b Aalborg University, Dep. of Civil Engineering, Sohngårdsholmsvej 57, 9000 Aalborg, Denmark

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ABSTRACT

Based on recent experiments carried out in wave basin on breakwaters with armour layer of rocks and cubes, this paper examines the dependence of the reflection coefficient on wave directional spreading and obliquity. Results suggest that long-crested and short-crested waves give similar reflection. The reflection coefficient is markedly dependent on the wave angle of incidence. The performance of formulae available in the literature is checked against the new dataset and a significant improvement is proposed by including the wave obliquity factor that appears in the traditional expression for the overtopping discharge.

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1. Introduction

The prediction of wave reflection from coastal structures is of high practical importance due to its adverse effects such as dangerous sea states at harbor entrances and intensified sediment scour which can lead to structure destabilization.

It is common in engineering practice to characterize the magnitude of wave reflection by means of the reflection coefficient K_r , which is defined as the ratio of the reflected to the incident wave height. A considerable number of studies were carried out to investigate wave reflection of normally incident waves on both smooth and rough slopes. These previous studies were based on physical model tests in flumes for long-crested waves and provided semi-empirical formulae checked against limited datasets. Most of these formulae related the reflection coefficient K_r to the surf similarity parameter ξ only, e.g. Battjes (1974), Seelig and Ahrens (1981), Postma (1989). Few approaches proposed to consider different parameters, the more adopted alternative to the surf similarity parameter ξ being the relative water depth, i.e. water depth to wave length ratio (Muttray et al., 2006; Davidson et al., 1996; Calabrese et al., 2009).

The reflection behaviour for various types of straight slopes, such as smooth structures, rock slopes (permeable and impermeable core), slopes with all kind of artificial armour units, has been analysed in depth by Zanuttigh and Van der Meer (2006, 2008). The authors provided an extensive reflection database and developed a new

formula for all types of slopes in design conditions, based only on ξ and on the roughness factor in the overtopping discharge formula γ_f that is well known from specific research (Bruce et al., 2009; Overtopping Manual, 2007).

The assumption of normally incident waves is often violated in a natural coastal environment. Wave refraction over complex bathymetry may result in significantly oblique angles of incidence at the structure. Until now, little is known about the reflection of oblique short-crested waves. Only the works by Van der Meer et al. (2005) and more specifically by Wang et al. (2005) dealt with wave reflection under oblique and short-crested waves limitedly to low-crested structures.

The aims of this work are to analyze wave reflection from breakwaters in 3D conditions, under head-on and oblique waves and to verify the performance of existing formulae for predicting K_r with varying wave obliquity and directional spreading.

The paper presents the experimental dataset, together with the measurement techniques and data processing. The effects of wave spreading and wave obliquity on the reflection coefficient are examined. Then the performance of existing formulae is checked against the dataset, and an expression accounting for wave obliquity is proposed. Some conclusions are finally drawn to provide engineers with a synthesis useful for design purposes.

2. The experimental dataset

2.1. The facility

3D hydrodynamic tests were performed in the directional wave basin of the Hydraulics and Coastal Engineering Laboratory at Aalborg

* Corresponding author. Tel.: +3905102093754; fax: +390512093263.

E-mail addresses: barbara.zanuttigh@unibo.it (B. Zanuttigh), tl@civil.aau.dk (T.L. Andersen).

University, DK. The basin is 12 m long (waves direction), 17.8 m wide and 1.0 m deep (Fig. 1). The wave generator is a snake-front piston type paddle system composed of 25 actuators with stroke length of 1.2 m, enabling generation of short-crested waves. The software used for controlling the paddle system to generate waves is AwaSys developed by the same laboratory (Aalborg University, 2007a). No active absorption on the wave paddles was used, but passive absorption was placed at the rear end of the basin and at both sides of the basin. The absorbing sidewalls were made of crates (1.21×1.21 m, 0.70 m deep) filled with sea stones with $D_{50} = 5$ cm. The areas outside the crates were left empty in all the tests. The 1:5 sloping beach placed opposite to the wave maker was made of gravel with $D_{50} = 1.5$ cm. Regular and irregular short-crested waves with peak periods up to approximately 3 s, oblique 2D and 3D waves can be generated with good results.

2.2. The tests

Two typical breakwater cross-sections were tested, one with rock and one with cubes. Off-shore slope was 1:2, in-shore slope was 1:1.5. The thickness of the armour layer was approximately $2 \cdot D_{50}$ for the rock case and $2.25 \cdot D_{50}$ for the cubes. The crown width was approximately $3 \cdot D_{50}$. The scheme of the tested cross section and the characteristics of the material adopted are shown in Fig. 2 and in Table 1 which are reproduced from Lykke Andersen and Burcharth (2009).

Four crest freeboards in the range 0.07 m–0.16 m were tested by varying the water depth in the range 0.54–0.45 m. Wave attacks included irregular waves with 2D and 3D Jonswap spectrum. Tested wave heights were in the range 5.3–15.6 cm and peak periods within 0.8–2.0 s. The length of the test series was around 1500 waves. The waves were in all tests generated from the Jonswap spectrum with a peak enhancement factor of 3.3 using a white noise filtering method.

In case of short-crested waves the Longuet-Higgins et al. (1963) cosines spreading function with the following definition was used:

$$D(f, \theta) = \frac{2^{2s-1}}{\pi} \cdot \frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \cdot \cos^{2s}\left(\frac{\theta-\theta_0(f)}{2}\right). \quad (1)$$

Two values of the spreading parameter s were tested (5 and 10) corresponding to a standard deviation of 34.5° and 25.0° respectively. The tested directional spreading represents the conditions close to the structure. In deep water the directional spreading would be higher due to refraction effects when traveling into shallower water. Tested conditions are listed in Table 2.

The generation of short-crested waves with an angle other than perpendicular to the wave maker leads to an asymmetrical spreading function and/or asymmetrical amount of spurious waves. As a consequence in order to test a large range of wave obliquity the structure was built in the basin at three different angles (see Fig. 1) with respect to the shoreline (and to the wavemaker): 5° , 25° and 52.5° . The configurations with the structure inclined at 5° and 52.5° allowed to examine wave obliquities between 0° and 10° and between 45° and 60° respectively. The configuration with the structure inclined at 25° was tested under perpendicular waves only.

No refraction and shoaling of waves on the foreshore took place as a flat bottom was used in all tests. The flat bottom limits testing to non-breaking waves on the foreshore.

2.3. Wave measurements and data processing

Tests were finalized to obtain wave overtopping discharge: the interested reader may refer to Lykke Andersen and Burcharth (2009).

Waves were measured with an array of seven gauges placed just in front of the structure, see Fig. 3. When testing 45° and 60° wave attack the model was close to the generator due to length limitations of the

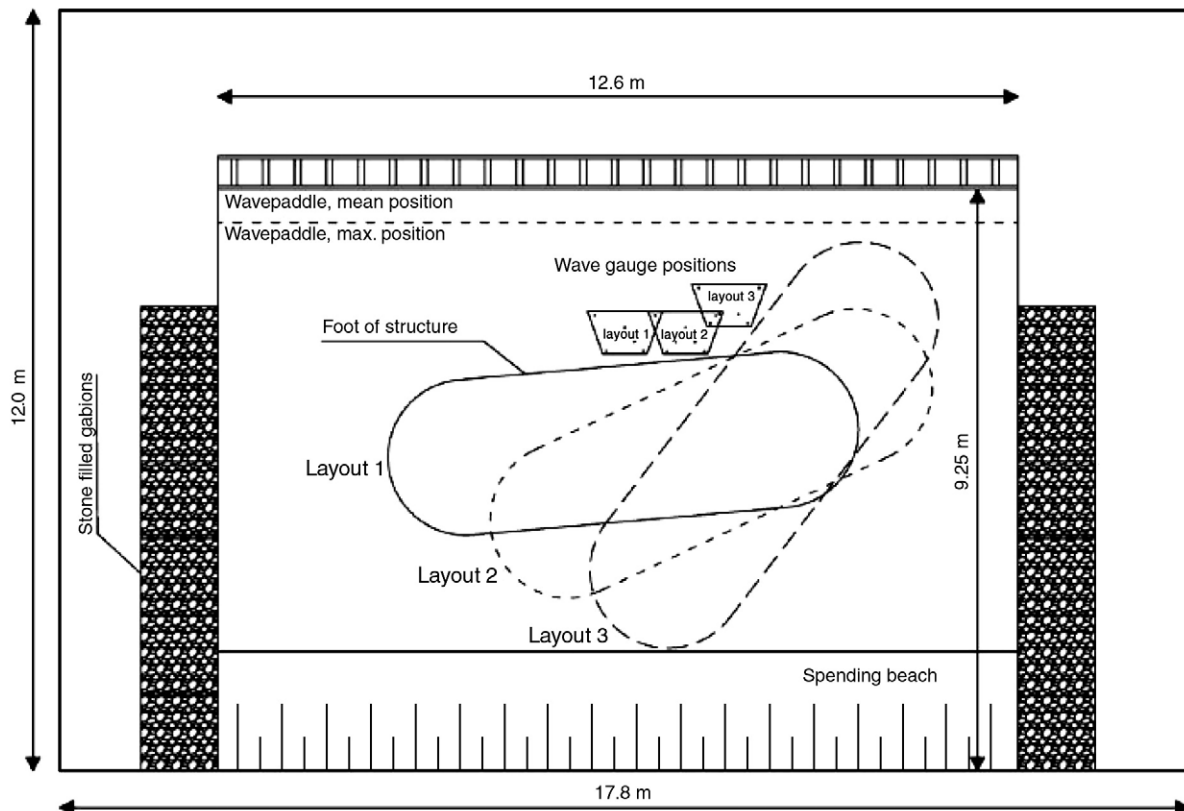


Fig. 1. Top view of the basin with position of the gauges and of the structure in the three tested configurations. From Lykke Andersen and Burcharth (2009).

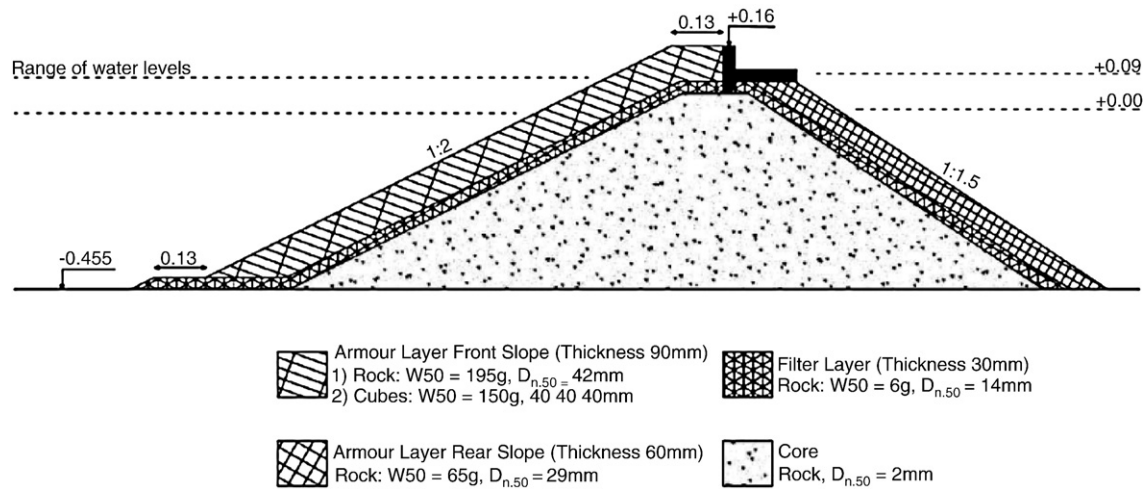


Fig. 2. Model cross-section with indication of layers and corresponding materials, measures in m where not specified. From Lykke Andersen and Burcharth (2009).

basin but standing waves did not develop due to the angle of the structure. In this setup the distance from the generator to the closest point of the structure toe was 1.3 m, which corresponded approximately to two times the water depth. The distance from the generator to the centre of the wave gauge array was approximately 1.8 m. In case of the 60° wave attack the position of the wave gauge array might thus lead to an underestimation of the reflection coefficient.

Resistance type wave gauges with nearly linear response of output voltage versus water level were used and performed properly during the experiments. All the wave gauges were calibrated every morning and repeatedly several times during the day and always whenever the water level was changed.

All signals were filtered using an analog low-pass filter with a cut-off frequency of 8 Hz. A digital filter with cut-off frequencies equal to $1/3 \cdot f_p$ and $3 \cdot f_p$ was applied to the wave signals, being f_p the peak wave frequency.

Directional analysis of the acquired data was performed with the WaveLab 2.94 software package (Aalborg University, 2007b) by means of the Bayesian Directional Spectrum Estimation Method (Hashimoto, 1988) which was proven to be accurate for a number of gauges exceeding 4 (Martinelli et al., 2003).

Fig. 4 compares the reflection coefficients measured in these tests with the data obtained for straight slopes in wave flumes, i.e. the data included in the homogenous reflection database by Zanuttigh and Van der Meer (2006). The data obtained in these new tests for both rock and cubes fall well within the corresponding database for rocks and armour units. The data cloud is indeed quite large: this can be explained because of measurement errors and/or differences in data processing but can also be due to the introduction of the Iribarren parameter ξ . In fact the use of ξ allows to incorporate different structure slopes but introduces some scatter since from the work by Postma (1989) it is well known that the wave period has more influence than wave height on the reflection behaviour.

3. Analysis of the reflection coefficient in 3D conditions

During storms, some waves will approach the structure perpendicularly; others will approach the structure obliquely. Based on the experimental findings for wave transmission by Van der Meer et al. (2005), wave reflection of a short-crested wave may be similar to that of a long-crested wave in case of a wave attack perpendicular to the structure ($\beta = 0^\circ$). The question to consider here is whether similar reflection results can still be found for the oblique waves.

Wave reflection coefficients for long-crested and short-crested waves are plotted in Fig. 5. If one directly compares the measured

values for the same angle of wave attack, long-crested waves result globally in lower values of K_r than short-crested waves, for both rocks and cubes. Indeed this difference is marginal and in the range of measured errors; it should be noted, however, that for the same directional spreading this difference is greater in cubes than in rocks. This fact may be explained with the different material composing the structure slope interacting with waves: the sharper shape of cubes can increase the differences in the reflection phase of waves hitting one side or one corner of the element and at the same time the smoother surface of cubes can reduce wave dissipation and consequently lead to an increase of the reflection intensity.

Maximum scatter can be seen both for rocks and cubes when $\beta = 25^\circ$, condition that may be related to the placement of the structure and measurement system in the wave basin.

In case of rocks, for which different spreading were tested, K_r tends to decrease with the increase of the directional spreading parameter s (and thus decrease of the standard deviation from mean direction).

Let us now consider the effect induced on the average reflection coefficient K_{rm} from wave obliquity β . Figs. 6 and 7 show long-crested and short-crested data together as a function of β . Void points are measurements (lighter grey for short-crested waves), solid points (black and white) are the average values of K_r for a given β , i.e. K_{rm} . For both rocks and cubes, K_{rm} shows the tendency to decrease with increasing β , following approximately a cosine function. This function was selected based on the findings by Wang et al. (2005) for rubble mound low-crested structures. Indeed the decrease of K_{rm} with β is less rapid for cubes than rocks.

The dependence of K_{rm} on β as indicated by this bulk analysis can be expressed as

$$K_{rm}(\beta) = K_{rm}(\beta = 0^\circ) \cdot \cos \beta, \quad \text{for rocks, if } 0 \leq \beta \leq 60^\circ. \quad (2)$$

$$K_{rm}(\beta) = K_{rm}(\beta = 0^\circ) \cdot \cos \beta^{2/3}, \quad \text{for cubes, if } 0 \leq \beta \leq 60^\circ. \quad (3)$$

Table 1

Armour material properties. From Lykke Andersen and Burcharth (2009).

	Armour rock	Armour cubes
Weight W_{50} [kg]	0.228	0.146
Density ρ [kg/m ³]	3060	2280
Nominal diameter $D_{n,50}$ [m]	0.042	0.040
$f_g = D_{n,85}/D_{n,15}$	1.30	1.00
Length ratio, l/b	1.96	1.00

Table 2

Overview of the wave attacks tested in the wave basin. H_s is the target significant wave height, R_c is the structure freeboard, T_p is peak wave period, h is water depth at the wave maker, s is the spreading parameter in Longuet-Higgins et al. (1963) distribution, Eq. (1).

MWD, °	s	R_c , m	h , m	#	H_s , m	T_p , s
0	None	0.07	0.545	22	0.053–0.108	0.91–1.90
0	None	0.10	0.515	20	0.069–0.129	0.98–1.90
0	None	0.13	0.485	21	0.072–0.138	0.98–1.97
0	None	0.16	0.455	15	0.085–0.140	1.14–1.90
0	5	0.07	0.545	22	0.051–0.105	0.98–1.83
0	5	0.10	0.515	15	0.064–0.123	1.00–1.65
0	5	0.13	0.485	20	0.078–0.125	1.11–1.97
0	5	0.16	0.455	14	0.086–0.134	1.11–1.83
0	10	0.07	0.545	0	–	–
0	10	0.10	0.515	8	0.070–0.127	1.00–1.42
0	10	0.13	0.485	11	0.080–0.126	1.02–1.90
0	10	0.16	0.455	7	0.087–0.136	1.14–1.90
10	None	0.07	0.545	12	0.057–0.107	0.88–1.83
10	None	0.10	0.515	8	0.082–0.128	1.11–1.38
10	None	0.13	0.485	12	0.081–0.138	1.11–1.90
10	None	0.16	0.455	7	0.091–0.134	1.14–1.83
10	5	0.07	0.545	22	0.048–0.106	0.93–1.77
10	5	0.10	0.515	16	0.077–0.126	1.11–1.60
10	5	0.13	0.485	18	0.074–0.127	0.98–1.90
10	5	0.16	0.455	14	0.087–0.139	1.11–1.90
10	10	0.07	0.545	0	0.000–0.000	0.00–0.00
10	10	0.10	0.515	6	0.077–0.126	1.11–1.83
10	10	0.13	0.485	0	–	–
10	10	0.16	0.455	0	–	–
25	None	0.07	0.545	21	0.065–0.117	0.95–1.90
25	None	0.10	0.515	18	0.081–0.138	1.05–1.71
25	None	0.13	0.485	21	0.079–0.139	1.07–2.05
25	None	0.16	0.455	17	0.090–0.151	1.09–1.77
25	5	0.07	0.545	21	0.062–0.118	1.02–1.83
25	5	0.10	0.515	17	0.071–0.129	1.14–1.77
25	5	0.13	0.485	19	0.082–0.139	1.11–2.05
25	5	0.16	0.455	16	0.085–0.148	1.11–1.90
25	10	0.07	0.545	0	–	–
25	10	0.10	0.515	0	–	–
25	10	0.13	0.485	0	–	–
25	10	0.16	0.455	0	–	–
45	None	0.07	0.545	27	0.070–0.121	0.97–1.90
45	None	0.10	0.515	18	0.093–0.142	1.16–1.90
45	None	0.13	0.485	18	0.092–0.151	1.16–1.90
45	None	0.16	0.455	12	0.110–0.164	1.25–1.97
45	5	0.07	0.545	25	0.065–0.116	1.00–1.90
45	5	0.10	0.515	19	0.079–0.138	1.19–1.83
45	5	0.13	0.485	19	0.095–0.147	1.22–1.97
45	5	0.16	0.455	17	0.103–0.153	1.28–1.97
MWD, °	s	R_c , m	h , m	#	H_{m0} , m	T_p , s
45	10	0.07	0.545	0	–	–
45	10	0.10	0.515	0	–	–
45	10	0.13	0.485	9	0.104–0.146	1.22–1.90
45	10	0.16	0.455	0	–	–
60	None	0.07	0.545	26	0.070–0.130	0.91–1.83
60	None	0.10	0.515	15	0.100–0.153	1.22–1.71
60	None	0.13	0.485	11	0.124–0.156	1.22–1.97
60	None	0.16	0.455	2	0.148–0.152	1.38–1.90
60	5	0.07	0.545	26	0.064–0.131	1.00–1.90
60	5	0.10	0.515	18	0.092–0.148	1.22–1.71
60	5	0.13	0.485	15	0.104–0.143	1.19–1.90
60	5	0.16	0.455	12	0.113–0.149	1.35–1.77
60	10	0.07	0.545	0	–	–
60	10	0.10	0.515	0	–	–
60	10	0.13	0.485	6	0.117–0.144	1.35–1.90
60	10	0.16	0.455	1	0.137	1.90

4. Prediction of the reflection coefficient in 3D conditions

For the purpose of comparing measurements and predictions, the following formulae were selected among others: Postma (1989), calibrated for rubble mound breakwaters with an impermeable core (Van der Meer, 1988); Zanuttigh and Van der Meer (2006, 2008), tested against a wide variety of slopes in design conditions ($R_c/H_s \geq 0.5$,

$H_s/D_{50} \geq 1$, $s_0 \geq 0.01$); Muttray et al. (2006), which was checked against experiments on a typical breakwater cross-section with an armour layer of accropodes; Calabrese et al. (2009), developed for rock permeable structures characterized by any kind of submergence. The formulae are recalled for convenience in Table 3.

It is worthy to note that the wave length appearing in the formula and by Calabrese et al. (2009), Calabrese et al. (2009) and by Muttray et al. (2006) was evaluated following the linear dispersion relation.

Another important remark is that the formula by Zanuttigh and Van der Meer (2006) was applied to rocks and to cubes by adopting the measured values of $\gamma_f = 0.39$ and $\gamma_f = 0.40$ respectively (Lykke Andersen and Burcharth, 2009) instead of the values of $\gamma_f = 0.55$ and $\gamma_f = 0.47$ suggested by Bruce et al. (2006, 2008) for two layers of rocks or cubes over an almost impermeable core.

Finally the results reported for Calabrese et al. (2009) do not account for the notional permeability P because the inclusion of $P = 0.4$ for a structure with armour, filter and core (TAW, 2002) leads to less accurate results. This can be explained because of the measured high permeability of the structure: the measured values of $\gamma_f = 0.39/0.40$ indeed are equal to the value of $\gamma_f = 0.40$ that literature suggests for rock homogenous structures (TAW, 2002; Overtopping Manual, 2007).

The results of the predictions are reported in Table 4 for rocks and in Table 5 for cubes. No one of the formulae available in the literature for predicting K_r can accurately represent the measured values if it is not properly accounted for the influence of wave obliquity and in minor part also for wave spreading.

It is proposed in the following to modify the existing formulae by including the obliquity factor that appears in the overtopping discharge formula γ_b (Overtopping Manual, 2007). This idea is based on the strict relation that physically exists between wave reflection and wave overtopping (i.e. the greater the reflection the lower the overtopping) as already proven by Zanuttigh and Van der Meer (2006, 2008).

The expression here adopted for the obliquity factor γ_b is given by Eqs. (9) and (10) in Lykke Andersen and Burcharth (2009) for long-crested waves and short-crested waves. By accounting for these equations, the predicted K_r is thus calculated as

$$K_r(\beta, s) = K_r(\beta = 0^\circ) \cdot \gamma_b = K_r(\beta = 0^\circ) \cdot (1 - 0.0077 \cdot \beta) \quad (4)$$

for long-crested waves and

$$K_r(\beta, s) = K_r(\beta = 0^\circ) \cdot \gamma_b = K_r(\beta = 0^\circ) \cdot (1 - 0.0058 \cdot \beta) \quad (5)$$

for short-crested waves respectively.

Results can be found again in Tables 4 and 5 for rocks and cubes.

Fig. 8 compares measurement and computations obtained from Zanuttigh and Van der Meer (2006) including Eqs. (4) and (5).

All the selected formulae, if properly corrected as suggested above, provide reasonable results. Indeed the formula of Calabrese et al. (2009) does not show any relevant dependence on wave obliquity and spreading.

More specifically, the overall performance of the formulae modified as in Eqs. (4) and (5) is described in terms of the rms error E_{rms} :

$$E_{rms} = \left[\sum_{k=1}^N (Xc_k - Xm_k) / N \right]^{0.5} \quad (6)$$

and of the Wilmott (1981) index

$$I_W = 1 - \frac{\sum_{k=1}^N (Xc_k - Xm_k)^2}{\sum_{k=1}^N [|Xc_k - \bar{Xm}| + |Xm_k - \bar{Xm}|]^2} \quad (7)$$



Fig. 3. Layout in shallow water wave basin at Aalborg University for testing waves with wave obliquity equal to 0° and 10°. From Lykke Andersen and Burcharth (2004).

where X_c and X_m are the computed/estimated and measured values respectively, and the overbar denotes the average. If I_w equals 1 there is a perfect agreement among computations/estimations and measurements whereas if I_w equals 0 there is no match.

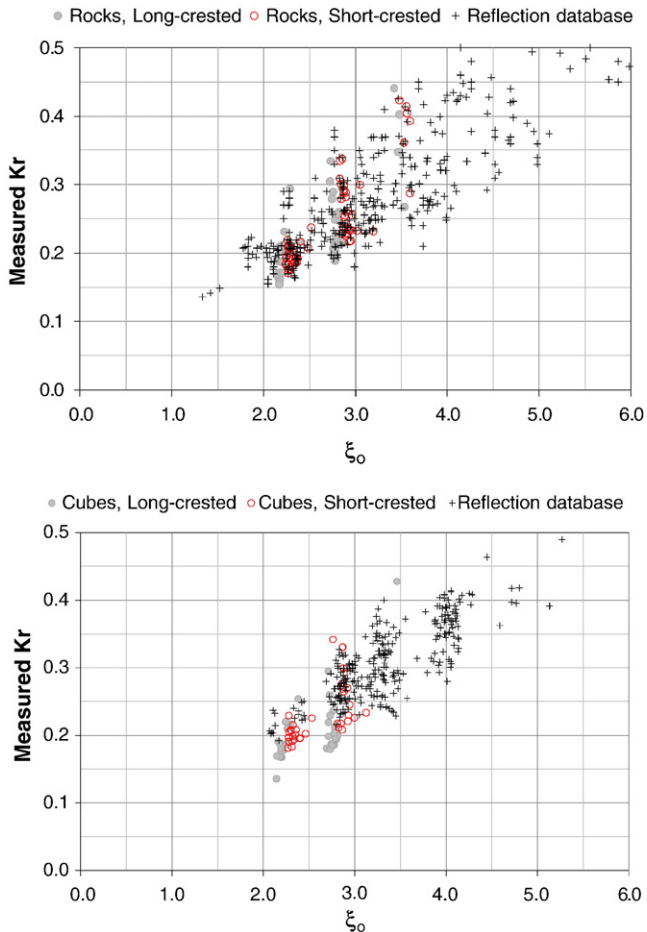


Fig. 4. The reflection coefficients obtained from rocks (at the top) and cubes (at the bottom) are compared respectively with the data for rock and armour unit contained in the reflection database (Zanuttigh and Van der Meer, 2006).

By looking at Tables 4 and 5 one can synthetically observe that

- Postma (1989) overestimates the measured values of K_r , leading to an E_{rms} of 6.04% for rocks and 6.26% for cubes (for which the scatter is greater),

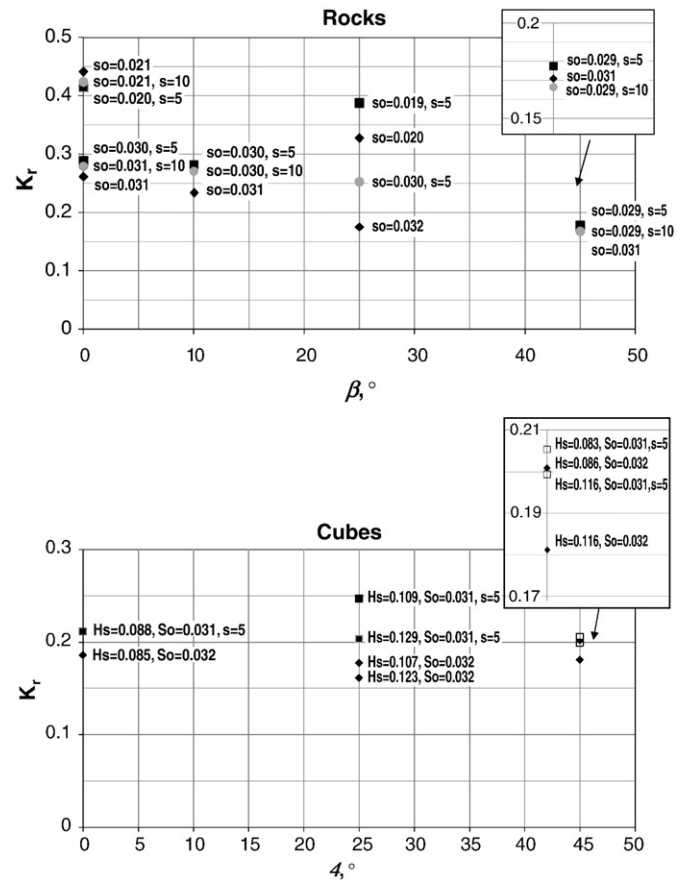


Fig. 5. Comparison of the reflection coefficient obtained in similar hydrodynamic conditions for long and short-crested waves with varying wave obliquity; rocks at the top and cubes at the bottom. Target significant wave height for rocks is equal to $H_s = 0.11$ m.

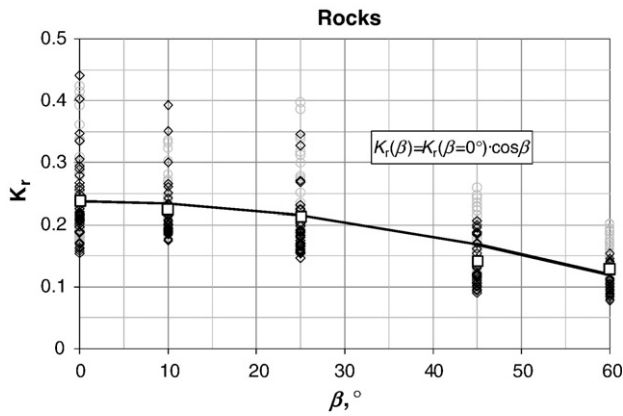


Fig. 6. Dependence of wave reflection from wave obliquity for rocks, long-crested (void dark-grey diamonds) and short-crested (void light-grey circles) waves. The squares (black contour and white filling) are the average values of the reflection coefficient K_{rm} for a given wave obliquity. The solid line is the cosine function.

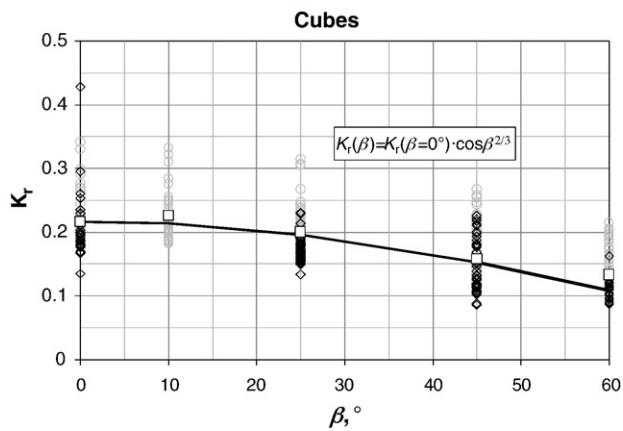


Fig. 7. Dependence of the wave reflection coefficient from wave obliquity for cubes, long-crested (void dark-grey diamonds) and short-crested (void light-grey circles) waves. The squares (black contour and white filling) are the average values of the reflection coefficient K_{rm} for a given wave obliquity. The solid line is the cosine function to the power of 2/3.

- **Muttray et al. (2006)** slightly underestimate the measured values of K_r , providing a very good accuracy both for rocks and cubes (E_{rms} equal to 3.74% and 3.46% respectively),
- **Zanuttigh and Van der Meer (2006)** tend to overestimate K_r and provide an accuracy similar to **Muttray et al. (2006)**, both for rocks and cubes (being E_{rms} equal to 4.06% and 4.28% respectively),

Table 3

Formulae selected for the prediction of wave reflection coefficient K_r . γ_f is the roughness factor in the overtopping discharge formula, h is water depth at the structure toe, H_s is the significant wave height, L_{op} is the deep water wave length based on peak wave period, D_{50} is the average stone diameter, ξ_0 is the surf similarity parameter based on spectral wave period ($T_m - 1.0$), α is the structure off-shore slope, R_c is the structure freeboard.

Reference	K_r
Postma (1989)	$0.15 \cdot \xi_0^{0.73}$
Muttray et al. (2006) for porous structures	$1 / (1.3 + 3h \cdot 2\pi / L_{op})$
Muttray and Oumeraci (2002), for impermeable slopes	$1 - (H_s / H_{s,crit})^{1.5} \cdot (1 - 2 / \pi) \quad \text{if } H_s / H_{s,crit} < 1$ $2 / \pi \cdot (H_s / H_{s,crit}) \quad \text{if } H_s / H_{s,crit} \geq 1$ $H_s / H_{s,crit} = L_{op} \cdot \sqrt{(2\alpha / \pi) \cdot \sin^2 \alpha} / \pi$
Zanuttigh and Van der Meer (2006)	$\tanh(a \cdot \xi_0^b)$ $a = 0.167 \cdot [1 - \exp(-3.2 \cdot \gamma_f)], \quad b = 1.49 \cdot (\gamma_f - 0.38)^2 + 0.86$
Calabrese et al. (2009)	$K_{r0} + r$ $r = 0.44 \cdot \tan \alpha \cdot \exp(-30.82 \cdot H_s / L_{op} - 1.3 \cdot P) \tanh(R_c / H_s)$ $K_{r0} = 6.35 \cdot \exp[1.85 \cdot \tan \alpha - 5.34 \cdot (h / L_{op})^{0.1}] - 0.28 \cdot \tan \alpha^{2.29}$ $0.0027 \cdot L_{op} / D_{50} \text{ if}$ $\text{Linear interpolation}$

Table 4

Overall performance (rms error and Willmot index) of the selected formulae for rocks by excluding (left value in each column) and including (right value in each column) γ_b as multiplier factor. The labels “LC” and “SC” denote respectively long and short-crested waves. Note that in **Calabrese et al. (2009)** the notional permeability is not included (the term containing P in the formula in **Table 3** is not included).

#		Postma (1989)		Muttray et al. (2006)		Zanuttigh and Van der Meer (2006)		Calabrese et al. (2009)	
		With γ_b		With γ_b		With γ_b		With γ_b	
E_{rms}									
183	LC	0.129	0.069	0.083	0.043	0.097	0.046	0.060	0.052
233	SC	0.107	0.054	0.065	0.033	0.079	0.036	0.047	0.048
416	Tot	0.116	0.060	0.073	0.037	0.087	0.041	0.053	0.050
I_W									
416	Tot	0.484	0.743	0.670	0.871	0.720	0.807	0.723	0.673

- **Calabrese et al. (2009)** slightly underestimate the measured values and essentially do not improve by including the expression of γ_b . The E_{rms} is 4.97% for rocks and of 4.78% for cubes.

Let us draw some further comments on the formulae here compared. **Postma (1989)** and **Zanuttigh and Van der Meer (2006)** both adopt the breaker parameter. Postma’s formula always leads to overestimation of K_r and even for rocks results in a greater scatter than **Zanuttigh and Van der Meer**.

The formula by **Muttray et al. (2006)** is a very simple formula, being based on water depth and wave length only, and thanks to the inclusion of γ_b fits very well the measured values of K_r . **Muttray et al. (2006)** and **Zanuttigh and Van der Meer (2006)** provide almost the same values of I_W , being both around 0.8 for cubes and in the range 0.8–0.9 for rocks.

The formula by **Calabrese et al. (2009)** underestimates K_r for the present structures and it shows a behaviour different from all the other formulae, providing almost the same accuracy with and without the corrections given by Eqs. (4) and (5).

The analysis performed so far shows that the formulae by **Zanuttigh and Van der Meer (2008)** and by **Muttray et al. (2006)** provide similar accuracy even if the first one is centred on structure roughness (through the factor γ_f) whereas the latter does not consider this effect. This fact can be explained because of the very similar value of γ_f for rocks and cubes (0.39 and 0.40 respectively) derived from the measurements by **Lykke Andersen and Burcharth (2009)**. Indeed, the performance of the formula by **Muttray et al. (2006)** resulted not sufficient when applied to the various straight and composite slopes included in the reflection database (**Zanuttigh et al., 2008**).

Table 5

Overall performance (rms error and Willmot index) of the selected formulae for cubes by excluding (left value in each column) and including (right value in each column) γ_b as multiplier factor. The labels “LC” and “SC” denote respectively long and short-crested waves. Note that in Calabrese et al. (2009) the notional permeability is not included (the term containing P in the formula in Table 3 is not included).

		Postma (1989)		Muttray et al. (2006)		Zanuttigh and Van der Meer (2006)		Calabrese et al. (2009)	
#		With γ_b		With γ_b		With γ_b		With γ_b	
E_{rms}									
140	LC	0.139	0.070	0.087	0.038	0.109	0.049	0.055	0.044
180	SC	0.123	0.057	0.066	0.032	0.082	0.038	0.044	0.051
320	Tot	0.130	0.063	0.054	0.035	0.094	0.043	0.049	0.048
I_W									
320	Tot	0.418	0.689	0.592	0.796	0.649	0.803	0.605	0.651

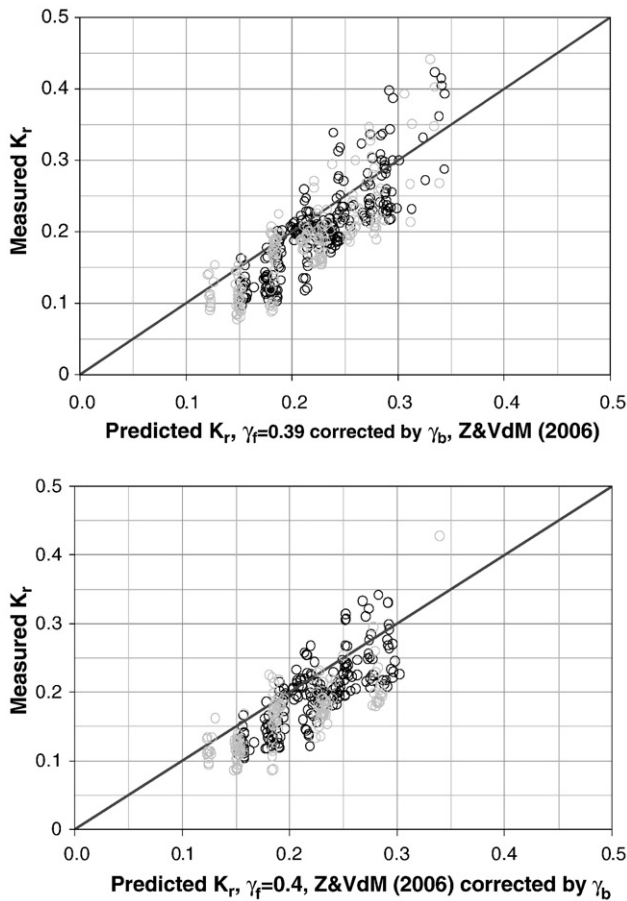


Fig. 8. Comparison among measured values of the reflected coefficient and predictions by Zanuttigh and Van der Meer (2006), at the top for rocks and at the bottom for cubes. Black circles correspond to short-crested waves and grey circles to long-crested ones.

5. Conclusions

This paper described the effects of wave obliquity and spreading on wave reflection from an experimental dataset of 736 tests performed in wave basin. The tests were carried out on breakwaters with a fixed typical cross-section and two types of armour layers: rocks and cubes.

Differences in wave reflection under similar tested conditions for short-crested and long-crested waves are more marked for cubes than for rocks, but indeed are small and comparable to measurement errors. Long-crested waves produce lower values of the reflection coefficient K_r than short-crested ones.

Both for rocks and cubes the average wave reflection coefficient K_{rm} for a given obliquity β shows a tendency to decrease with increasing β . For rocks, the decrease can be represented through a cosine function as already found for permeable low-crested structures by Wang et al. (2005), Eq. (2). For cubes, the decrease is less rapid than for rocks and can be approximated through a cosine function to the power of 2/3, Eq. (3). Eqs. (2) and (3) can represent the trend of K_{rm} , providing that $K_{rm}(\beta=0^\circ)$ is known.

Few formulae existing in the literature for predicting the reflection coefficient were selected to be checked against the 3D dataset: Postma (1989), Zanuttigh and Van der Meer (2006), Muttray et al. (2006) and Calabrese et al. (2009). The correction of the predictions by means of the obliquity factor γ_b , Eqs. (4) and (5) where γ_b is given by the expressions derived by Lykke Andersen and Burcharth (2009), provides a substantial improvement of the performance of almost all formulae. The formulae by Muttray et al. (2006) and Zanuttigh and Van der Meer (2006) with the inclusion of γ_b result in a particularly accurate prediction of K_r , being E_{rms} around 4% and I_W in the range 0.8–0.9.

Notations

D_{50}	nominal rock diameter or typical armour unit size
E_{rms}	rms error
H_s	significant wave height at the structure toe
h	water depth at the structure toe
I_W	Willmot index
K_r	reflection coefficient
K_{rm}	average values of K_r for a given wave obliquity
L_{op}	deep water wave length based on peak wave period
R_c	structure freeboard
s_o	wave steepness based on wave spectral period at the structure toe
$T_m - 1.0$	wave spectral period at the structure toe
α	off-shore structure slope
γ_b	obliquity factor in the overtopping discharge formula
γ_f	roughness factor in the overtopping discharge formula
ξ_o	breaker parameter based on s_o

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