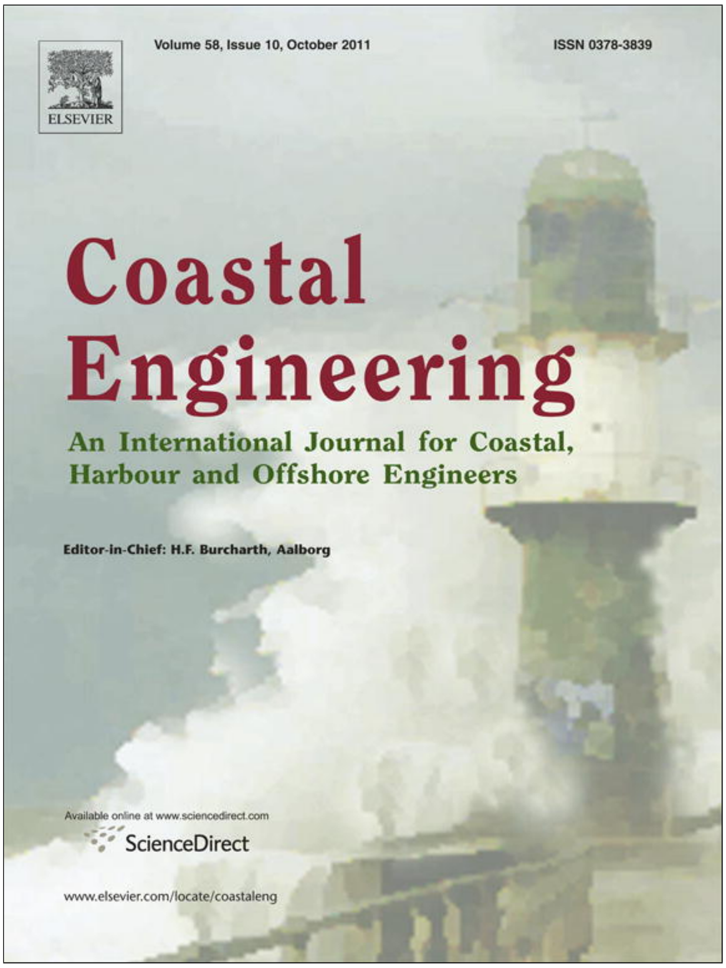
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Equilibrium scour depths around piles in noncohesive sediments under currents and waves

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ABSTRACT

A universal formula for the estimation of equilibrium scour depth around a single cylindrical pile under the action of steady currents, tidal and short waves is presented.

Following a dimensional analysis, the controlling parameters of scour was found to be the Keulegan–Carpenter number and the ratio between the flow velocity and the critical velocity for the initiation of motion of the sediment particles, u/u_c. Based on this finding a new formula for the prediction of equilibrium scouring depths was derived in this paper.

Measurements of the scouring process and the action of waves and/or currents from literature are used with KC and u/u_c ranging between 6 and 10^5, and 0.6 and 4.5, respectively for the parameterization and verification of the newly proposed formula.

The predicted scour depths and the investigated measurements show clearly the influence of the bed properties which have not been reported before in the literature. Predictions using the new approach have been compared with previous measurements and other formulations presented in the specialized literature. The proposed empirical formula accounts for the effect of waves, currents and sediment characteristics on the scouring process and shows better results than previous ones.

1. Introduction

Scouring is one of the main failures of water works in inland and coastal waters. It occurs if the critical flow condition for the initiation of sediment motion at the pile foundation is exceeded by the ambient current. In case of cylindrical piles this occurs when the undisturbed upstream current depth-averaged velocity exceeds about 50% of the critical velocity.

In the specialized literature, numerous investigations have been presented on scour at cylindrical bridge piers in steady currents, distinguishing between clear-water and live-bed scour in sands and gravels. Scour formulas have been proposed for estimation of the maximum scour depth (Breusers et al., 1977; Dey, 1999; Melville and Sutherland, 1988; Richardson, 1987; Zanke, 1982, a.o.) and of time-dependent maximum scour depth (Dey, 1999; Melville and Chiew, 1999; Mia and Nago, 2003; Oliveto and Hager, 2002; Yanmaz and Altinbilek, 1991; Zanke, 1982). Others concentrated on the flow field around the cylinder, via experimental measurements (Dey and Raikar, 2007; Graf and Istiarto, 2002; Graf and Yulistiyanto, 1998; Unger and Hager, 2007) and numerical experiments (Gobert et al., 2010; Kirkil et al., 2008, 2009; Rolound et al., 2005). Link et al. (2008) and Diab et al. (2010) presented detailed measurements of developing and equilibrium scour holes around cylinders in fine sand and gravel, respectively. Scour manuals and design guidelines offer a comprehensive description of scour at cylinders in steady currents (e.g. Breusers and Raudkivi, 1991; Hoffmans and Verheij, 1997; Lagasse et al., 2001; May et al., 2002; Melville and Coleman, 2000; Richardson and Davis, 2001).

For the estimation of scour depth around piles in coastal waters, in addition to currents, wave action must also be considered. However, for tidal currents as well as currents and waves (Das, 1970; Dey et al., 2006; Kawata and Tsuchia, 1988; Sumer and Fredsoe, 2001a,b; Sumer et al., 1992, 2007; Zanke, 1982), a lack of research on scouring is detected.

In this paper the datasets published by Zanke (1982), Sumer et al. (1992) and Prepernau et al. (2007) are revisited for the development of a new design formula for estimation of equilibrium scour depth around a single pile under the action of current and waves. First, the controlling parameters of equilibrium scour around piles under the action of current and waves are presented and discussed. Next, previous experimental results are summarized for the development of a new unifying formula for estimation of equilibrium scour depth. Finally, performance of the new formula is evaluated by comparing its

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predictions with well documented measurements and prediction of other formulas.

2. Controlling parameters of equilibrium scour depth

Equilibrium scour depth around piles is expected to be controlled by fluid (density ρ_{fluid}, gravity g, viscosity ν), flow (velocity u, depth h, gravity g, wave period T), sediment (density ρ_{sediment}, particle diameter d, critical velocity for initiation of motion u_c), and pile (diameter D). Previous investigations (Dey et al., 2006; Sumer and Fredsoe, 2002; Sumer et al., 1992; Zanke, 1982) consistently reported the existence of a transition range between the current and the wave scour. The ranges have been defined in terms of the Keulegan–Carpenter number. In case of unidirectional flows and long period of alternating flows, i.e. when KC>100, the horseshoe vortex controls the scouring process. Vortex shedding becomes of importance when 6<KC<100. With decreasing KC the effect of the vortex shedding increases and that of the horseshoe vortex decreases. The formation of the horseshoe vortex is suppressed when KC <6 (Sumer et al., 1997). When sediment sizes are in the very fine range, lee side cyclones might uplift sediment particles into suspension. In dimensionless form, the functional relation between relative equilibrium scour depth H/D and the other parameters is:

$$\frac{H}{D} = f\left(\frac{u}{u_c}\right) \tag{1}$$

in case of steady currents and, when also or only waves are present

$$\frac{H}{D} = f\left(\frac{d_0}{D}, \frac{u}{u_c}\right) \tag{2}$$

where d_0 is the near bed horizontal water displacement during a half wave period, i.e. the double horizontal wave orbital amplitude. As d_0/D can also be expressed by the so called Keulegan–Carpenter number, also holds

$$H/D = f(KC, u/u_c) \tag{2a}$$

where u/u_c can also be replaced by u^*/u_c^*.

According to Hjulström curve (1935), the depth averaged critical velocity for initiation of motion is expressed as (Zanke, 1978):

$$u_c = 1.4\left(2\sqrt{\rho' g d} + 10.5\frac{\nu}{d}\right) \tag{3}$$

where $\rho' = (\rho_s - \rho)/\rho$ is the relative density. The critical velocity at any distance from the bed may be calculated using the logarithmic velocity distribution law. According to sediment transport theory, the bed load capacity is related to $(u^{*2} - u_c^{*2})^{\alpha}$ where α is an exponent between 1.5 and 2.0. As example, Meyer-Peter and Mueller (1948) suggest a value of $\alpha = 1.5$. Note that for a given KC the relative flow strength u/u_c must have remarkable effects on the scouring process in steady flow as well as under waves.

3. Previous experimental work

3.1. Experiments by Zanke (1982)

Experiments by Zanke (1982) were carried out in a recirculating laboratory flume at the Franzius Institute, University of Hannover, Germany. The flume was approximately 50 m long, 0.6 m wide and 1.25 m deep. It could be run with unidirectional flow and/or with alternating, oscillatory flow generated by 4 layers of propeller pumps which were controlled electronically. Also the measuring device was computer controlled. Scour depth was measured in real time during full flow action by a bottom follower which held a constant distance to the actual sediment bed. Scour around two piles was investigated with D = 6.5 and 9 cm in two sediment materials, quartz sand with a mean diameter $d_m = 0.24$ mm and a light weight material Hostyrene, mean diameter $d_m = 2.4$ mm with a density 1035 kg/m^3. The critical velocities for beginning of sediment motion were observed directly and are well represented by Eq. (3). Current velocities were measured with a Prandtl tube. Experimental runs duration varied from 4 h to 10 days, depending on the time required to approach equilibrium. In cases where equilibrium still was not reached, equilibrium scour depth was calculated by a semi-analytical approach for the time development which was verified by cases that approached equilibrium during test.

Fig. 1 shows the equilibrium scour depth, H/D over the relative amplitude, d_0/D for different ratios of u/u_c. H is the equilibrium scour

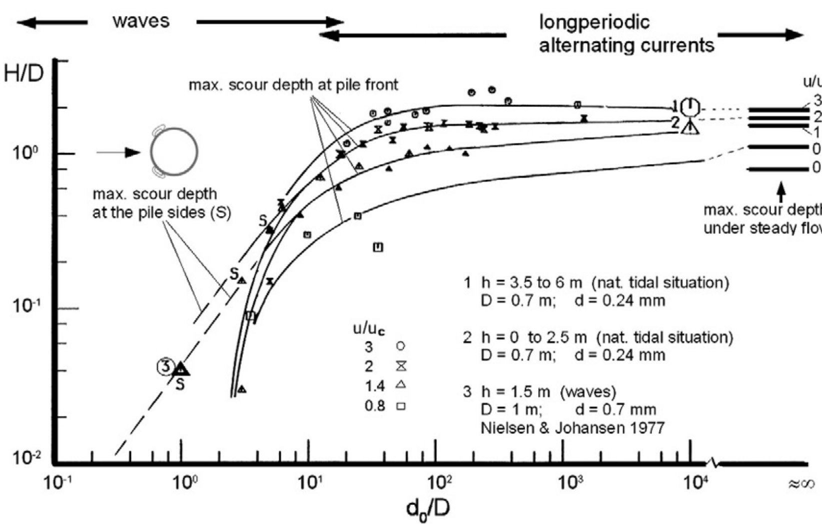

Fig. 1. Equilibrium scour depth, H/D over the relative amplitude, d_0/D. Data marked with '1'and '2' represent natural tidal situations (Zanke, 1982).

depth, D is the pile diameter, and d_0 is the double amplitude of the water displacement near bed during a half wave period:

$$\frac{d_0}{D} = \frac{KC}{\pi} \tag{4}$$

with KC being the Keulegan–Carpenter number

$$KC = \frac{uT}{D} \tag{5}$$

where u in case of waves is the undisturbed maximum orbital velocity near the bed in the far field and T the wave period.

Note that no scour at the pier front developed under waves when $d_0/D_{crit}<2$, respectively KC<6. Instead, when 0<KC<6, scours develop at the pier sides at azimuthal half planes forming an angle equal 40° with the main flow direction. The scouring at the sides was also observed during the first phase of scour evolution at also higher KC but then it would only be an intermediate effect. Later in the tests, scouring surrounded the pile front. The duration of flow action needed for the change from maximum scour depth at the pier sides to maximum scour depth at the pier front increases with the absolute pile diameter. Therefore, in case of large pile diameters, in nature maximum possible front scour may not be reached in each case.

During the scouring process the upstream scour slopes became significantly steeper than the sediment angle of repose mainly due to the action of the horseshoe vortex that supports the slope. After the flow was stopped scour slopes were no longer supported by the horseshoe vortex and thus side slides occurred filling the scour hole and reducing the maximum scour depth. Fig. 2 shows the measured scour depth difference ΔH/D, before and after flow stopped for different velocity ratios u/u_c.

In case of unidirectional flows and long period alternating flows, the scour is dominantly produced at the pier front by the effect of horseshoe vortex. When d0/D resp. KC becomes small, the time to develop a horseshoe vortex becomes also small. As lined out by Sumer et al. (1992), no horseshoe vortex forms when KC<6. For KC>6, "…the scour increases with increasing KC number. This increase is partly due to the increased extension of the lee wake and partly due to the increased presence and strength of the horseshoe vortex with increasing KC number." (Sumer et al., 1992). In the lee wake, behind the pile, the effect of vortex shedding is accompanied with cyclones. After Sumer et al. (1992) and later Sumer and Fredsoe (2002, pp. 190–191) and Dey et al. (2006), for scouring under waves, vortex shedding becomes of importance when KC<100. With the decreasing effect of the horseshoe vortex for lower KC, also the effect of stop of flow on measured scour depth decreases. Therefore, as a first approach, it is recommended here to take this into account by correcting the error ΔH/D from Fig. 2 by the ratio of $(H/D_{steady})/(H/D_{wave})=f(KC)$. This factor can be read from Fig. 6 for the different u/uc.

It is suspected that the relation of the scouring effects of the horseshoe vortex on one side and the vortex shedding on the other side depends not only on KC but also on the sediment characteristics as the suspendability strongly affects the scouring effect of the lee side cyclones.

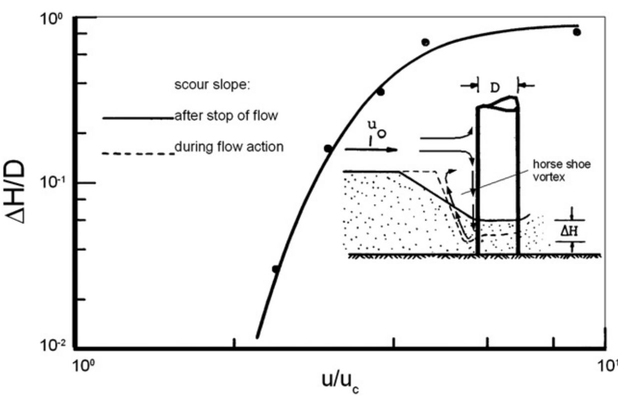

Fig. 2. Measured scour depth difference before and after flow stopped for different velocity ratios of u/u_c.
Modified after Zanke, 1982.

3.2. Experiments by Sumer et al. (1992)

Sumer et al. (1992) carried out experiments on wave scour in three different laboratory flumes at the Technical University of Denmark. Measurements of flow velocity were carried out using a Laser Doppler Anemometer (LDA). Not only their own data but also the experimental data of Das (1970) and Kawata and Tsuchiya (1988) for the development of a scour depth design curve under waves were used in their analysis.

Sumer et al. (1992) pointed out that no direct observation of the scour was possible because of a high sediment concentration during their measurements. Therefore, every 5 min the flow was stopped and the scour depth was measured. The data mentioned above were fitted by Eq. (6):

$$\left(\frac{H}{D}\right)_{waves} \approx 1.3\{1-EXP[-0.03(KC-6)]\}. \tag{6}$$

3.3. Experiments by Prepernau et al. (2007)

Prepernau et al. (2007) carried out measurements on wave scour in the Hannover Great Wave Kanal, GWK, using a pile of 0.55 m diameter. Bed material was sand with a mean diameter of $d=0.33$ mm with $u_c\approx0.3$ m/s. Four experimental series were carried out with wave heights ranging 0.75<Hs<1.0 m, wave periods 5.05<T<8.66 s, Keulegan–Carpenter numbers 11<KC<39 and a water depth of h=2.1 m. Measurements were performed after a certain number of waves when the flow was stopped. Table 1 shows measured scour depth by Prepernau et al. (2007) and corrected according to Fig. 2.

4. A general formula for equilibrium scour depth

With respect to the foregoing, a revised design formula for the equilibrium scour depth is developed. The basic idea is to treat the wave effect via the reduced length of the flow path along the pile: In case of steady flow, this length tends to infinite and under very long waves, scours develop like in steady flow. In case of waves with a shorter period, the relative water displacement d_0/D becomes finite and now two scours develop at a pile. The one in the luff side deepens while the one at the lee side is partly filled up with the material from the other, deepening scour. Luff and lee change permanently with the wave period. So it can be stated that the scour depth is controlled by d_0/D. As described above, additionally the sediment characteristics are of importance.

4.1. Approach for steady flow conditions

Zanke (1982) developed a semi analytical solution for equilibrium scour depth H/D. This solution gives the scour depth which occurs during the flow event but not that what may be measured after stop of flow (see above). As in this solution H/D is given implicitly, it requires iterations. A good fit without the need of iterations is Eq. (7), which is plotted in Fig. 3:

$$\left(\frac{H}{D}\right)_{\text{steady flow}} \approx 2.5\left(1-0.5\frac{u_c}{u}\right) \tag{7}$$

In the solution shown in Fig. 3, which is based on the measurements by Zanke (1982), maxH/D increases monotonously

Table 1
Data taken from Prepernau et al. (2007) for scour depth after stop of wave action and estimated equilibrium scour depth corrected according to Fig. 2.

1	2	3	3	5	6	7	8
Test	u_{max} (m/s)	u_{max}/u_c	KC	Measured equilibrium H/D after stop of waves by Prepernau et al.	Correction ΔH/D from Fig. 2	Correction factor on ΔH/D with respect to efficiency of horseshoe vortex	Estimated equilibrium (H/D)max during wave action
1	1.34	4.5	11	0.17	0.6	0.24/2 = 0.12	0.17 + 0.12 0.6 = 0.24
2	1.8	6.0	20	0.48	0.8	0.56/2.1 = 0.266	0.48 + 0.266 0.8 = 0.61
3	1.9	6.3	23.5	0.55	0.85	0.72/2.2 = 0.33	0.55 + 0.33 0.85 = 0.83
4	2.4	8.0	33.8	0.72	0.88	0.97/2.25 = 0.41	0.72 + 0.41 0.88 = 1.08

with u/uc in ripple forming sandy sediments. Observations reported by Link (2006) for sediment with sizes in the range of sand also indicated that the measured end scour depth increased with u/u_c ratio. Experiments presented by Sheppard and Miller (2006) also showed scour depth increasing with u/uc. On the other hand, several results, also from measurements, show a maximum of H/D near $u/u_c = 1$, i.e. under the clear water condition (see e.g. Melville and Coleman, 2000, Fig. 6.7 or Sumer and Fredsøe, 2002, Fig. 3.25). Under the live bed condition (u/uc>1) in sandy, ripple forming sediments they observed at first a decrease of maxH/D with increasing u/u_c. Later, with a further increase of u/u_c, they measured again an increase of maxH/D with a second maximum when the ripples are washed away.

For design purposes, we assume that both results are valid, i.e. as processes which are not fully understood control the effect of ripples on scour depth. As long as this discrepancy is not solved, we recommend the use of the deeper scour for design purposes.

4.2. Scour induced by waves

For the case of low values of KC, the scour produced by waves will depend on the probability of the sediments, to leave the scour but not to come back into the scour after the turn of the flow. As the erosive intensity of the flow directly at the pile front is about twice than in the far field and is generally highest in the horseshoe vortex, the probability to sediments being moved back into the developing scour depends on the sediment displacement. This is somehow proportional to the sediment displacement during a half wave period and is indicated by 'x' in Fig. 4.

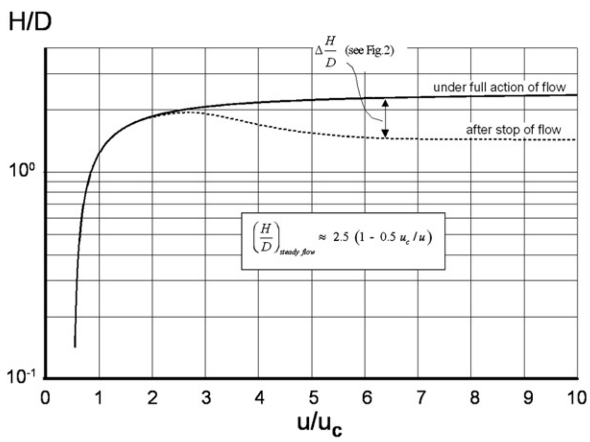

Fig. 3. Equilibrium scour depth in steady currents for KC → ∞ after Eq. (7) (additionally the scour depth after stop of flow according to Fig. 2 is demonstrated by the dashed line).

By approximating the sediment displacement by $x = \alpha D + \beta H/\sin\varphi$ on one hand (Fig. 4) and $x \sim d_0 = KC \cdot D/\pi$ (Eq. (4)) on the other hand one arrives at $KC \sim \pi\alpha + H/D \cdot \pi\beta/\sin\varphi$. With $\alpha = 1.9$ and the treatment of β and φ as constants, after rearrangement yields

$$\left(\frac{H}{D}\right)_{wave} \sim \underbrace{KC-6}_{KC_{eff}}. \tag{8}$$

Note that Eq. (8) is the main term of Eq. (6) and is equivalent to

$$\left(\frac{H}{D}\right)_{wave} \sim \underbrace{\frac{d_0}{D} - 6/\pi}_{(d_0/D)_{eff}} \tag{8a}$$

Under this assumption, for waves with low KC numbers, the scour depth is dominated by KC which describes the water displacement under waves relative to the pile diameter, D. Additionally, Eqs. (8), (8a) express that for the initiation of scouring, a minimum or say a critical value of KC = 6 respectively $d_0/D \approx 2$ is required.

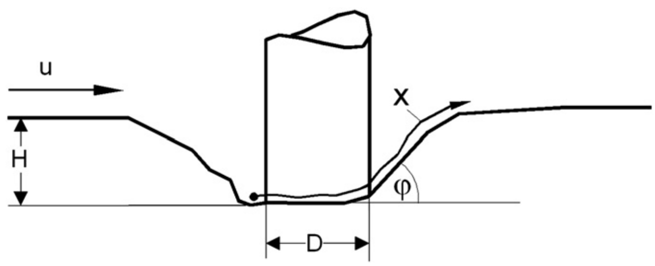

Fig. 4. Definition of sediment displacement, x.

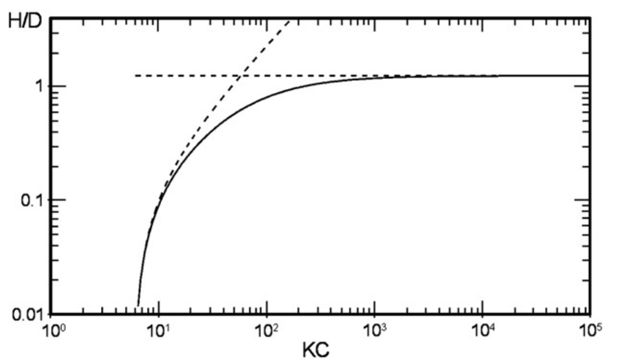

Fig. 5. Demonstration of Eqs. (7, horizontal dashed line), (10, left dashed) and (12, solid line) for $u/u_c = 1$.

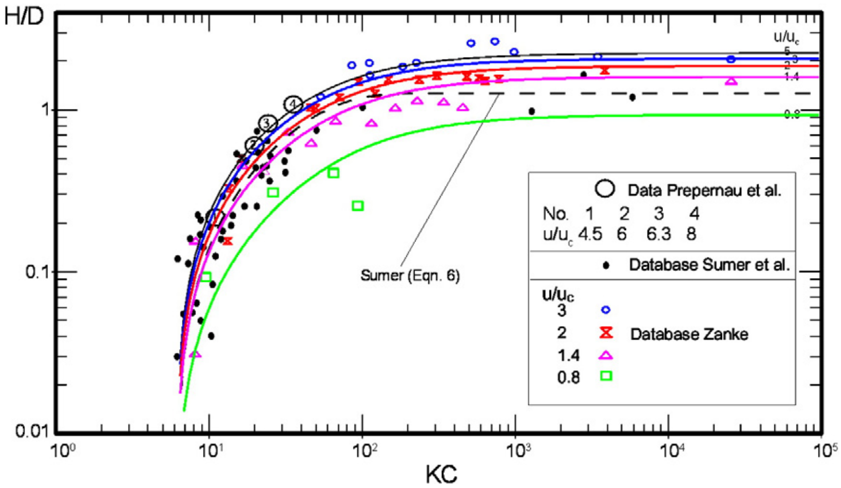

Fig. 6. Equilibrium relative scour depth H/D versus KC number for different u/u_c according to Eq. (12).

However, from the view of sediment mechanics, the water displacement is of only indirect importance but the displacement of sediment particles is of immediate importance here. After Zanke (2001), the slip between water near bed and sediment for an undisturbed flat bed can be described by

$$\frac{u_{sediment}}{u_b} = \left(1-0.7\frac{u_c^*}{u^*}\right) \text{ or } \frac{u_{sediment}}{u_b} \approx \left(1-0.7\frac{u_c}{u}\right) \tag{9}$$

where '*' denotes shear velocities and u_b is the near bed velocity, here the near bed orbital velocity u. Similar solutions for the slip between water and sediment refer to Yalin (1957), Luque (in Karim and Kennedy, 1983) and Engelund and Fredsoe (1976). As at the pier front $u_{c,eff} \approx 0.5 u_{c,plane\ bed}$, for the scour process at piles

$$slip_{pier} = (1-0.35u_c / u). \tag{9a}$$

Thus, for small KC, say KC<10, the depth of wave scour should be controlled by 'slip·(KC-6)'. Under this prerequisite, the following extension of Eq. (8) fits the wave scour depth with good agreement to the data in Fig. 1:

$$\left(\frac{H}{D}\right)_{wave} \Big/ \left(\frac{H}{D}\right)_{steady} = 0.03\cdot(1-.035u_c / u)\cdot(KC-6) \tag{10}$$

The right hand side of Eq. (10) can be understood as the relative sediment displacement during a half wave period which is effective for wave scour development, x_{eff}. Thus

$$x_{eff} = 0.03\cdot(1-035u_c / u)\cdot(KC-6) \tag{10a}$$

or in terms of d_0/D

$$x_{eff} = 0.03\pi\cdot(1-0.35u_c / u)\cdot(d_0 / D-1.91). \tag{10b}$$

4.3. General solution

While Eq. (7) is valid for large values of say KC>10,000, Eq. (10) fits the case of small KC, say KC<10. The transition between these extremes can be described by a transition function which describes the reduced flow path according to the assumptions made in Section 4.2 via the factor of x_{rel}. For small KC, x_{rel} must approach x_{eff} while for large KC must become $x_{rel} \rightarrow 1$:

$$x_{rel} = \frac{x_{eff}}{1 + x_{eff}}. \tag{11}$$

Introducing the wave effect on scouring, x_{rel}, into Eqs. (7), (12) results:

$$\frac{H}{D} = 2.5\left(1-0.5\frac{u}{u_c}\right)x_{rel}. \tag{12}$$

This formula for the equilibrium scour depth H/D covers wave induced scour, as well as scour generated by long alternating flows as in tidal areas, and scours which develop in case of steady flows like in inland rivers. Eqs. (7), (10) and (12) are demonstrated in Fig. 5 for the case of $u/u_c = 1$.

Fig. 6 shows values of H/D over KC measured by Zanke (1982), Sumer et al. (1992), Prepernau et al. (2007) and calculated with Eq. (12).

Note that Eq. (12) covers the whole range of *KC* numbers, from short waves to steady flow conditions.

4.3.1. Example

Example: KC = 40		
u/u_c	H/D	
	This paper	Sumer et al.
0.8	0.34	0.83
1.0	0.5	0.83
2.0	0.86	0.83
5.0	1.1	0.83

4.3.2. Scale effects

Possible scale effects especially with respect to ripples which have developed during laboratory tests have been discussed by Sumer et al. (1992). The authors came to the result that ripples are not an essential factor in the scour process.

5. Summary and conclusions

Zanke (1982) and Sumer et al. (1992) proposed design curves for the equilibrium scour depth under the action of alternating currents and waves. These two papers are based on data collected from totally different experiments. Remarkably, very similar results were obtained, indicating a minimum Keulegan–Carpenter number for the development of scour under the action of waves. Also a widely agreement in the general course of the design curves is observed.

In this paper, a new formula for estimation of maximum scour depth under the action of current and waves was proposed. This formula covers the whole range of waves, alternating flows and steady currents, and takes into account not only the currents but also the effect of the sediment characteristics on scouring. The new curve has been fitted on an extensive dataset, where it is clearly shown that the influence of the sediment characteristics has to be taken into account.

Additionally, this paper highlighted that in the case of live-bed scour the depth measurements have to be taken during developing experiments. If the measurements are taken after flow stop, they will result in reduced scour depths. As the regarding misinterpretation of the measured scour depth when the flow was stopped may be of the order of one H/D, this is of high importance. This effect was taken into account in the proposed scour formula and should be taken into account when future measurements are considered.

The new suggested formula lies on the safe side for estimation of design values for sour depths in marine environment.

Symbol Table

D	(m)	= pile diameter
d	(m)	= sediment grain size
d_m	(mm)	= mean grain diameter of a sediment mixture
d_0	(m)	= length of near bed water displacement during a half wave period = double of amplitude of horizontal water motion
g	(m/s^2)	= gravitational acceleration
H (or S)	(m)	= equilibrium scour depth = maximum scour depth
h	(m)	= water depth
KC		= Keulegan–Carpenter number = uT/D = $d_0/D\pi$
T	(s)	= wave period
t	(s)	= time
u	(m/s)	= max. near bed velocity of wave induced orbital currents resp. mean velocity in case of steady currents
u_b	(m/s)	= near bed velocity
u_c	(m/s)	= critical velocity for beginning of sediment motion (here defined by Eq. (3))
ρ'		= relative density = $(\rho_s - \rho)/\rho$
ρ	kg/(m^3)	= density of fluid
ρ_S	kg/(m^3)	= density of sediment
ν	(m^2/s)	= kinematic viscosity

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