# Wave Spectrum Retrieval from Airborne Sunglitter Images

Maria Yurovskaya[a,b], Nicolas Rascle[c,d], Vladimir Kudryavtsev[a,b], Bertrand Chapron[b,d], Louis Marie[d], Jeroen Molemaker[d]

[a]*Federal State Budget Scientific Institution "Marine Hydrophysical Institute of RAS", Sevastopol, Russia*
[b]*Satellite Oceanography Laboratory, Russian State Hydrometeorological University, Saint Petersburg*
[c]*Division de Oceanologia, Centro de Investigación Científica y de Educación Superior de Ensenada, Baja California, Mexico*
[d]*Institut Français de Recherche pour l'Exploitation de la Mer, Plouzané, France*

## Abstract

The method and algorithm development to retrieve the two-dimensional wave spectrum from airborne sunglitter photographs are presented. Based on a linear transfer function deduced from the shape of brightness distribution in the sunglitter zone, the absolute wavenumber elevation spectrum does not require any additional assumption or information about sky brightness, wind or wave energy. A step by step algorithm is given and applied to airborne images taken during an experiment in the Gulf of Mexico. Retrieved spectra agree well with nearby NDBC buoy data, both for spectrum shape, level and energy angular distribution. The 180-degree wave direction ambiguity, inherent to image-derived spectra, is eliminated by using cross-correlation analysis between two consecutive images. A case study corresponding to the spectral evolution with increasing distance from shore in slanting-fetch conditions is then considered. Energy level and peak position transformation agree with established approximations and laws of wind-sea development. The technical requirements (flight altitude, image resolution, view angles, etc) and applicability of the suggested methodology are discussed. These results demonstrate the potential efficiency of high resolution sea state monitoring from drones or light aircrafts using sunglitter imagery.

*Keywords:* sunglitter, sea surface waves, directional wave spectrum, aerial photography, field measurements, remote sensing observations, high resolution, drone

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## 1. Introduction

For a wide range of applications, such as coastal management, the design and operational safety of harbours, ships, and offshore structures, a precise knowledge of the directional spectrum of ocean waves is needed. The directional wave spectrum describes the distributed energy contributions from waves propagating in different directions with different wavelengths. It is key to help determine the consequences of interactions between waves and other structures, i.e. breakwaters and offshore structures, but also to evaluate wave-induced upper ocean transport and erosion processes.

Significant advances have thus been made to estimate these directional wave statistical properties. Today, a large number of measuring devices, working on different principles, are available (e.g. Herbers et al., 2012). Yet, the directional and frequency response of these systems may often be limited and not sufficient to fully resolve directional surface wave spectra. Further, requirements for near-simultaneous, high spatial resolution observations, to provide more direct directional wavenumber measurements of the local surface field over entire regions, has attracted the attention on remote sensing technologies. To complement sparse in-situ buoy measurements, techniques can include sea level radars (coastal HF radars, Barrick and Lipa, 1985), microwave and marine X-band radars (Senet et al., 2008; Nieto et al., 2004), scanning altimeter and lidar high-resolution topography instruments from airplane platforms (Walsh et al., 1998; Melville et al., 2016), and also synthetic aperture or rotating real-aperture airborne radar instruments (Caudal et al., 2014). As well, photographs of the ocean surface have long been proved to contain quantitative information about ocean surface slope statistics (e.g. Barber, 1949; Cox and Munk, 1956), to help infer directional spectra of surface waves (Stilwell, 1969; Stilwell and Pilon, 1974). Today, with the significant cost reduction and improvement of both instruments and drones, the photograph techniques may become more widely used to observe and monitor surface waves at regional or coastal scales.

Since almost two centuries (Spooner, 1822), it has been understood that the shape of the sunglint on the sea surface contains information on the statistical properties of wave slopes. Airborne and satellite sunglint images at medium ($\sim$ 1 km) resolution have then been used to precisely estimate sea surface slope statistical properties (Cox and Munk, 1956; Breon and Henriot, 2006), and modulations by various dynamical ocean processes like currents and fronts, internal waves, or surface slicks (Barber, 1954; Apel et al., 1975; Hennings et al., 1994; Kudryavtsev et al., 2012; Kudryavtsev et al., 2012; Rascle et al., 2016, 2017). At higher ($\sim 1 - 10$ m) resolution, glitter modulations are more directly connected to the wavy surface. Indeed, wave contrasts on the image result from the modulation of sun reflected radiation by individual tilting wave slopes, and those can be used to estimate the wave directional elevation spectrum (Stilwell and Pilon, 1974; Monaldo and Kasevich, 1981).

To derive wave elevations from these brightness variations, a transfer function must thus be determined. Using airborne photographs, this task is eased, as the overall sunglitter shape can be captured, to help directly infer a linear transfer function (Bolshakov et al., 1988). Adapted to a satellite configuration, such a method was successfully applied (Kudryavtsev et al., 2017a,b) to reconstruct the spectrum of long (energy containing) waves from satellite sunglitter images, taking advantage of the high resolution and specific viewing geometry of the radiometers on-board the satellite Sentinel-2.

In this paper, we further dwell on this capability of airborne sunglitter imagery to provide the overall glitter pattern. As mentioned above, this property provides direct means to determine a linear transfer function. Our motivation is then to further assess how robust is our proposed methodology to efficiently provide quantitative estimates of the directional wave spectrum, including energy containing waves and also short waves. The development is specific to airborne measurements and applied to data collected over a coastal area in the northern Gulf of Mexico.

The paper structure is as follows. The experiment is described in Section 2; theory and spectrum reconstruction algorithm are presented in Section 3; method implementation and validation are given in Section 4; the results of the study of wave development and transformation with fetch are presented in Section 5, and finally, the discussion of method applicability and some recommendations on experimental setup are suggested in Section 6.

## 2. Experiment and Data

The airborne sunglitter images were obtained on Jan. and Feb. 2016 during the Lagrangian Submesoscale Experiment (LASER), where a large number ($\sim 1000$) of surface drifters were deployed to study surface dispersion within the Gulf of Mexico (D'Asaro et al., 2018; Rascle et al., 2017), close to the site of the Deep Horizon oil platform accident in 2010 (Fig. 1, a). The images were acquired from airplane (a Partenavia P.68) flying at altitudes up to 3000 m.

The visible light intensity was measured by two panchromatic cameras (JAI BM-500GE) equipped with a 5 mm focal length low distortion lens to ensure a large field of view. The cameras setup is sketched in Figure 1, b. To capture the sunglint, the two cameras were arranged symmetrically about the airplane nadir with a pitch of $\pm 35°$ for the forward/aftward cameras. The camera aperture angles are $80° \times 70°$ along-track and across-track, respectively, with $2456 \times 2058$ pixels in the respective directions. For a flight altitude of 1000 m, this leads to a ground resolution from 0.5 m to 6 m. The cameras acquired images at 2 Hz. The images were geolocated using an internal motion unit Applanix POS AV V610.

We selected cases corresponding to measurements made during flights with trajectories close to National Data Buoy Center (NDBC) buoy locations, to benefit from synchronous wind and wave ancillary data. A step by step algorithm is provided for images obtained close to NDBC 42012 in developed wind sea conditions on 11-Feb-2016 (green star on Fig. 1, a). Further we analyze the wave evolution on 23-Jan-2016, when sunglitter images were acquired (in cloudless regions) at different distances from the shore along the

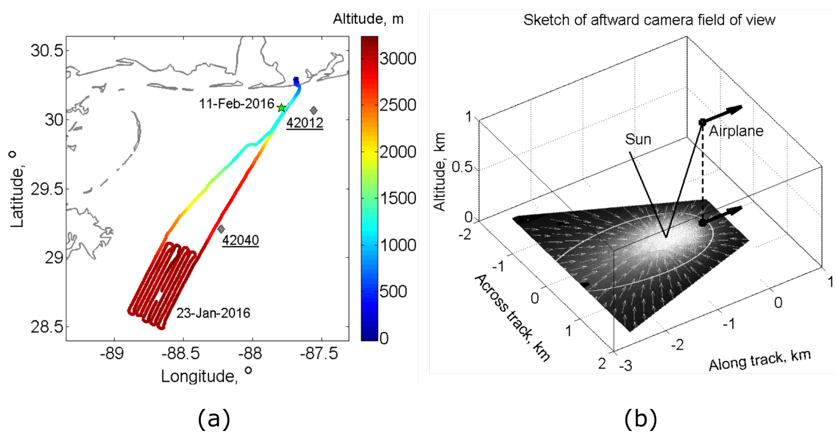


Figure 1: (a) The observation area with NDBC buoy locations (gray diamonds). Green star shows location of the analysis on 11-Feb-2016 (section 4), colors are the tracks of the 23-Jan-2016 flight (section 5). (b) Sketch of the field of view of the afterward camera, for a flight altitude of 1000 m. Here we show the special case of the sun exactly at the rear of the airplane when the specular sun spot is at the center of the camera field of view. The ellipse is the contour $Z_n^2 = s^2$ (see the notifications below). The white arrows show the orientations of the transfer function gradient, $G_{zi}$.

plane tracks shown in Fig. 1, a.

## 3. Theoretical Background

Based on the classical model of the sea surface brightness formation in the visible range (Cox and Munk, 1956), the intensity in each pixel of sunglitter image is proportional to the sun reflected radiance, or the energy brightness of the surface (the spectral energy flux per unit area per unit solid angle):

$$N = \frac{\rho E_s}{4 \cos\theta \cos^4\beta} P(Z_1, Z_2), \qquad (1)$$

where $P$ is the probability density of two slope components, $Z_1, Z_2$, satisfying the conditions of specular reflection:

$$Z_1 = -\frac{\sin\theta_s \cos\phi_s + \sin\theta \cos\phi_\nu}{\cos\theta_s + \cos\theta}$$
$$Z_2 = -\frac{\sin\theta_s \sin\phi_s + \sin\theta \sin\phi_\nu}{\cos\theta_s + \cos\theta}, \tag{2}$$

$\theta$ and $\theta_s$ are zenith angles for the camera and the sun, respectively, $\phi_\nu$ and $\phi_s$ are corresponding azimuth angles, $\rho$ is the Fresnel reflection coefficient, $E_s$ is the solar radiance, $\tan\beta = \sqrt{Z_1^2 + Z_2^2}$.

Local modulations of $B = N\cos\theta/\rho$, or equivalently, of $P$, can arise for two reasons: variations of the slope statistics mostly governed by changes of mean square slope (MSS) due to different upper ocean processes (fronts, internal waves, surface slicks, etc), or the tilting of the ocean surface while a long wave is propagating. The latter can also lead to a short wave (and thus, MSS) modulation along the wave profile. As demonstrated by Bolshakov et al. (1988) and Kudryavtsev et al. (2017a), one can ignore these MSS modulations in the vicinity of brightness contrast inversion zone, i.e. $0.5 < Z_n^2/s^2 < 2$, where $Z_n^2 = Z_1^2 + Z_2^2$, and $s^2$ is the surface MSS to the first order estimated from the assumption of Gaussian brightness and slope distribution as $s^2 = -2\overline{Z_n} \cdot \overline{B}/(\partial B/\partial Z_n)$. The brightness variation due to the long wave propagation then writes:

$$\tilde{B} = B(Z_1 + \zeta_1, Z_2 + \zeta_2) - B(Z_1, Z_2) = \frac{\partial B}{\partial Z_i}\zeta_i \equiv G_{zi}\zeta_i, \tag{3}$$

where $\zeta_{1,2}$ are the components of tilting wave slope. $G_{zi}$ is the transfer function, relating brightness and slope variations. This transfer function is then determined as the brightness gradient in specular slope space and can be obtained through the observed brightness gradients:

$$G_{z1} = (G_2 Z_{2,1} - G_1 Z_{2,2})/\Delta$$
$$G_{z2} = (G_1 Z_{1,2} - G_2 Z_{1,1})/\Delta, \tag{4}$$

where $G_i = \partial B/\partial x_i$, $Z_{i,j} = \partial Z_i/\partial x_j$, $\Delta = Z_{1,2}Z_{2,1} - Z_{1,1}Z_{2,2}$.

From (3), the relation between elevation and brightness spectra thus writes :

$$S_\zeta(\mathbf{k}) = S_B(\mathbf{k})/(G_{zi}k_i)^2. \tag{5}$$

The linear combination of wave vector components in the denominator of (5) vanishes in a direction perpendicular to the gradient direction. Close to this direction, the spectrum cannot be simply retrieved. As suggested by Bolshakov et al. (1988) and also Lupyan (1988), this singularity can be eliminated, by using several image fragments with different gradients $G^n_{zi}$, but statistically identical wave spectrum, $S^n_\zeta(\mathbf{k}) = S_\zeta(\mathbf{k})$. As sketched in Fig. 1, b, where the typical distribution of $G^n_{zi}$ orientations is shown, the vectors converge towards the sunglitter center, changing their direction from 0° to 360° around it. Brightness spectra taken from fragments with different vector orientations can then be averaged, to obtain the elevation spectrum without any singularity:

$$S_\zeta(\mathbf{k}) = \sum_{n=1}^{N} S^n_B(\mathbf{k}) / \sum_{n=1}^{N} (G^n_{zi}k_i)^2. \tag{6}$$

As described, the considered methodology is self-consistent, solely based on the transfer function estimation from the observed shape of solar glint. For airborne photography, the following steps must then be taken:

- gradients $G_i$, $Z_{i,j}$ are determined from the smoothed sunglitter pattern and known geometrical parameters;
- a transfer function, $G_{zi}$, is calculated using (4);
- several image fragments are selected in different image parts, still in the vicinity of contrast inversion zone, and their brightness spectra are calculated;
- the absolute directional wave elevation spectrum is derived from the sum of brightness spectra and transfer function field, using expression (6);
- 180-degree wave direction ambiguity can be removed using cross-correlation analysis of two consequent images.

A detailed example of airborne sunglitter image processing is given below.

## 4. Method Implementation

### *4.1. Image preprocessing*

On 11-Feb-2016, a snapshot of the sea surface (Fig. 2, a) was extracted close to the location of the NDBC buoy number 42012 (Fig. 1, a). The wind was about 9 m $s^{-1}$ blowing from South-West, and peak waves of about 40 m wavelength were propagating from the same direction.

The above procedure must be applied to a brightness field predominantly formed by the sunlight reflections from the sea surface. Besides the image projection onto the sea surface plane (Fig. 2, b), a preparatory step is to consider an intensity correction to possibly account for extraneous factors hampering the image brightness. We neglect any vignetting effect and consider the image intensity proportional to the energy surface brightness, $N$. First, the sky reflection and scattered radiation can contribute to the image brightness. Cox and Munk (1956) report corresponding dependencies on incidence angle by considering intensities from regions far outside the glitter. A similar procedure is to use the darkest column of the photograph (the right one in the example on Fig. 2, a). Given the viewing geometry and neglecting the sunglitter contribution within this darkest line, the incidence angle dependency of the background radiance can be estimated. A corresponding polynomial approximation, Fig. 2, c, is then assumed to extend over the whole 2D image, and further subtracted. Nevertheless, in all considered examples, we do not use parts of the images with $\theta > 50^\circ$, areas over which the impact of scattered radiation rapidly grows (Cox and Munk, 1956), and the assumption (3) loses its validity.

According to (1), the detrended field, $N - N_{back}$ (not shown), should be multiplied by $\cos\theta/\rho$, shown in Fig. 2, d. Values of $\cos\theta/\rho$ differ up to 4-5 times on the opposite image borders with incidence angles $25^o$ and $60^\circ$. This operation suppresses the brightness of the distant zone and shifts the sunglitter center towards the edge corresponding to the lowest incidence angle (compare Fig. 2, b and Fig. 2, e). The mean brightness field, $B_0$ (Fig. 2, f), is then derived by smoothing $B = (N - N_{back})\cos\theta/\rho$ using a moving average filter, with a window size depending on the image resolution (about several lengths of dominant wave). All the algorithm steps then apply to the brightness variation field, $B - B_0$.

### *4.2. Spectrum validation*

A fragment of the brightness variation field is shown Fig. 3, a. Fragments are taken between the two ellipses indicating the zone $0.5 < Z_n^2/s^2 < 2$, and above the line $\theta = 50°$.

Fig. 3, b displays the sum of directional brightness spectra. As expected, the resulting transfer function, $\sum(G_{zi}^n k_i)^2$ (Fig. 3, c), does not vanish in any particular direction, but tends to zero in the wavenumber plane center. This may enhance noise level and errors at the lowest wavenumbers. After application of the transfer function, Fig. 3, d, both brightness and wave elevation spectra possess a distinct spectral peak, visually corresponding to the waves observed on the fragment, Fig. 3, a, but the angular distribution of the elevation spectrum is apparently broader, possibly revealing waves moving closer to zonal (eastward or westward) directions.

The comparison with the nearby NDBC buoy wavenumber directional spectrum (Fig. 3, d), calculated with the use of the maximum entropy method (Lygre and Krogstad, 1986) and linear dispersion relation for gravity waves, gives a satisfactory agreement of 2D energy distribution. Notice that in contrast to NDBC data that provides a “true” directional spectrum, the spectrum retrieved from the image is folded ($S(\phi) = S(\phi) + S(\phi + 180°)$) having a 180-degree ambiguity in wave direction.

Omnidirectional spectra are compared in Fig. 3, f, and give an excellent agreement of peak position and its energy level. Energy underestimation

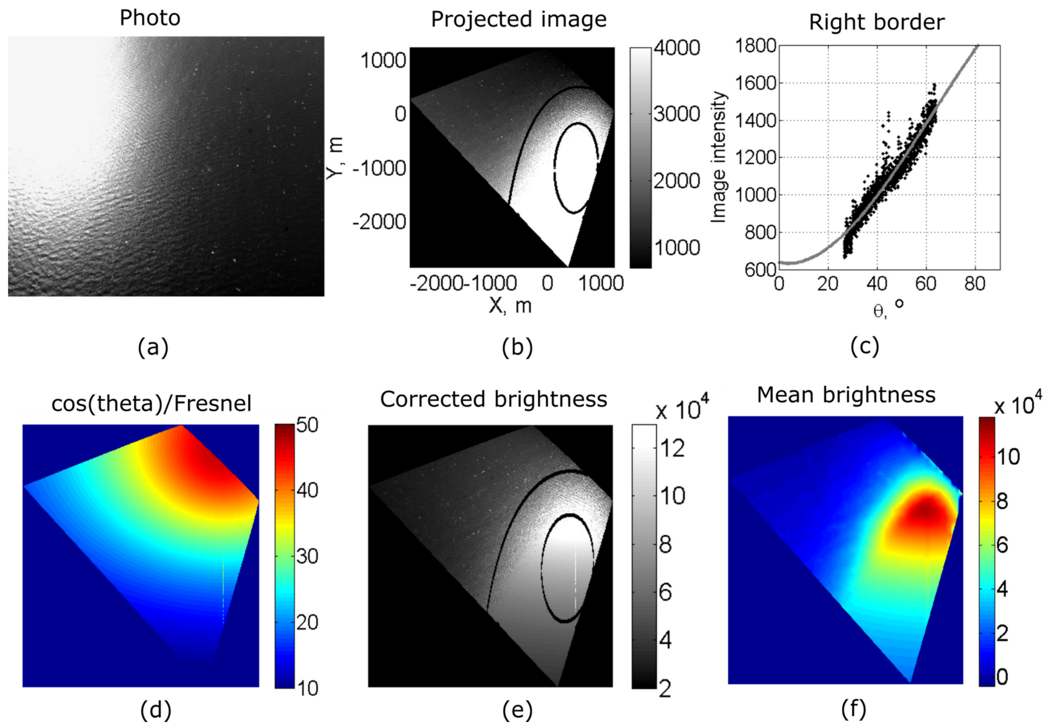


Figure 2: (a) An airborne snapshot of a sea surface; (b) image projected on the sea surface plane (x-label is to the East, y-label is to the North, two ellipses determine the zone $0.5 < Z_n^2/s^2 < 2$); (c) pixel intensities for the left column of a photo (dots) and their polynomial approximation indicating the background radiation; (d) $\cos\theta/\rho$ field; (e) $B = (N - N_{back})\cos\theta/\rho$ field; (f) mean brightness field, $B_0$

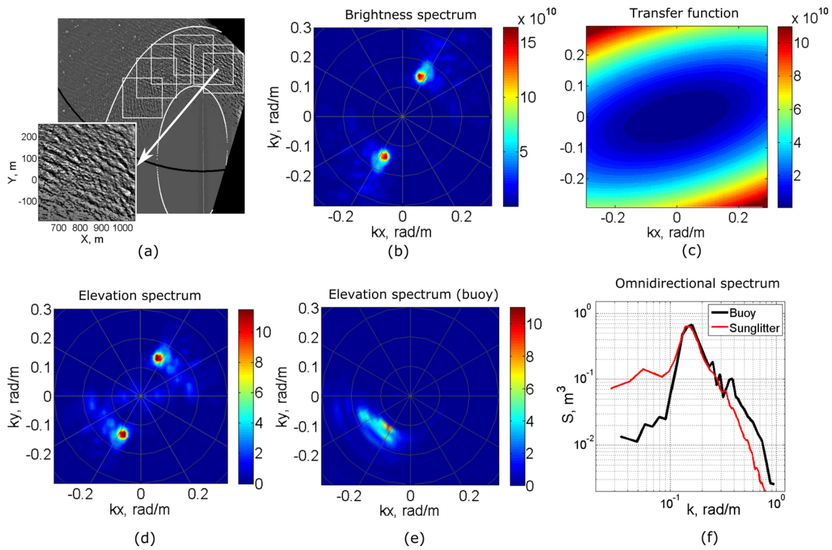


Figure 3: (a) Square fragments (450 m size) of brightness variation field, $(B - B_0)$, taken for spectrum retrieval. Two ellipses determine the zone $0.5 < Z_n^2/s^2 < 2$, black line is $\theta = 50°$; (b) The sum of brightness spectra; (c) the transfer function, $\sum_{n=1}^{N}(G_{zi}^{n}k_i)^2$; (d) retrieved from (6) elevation spectrum; (e) NDBC buoy data directional spectrum (42012); (f) Omnidirectional spectra comparison.

of retrieved spectrum at wavenumbers $k > 0.3$ rad/m can be explained by the actual (not interpolated) image resolution and smoothing of features smaller than 10 m. The noise level at $k < 0.1$ rad/m depends on the $B_0$ calculation (the smaller the filter window size, the lower the spectrum), and is also controled by the singularity of a transfer function around $k = 0$ rad/m. Energy distribution of waves in a range 20 m - 60 m is reliably reproduced.

### *4.3. Wave direction ambiguity*

In our cases, the camera acquired images every 0.5 s. Consecutive snapshots can then be analyzed to remove the wave propagation directional ambiguity (Fig. 3, d), as already demonstrated for satellite measurements (Kudryavtsev et al., 2017a; De Michele et al., 2012). Two images of the same square region of the sea surface taken with a $\Delta t$ =0.5 s time difference are shown on Fig. 4, a-b. Their spectral coherence, $< \hat{I}_2 \hat{I}_1^* >^2 / (< \hat{I}_1 \hat{I}_1^* >< \hat{I}_2 \hat{I}_2^* >)$, hat means the Fourier transform, a star is complex conjugation, and phase, $\arg(< \hat{I}_2 \hat{I}_1^* >)$, are given in Fig. 4, b-c. The coherence level is high, up to 1, in the direction where the waves are observed. The phase spectrum is a-priory asymmetric, and the positive phase shift corresponds to the direction "from" in a case when $I_1$ is taken earlier than $I_2$. Thus, the wave system is moving from South-West, to agree with the NDBC directional spectrum (fig 3, b).

Airborne image time series can further be used to estimate ocean surface currents from the dispersion of the detected gravity waves (e.g. Dugan and Piotrowski (2003)). Taking a transect in the phase spectrum, $\Delta\Phi$, along a direction corresponding to maximum coherency, marked with a dashed line in Fig. 4, the dispersion can be evaluated for the relative projection of the phase velocity: $c(k) = \frac{\Delta\Phi/\Delta t}{k}$. As obtained, Fig. 4, e, experimentally derived points lie very close to the standard prediction, $c = \sqrt{g/k}$, even at large wavenumbers for which the elevation spectral analysis is less reliable. This indicates the absence of surface current, or at least its component along the chosen direction, in the region of observation.

## 5. A Case Study: Spectrum Evolution with Fetch

On 23-Jan-2016, an experiment to study wave transformation at varying distance from the shore was conducted. The airplane moves seawards across

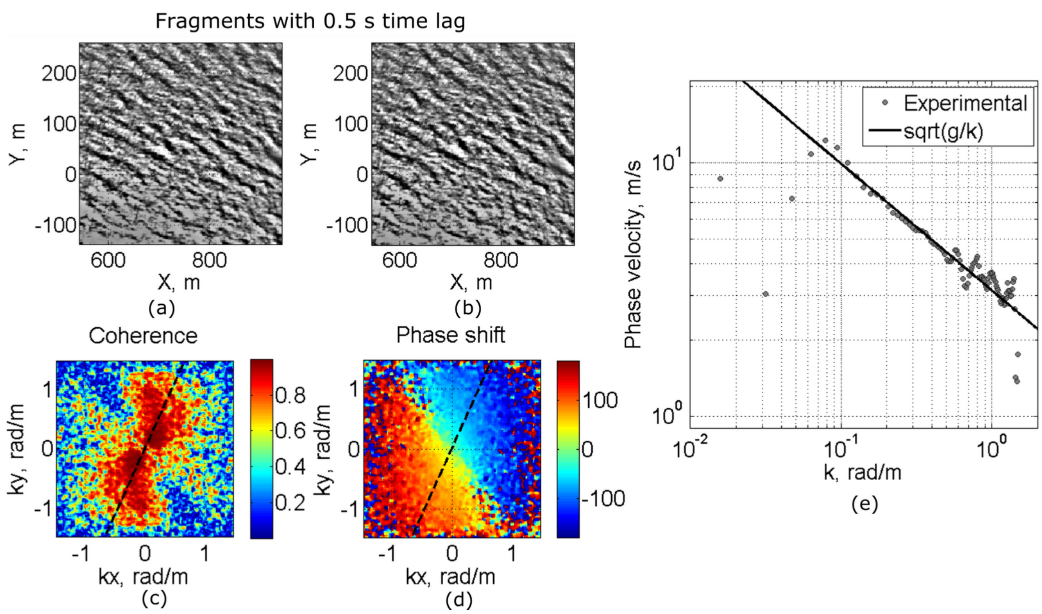


Figure 4: (a)-(b) Snapshots of the same location taken with 0.5 s time shift; (c) coherence of two brightness fields; (d) phase shift; wave direction (from) corresponds to positive values of a phase shift; (e) dispersion relation calculated from a phase shift along the line of coherence maximum (dashed lines in (c) and (d) plots).

the NDBC 42012 and 42040 locations (Fig. 1, a, and Fig. 5). The flight started at 19:20 UTC in clear sky conditions, but at 19:31 the plane entered a cloudy zone with gleam areas too small to estimate the wave spectrum. Yet, two images at 19:50 could be exploited. On the way back, at 23:00, camera pitch and sun elevation angle didn't satisfy the condition $Z_n^2/s^2 < 2$ and clouds were still hindering the glitter. As a result, only one image fragment from the glint periphery could be used, with relatively low reliability.

The wind speed and direction (in nautical system) around the time of acquisition are plotted in Fig. 6. Wind was blowing from the North-West, slightly rotating clockwise and calming down from 12 m s$^{-1}$ to 10 m s$^{-1}$, accordingly to NDBC 42040 data. The slow clockwise wind rotation took place during the previous two days, starting to blow from South, then West, before finally subsiding to 3 m s$^{-1}$ from North on 24-Jan.

The two-dimensional slope spectra ($Sk^2$) from the buoys are shown in Fig. 6. The slope spectra, reconstructed from the airplane images are shown in Fig. 7, for the points marked by red squares on the map of Fig. 5. Many different wave systems co-exist in the area (see sketch Fig. 5).

First, there is a long ($k \sim 0.05$ rad m$^{-1}$) swell from West-South-West, probably originating from West of the Mississipi delta and entering the area from the South-West. This swell is well observed at buoy 42040 (Fig. 6, bottom right) and on the airplane spectra at 22:50 (Fig. 7). The North of the bay is probably partially sheltered from that swell, which is weaker seen on the 42012 buoy data (Fig. 6, bottom left) as a Southern swell, though a well-distinguished peak is resolved on omnidirectional spectrum (black and gray curves in Fig. 8 before 19:31). Also, that swell is not properly resolved by the smaller image fragments (due to lower plane altitude, see Fig. 1) used around buoy 42012.

Second, there is the wind sea at short wavenumbers ($k > 0.1$ rad m$^{-1}$). The peak of the wind sea is slightly more from the North than the wind direction (marked by a white dashed line in Fig. 6 and Fig. 7), both on buoy data and on the airplane spectra. Third, there is a series of spectral peaks from West to North-West (see before 19:31 on Fig. 7, see also buoy data on Fig. 6). Those peaks are typical of slanting fetch conditions (Ardhuin et al., 2007, e.g.), where the wind sea separates between subsystems, the high-frequency remaining downwind whereas waves at relatively lower frequency develop and propagate in the slanting fetch direction (along-shore). Very close to the shore (before 19:25 on Fig. 7), those slanting short waves even dominate the wind sea spectrum.

From the analyzed spectra, the main tendency is a gradual peak shifting towards lower wavenumbers and a corresponding energy growth with the fetch distance. These effects are better identified in omnidirectional spectrum evolution, Fig. 8. The figure presents angle-integrated surface elevation spectrum (red) together with buoy-derived spectra at 19:00 (t1, black) and 20:00 (t2, gray) for NDBC 42012, and at 21:00 (t1) and 23:00 (t2) for NDBC 42040. To help the interpretation, empirical model spectra, as suggested by Donelan et al. (1985) and Babanin and Soloviev (1998), are displayed, for different fetches (given in figure titles). Fetches are calculated as the distance to the line passing through alongshore islands (bold green on Fig. 5) in the direction of the wind taken from the nearest buoy. They are further corrected to account for the direction of the spectral peak mostly deviating from NDBC wind direction. At small fetches (below 5-10 km), the spectrum is rather variable in energy level and peak position, also probably due to the changing bottom topography and consequent refraction effects. The wind wave peak is hardly distinguished and only starts to be clearly obtained at fetch about 10 km. At 19:25:31, the fetch value is close to the one captured at NDBC 42012 location (see Fig. 5). The respective spectra are then found very close (compare black and red curves in Fig. 8). The evolution then continues and closely follows Donelan et al. (1985) and Babanin and Soloviev (1998) predictions. Despite the low reliability for the sunglitter-derived spectrum at 22:50 (the last subplot), a good agreement is found with both model and buoy data taken at approximately the same fetch.

To generalize the wind sea peak transformation, we present (Fig. 9, a) the dependency of dimensionless peak frequency, $f_p u_{10}/g$, and dimensionless energy, $Eg^2/u_{10}^4$, estimated as the spectrum integral around the wind wave peak and shorter waves, on dimensionless wave fetch, $Lg/u_{10}^2$. Comparison is made with other data collected by Babanin and Soloviev (1998). As obtained, results are consistent with the cited approximations, except for the wave energy at the near-shore points. For these cases, the wind peak wavelength is not far from the camera resolution.

The present data, unfortunately, cannot trace any pronounced tendency for the angular distribution evolution. This is due to the presence of several swell peaks much stronger than the wind ones, and inaccurate data at large fetches, where the wind peak dominates. Yet, the average angular distribution around the peak wavenumber (Fig. 9, b) does not contradict the dependency, $S(k_p) = 0.5\beta/\cosh^2(\beta\phi)$, $\beta = 2.28$, reported by Donelan et al. (1985), confirming that multi-modal spectrum structure provides some broadening at the angles far from $\phi = 0$ (peak position).

## 6. Method Applicability and Constraints

As demonstrated, the proposed spectral reconstruction robustly applies when several requirements are satisfied.

The photograph should contain a part of sunglitter ellipse, $Z_n^2 = s^2$, within the camera incidence angle range $\theta < 50^o$. The area must be large enough to provide sufficient angle diversity between the transfer function vectors that are about normal to the ellipse. This ensures to properly eliminate the singularity of the transfer function. The brightness of the observed area should not be saturated. A saturation shortens the range of slope values. Clouds are also to avoid. Cloudiness, or other inhomogeneities, impact the estimation of the mean brightness characteristics.

The question of the impact of wave breaking is still open. Breakers can appear on the image as bright spots, to possibly distort the retrieved slope/elevation distribution. Under high-wind conditions, individual breakers shall be excluded, and individual breaking crests possibly interpolated.

Coming back to the part of the sunglint where the spectrum can be derived, i.e. $0.5 < Z_n^2/s^2 < 2$ and $\theta < 50^o$, a simplified one-dimensional analysis leads to a necessary condition for the camera zenith angle: $\beta_1 < |\theta - \theta_s| < \beta_2$, where $\beta_1 = 2 \arctan \sqrt{0.5 s^2}$, $\beta_2 = 2 \arctan \sqrt{2 s^2}$ with $s^2 = 0.003 + 0.00512 U_{10}$ (Cox and Munk, 1956). Close to the camera nadir direction (Fig. 1), the distance between the two curves represents the longest wavelength being de-

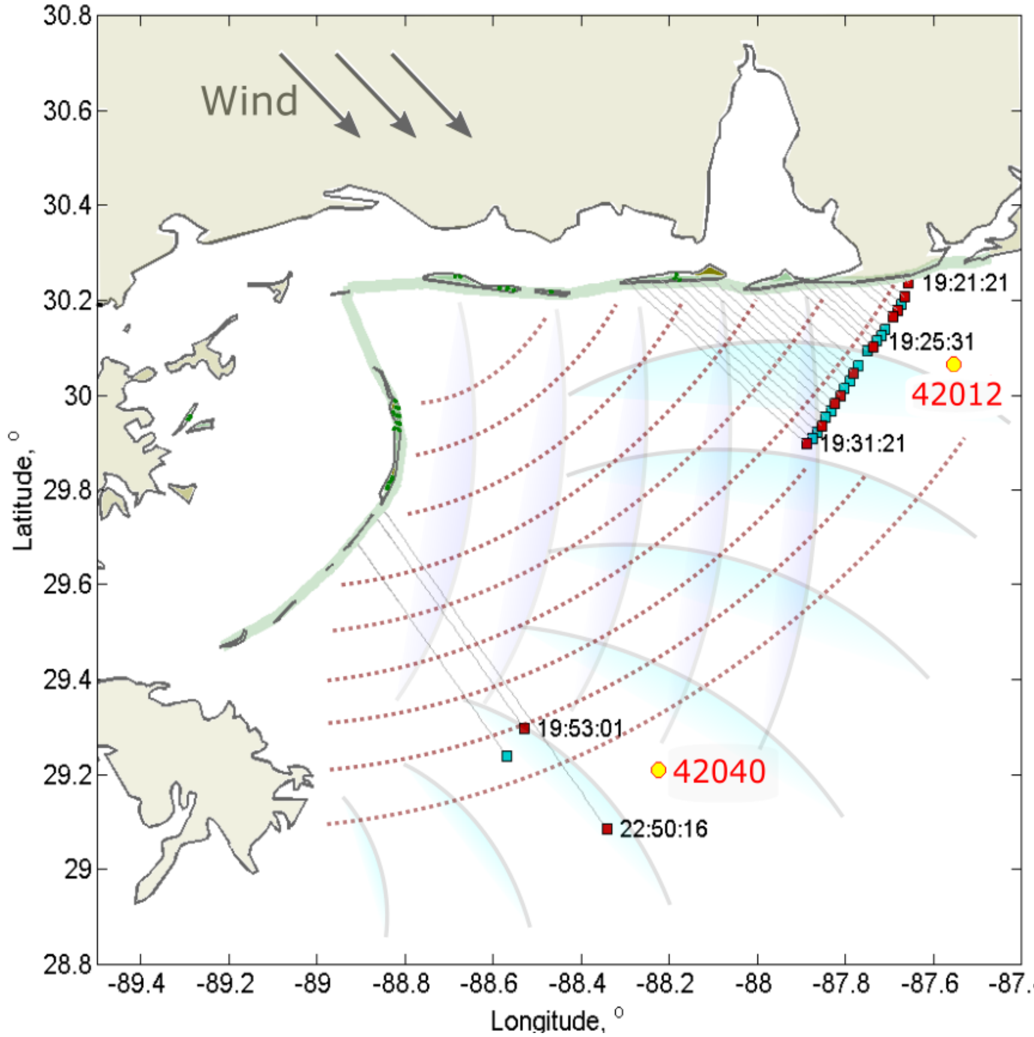


Figure 5: Locations of the images acquired on 23-Jan-2016 near NDBC buoys 42012 and 42040, and schematic wave systems orientations.

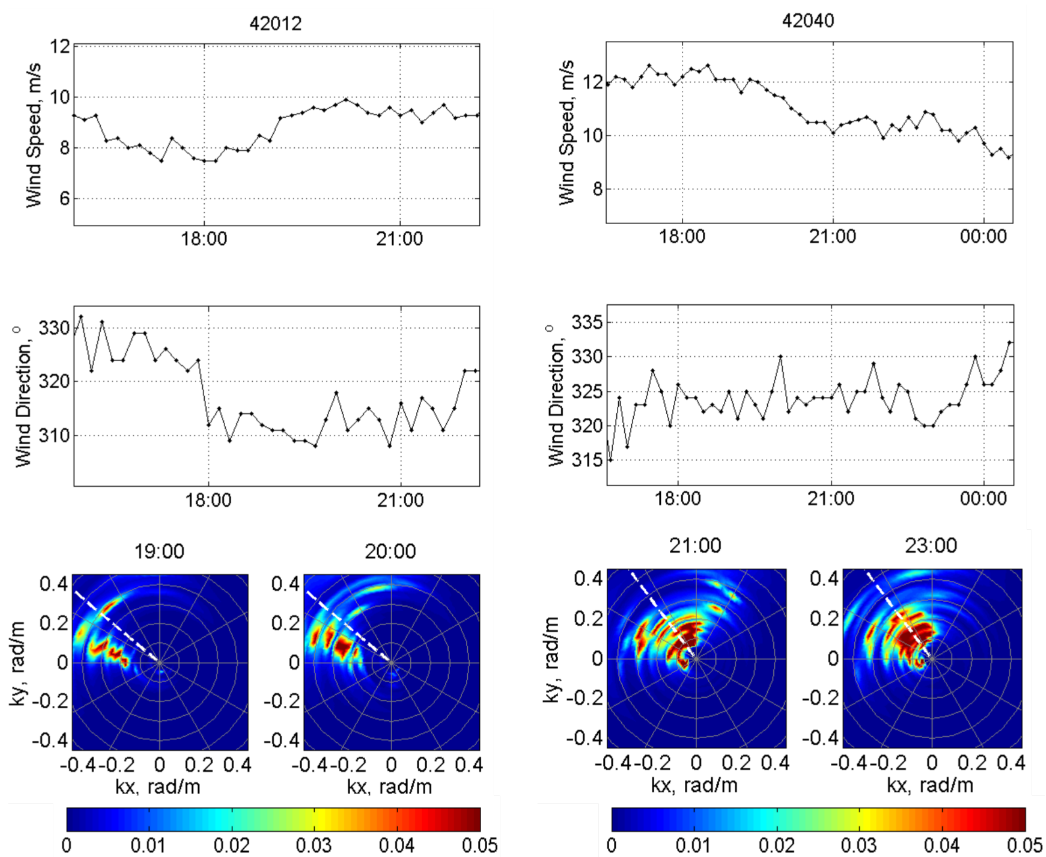


Figure 6: Windspeed, wind direction and directional slope spectra from NDBC 42012 and 42040 buoys around the time of airplane flight. Wind and wave directions are ''from" in nautical system.

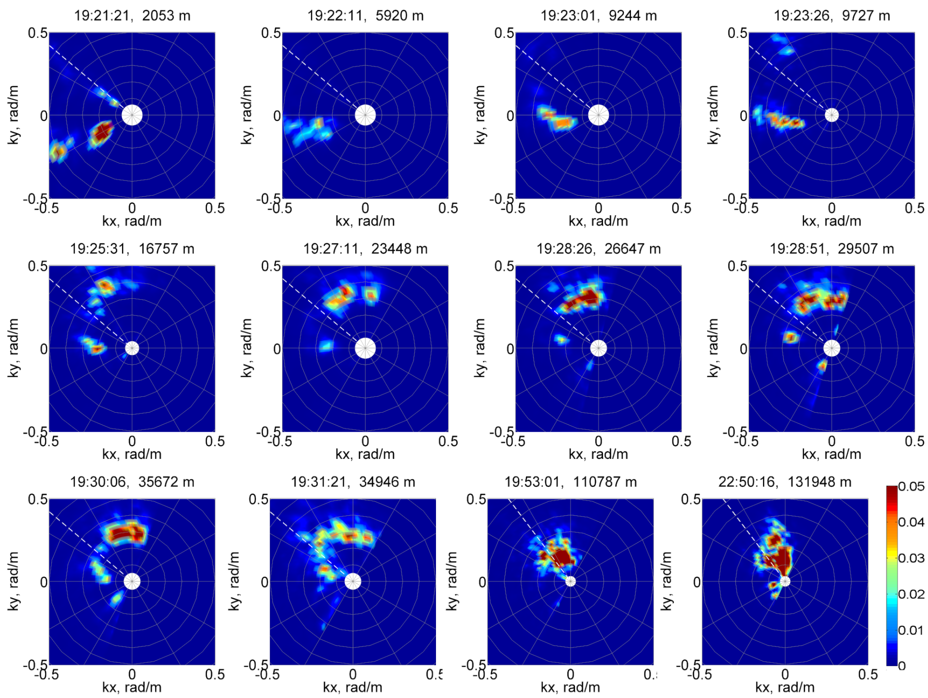


Figure 7: Directional slope spectra at the points marked by red squares on fig 5. White line is the wind direction from buoy data (trigonometrical system). Spectrum develops in presence of swell from West. Wind wave peak grows and shifts towards low wavenumbers, slightly deviates from NDBC wind direction (actually the wind also changed its direction).

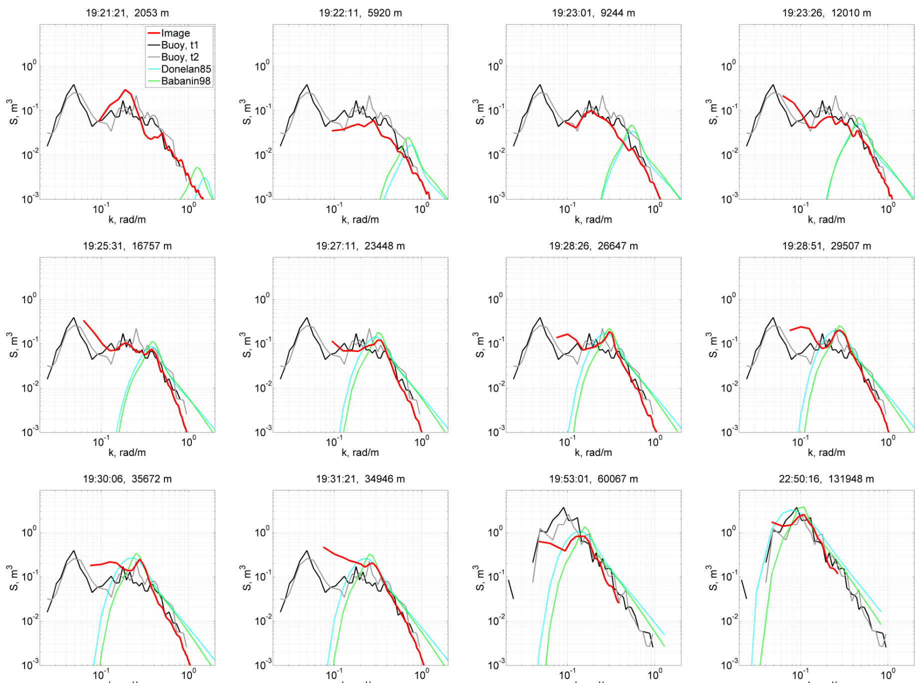


Figure 8: Omnidirectional spectra at the points marked by red squares on fig 5. Wind peak grows and shifts towards lower wavenumbers in consistence with Donelan et al. (1985) and Babanin and Soloviev (1998) spectra. Blue (Babanin and Soloviev, 1998) and green (Donelan et al., 1985) curves are given for the wind speed taken from the nearest buoy (42012 or 42040) and the fetch is corrected accounting for the spectrum wind wave peak direction estimated from Fig. 7.

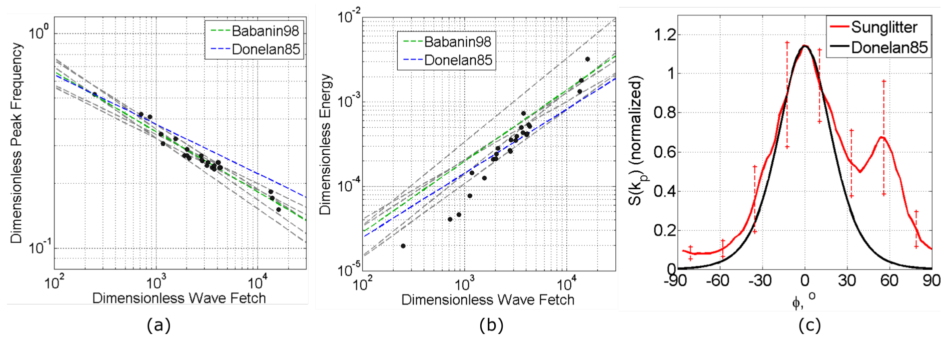


Figure 9: (a) Dimensionless peak frequency vs. dimensionless fetch. Black circles – experimental points (all retrieved spectra), dashed lines – approximations from other authors (Davidan, 1980; Babanin and Soloviev, 1998; Donelan et al., 1985; Kahma, 1981; Dobson et al., 1989; Wen et al., 1989; Ewans and C. Kibblewhite, 1990) for the wind speed 9 m $s^{-1}$; (b) Dimensionless peak frequency vs. windsea demensionless variance with the same notations; (c) angular function suggested by Donelan et al. (1985) and ensemble average wave energy distribution around the peak wavenumber ($0.75k_p < k < 1.25k_p$); the length of vertical lines is equal to standard deviation.

tected,

$$d_{long} = H\left[\tan(\theta_s - \beta_1) - \tan(\theta_s - \beta_2)\right], \quad (7)$$

where $H$ is the plane altitude.

The shortest wavelength being detected depends on the camera technical parameters, the camera view angle, $\gamma$, and the image pixel size, $N_p$. The 1D spatial resolution, the Nyquist wavelength, in and around the vicinity of the lens optical line-of sight axis, reads:

$$d_{short} = \frac{4H\tan(\gamma/2)}{N_p\cos\theta}, \quad (8)$$

for an altitude $H$ and zenith angle $\theta$.

Values of $d_{long}$ for different sun zenith angles and different wind speeds, and of $d_{short}$ for $N_p = 1000$, different camera view angles and two boundary camera zenith angles $\theta$ (nadir and $50^o$), are presented in Fig 10, a, b, as functions of camera altitude. It summarizes the range of wave scales that can be resolved from a sunglitter photograph. In practice, this range is significantly reduced. For the upper limit, a reliable spectrum estimation shall require a window size to encompass at least three wavelengths, or more, when considering the singularity of the transfer function around zero. The practical maximum wavelength is thus much shorter than $d_{long}$. As well, the estimate (8) is usually very optimistic compared to the real optical resolution possible to achieve. The estimate stands for the case of perfect lens focusing and the absence of any image blurring due to airplane movements and camera jitters. As shown, Fig. 3, f, and Fig 8, measurements from an altitude $H \simeq 1-1.5$ km, with $N_p \simeq 2000$, $\theta_s \simeq 45^o$, $\gamma = 80^o$, provide a wave spectrum reliably defined for waves between 10-20 m to 50-60 m. The

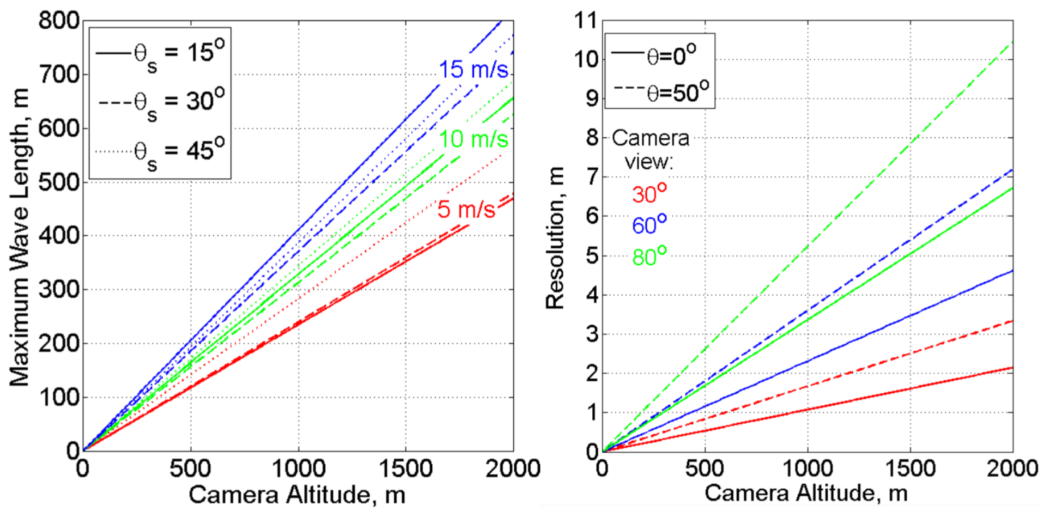


Figure 10: Left: the longest waves which can be observed within the useful part of the sunglint ($0.5 < Z_n^2/s^2 < 2$), as function of the camera altitude $H$. Three different sun zenith angles ($\theta_s = 15, 30, 45^o$) and three different wind speeds ($5, 10, 15$ m s$^{-1}$) are used for the calculations. Right: surface resolution as a function of camera altitude for the image size $N_p = 1000$ pixels, different camera view angles and two camera zenith angles ($\theta = 0^o$ and $\theta = 50^o$)

initial expectation gives a range between 3 m and 300 m. Nevertheless, these nominal estimates are useful to guide experiments and analysis for different conditions.

As a final remark, we note that, to retrieve spatio-temporal wave characteristics, the requirement of a perfect sunglint is less strict. Indeed, those characteristics can be derived directly from the surface brightness field and do not need the surface elevation spectrum. The previous stringent requirements apply to robustly retrieve the wave elevation spectrum from a sunglint photograph. Other spatio-temporal wave characteristics, such as the determination of a surface current-induced Doppler shift in the dispersion relation, merely needs to follow individual wave crests. As such, it can be applied further away from the sunglint, or even using the sky glint. Yet, a perfect geolocation might be required to accurately estimate wavelengths and shifts, and it is therefore recommended to work with images at small incidence angles.

## 7. Conclusion

In this paper, the method and algorithm development are presented to retrieve the two-dimensional wave spectrum from airborne sunglitter photographs. The implementation is demonstrated with airborne sunglitter photographs acquired during an experiment in the Gulf of Mexico. A linear transfer function to relate the image brightness variations to surface elevations is simply deduced from the shape of the glint. The singularity in wavenumber space, inherent to this approach, is eliminated by using several image fragments corresponding to different directions of the transfer function gradient. This was earlier suggested by Bolshakov et al. (1988) and Lupyan (1988), and it was also applied to satellite observations by Kudryavtsev et al. (2017a). Following this methodology, the absolute wavenumber elevation spectrum does not require any additional assumption or ancillary information about the sky brightness, wind or wave energy.

As also demonstrated, a cross-correlation analysis between consecutive photographs, taken with a small time lag (0.5 s), resolves the 180-degree ambiguity to provide the wave direction. Further, using a transect in the resulting phase spectrum gives an estimate of the wave dispersion along the propagation direction. As tested, comparisons between retrieved spectra and nearby NDBC buoy estimates are in good agreement, for both the spectral level and energy angular distribution.

A case study corresponding to the wave spectral evolution with increasing distance from shore in slanting-fetch conditions has then been considered, and further provide convincing evidence of the applicability and validity of the proposed method. Indeed, energy level and peak position transformation agree well with established approximations and laws of the wind-sea development, and quantitatively compare with previous experimental data and model predictions (Donelan et al., 1985; Babanin and Soloviev, 1998; Ardhuin et al., 2007).

In the context of today's rapidly growing technologies and the development of relatively simple remote controlled measurements from drones, the straightforward step-by-step proposed algorithm shall provide efficient means to renew and enhance the interest of aerial photographs of ocean sunglint patterns to infer quantitative information about surface wave characteristics and related rapid transformations over coastal areas.

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