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A reappraisal of ocean wave studies

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[1] A reappraisal of wave theory from the beginning to the present day is made here. On the surface, the great progress in both theory and applications seems to be so successful that there would be no great challenge in wave studies anymore. On deeper examination, we found problems in many aspects of wave studies starting from the definition of frequency, the governing equations, the various source functions of wave models, the directional development of wind wavefield, the wave spectral form and finally the role of waves as they affect coastal and global ocean dynamics. This is a call for action for the wave research community. For future research, we have to consider these problems seriously and also to examine the basic physics of wave motion to determine their effects on other ocean dynamic processes quantitatively, rather than relying on parameterization in oceanic and geophysical applications.

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1. Introduction

[2] Ocean wave studies actually have two distinctive sources as reviewed earlier by *Craik* [2004]. Briefly, the first theoretic study of water waves was produced by *Airy* [1841a, 1841b]. Using potential theory, he obtained the linear solution as a progressive sinusoidal wave train. Airy's solution was soon extended by *Stokes* [1847] to higher order, through perturbation expansion based on the assumption of small surface slope of the waves. Stokes's solution has firmly established the wave of permanent shape that would last till the 1960s, when *Phillips* [1960] found that weak nonlinearity could make the wave unsteady and evolve into totally different states. The weak nonlinear wave theory was immediately generalized to spectral representation by *Hasselmann* [1962, 1963a, 1963b], which was even viewed as 'Wave Turbulence' description by *Newell and Rumpf* [2011]. Therefore, Phillips' seminal study of weakly nonlinear wave-wave interaction could be regarded as the starting point of modern water wave studies.

[3] Contemporaneous with Airy and Stokes' studies, *Russell* [1844] reported an entirely different type of wave that is also of permanent shape but nonperiodic. The wave exists only in localized regions and is capable of traveling over long distances without change in shape. *Russell* [1844] reported his finding of this solitary wave, but the report drew deep suspicion from both Airy (then the Plumian Chair of Astronomy at Cambridge University and also the Astronomer Royal) and Stokes (then the Lucasian Professor of Mathematics at Cambridge University) because the phenomenon seemed to be at odds with the hydrodynamics theory known then. On the preponderant objections from the well-established authorities in mathematics and physics (in the persons of Airy and Stokes), the periodic waves of permanent shape became the mainstream of wave studies at that time. The solitary wave phenomenon, on the other hand, was almost relegated to oblivion until the 1870s when Boussinesq and Lord Rayleigh resuscitated it by providing some support based on hydrodynamics. Then, *Korteweg and de Vries* [1895] established the governing equation for solitary wave and periodic cnoidal solutions for propagation in canals, known as the KdV equation

$$\boldsymbol{u_t} + \boldsymbol{u}\boldsymbol{u_x} + \boldsymbol{u_{xxx}} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{u}$ is the velocity field in terms of the temporal and spatial variables, $\boldsymbol{t}$ and $\boldsymbol{x}$. What is truly amazing was the discovery by *Zabusky and Kruskal* [1965] of the exact closed form solution of this nonlinear equation, which they named as solitons. As solitary waves could propagate without interference from co-existing waves, they behave like particles, hence the name solitons. Later, it was shown (for example, by *Infeld and Rowlands* [1990]) that starting from the KdV equation given in (1), through perturbation analysis, one could arrive to a form of nonlinear Schrödinger equation

$$\boldsymbol{i}\left(\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{t}} + \boldsymbol{c}_{og}\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{x}}\right) + \frac{\boldsymbol{1}}{\boldsymbol{2}}\left(\frac{\partial^2 \omega_0}{\partial \boldsymbol{k}_0^2}\right)\frac{\partial^2 \boldsymbol{a}}{\partial \boldsymbol{x}^2} + \frac{1}{24\boldsymbol{k}_0}\boldsymbol{a}^2\boldsymbol{a}^* = \boldsymbol{0}, \tag{2}$$

with $\boldsymbol{a}$ as a complex valued envelope of the velocity field with the asterisk indicating the complex conjugate

$$\boldsymbol{u} = \frac{\boldsymbol{a}}{2}e^{i\theta_0} + \ldots(\textit{higher order terms})\ldots \tag{3}$$

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and $\boldsymbol{\theta}_0$ is the phase function so that

$$\frac{\partial\theta_0}{\partial \boldsymbol{x}} = \boldsymbol{k}_0\,; \quad \frac{\partial\theta_0}{\partial \boldsymbol{t}} = -\omega_0, \tag{4}$$

which stand for wave number and frequency respectively. It should be pointed out that, in equation (2), there is a dispersion relation derivative term, which has opposite signs for deepwater and for shallow water waves. The sign difference would give different stability conditions. Some discussions were given in *Infeld and Rowlands* [1990], for example. But details of the stability condition are much more complicated and depend on the coupling between dispersion and amplitude. Systematic studies are needed.

[4] Nevertheless, after establishing the nonlinear Schrödinger equation connection, in quick succession, *Zabusky and Kruskal* [1965] studied the behavior of soliton numerically, and *Gardner et al.* [1967] discovered the inverse scattering transform method to give analytic solution to a class of nonlinear differential equations including the Korteweg-De Vries equation, nonlinear Schrödinger equation, Camassa-Holm equation, Sine-Gordon equation, Ishimori equation and Toda lattice. The solitary wave dynamics actually have much richer mathematical properties. It will be shown that even the amplitude variations of the periodic waves over a long period of time are approximately controlled by the soliton type solution. Recently, *Osborne* [2010] has summarized the soliton type of nonlinear waves for specific ocean wave applications.

[5] Meanwhile, *Zakharov* [1968] proved that the envelope of finite amplitude deepwater surface waves would be governed by the nonlinear Schrödinger Equation through rigorous Hamiltonian calculation. Independently, *Yuen and Lake* [1975], *Dysthe* [1979], *Mei* [1983] and *Infeld and Rowlands* [1990] had also reached the same conclusion through perturbation analysis. Thus the two types of wave actually come back together to explain the ocean wave phenomena jointly. With these developments, the wave studies have become rich in both mathematical and physical aspects.

[6] On the surface, the state of theoretical wave studies seems to be complete by now. On the practical side, various wind wave models have made steady progress to the point that prediction of the sea state could be made with amazing accuracy [*Komen et al.*, 1994; *Lavrenov*, 2003]. Thus far, both the theoretical studies and practical applications seem to be completed. These successes made further progress in wave studies seemingly insignificant. The golden age of wave research in the 1980s was long gone and over.

[7] At the recent WISE Meeting (The 18th Waves in Shallow Environments (WISE) 2011, May 22–26, 2011, Qingdao, China), participants collectively lamented the lack of adequate attention on and financial support for wave studies in the recent past and especially at the present moment. We ventured some of our views at the occasion and were asked by the co-organizer, Professor Fangli Qiao, and WISE Chairman, Dr. Luigi Cavaleri, to summarize our remarks at the meeting for the record. This paper is an embellished version of our impromptu remarks. It is a personal reappraisal, tainted by the prejudices and shortcomings of our personal experiences and knowledge limitations, rather than a comprehensive review. In light of the recent developments in nonlinear and nonstationary data analysis and the progresses in incorporation of wave to large scale geophysical phenomena, we believe our remarks might serve as a new impetus for future wave studies. One of the key ideas in our new view was that we found the correct way to define frequency, which is somewhat irreconcilable and incompatible with the traditional ones based on physical intuition or available mathematical analysis tools. That revelation had taken us to a totally different arena of research deeply related to nonlinear and nonstationary processes. Though such processes are not confined to wave study, they underlie much of it. We do feel strongly that if we examine the state-of-the-art wave studies in detail and think through the fundamental physics (not mathematics), we could find many outstanding physical problems and worthy topics to spend our energy on covering topics from understanding of basic physics to practical applications. We hope these topics might attract the attention of funding agencies as well as the budding ocean scientists and applied mathematicians.

[8] We believe that those problems are crying for more research and understanding, albeit from a totally different standpoint. To begin with, ocean wave studies are usually expressed in terms of spectrum. Consequently, wavefield data are also represented by and reported in the form of spectra. Then, what kind of spectra should we use, in terms of frequency or wave number? How should we define frequency or wave number? Is directional distribution as important as mean wave energy? Do we know enough of the directional wind wavefield evolution? Are the governing equations of the wavefield derived in a mathematically self-consistent and physically sensible way? Are the physics of the wave fully represented by the governing equation currently employed? And finally, is wave study just for waves? What are the roles of waves in the framework of other ocean dynamic processes, especially the large scale ocean dynamics? What are the roles of waves in the complicated coastal zone? These are some of the questions unanswered in our minds; they are also the topics we deem to be worthy of attention for all ocean scientists and funding agencies. Such questions will be discussed in this review. After this brief introduction, a critical examination of the definition of frequency will be introduced in section 2. The governing equations will be discussed in section 3, followed by a review of wind-wave modeling in section 4 and an examination of the role of ocean wave in large scale ocean dynamics in section 5. Finally, some of our own observations and reflections on the evolution of wave studies over the last 40 years and future research directions, from our personal prospective, will be given as a conclusion of this review. We hope the reader will not treat this review as pontification but rather as a call for action.

2. Frequency and Wave Frequency Downshift

[9] Frequency is a fundamental and very useful quantity for any study of oscillatory phenomena. In fact, frequency is such a powerful concept that after Fourier's pioneer work, people tend to think of any motion in terms of frequency, for any function (under very general restrictions) could be expanded into Fourier series and each term seems to represent a simple harmonic component with constant amplitude and frequency. This is even more useful and powerful when

the phenomena we study are stochastic processes such as in a random wavefield. Powerful as Fourier analysis is, it is essentially a mathematical physics tool. Can we use it to analyze and understand the underlying physical process of any natural phenomenon even if the motion such as in the wavefield is nonlinear and only nearly periodic? Can we deduce meaningful physical interpretations based on Fourier analysis results? We think the answer depends on the statistical properties of the phenomenon in question. The answer is definitely negative, if the phenomena are nonlinear and nonstationary. As discussed by *Huang et al.* [1998, 2009], Fourier frequency is a physically meaningful representation only if the phenomenon is both linear (to allow superposition) and stationary (to allow constant amplitude and frequency values for all time). The recent study by *Rilling and Flandrin* [2008] raised an even more intriguing question: Whenever we use Fourier to represent linear and regular oscillations, are we studying physics or mathematics? For two strictly monotonic sinusoidal waves representing monotonic sounds with closely spaced frequencies, ω_1 and ω_2, for example, there could be two different views. Indeed, mathematically there are two different but equally valid expressions:

$$x(t) = \sin\omega_1 t + \sin\omega_2 t = 2\sin\left(\frac{\omega_1+\omega_2}{2}\right)\cos\left(\frac{\omega_1-\omega_2}{2}\right). \quad (5)$$

[10] Should we treat it as two separate tones as in the first expression? Or should we treat it as one single tone $(\omega_1 + \omega_2)/2$ with slowly varying amplitude as in the second expression? Their study indicates that two close tones are perceived naturally by human ears as a single tone rather than the two constituent components. Furthermore, if one utters a constant frequency sound but with regular fluctuation in volume, is he/she producing one frequency or three with a main peak plus two sidebands? The difference is totally in physical perceptions. Consequently, even for a strictly stationary signal, Fourier component view might not be the best answer, if we are interested in the details of physics rather than mathematics. All the other data analysis methods, such as wavelet analysis [*Daubechies*, 1992] or Wagner-Ville distribution [*Flandrin*, 1999] are also developed to alleviate the time-frequency variation problem, or nonstationary problem. All the methods are based on the a priori existence of Fourier like bases. While they might offer some improvements for nonstationary problems, they suffer the same fundamental restriction as Fourier analysis. Now, let us turn our attention to frequency.

[11] In a physics class, the frequency, ω, is simply defined as

$$\omega = \frac{1}{T}, \quad (6)$$

where T is the period of the wave. As discussed by *Huang et al.* [1998, 2009], this definition for frequency is correct only dimensionally. It represents a mean value over one wavelength and is therefore too crude to be useful for studying the dynamics of any phenomena involving nonlinear nonstationary processes, such as water waves.

[12] As discussed by *Huang et al.* [1998, 2009, 2011a], for nonlinear phenomena, we have to describe nonlinear distorted wave profiles. Let us take the simplest nonlinear system given by

$$\begin{aligned} &\frac{d^2x}{dt^2} + \alpha x + \varepsilon x^{n+1} = f(t) \textit{ could be written as} \\ &\frac{d^2x}{dt^2} + x(\alpha + \varepsilon x^n) = f(t), \end{aligned} \quad (7)$$

in which α and ε are constants. For a nonlinear oscillatory system, the motion is equivalent to a spring with variable spring constant. In fact, the term in the parenthesis is equivalent to the frequency squared. Indeed, the frequency of the system should be ever changing even within a single swing of the pendulum, which is defined by *Huang et al.* [1998, 2011a] as intrawave frequency modulation. For $\alpha = 1$ and $n = 1$, equation (7) becomes a quadratic nonlinear oscillator as given in *Huang et al.* [1998, 2011a]. For $\varepsilon = 0.375$, the wave profile and the corresponding instantaneous frequency are given in Figure 1. The up-down asymmetry and the sharp crests and rounded troughs wave form are the consequence of odd value of n and positive ε. The intrawave frequency modulation pattern, once each wave, confirms the nonlinearity order to be quadratic. The characteristic of this wave form could be captured by a simple $n = 1$ intrawave modulation model as

$$x(t) = \cos(\omega t + \varepsilon \sin \omega t), \quad (8)$$

which would have an instantaneous frequency

$$\Omega(t) = \omega(1 + \varepsilon \sin\omega t). \quad (9)$$

[13] This value is ever changing. This is the nature representation of the wave frequency. But one could expand the expression given in equation (8) as

$$\begin{aligned} x(t) &= \cos(\omega t + \varepsilon\sin\omega t) \\ &= \cos\omega t\cos(\varepsilon\sin\omega t) - \sin\omega t\sin(\varepsilon\sin\omega t) + \ldots \\ &= -\frac{\varepsilon}{2} + \cos\omega t + \frac{\varepsilon}{2}\cos 2\omega t + \ldots, \end{aligned} \quad (10)$$

which bears striking similarity with the Fourier expansion of the Stokes wave. Obviously, the same wave form could have two different presentations depending on the method one choose to use. With Fourier analysis, we would be forced to resort to harmonics as given in equation (10). If the original wave happens to be dispersive, such as deepwater surface waves, then all the harmonics would have to travel at the phase velocity of the fundamental in order to maintain the integrity of the wave form and cease to be dispersive. Consequently, no matter how high the order of the harmonics is, the phase velocity of that component will have to be a constant commensurate with whatever the fundamental frequency it is associated with. This would force the wave with a given frequency, but for being harmonics of different fundaments, to propagate at different phase velocity. Thus, none of the harmonics can be physical waves, but are mathematical artifacts rather than true physical wave components. Therefore, all harmonics should not be meaningful physical quantities.

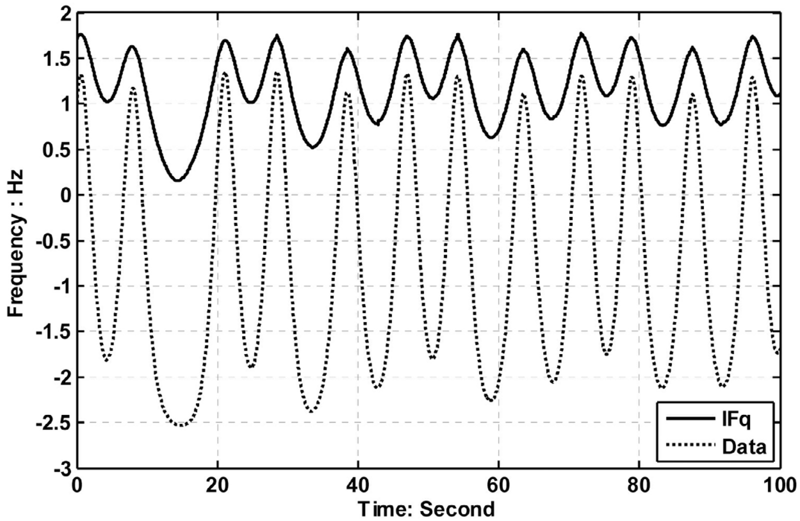

Figure 1. The intrawave frequency modulation for a nonlinear wave based on Stokes model. The dotted line is the model wave profile as the solution from equation (7) with $\boldsymbol{n} = 1$ and $\boldsymbol{\varepsilon} = 0.375$ that shows the typical water wave characteristics with sharp crests and round trough. The solid line is the instantaneous frequency. The once per wave cycle intrawave modulation indicates the quadratic nonlinearity in the model equation causing the asymmetric wave from distortion.

[14] Indeed, in the theoretical study of wave motion, the most important governing equations are certain conservation laws. The simplest and the most fundamental one is the kinematic conservation: In general, for any wave motion, there must be a smooth phase function, $\boldsymbol{\theta}$, so that we can define wave number, $\boldsymbol{k}$, and frequency as follows:

$$k = \frac{\partial \theta}{\partial x}, \text{ and } \omega = -\frac{\partial \theta}{\partial t}, \tag{11}$$

as a generalization of the expression in equation (4). Therefore, by cross differentiation, we have

$$\frac{\partial k}{\partial t} + \frac{\partial \omega}{\partial x} = 0. \tag{12}$$

[15] No wave motion can violate it. For it to hold, it is obvious that both the wave number and the frequency have to have instantaneous values and also be differentiable. The constant wave number and frequency defined through Fourier analysis certainly satisfy the kinematic conservation, but that would be a trivial condition. The only frequency that could satisfy the requirement for the kinematic conservation would be the instantaneous frequency [*Huang et al.*, 1998, 2009, 2011a]. Only with instantaneous frequency can we describe the richness of variation in frequency of the nonlinear and nonstationary waves, where the intrawave frequency modulation is the rule rather than the exception.

[16] Let us consider a simple but real physical example, the water waves. The instantaneous frequency variation of a mechanically generated regular wave train of 2.5 Hz with initial wave steepness at $\boldsymbol{a_0 k_0} = 0.2$ is given in Figure 2. Here the instantaneous frequency, computed with the Hilbert-Huang transform [*Huang et al.*, 1998], is not gradual at all. The values fluctuate noticeably over a sizable range and tend to be slightly higher at the wavefront. In comparison with the Morlet wavelet analysis results, as superposed here, the instantaneous frequency result is sharp and clear with no smearing and smoothing from the integral transform used in wavelet. Statistically, the instantaneous frequencies for all the waves at this particular station are coherent and differ significantly from the constant values as shown in Figure 3. The evidence shows that the nonconstant frequency is not a fluke, but a fact.

[17] The similarity among the nonlinear oscillator given in equation (7), the simple mathematical model given in equation (8), and the laboratory waves data given in Figure 2 indicate that the rich dynamics could be revealed by using the proper new method that does not depend on the linear and stationary assumptions and is not based on integral transform on a priori basis. If we accept the instantaneous frequency as a valid way to represent frequency in wave study, we have to ask the next question: What is a frequency downshift? How does it happen?

[18] From an anthropocentric view, the well-known phenomenon of wave evolution under persistent wind from short wavelengths and low energies at short fetches to long and more energetic ones at longer fetch is labeled as 'wave growth.' Even without wind-forcing, the waves are observed [*Huang et al.*, 1996, 1998, 1999] to evolve and become longer though. Traditionally, such a process is described as a gradual and slow evolution governed by weak nonlinear wave-wave interactions discovered by *Phillips* [1960] and subsequently extended to spectral representation by *Hasselmann* [1962, 1963a, 1963b] through a sixfold Boltzmann integral:

$$\frac{\partial \boldsymbol{N}(\boldsymbol{k}_1)}{\partial \boldsymbol{t}} = \frac{\partial \boldsymbol{N}_1}{\partial \boldsymbol{t}} = \int\int\int_{\boldsymbol{k}_2\boldsymbol{k}_3\boldsymbol{k}_4} \boldsymbol{Q}\{(N_1+N_2)N_3N_4 - (N_3+N_4)N_1N_2\} \cdot \boldsymbol{\delta}(\omega_1+\omega_2-\omega_3-\omega_4)\boldsymbol{\delta}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_4)\boldsymbol{dk}_2\boldsymbol{dk}_3\boldsymbol{dk}_4 \tag{13}$$

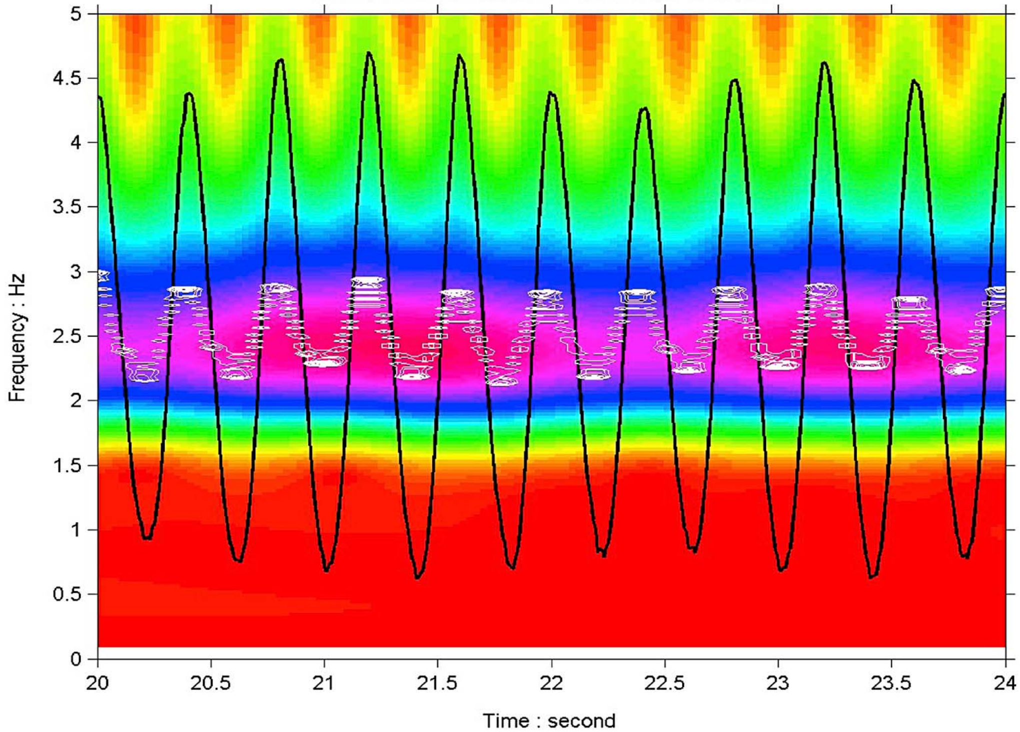

Figure 2. Section of mechanically generated water data: The black line is the wave profile, superposed on the colored Morlet wavelet analysis result and the Hilbert instantaneous frequency (white line). Note the frequency resolution of Hilbert spectrum is much sharper than the wavelet analysis. Again, the intrawave modulation is similar to the modeled one given in Figure 1. The asymmetric wave profile and once per wave intrawave modulation suggest that the nonlinearity is of the quadratic order.

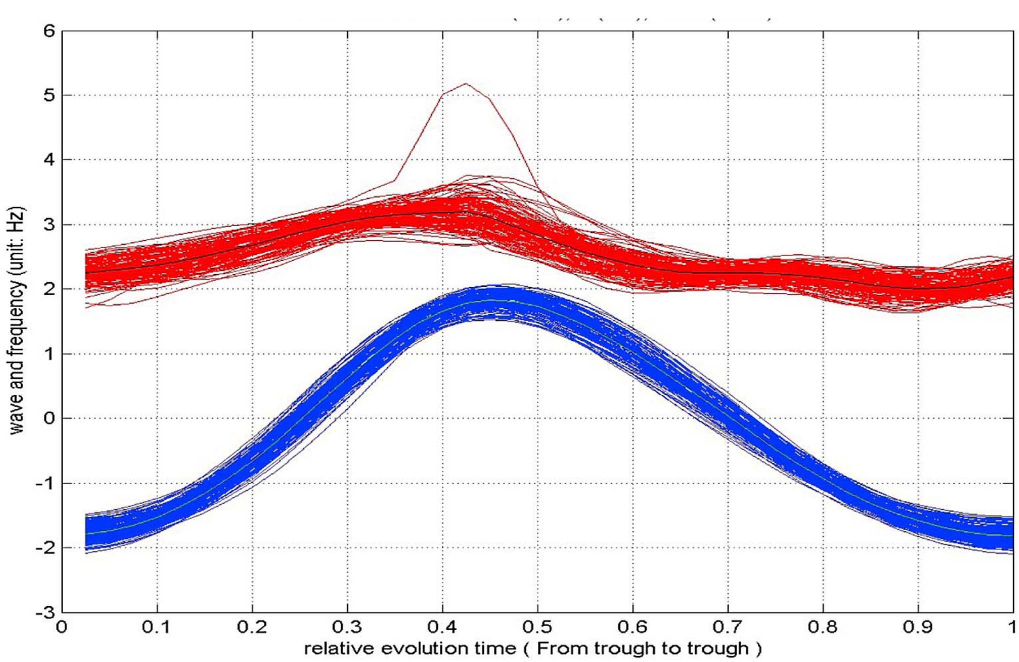

Figure 3. The phase averaged wave profile and instantaneous frequency, IF, for the case given in Figure 2. Here the asymmetric profile and highly modulated instantaneous frequency are obvious and consistent facts.

in which $\boldsymbol{N}$ is the action spectral density, $\boldsymbol{Q}$ a complicated function of the directional wave numbers, $\boldsymbol{k_1 \ldots .k_4}$, and $\boldsymbol{\delta}$ the Dirac delta function. We will return to the details of nonlinear wave-wave interaction later. Suffice here to say that this description calls for gradual and continuous change in the wavelength. The difficulty and deficiency of that approach had been discussed and an alternative approach also proposed by *Huang et al.* [1996, 1998, 1999]: the wave fusion. Based on observations, wave fusion processes are discrete, local and abrupt. Waves grow longer not through a gentle lengthening of wave individually or spectrally. Lengthening of wave would violate the limitation on propagation speed imposed by the dispersion relationship and the kinematic conservation law. The only possible process is fusion of waves locally, discretely and abruptly: with 2 waves fusing into 1, 3 waves into 2 and $\boldsymbol{n}$ waves into $\boldsymbol{(n}$-1$\boldsymbol{)}$. The fusion of waves is not new; it had been observed as 'missing crest' by *Yuen and Lake* [1975], 'crest-pairing' by *Ramamonjiarisoa and Mollo-Christsen* [1979], and 'mutual coalescence' by *Hatori and Toba* [1983]. All the past studies had provided only qualitative pictorial descriptions of the processes involving two waves fusing into one on the time axis only. *Huang et al.* [1996] were the first ones to quantify the process based on phase angle analysis through Hilbert transform and described the $\boldsymbol{n}$ to $\boldsymbol{(n}$-1$\boldsymbol{)}$ variations with $\boldsymbol{n}$ equal 2, 3 and 4. Presumably, the process will go on to higher number of waves. An example of how the waves evolve is given in Figures 4a and 4b. In Figure 4a we can see the fusion of two waves into one occurring at 59th second. In Figure 4b, we can see 3 waves fusing into 2 (around 28 to 30 and 31 to 32 s) and 2 waves fusing into 1 again (around 27 to 28 s). The phase function scale is not given, but the magnitude is roughly around 2 radians per division, or the drop in value across 27 to 28 s would be around $\boldsymbol{2\pi}$. Details of the results can be found in *Huang et al.* [1996, 1998, 1999]. Technically, the fusion process violates wave conservation in the traditional view. What kind of equations could describe such discrete, local and abrupt changes is a question begging for an answer. But, it seems to be self-evident that the present gradual spectral representation of weak nonlinear wave-wave interactions could not be the solution. As demonstrated by *Huang et al.* [1996], at the initial fusion stage, the spectral form only indicates sideband instability, with the two sidebands slightly asymmetric. There is no indication of local frequency change, as Fourier analysis should not be applied to this unsteady condition. Critically, we have to ask during the local and discreet fusion process, what is conserved? Would this violate the wave conservation? How about the energy, action and even action flux conservation? We need a new method of data analysis and a new view and new understanding of the crucial physical evolution process in the wind wave modeling. Next, let us turn to the problems of the governing equations.

3. Governing Equations

[19] The modern view of random ocean wavefield dynamics is based on the weak nonlinear interactions [*Phillips*, 1960; *Hasselmann*, 1962, 1963a, 1963b; *Zakharov*, 1968], which was designated by *Newell and Rumpf* [2011] as the wave-turbulence analogy. The closure of this system is the resonant interactions. The weakly nonlinear wave-wave interactions as given by equation (13) could happen only when

$$\begin{aligned} \boldsymbol{k}_1 + \boldsymbol{k}_2 &= \boldsymbol{k}_3 + \boldsymbol{k}_4; \\ \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 &= \boldsymbol{\omega}_3 + \boldsymbol{\omega}_3. \end{aligned} \tag{14}$$

[20] Furthermore, the wave numbers and frequencies involved in the interactions will have to satisfy the dispersion relationship

$$\boldsymbol{gk_i = \omega_i^2 \; for \; all \; i}, \tag{15}$$

in which $\boldsymbol{g}$ is the gravitational acceleration. With these conditions satisfied, the interacting waves would have their wave numbers following the famous ***figure-8*** locus as derived by *Phillips* [1960]. The reason these conditions are imposed is to guarantee the participating waves follow the dispersion relationship and be physically meaningful waves. There are two difficulties in this condition: First, the simple resonant phenomena are special condition for linear systems. When oscillation amplitude becomes finite, the system response could be very complicate. A systematic understanding of the response to strong forcing does not exist. It could involve hysteresis, bifurcation, sub-harmonics and even chaos. Furthermore, in a random wavefield, persistent periodic forcing would not exist. Whether the resonant condition is satisfied, or what are the detailed responses are problems worthy of our attention. Second, the weakly nonlinear wave-wave interactions are formulated in terms of Fourier spectra, which contain all orders of harmonics that are not dispersive. Indeed, field measurements of the dispersion relation (*Beal* [1991] and many other independent experiments) indicated that the dispersion relation is only true at a very narrow range of frequencies and wave numbers near the energy containing peak. There is no guarantee that the dispersion relation holds for all the spectral components, especially for the high frequency and wave number ranges and there is no method to separate the physical waves from the nonphysical harmonics. Indeed, we will show later that the high frequency range of the Fourier spectra of random wavefield consists most of harmonics. Even the theoretical derivation of the dispersive relationship [*Huang and Tung*, 1976] depends on the spectral representation of the waves assumed to be free waves. Therefore, the validity of the spectral representation should be carefully evaluated. A potential research topic should be to define the wave spectral form in terms of the instantaneous frequency rather than the Fourier frequency. With such a spectrum, all waves are physical. Then, the evaluation of the weak nonlinear wave-wave interaction might yield different results. But even with this change, it is still doubtful whether the new formulation could fully account for the discrete fusion phenomenon.

[21] An alternative approach for the governing equation is to study the wave envelope rather than the actual wave surface. *Zakharov* [1968] proved that the envelope of finite amplitude water waves would be governed by the nonlinear Schrödinger Equation through rigorous Hamiltonian calculation. Independently, *Yuen and Lake* [1975] and *Mei* [1983] had also reached the same conclusion through perturbation analysis. Subsequently, *Dysthe* [1979] derived the nonlinear

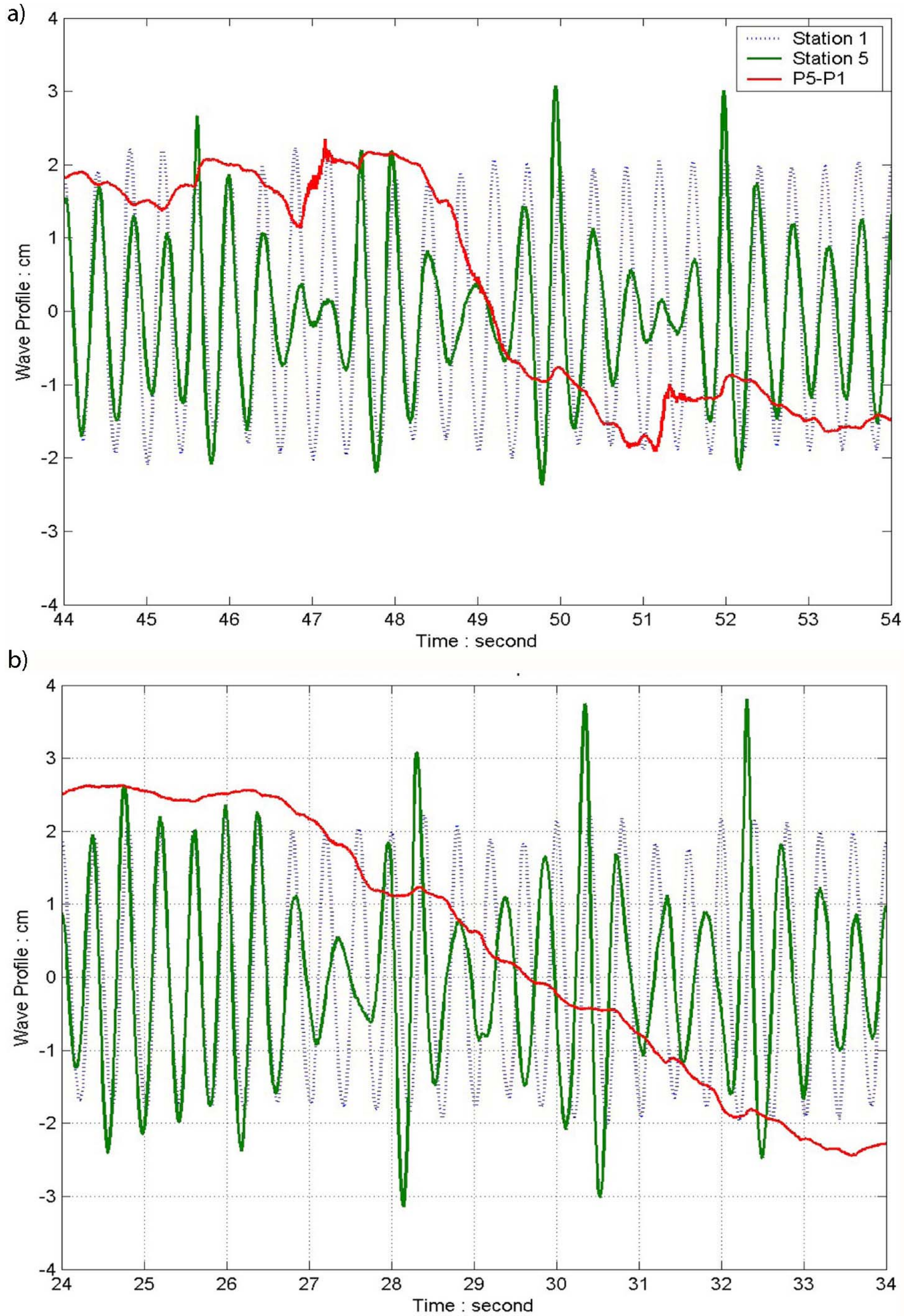

Figure 4. (a) The wave fusion process in action at station number 5. The dotted line gives the wave condition at station number 1 serving as a reference here. The solid green line is the wave profile at station number 5. The wave fusion occurs around the 49th second of this record, where two waves fuse into one and the phase values (red line) jump 2π. (b) The wave fusion process in action at station number 6. Same as in Figure 4a but at a later stage: the fusions occur at 27–28th, 28–30th and 31–33th second, where '2 to 1' and '3 to 2' fusion processes could be seen.

Schrödinger equation to the third order by perturbation analysis from the governing equation of a perfect fluid with the nonlinear boundary condition at the free surface. Thus, the two types of waves discovered separately by Russell and Stokes are finally re-unified. The solitary wave equation could also be used to govern the deep water periodic waves, if the solitary wave is treated as the envelope of the underlying aperiodic carrier wave train [*Zakharov*, 1968]. As the nonlinear Schrödinger Equation has been studied extensively in physics in connection with plasma flow [*Infeld and*

Rowlands, 1990], this new approach tapped a rich vein of mathematical physics and initiated a golden age of wave study in the 1970s–1980s. Even though results such as the Fermi-Pasta-Ulam recurrence, an experimental *tour de force*, had been shown to exist by *Tulin and Waseda* [1999], the overwhelming number of studies, however, produced downshifting of the wave frequency. For example, the first experimental results reported by *Lake et al.* [1977] as the proof of Fermi-Pasta-Ulam recurrence all showed frequency downshift. The recurrence could be achieved easily in numerical modeling when the periodic boundary condition is usually imposed. Experimentally, the recurrence is hard to achieve and could only occur under exceptional carefully controlled conditions as implemented by *Tulin and Waseda* [1999]. Downshifting is a rule, not an exception.

[22] All these results, recurrence or downshift, are interesting and impressive, but they also suffer from a common flaw: the carrier waves are assumed to be purely sinusoidal with constant frequency up to the third order. For example, the most complete derivation of the Schrödinger equation through perturbation analysis by *Dysthe* [1979]. Assuming water to be incompressible, inviscid, without surface tension and of infinite depth, we should have a potential flow governed by the following set of equations:

$$\left.\begin{array}{ll}\nabla^2\phi = \mathbf{0} & z \le \varsigma \\ \dfrac{\partial\varsigma}{\partial t} + \dfrac{\partial\phi}{\partial x}\dfrac{\partial\varsigma}{\partial x} + \dfrac{\partial\phi}{\partial y}\dfrac{\partial\varsigma}{\partial y} = \dfrac{\partial\phi}{\partial z} & \\ \dfrac{\partial^2\phi}{\partial t^2} + g\dfrac{\partial\phi}{\partial z} + \dfrac{\partial}{\partial t}(\nabla\phi)^2 + \dfrac{1}{2}\nabla\phi\cdot\nabla(\nabla\phi)^2 = \mathbf{0} & z = \varsigma.\end{array}\right\} \quad (16)$$

Again, assuming slow evolution of wave train, Dysthe developed the velocity potential, ϕ, and the surface elevation, ζ, as

$$\begin{aligned}\phi &= \overline{\phi} + Ae^{kz}e^{i\theta} + A_2e^{2kz}e^{2i\theta} + \ldots..\\ \varsigma &= \overline{\varsigma} + Be^{i\theta} + B_2e^{2i\theta} + \ldots..,\end{aligned} \quad (17)$$

and eventually arrived at the classic expression,

$$\begin{aligned}2i\left(\frac{\partial a}{\partial t} + \frac{1}{2}\frac{\partial a}{\partial x}\right) + \frac{1}{2}\frac{\partial^2 a}{\partial y^2} - \frac{1}{4}\frac{\partial^2 a}{\partial x^2} - a|a|^2 = -\frac{1}{8}i\left(6\frac{\partial^3 a}{\partial x\partial y^2} - \frac{\partial^3 a}{\partial x^3}\right)\\ + \frac{3}{2}ia\left(a\frac{\partial a^*}{\partial x} - a^*\frac{\partial a}{\partial x}\right) + \frac{1}{2}i|a|^2\frac{\partial a}{\partial x} + a\left(\frac{\partial\overline{\phi}}{\partial x} - i\frac{\partial\overline{\phi}}{\partial z}\right),\end{aligned} \quad (18)$$

in which a is a complex valued envelope with the asterisk indicating the complex conjugate. Inclusive as it is, one should notice that the conspicuous absence in the equation are the variations of frequency and wave number. In deriving this expression, the frequency and wave number are assumed to be strictly constant. In other words, the underlying carrier waves are pure sinusoidal waves and harmonics with only amplitude modulations. Such assumption is totally artificial, for pure sinusoidal water surface wave could not exist physically nor satisfy the equation of motion and boundary conditions mathematically.

[23] If we relax the constant frequency restriction slightly, the governing equation would change to very different forms. The culprit is the dispersion relation, for the small amplitude approximation used in most case is not strictly correct. *Fornberg and Whitham* [1978] proposed

$$\omega = \omega_0(k) + \omega_2(k)A^2 - \frac{1}{2}\omega_{0kk}A_{xx}A^{-1}. \quad (19)$$

[24] This would cause considerable complication on the governing equation. The nonlinear dispersion is precisely the reason that the soliton form would not last for long time. Even for a simpler case of the shallow water waves governed by the KdV equation given in equation (1), the results are already very complicated. Here, we could follow the development given in *Infeld and Rowlands* [1990] for shallow water waves, but with the restrictions on the frequency and wave number variations relaxed systematically. We start with

$$u = ae^{i\theta} + u_2(\theta, x, t) + \ldots., \quad (20)$$

with a, k, and ω all functions of position and time. When we assume both the frequency and wave number to be constant up to the third order, we have

$$i\left(\frac{\partial a}{\partial t} + c_{og}\frac{\partial a}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial^2\omega_0}{\partial k_0^2}\right)\frac{\partial^2 a}{\partial x^2} + \frac{1}{24k_0}a^2a^* + \gamma a = 0, \quad (21)$$

with γ as an arbitrary constant.

[25] Following the same derivation, when we relax the restriction on the frequency and wave number to second order, we would have extra terms in the following governing equation with the first derivative of wave number nonzero:

$$\begin{aligned}&i\left(\frac{\partial a}{\partial t} + c_{og}\frac{\partial a}{\partial x}\right) + \frac{1}{2}\omega_{0kk}\frac{\partial^2 a}{\partial x^2} - 3\omega_{20}a^2a^*\\ &\quad + \frac{i}{2}\omega_{0kk}a\frac{\partial k}{\partial x} + \frac{1}{6}\omega_{okk}aC = 0;\\ &\frac{\partial C}{\partial t} + \frac{1}{2}a^*\frac{\partial a}{\partial x} = 0;\ \textbf{and}\\ &\frac{\partial k}{\partial t} + \omega_{0k}\frac{\partial k}{\partial x} = -\omega_{20}\frac{\partial aa^*}{\partial x},\end{aligned} \quad (22)$$

with C as an arbitrary constant. Now, when we relax the restriction further for the frequency and wave number to be constant only to the first order, we would have more extra terms as in

$$\begin{aligned}&i\left(\frac{\partial a}{\partial t} + c_{og}\frac{\partial a}{\partial x}\right) + \frac{1}{2}\omega_{0kk}\frac{\partial^2 a}{\partial x^2} - 3\omega_{20}a^2a^* + \frac{i}{2}\omega_{0kk}a\frac{\partial k}{\partial x}\\ &\quad - 3\frac{\partial k}{\partial x}\frac{\partial a}{\partial x} - a\frac{\partial^2 k}{\partial x^2} + \frac{1}{6}\omega_{okk}aC = 0;\\ &\frac{\partial C}{\partial t} + \frac{1}{2}a^*\frac{\partial a}{\partial x} = 0;\ \textbf{and}\\ &\frac{\partial k}{\partial t} + \omega_{0k}\frac{\partial k}{\partial x} = -\omega_{20}\frac{\partial aa^*}{\partial x}.\end{aligned} \quad (23)$$

[26] Given the observed variation in the instantaneous frequency of a periodic wave train; it would be hard to justify

even the second order smoothness of the instantaneous frequency here. Therefore, it would be difficult to reconcile any form of the nonlinear Schrödinger equation, without considering the strong variation of frequency and wave number, as a valid governing equation for nonlinear water waves, especially when frequency and wave number are defined through equation (11). As the whole approach is based on the existence of a phase function to define both frequency and wave number for their instantaneous values, it behooves us to consider the real change of them beyond the zero-th order. Any wave equation without considering the variations of frequency and wave number are not really physical. Consequently, most of the theoretical wave studies should be regarded as a mathematical exercise and inquiry rather than an investigation of the physical phenomena. Indeed, most of the best results of the wave analysis are published by applied mathematicians as summarized by *Whitham* [1974], *Phillips* [1977], *Mei* [1983] and *Infeld and Rowlands* [1990]. Based on our review so far, it seems that the explanation for the real interesting physics remains elusive. It is time that we bring back the physics into our serious consideration. Other than the theoretical study of waves, an applied aspect of wave modeling has also received a lot of attention. That is what we will examine next.

4. Wind Wave Modeling

[27] Wind wave modeling had been a subject of applied research since the 1940s, when Allied Forces were planning the Normandy landing during WW II. After the war, commercial activities calling for ship routing, ship building, coastal and ocean engineering projects had given wave modeling another push. Various numerical models were advanced to fill the needs. Most recent efforts were summarized by *Komen et al.* [1994] and *Lavrenov* [2003]. Contrary to most claims, wave modeling is not based on dynamical equations but on simple energy or action balancing ones:

$$\left[\frac{\partial}{\partial \boldsymbol{t}} + (\boldsymbol{c_g} + \boldsymbol{U}) \cdot \frac{\partial}{\partial \boldsymbol{x}} - \boldsymbol{k} \cdot \frac{\partial}{\partial \boldsymbol{k}}\right] \left(\frac{\boldsymbol{F}(\boldsymbol{k}; \boldsymbol{x}, \boldsymbol{t})}{\boldsymbol{\omega}}\right) = \boldsymbol{S_{in}} + \boldsymbol{S_{nl}} + \boldsymbol{S_{ds}}, \tag{24}$$

in which the left hand side is the standard action conservation equation with $\boldsymbol{F}$ as the directional wave number spectrum, a function of wave number, $\boldsymbol{k}$, position, $\boldsymbol{x}$, and time, $\boldsymbol{t}$; $\boldsymbol{\omega}$ is the intrinsic frequency; and $\boldsymbol{S}$ stands for the various source functions including input from the wind, $\boldsymbol{S_{in}}$, nonlinear interactions, $\boldsymbol{S_{nl}}$, and dissipation from breaking, $\boldsymbol{S_{ds}}$. As it stands, we have made the tacit assumption that the source functions are independent and could be linearly superposed. Such an assumption seems reasonable, but it is not strictly correct. Laboratory observations have indicated that surface wind stress could severely interfere with nonlinear interaction [*Bliven et al.*, 1986]. It is also well known that surface wind stress could enhance wave breaking [*Phillips and Banner*, 1974]. On a parametric model, the effects of enhanced breaking have been conducted by *Huang* [1986]. The results are consistent with laboratory and field experimental results. Quantification of the relevance of these phenomena in the field has never been seriously investigated and quantified. Based on the reliance of the assumptions used in wave modeling, it is imperative that such detailed experimental validation be conducted.

[28] All these reservations notwithstanding, most of the validations of the modeling efforts were carried out on the wave amplitude, in terms of significant wave height. The results seem to have been so successful that wave prediction has become routine operation of meteorological agencies in many countries. The reality is quite different. In the action balance equation, all the source functions are heavily parameterized and tuned specifically for the location where the prediction is applied. Each parameterization has employed simplification and idealization to gloss over the complicated real physics. For example, the nonlinear source seems to have the firmest theoretical foundation, yet even here problems still exist; the dispersion relationship is simply another one of them as discussed above. Furthermore, the terms is usually simplified and parameterized to save computing time. The most serious problem is on the dissipation source term.

[29] Now let us discuss the problems on dissipation source term. Obviously, the most effective dissipation process for wave motion is wave breaking. Unfortunately, our understanding of the breaking processes is still limited, in spite of the progresses summarized by *Melville* [1996] and *Babanin* [2009, 2011] over the last twenty years. As a result, most of the modelers would agree on the fact that the dissipation source is the least understood one among the source functions. It is generally accepted that the dissipation source should be of the general form

$$\boldsymbol{S_{ds}} \sim \boldsymbol{F^n}(\boldsymbol{w}, \boldsymbol{k},\ \boldsymbol{\theta})$$

with the exponent $\boldsymbol{n}$ still unspecified, as given in *Babanin* [2009]. Various forms have been proposed, for example, *Polnikov* [2010] used

$$\boldsymbol{S_{ds}} = \boldsymbol{m}\, \boldsymbol{C}(\boldsymbol{\omega}, \boldsymbol{\theta}, \boldsymbol{\omega_p})[\boldsymbol{\beta_{dis}}, \boldsymbol{\beta}(\boldsymbol{\omega}, \boldsymbol{\theta}, W)]_{\max} \frac{\boldsymbol{\omega}^6}{\boldsymbol{g}^2} \boldsymbol{F}^2(\boldsymbol{\omega}, \boldsymbol{\theta}), \tag{25}$$

in which $\boldsymbol{m}$ is a constant to be determined; $\boldsymbol{C}(\boldsymbol{\omega}, \boldsymbol{\theta}, \boldsymbol{\omega_p})$ describes the dissipation at the range of spectral maximum by its own formula in terms of frequency, $\boldsymbol{\omega}$, and angular spreading, $\boldsymbol{\theta}$, with the subscript $\boldsymbol{p}$ indicates the spectral peak value; $\boldsymbol{\beta}$ is the input source function; $\boldsymbol{\beta_{dis}}$ is a background control energy level for nonzero dissipation; and $\boldsymbol{F}(\boldsymbol{\omega}, \boldsymbol{\theta})$ is the directional spectrum. But the more popular form is due to *Donelan and Yuan* [1994] used in WAM model [*Komen et al.*, 1984]:

$$\boldsymbol{S_{ds}} = \boldsymbol{C_{ds}} \left(\frac{\hat{\alpha}}{\hat{\alpha}_{PM}}\right)^{\boldsymbol{m}} \left(\frac{\boldsymbol{\omega}}{\overline{\boldsymbol{\omega}}}\right)^{\boldsymbol{n}} \boldsymbol{\omega} \boldsymbol{F}(\boldsymbol{k}), \tag{26}$$

in which $\boldsymbol{C_{ds}}$, $\boldsymbol{m}$, $\boldsymbol{n}$ are fitting constants,

$$\hat{\alpha} = \boldsymbol{m_o} \overline{\boldsymbol{\omega}}^4 / \boldsymbol{g}^2,$$

$\boldsymbol{m_o}$ is the total energy density of the wavefield, the ratio of α actually measures the overall steepness of the wavefield, and the subscript PM stands for the Pierson-Moskowitz spectrum. *Donelan and Yuan* [1994] based their model on

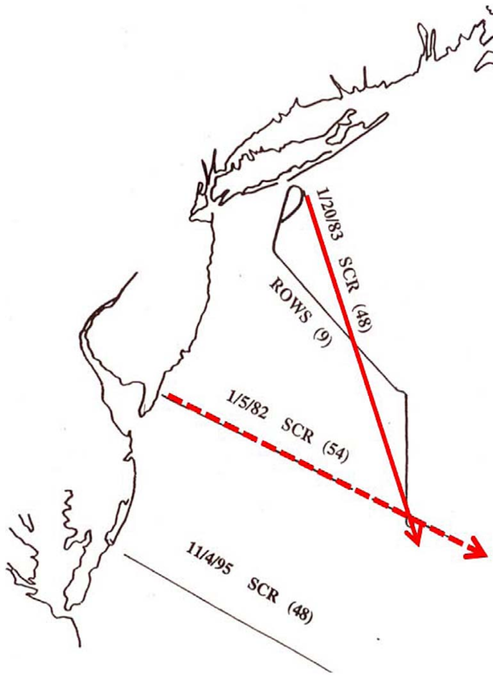

Figure 5. The flight tracks of the Surface Contour Radar experiments that were conducted by *Walsh et al.* [1985, 1989].

the observed of geometric similarity of the whitecaps and the underlying waves from the laboratory airfoil induced steady breakers of *Duncan* [1981]. Yet, it is well known that breakers are not steady in reality. *Donelan and Yuan* [1994] proposed still another variation of the dissipation source function based on probability density function of the wavefield:

$$\boldsymbol{S_{ds}} = \boldsymbol{C_{ds2}}\boldsymbol{P}(\phi(\omega))\left(\frac{\omega}{\omega_z}\right)^2 \boldsymbol{F}(\boldsymbol{k}), \tag{27}$$

where $\boldsymbol{C_{ds2}}$ is another constant, $\boldsymbol{P}(\phi(\omega))$ is a complicated function of the wave frequency spectrum, $\phi(\omega)$. Both of their results are linearly dependent on the spectrum $\boldsymbol{F}(\boldsymbol{k})$.

[30] The contrast of among equations (25), (26) and (27) speaks volumes. If all of them works well in their respective wave model, we could only attribute the success to the tuning of the to be determined constants and empirical functions in the formulae. True physics could not be represented by different functional forms and still be precise and all correct.

[31] The core of the problem for the present uncertainty state is our poor understanding of the breaking mechanism. Of course, wave breaking is due to the wave form instability. Currently, there are two approaches to represent the results: The first one attributes breaking purely to the wave instability as in all the above equations, for there is no wind condition involved directly. Even the ones attributed breaking on phenomenological whitecaps distribution as in equations (25) and (26) were expressed in terms of wave parameters. But the wind effects on breaking could not be neglected. It could be so strong that breaking could occur on otherwise smooth wave surface as described in *Stoker* [1992]. Even at the active breaking stage, the severity of breaking could be drastically different all depending on the wind condition [*Babanin*, 2009]: Wind would increasing the breaking probability but decrease the breaking severity, presumably due to the combined wind effects both on the wave group modulation [*Bliven et al.*, 1986] and the enhanced breaking due to the surface drift current [*Banner and Phillips*, 1976; *Phillips and Banner*, 1974]. Indeed wind condition is so important that the passive microwave radiometry sensing of the oceanic surface wind is based empirically on the brightness temperature from the whitecap coverage alone. Such approach was also used by some ocean scientists to relate the whitecaps to wind speed only [*Monahan and Muircheartaigh*, 1980], an approach that produced wildly scattered results of limited practical use.

[32] To parameterize breaking with a combination of both wave and wind is essential, but it might be too difficult to implement right now. An alternative is to parameterize the breaking directly by measuring breaking statistics as proposed by *Phillips* [1985]. This approach was tried recently by *Kleiss and Melville* [2011], where the breaking is measured in term of unit length of breaking front per unit area as required by the theory proposed by *Phillips* [1985]. Unfortunately, they found that the whitecaps could appear in different stages: the active breaking stage, the mature breaking event and the advecting foam patch, which would make the parameterization and quantification difficult. The seminal work by *Phillips* [1985] and the pioneer experiments by *Kleiss and Melville* [2011], however, had pointed out a new direction but also brought new complications of our understanding on the breaking processes. To use it eventually in wave modeling would depend on many physical properties of the ocean wavefield that need to be quantified. It looks like we have to rely on empirical fitting and tuning for the foreseeable future. Right now, the dissipation source term also serves to catch all the uncertainty of input source function, for the difference between the two represents the net wavefield energy content. It seems to us there is a long way to go in this particular aspect of wave modeling. As all the dissipation source formulae indicated, the dissipation depends on the detailed wave directional spectral form, which will be our next topic.

4.1. Directional Development of Wind Waves

[33] All the wave prediction models produce acceptable wave amplitude results, from the simplistic one, using a coefficient times the wind speed squared, to the most sophisticated ones, with complicated wave-wave interactions and wave generation/dissipation functions. The wave directional spectral form should be the crucial quantity to specific the wavefield, not the wave amplitude as indicated by the wave modeling research discussed above. Unfortunately, the directional spectral form has never been seriously tested for lack of observational data, mainly due to the fact that the directional spectrum is extremely hard to measure. For example, in the famous JONSWAP experiment, only wave elevation data were collected as a time series to produce frequency spectra. Most of the available field directional wave measurements were the result of pitch-and-roll buoys, a variation of the original floating buoy used by *Longuet-Higgins et al.* [1963]. Such results could yield only four directional parameters consisting of the combination of **cos*θ*, sin*θ*, cos2*θ***

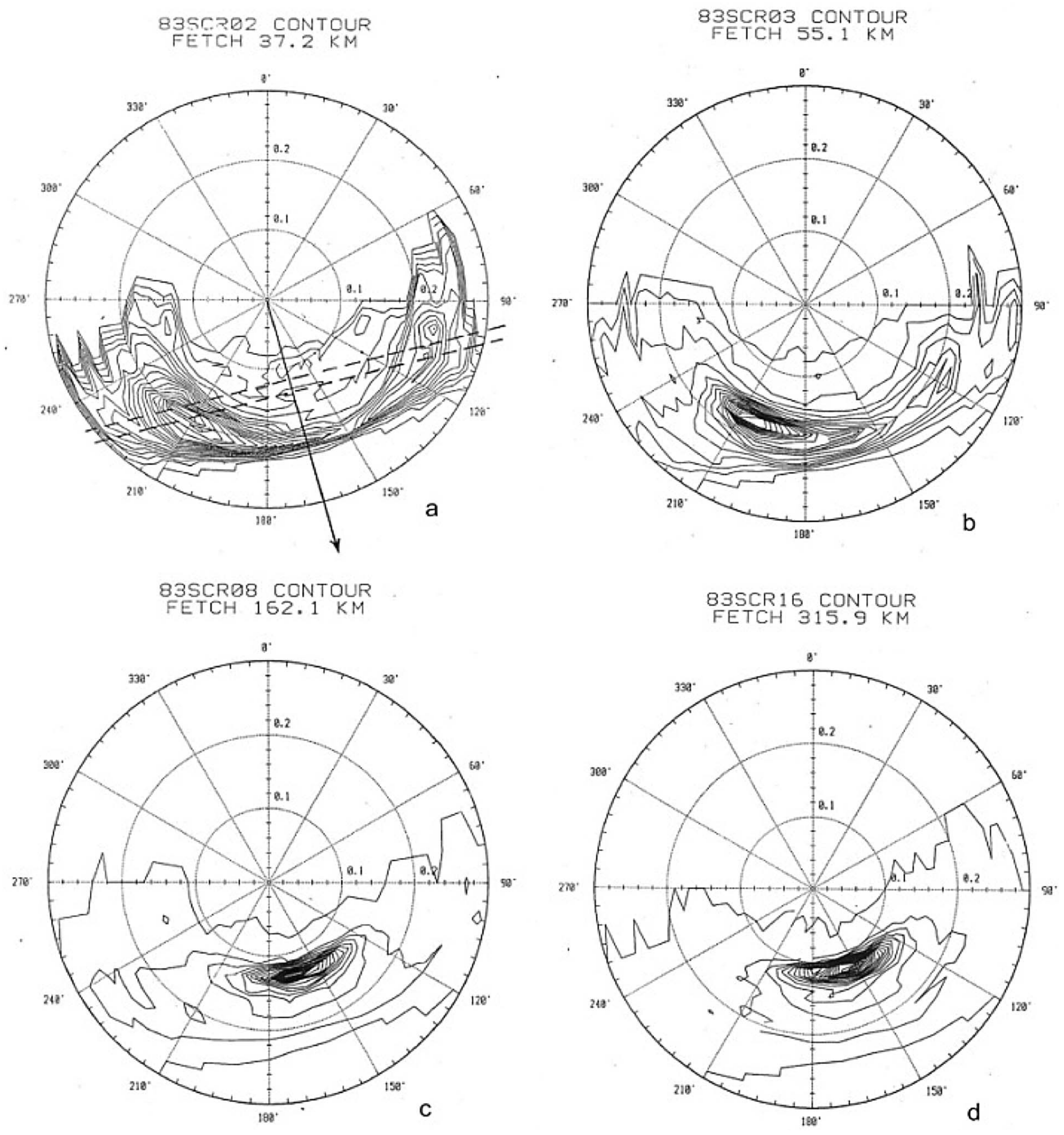

Figure 6. The evolution of wave directional spectra as a function of fetch along the track 1/20/83 SCR (48) as indicated in Figure 5. The directional distribution of the wave energy is clearly bimodal for short fetch (37.2 km) in agreement with Phillips' resonant theory. The two directions merge gradually at longer fetch (55.1 km) and eventually into a uni-modal spectrum downwind at long fetches (around 150 km and more). The dashed lines indicate the matching condition between the wind speed and the phase velocity of the energy containing waves given in equation (28).

and $\sin 2\theta$. Therefore, their power to depict the true wavefield characteristics and their directional distribution is very limited. Perhaps, the in situ measurement of wave directional spectrum as conducted by *Donelan et al.* [1985] using an array of wave probes is more detailed. Yet, the geometry of the probe array and the requirement of stable platform again imposed a limitation its applications. So far, the best measurements of directional wave development are all from remote sensing techniques. To understanding the wind wave generation process, the observations have cover various stages of development as a function of fetch. Open ocean data might be useful for understand the general oceanic conditions; with fetch ill-defined and complicate existing wave conditions, those data are of limited use in understanding the

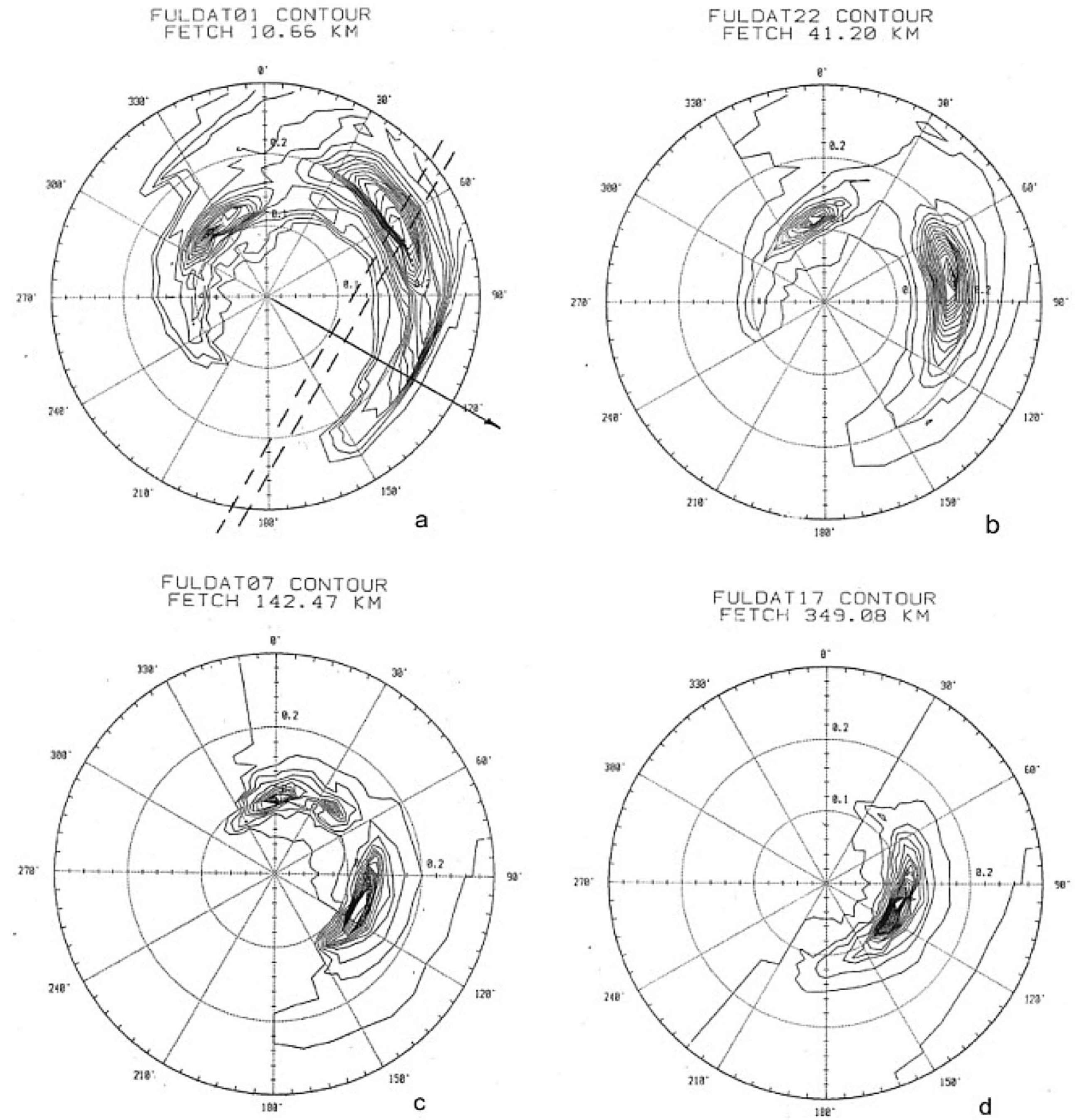

Figure 7. The evolution of wave directional spectra as a function of fetch along the track 1/5/82 SCR (54) as indicated in Figure 5. The directional distribution of the wave energy is not bimodal at short fetch (10.66 km), and it is not in the direction of the wind either. In this case, the wave energy radiating from Delaware Bay might have played a role. The existence of swells is also shown at stations near the coast. The energy is again centered in the direction in agreement with Phillips' resonant theory given in equation (28) as indicated by the dashed lines. With increasing fetches, the wave energy eventually merges to the down wind direction at around 150 km again.

wind wave generation mechanism and processes. A good example of detailed fetch limited wave generation process was reported by *Long et al.* [1994] based on the Surface Contour Radar developed by *Walsh et al.* [1985, 1989]. The data covered a fetch up to 500 km along a steady offshore wind with the flight track as shown in Figure 5. These measurements represent the most stringent tests available so far. Directional spectra for two selected flight tracks at different fetches are given in Figures 6a–6d and Figures 7a–7d. The directional energy distribution is nothing like any of the model functions proposed, and the direction evolution does not bear any semblance to the theoretical model based on

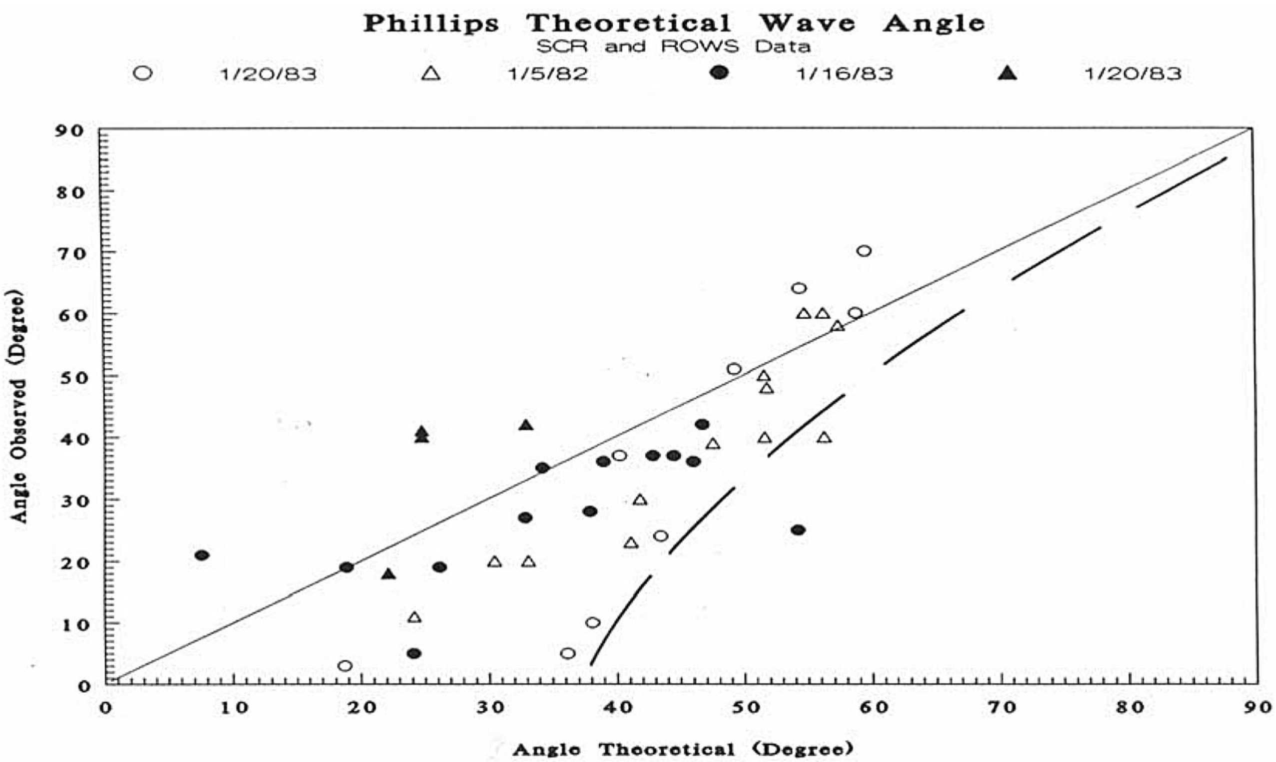

Figure 8. The summary of the directional distribution of the wave energy as compared with Phillips' resonant theory given in the solid line. The agreement is good, except the theoretically predicted values are slightly higher than the observed ones.

weak nonlinear interactions given in *Lavrenov* [2003], for example. The critical differences are both in detailed geometrical patterns and in dynamics. At the initial development stage, the waves are not propagating in the direction of the wind but in a direction roughly in agreement with Phillips's resonant wind wave generation theory [*Phillips*, 1957, 1977]:

$$\omega = \vec{k} \cdot \vec{U} = kU\cos\theta \;\text{ or }\; \theta = \cos^{-1}\frac{c}{U}, \tag{28}$$

with c as the phase velocity of the wave and U as the wind velocity at the height of 10 m. The same mechanism is also important in *Miles* [1957, 1959a, 1959b, 1960, 1962] shear instability theory, but the salient point seemed to have been ignored in some of the wave model, where seeding of waves in the direction of wind is usually implemented.

[34] As the waves develop, the wavelength grows, the phase velocity increases, and the direction of the wavefield also turns according to the resonant condition given in equation (28). Again, these are not exception cases, as the characteristics show up in any accurate and detailed measurements. The turning of the wave direction is not due to the wind direction change. The experimental conditions were carefully selected and monitored so that the wind conditions were steady as discussed by *Walsh et al.* [1985, 1989]. All the observed values for the flight tracks given in Figure 5a on the angle between the waves at the peak of the spectra with the wind data are summarized as in Figure 8. It should be pointed out that none of the examples give a pair of perfect symmetric lobes as dictated by equation (28). The one-sided case in Figure 7 is especially interesting, which might hint that the physical and preexisting conditions could have decisive influence of the final wavefield. In this case, the waves radiating from the Delaware Bay could be the initial seedling surface disturbance that favored the waves in that direction. The interesting fact, however, is that, as the wavefield evolves, the phase velocity still matches up according to the value given by equation (28), pointing to the relevance of the resonance condition.

[35] The relationship given in equation (28) is true in almost all cases. This result is reasonable, for the waves at the initial stage are usually short crested. The short crestedness can only result from two interacting wave trains as suggested by *Phillips* [1957], known as the resonant theory. The direction of the wave will eventually merge into the direction along that of the mean wind only when the waves have grown sufficiently and become long crested so that the phase velocity matches up with that of the local wind. It is not the picture given in the existing wave model of having the initial wave as long crested and in the direction of the wind as depicted in Figure 9, where growth is determined by steady evolution of the wave frequency and wave number along the wind direction. The true picture is much more complicate and dynamically interesting: waves grow and turn and the wave crests eventually change from short to long as the waves mature.

[36] Unfortunately, no systematic study of wave directional development has been conducted over the last twenty years, as if the wave amplitude is all important and the wave direction is irrelevant. Nothing could be more mistaken than this view. As detailed wave form geometry could produce 'wave drag on wind' effect [*Janssen*, 1989; *Donelan and Dobson*, 2001], waves should be the most important factor

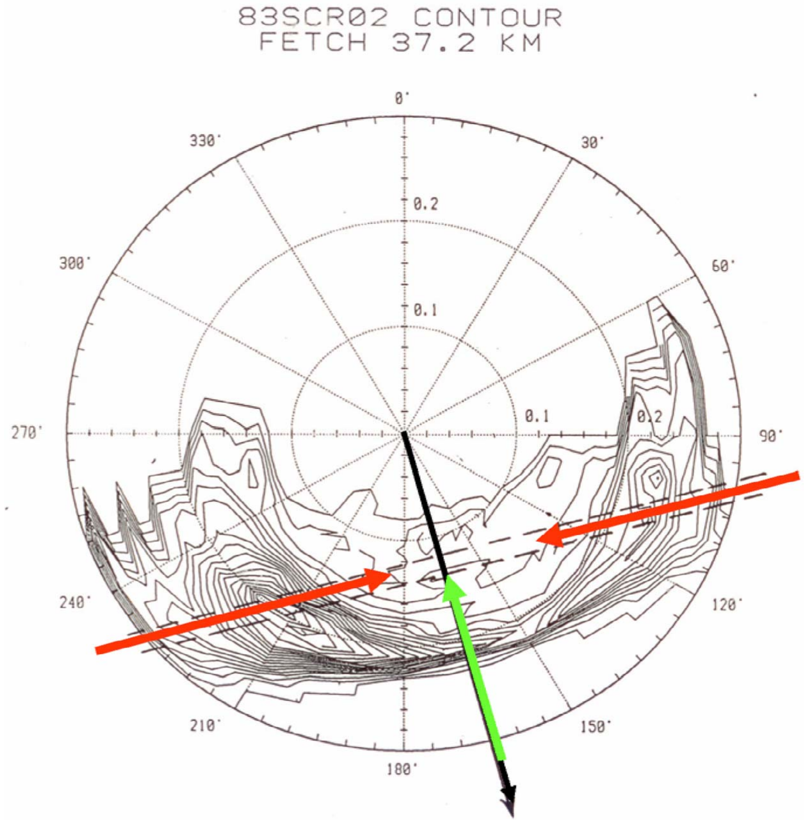

Figure 9. The difference between the observed wave directional development and the wave model: Most wave models call for 'wave seeding', which is essentially adding wave energy at the initial stage down wind. As the wavefield develops, the wave number (or frequency) decreases, but always in the down wind direction. The actual observed of the wave development process is different: it starts from initial stage along off-wind direction(s) and gradually merge into the down wind direction.

in determining the sea surface stress, which in turn determines the momentum flux and current across the air-sea interface. Indeed the situations when the mean wave propagation direction does not align with the local wind are common occurrences [*Friehe et al.*, 2001], especially during turning and variable wind conditions. This is precisely why wind stress is a tensor quantity rather than a simple vector or scalar. Therefore, to determine surface stress accurately, one would have to consider the wave direction especially when nonalignment of the surface wind with the wave is a problem. This is critical for the coastal region, where the sea state would be determined by offshore winds in the generating area. When waves propagate into coastal waters as long waves and swells, they would be independent of the local wind. Then the nonalignment observed by *Donelan and Dobson* [2001] would be routine rather than exceptional. The complicated coupling of wind, wave and current as an urgent and challenging problem has started to attract attention. The urgency of the problem would be addressed in the forthcoming SCOR Working Group 111 Report [*Mooers et al.*, 2012].

4.2. Wave Spectrum Shape

[37] Almost all the wave computations are based on the wave spectral function. For example, the most popular dissipation source function used in WAM is parameterized in terms of Pierson-Moskowtiz spectrum. The latest effort to model surface stress by *Ting et al.* [2012] is based on JONSWAP spectrum. Although all wave models finally produce directional spectra, the effects of using some model spectral forms might be problematic. How can one be sure that the final spectrum produced by the model would match up with the spectral form used in the source function? If they do not match, then are the model results still valid? Furthermore, all the spectral forms are Fourier based and empirically determined. At the best, they are phenomenological description. Let us consider the most popular form, the JONSWAP spectrum. It started from a simple equilibrium range spectrum proposed by *Phillips* [1958] based on dimensional analysis. Later, the simple form was expanded by *Pierson and Moskowitz* [1964]. A further extension was made by *Hasselmann et al.* [1973, 1976] after the famous JONSWAP experiment. It finally assumed a very complicated form:

$$\phi(\omega) = \frac{\beta g^2}{\omega^5} \exp\left\{-\frac{5}{4}\left(\frac{\omega_o}{\omega}\right)^4\right\}\gamma^{\varepsilon_\omega}$$
$$\textit{with}$$
$$\gamma = \frac{\phi(\omega)_{\max}}{\phi(\omega)^{PM}_{\max}},\ \varepsilon_\omega = \exp\left[-\frac{(\omega-\omega_o)^2}{2\eta^2\omega^2}\right], \qquad (29)$$
$$\eta = \begin{cases}\eta_a, & \omega \le \omega_o,\\ \eta_b, & \omega > \omega_o,\end{cases}$$

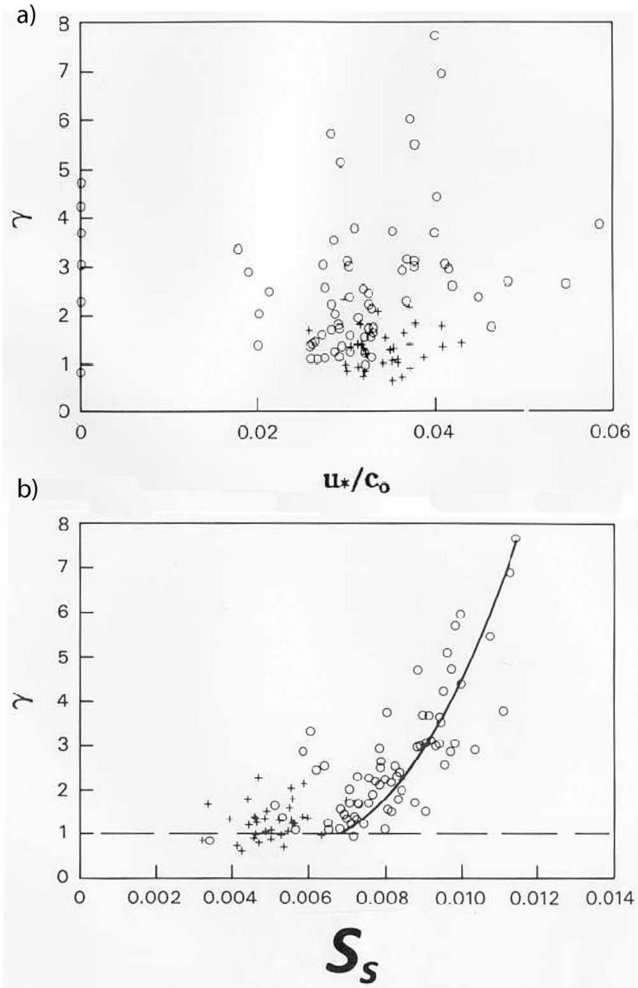

Figure 10. (a) The variation of the peak enhancement parameter in JONSWAP spectral model as a function of nondimensional frequency, u*/c$_o$. The widely scattering of the data indicates the irrelevance and inefficiency of the parameter used [*Huang et al.*, 1990]. (b) The same data as given in Figure 10a plotted against the significant slope of the wavefield. If wave spectral evolution is indeed controlled by nonlinear wave interaction processes, then a parameter measuring the nonlinearity of the wavefield should be used. Indeed, the parameter of significant wave slope caused the scattering parameter to collapse into a much narrow region [*Huang et al.*, 1990].

where PM stand for the Pierson-Moskowitz Spectrum [*Pierson and Moskowitz*, 1964], which is essentially the JONSWAP spectrum without the peak enhancement term, γ; the subscript 0 indicates the value at the spectral peak; β is the Phillips equilibrium range constant. From the form, we can see the genesis of the spectral form clearly: from Phillips equilibrium range to Pierson-Moskowitz, and from Pierson-Moskowitz to JONSWAP. At the final form of JONSWAP, the spectral function has five constants that need to be determined empirically or parametrically. Unfortunately, all attempts to parameterize them as functions of the nondimensional fetch, gX/u_*^2, or the nondimensional peak frequency, u_*/c_o, with u_* as the frictional velocity, met with dismal results as shown in Figures 10a and 10b. The failure to achieve a reasonably coherent pattern has been discussed by *Huang et al.* [1990]. This forced the users to assume a mean state, which makes real applications of the spectrum artificial and difficult. Other than the practical reasons, there are also many theoretical grounds for eschewing this spectral form: To begin, the asymptotic form of the Pierson-Moskowitz form was based on *Phillips*' [1958] fully developed equilibrium spectrum. But later research by *Toba* [1973] and *Phillips* [1985] has firmly repudiated the **−5** power form. Indeed, not all sea states are in equilibrium. Furthermore, the introduction of peak enhancement is to accommodate the presumed internal nonlinear wave-wave interaction effects. If the internal nonlinear wave-wave interaction is the true mechanism, it should not be a function of external variables at all. In terms of the internal parameter of significant slope the scattering of data collapsed to a narrow region as shown in Figure 10b. For application, a simpler and easily adoptable empirical alternative is available. Such a simpler and versatile spectral form has been proposed by *Huang et al.* [1981] and designated as the Wallops spectrum:

$$\phi(\omega) = \frac{\beta g^2}{\omega^m \omega_0{}^{5-m}} \exp\left\{-\frac{m}{4}\left(\frac{\omega_o}{\omega}\right)^4\right\}, \tag{30}$$

where

$$m = \left|\frac{\log\left(2\pi^2 S_s^2\right)}{\log\ 2}\right| \ and\ S_s = \frac{\left(\overline{\zeta^2}\right)^{1/2}}{\lambda_o}. \tag{31}$$

[38] In this simple form, even the Phillips constant, β, can be shown to be a function of m, which is in turn, a function of the significant slope, S_s, a measure of nonlinearity of the wavefield, given in equation (31) as

$$\beta = \frac{(2\pi S_s)^2 m^{\frac{1}{4}(m-1)}}{4^{\frac{1}{4}(m-5)}} \frac{1}{\Gamma\left(\frac{1}{4}(m-1)\right)}. \tag{32}$$

[39] The agreement of all field and laboratory data with this formula is shown in Figures 11a and 11b. Thus the whole spectral form is determined by the internal parameter of the significant slope, S_s, which depends only on two variables: the ***RMS*** wave elevation and the energy containing wavelength, λ_o, corresponding to the wavelength at the peak of the spectrum. The external environmental variables are by and large irrelevant. The significant slope of the wavefield turns out to be an extremely useful nondimensional number to parameterize even all the parameters in JONSWAP spectrum as reported by *Huang et al.* [1990]. If wave spectral 'form' is truly controlled in some fashion by nonlinear wave-wave interactions as proposed by the WAM group [*Komen et al.*, 1994], it is only reasonable to expect the spectral form to be determined by the internal parameter, the significant slope, rather than the external ones. For example, the peak enhancement parameter is rendered much more coherent with internal parameter in Figure 11b than the external parameter in Figure 11a. Though the internal parameter does not eliminate all the data scattering, it reduces the scattering considerably and also offers a much more direct measure for the nonlinearity of the wavefield. If one

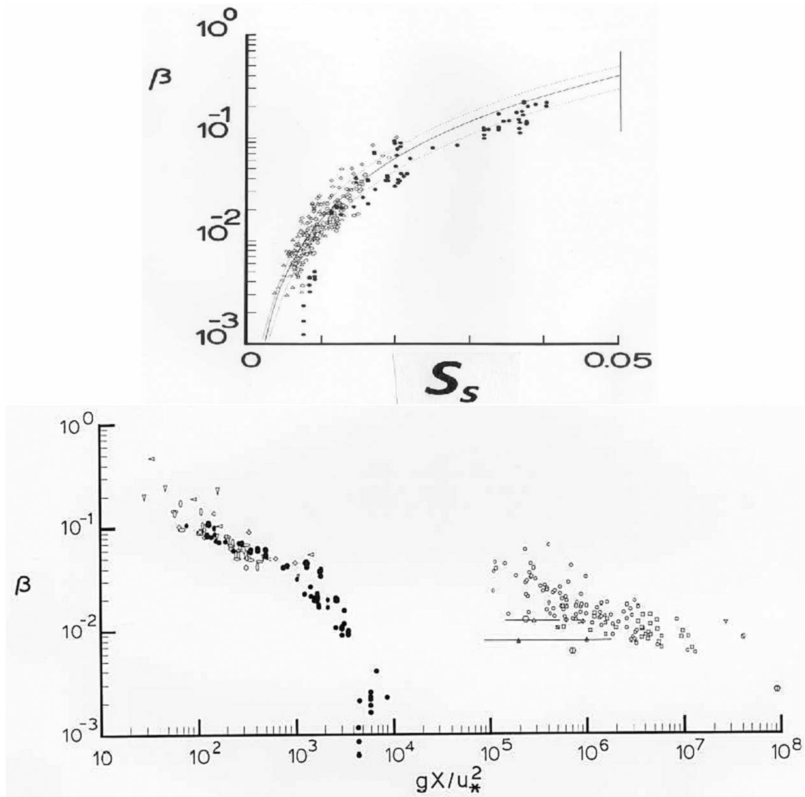

Figure 11. (a) The Phillips equilibrium range constant as a function of significant slope derived by *Huang et al.* [1981], where both laboratory and field data all collapse along the single theoretical line given by equation (32), indicating that the nonlinear effects are indeed a controlling factor in determining the wave spectral shape [*Huang et al.*, 1981]. (b) The same Phillips equilibrium constant presented as a function of nondimensional fetch as adopted in most popular studies indicated the irrelevance and inefficiency of the approach again. Furthermore, the data also indicate that it would be impossible to use the nondimensional fetch as a parameter to represent the so-called 'constant' [*Huang et al.*, 1981].

would like to relate the internal variable to the external ones, such as wind and fetch, for example, that could be implemented too. It is well known that the nondimensional energy and fetch have a very tight relationship. But fetch would be challenging quantity to determine in the open oceans. Increasingly, wave spectral forms are computed through models; it seems that wave spectral models are no longer needed. Yet, with the many parameterization schemes involved and other practical problems we face in engineering, for example, a simple model function could still find use in many applications [*Huang et al.*, 1990].

[40] All the above empirical spectral forms are still Fourier based. The asymptotic from of the Wallops spectral from is based on the ratio of fundamental to its second harmonics. Therefore, this spectral form indicates that other than the region near the peak, the spectral content is dominated by harmonics. As discussed above, it is impossible to separate the energy density of bound waves from that of free waves in Fourier frequency space, the dynamics of free and bound waves being so different: Free waves are true physical entities; whereas bound waves are mathematical artifacts, deprived of physical properties and physical meaning when considered as individual waves. The only meaning they have is in their sum. Therefore, it might be desirable to develop spectral form based on instantaneous frequency as proposed by *Huang et al.* [2011b].

[41] It should be pointed out that mathematically, there is nothing wrong to perform any computation in terms of Fourier expansion even for nonlinear problems. The problem arises when one inject physics into the formulation and assign physical meaning to each individual Fourier component terms. For water wave studies, this practice is especially troublesome: not all the components are free waves. Yet, only the free waves obey the dispersion relationship but not the bounded harmonics. The dispersion relationship happens to serve as the closure of the all-important wave-wave interactions. Thus any computation of resonant wave-wave interaction energy transfer over the whole spectral range, containing all orders of harmonics based on Fourier based spectral form [*Komen et al.*, 1994; *Lavrenov*, 2003], should be re-examined carefully. The task would not be easy, but the need seems vital.

5. The Role of Ocean Waves in Large Scale Geophysical Fluid Dynamics

[42] As part of ocean sciences, wave patterns has appeared almost in every logo of ocean related institutes and enterprises, but most of the serious scientific studies of large scale

oceanic and geophysical phenomena treat waves as a superfluous nuisance. Part of the reasons is that waves were thought to be of small scales, therefore, irrelevant; the other part is the wave studies had been confined to studying waves for the sake of understanding the dynamics of waves only. The expression, *l'art pour l'art*, is certainly true, for the existence of pure art is for aesthetics only. Sometimes, endeavors in science could indeed have an aesthetic component. Historically, most scientific investigations were inspired by the societal needs: for example, the rise of thermodynamics was certainly related to industrial revolution. Therefore, science should also answer certain calls for relevance to our lives, with definite objectives, goals and even utilities at the end. Fascinating as wave phenomena are, organized large scale wave study could not exist in isolation or vacuum and would be an exercise in futility without clear final justifications and practical applications. Though wave motion is small in scale, to disregard the wave contribution to large scale oceanic phenomena is a blind spot in ocean science. Indeed, waves are important for ocean and climate sciences when all the fluxes of mass, momentum and energy at the air-sea interface are items of crucial concern [*Wunsch and Ferrari*, 2004; *Cavaleri et al.*, 2012]. Take the mass flux for example: One of the critical issues of global warming is the ability of the ocean to function as a sink for both heat and CO_2. Studies of gas transfer across the air-sea interface depend critically on sea states [see, e.g., *Komori et al.*, 2011]. Furthermore, waves are the most energetic motion at the ocean upper ocean layer. Therefore, it should be the key factor in studying the oceanic mixing processes, such as the formation of the mixed layer, which is an energy balance problem. The present poor state of our understanding of oceanic mixing process at the upper layer could easily attribute to the poor parameterization scheme without proper wave inputs. Unfortunately, our attention of waves is mostly centered on coastal and ocean engineering, ship design, building and routing. For such applications rudimentary quantities such as significant wave height could go a long way, and the present state-of-the-art wave models might be sufficient. When we look beyond the immediate applications, we would need more sophistic and detailed information of waves in large scale oceanic and geophysical phenomena such as the general circulation of the world's oceans.

[43] A recent review by *Wunsch and Ferrari* [2004] clearly states that the role of waves in vertical mixing of momentum and energy in the whole ocean is of critical importance. The standard way is to parameterize fluxes across the air-sea interface is in terms of surface wind. Further mixing in the ocean water column is usually expressed in terms of Eddy Diffusivity. For vertical momentum, we have $\boldsymbol{K_v}$:

$$\boldsymbol{K}_v = \frac{-\langle \boldsymbol{u'w'} \rangle}{\frac{\partial \boldsymbol{U}}{\partial z}}, \tag{33}$$

in which $\langle \boldsymbol{u'w'} \rangle$ is the Reynolds stress and $\boldsymbol{U}$ is the mean shear current. In practice, the values used for $\boldsymbol{K_v}$ are determined as ad hoc constant, through empirical formulae [see, e.g., *Thorpe*, 2005] or turbulence closure scheme [*Mellor and Yamada*, 1982]. None of the available approach had invoked the wave parameters. Recently, *Yuan et al.* [1999] and *Qiao et al.* [2004, 2008, 2010] have demonstrated the importance of surface waves quantitatively through wave induced mixing, especially the nonbreaking ones. This is somewhat counter intuitive, for the breaking waves are thought to be the main source of turbulence energy at the upper layer of the ocean [*Thorpe*, 2005]. But the turbulent energy would be dissipated within a thin layer of the order of the wave amplitude. For the mixing effect to penetrate to great depth, the energy has to come from other motions. Surface waves are the primary sources. Using a mixing length analogy, *Yuan et al.* [1999] and *Qiao et al.* [2004, 2008, 2010] have formulate the wave induced eddy viscosity, $\boldsymbol{B_v}$, as

$$\boldsymbol{B}_v = \iint_{\vec{k}} \varphi(\vec{k})\, \mathbf{exp}(2\boldsymbol{k}z) d\vec{\boldsymbol{k}} \frac{\partial}{\partial z} \left[\iint_{\vec{k}} \omega^2 \varphi(\vec{k})\, \mathbf{exp}(2\boldsymbol{k}z) d\vec{\boldsymbol{k}} \right]^{1/2}, \tag{34}$$

where ϕ is the wave number spectrum. To give a physical interpretation of this eddy viscosity, we can assume the wavefield consists of a mono-chromatic wave train of amplitude, $\boldsymbol{a}$, frequency, ω, and wave number, $\boldsymbol{k}$, then

$$\boldsymbol{B}_v = \boldsymbol{a}^3 \boldsymbol{k} \omega \; \mathbf{exp}(3\boldsymbol{k}z) = \boldsymbol{a u}_s \mathbf{exp}(3\boldsymbol{k}z), \tag{35}$$

in which $\boldsymbol{u_s = a^2 k^2 \omega / k}$ is the Stokes drift. Thus the mixing is caused by the interaction of the water mass particle motions and the slow drift current associated with the waves. This simple expression had been validated by laboratory [*Dai et al.*, 2010] and filed [*Huang and Qiao*, 2010] observations of mixing. Thus, the relevance of wave motion in mixing is firmly established.

[44] The most important aspect of the wave induced mixing is that the mixing effects could penetrate to great depths (of the order of the wavelength) beyond the immediate surface breaking influenced layer (of the order of the wave amplitude). To implement this approach, the general circulation model would have to run with a coupled wave model [*Yuan et al.*, 1991]. *Qiao et al.* [2008] have used this wave associated eddy viscosity and produced a drastically different global surface temperature structure, as shown in Figures 12a and 12b. Here the wave induced mixing had changed the surface temperature structure on a global scale. It offers a correction of insufficient mixing through the turbulence closure scheme and eliminates the structured surface temperature bias and effectively removed the systematic gradient of the error distribution. As a result, the global surface temperature pattern agrees much better with the observed climatologic data. Clearly, the wave's role has global large scale and long-term implications for the earth's climate. To implement Qiao's computations, significant wave height is not enough. They need the whole wave spectrum.

[45] We can take another step further. Let us take the studies of hurricanes as another example. The current models rely heavily on parameterizations. Unfortunately, most parameterization schemes are empirical and the databases represent conditions decidedly less drastic than that under real hurricanes. As a result, the validity of the parameterization schemes had been stretched to their limits. Furthermore, the constantly changing hurricane path and the swirling windfield necessitate perpetual adjustments of the wind and waves that would never reach equilibrium. These

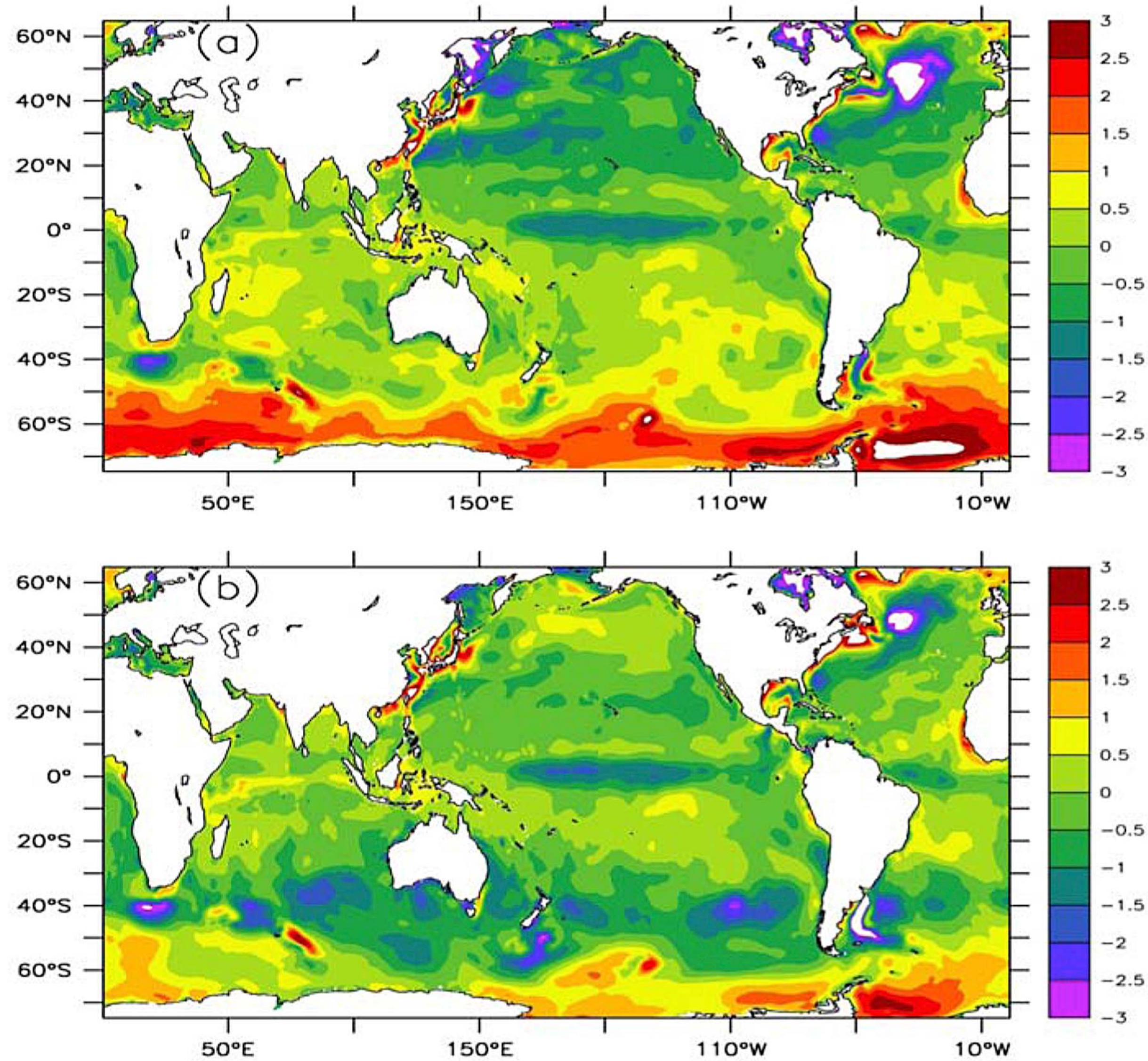

Figure 12. The difference of global ocean surface temperature pattern between climatological data based on *Levitus* [1982] and different General Circulation Models. (a) The Princeton Ocean Model based GCM with mixing controlled by the turbulence closure scheme proposed by *Mellor and Yamada* [1982]. (b) The same GCM model as in Figure 12a with the additional wave associated mixing as given in equation (34). Notice the large deviation in Figure 12a and systematic north-south gradient of deviation all indicating insufficient mixing. The additional wave induced mixing has substantially improved the agreement between model and observed data in Figure 12b.

complications made the skill of hurricane intensity prediction formidable and difficult to acquire. A recent study by *Chen et al.* [2007], for example, demonstrated the critical importance of the wind-wave-current fully coupled model of hurricane and showed dramatic improvements in intensity prediction. This work is just beginning. More future work is urgently needed. Here wave studies would be an essential part, for the surface condition controls all the fluxes and the upper layer mixing too. The studies would not be easy, for the data we need would have to be in great detail, but it is imperative that we take this challenge. Here, the simple significant height would not be sufficient; we need the full wave directional spectrum down to short wavelengths, where momentum transfer is taking place. To achieve this eventual goal, we also need better theoretical understanding of wind wave generation mechanisms, better observation tools and better physical understanding for parameterization as well as model building.

6. Conclusions

[46] It is not uncommon to see research scientists reporting and displaying their results on wave prediction in an ocean wave related meeting. The usual claims are on the accuracy of their predicted wave height not only locally, but also

globally to be within a few percent compared to the observations. Whatever is wrong, as the claims go, could be attributed to the error and problem of wind field predictions. Two extreme positions could be drawn from those claims:

[47] 1. The claims are true. The wave prediction problem is solved and therefore, no more research would be needed, for who will throw money on research to improve the wave height prediction for a couple of percent, while other urgent problems, such as climate change, are crying for attention?

[48] 2. The claims are false. It is just an illusion, for wave height prediction is a relatively trivial problem: wind wave energy might be simply related to the surface wind speed. Information on wave height alone is far from enough for scientific and practical applications. There are many other problems such as wave propagation direction, directional distribution, wave frequency, wave spectrum shape and associated evolution stages are also important for various applications and need to be solved too. The problem is far more involved than tightening up the wind forecasting.

[49] The truth is somewhere in the middle. Indeed for operations as well as scientific research, wave height is not that informative. A swell field could have the same mean wave height as a freshly generated sea, but their dynamics and dynamic implications are very different. Even something as simple as ship routing, wave direction is as critical a parameter as, if not more than, wave height. Scientifically, wave direction relative to the local wind is critical in determining the surface wind stress (a tensor, not a vector). And surface stress is the crucial input for general circulation, coastal dynamics and hurricane modeling. Large waves dominate wave height, but small waves determine the various fluxes across the air-sea surface. What about wave breaking and spray [*Babanin*, 2011] as a source function for wave dissipation and mixing of the upper ocean mixed layer? Based on our present knowledge, we know very little about the answers to those problems. While showing off our accomplishments is important, emphasizing our ignorance is even more essential. We should realize that research funding is not allocated as prizes to reward accomplishments; it is designated to address unknown and advance our knowledge. Only by revealing our ignorance can we strive to overcome it by our research efforts and make true progress.

[50] Based on our personal assessments, we are far from the nirvana stage of having solved the wave prediction problem, for there are too many physical processes that we simply have glossed over with parameterizations. Von Neumann has famously said: with four parameters he could fit an elephant; with five he could even make its trunk swing. We should not be too smug with the skillfully tuned results of wave height prediction by using so many parameters in the process. We believe that knowing what we do not know should be the beginning. The true and fundamental study of ocean wave physics is still ahead of us. Therefore, we should improve our understanding of the underlying physical processes in order to elucidate the parameterized processes and try to minimize the numerous parameterizations or even eliminate them.

[51] Meanwhile, we have also to establish the relevance of wave study. The last golden age of wave studies in the 1970s and 80s were ushered in partly by the nonacoustic antisubmarine warfare (ASW) and partly by the development of satellite remote sensing of the oceans. Although the ASW part might be over, the remote sensing of the ocean environmental problem is very much with us. Independent of remote sensing, we need to model the ocean in order to address the role of the oceans in global climate change studies. The ocean plays a key role in climate change and global warming problem, yet they are poorly modeled, for lack of data, physical understanding and proper parameterization. As the recent review by *Wunsch and Ferrari* [2004], *Cavaleri et al.* [2012] and the research work of *Qiao et al.* [2004, 2010] have illustrated, ocean waves are an essential part in the dynamics of large scale general circulation problem. In the frame of large scale geophysical fluid mechanical phenomena, we should not study wave for wave's sake; we have to include waves to make the mixing and fluxes correct. This is crucial for the climate system. Additionally, the coastal ocean dynamics is another area that needs urgent attention and a fertile ground for future research [*Mooers et al.*, 2012]. A full of 7% of the ocean surface could be classified as coastal zone, and 40% of the world population lives within 100 km of the coast. According to *The Encyclopedia of the Earth* (http://www.eoearth.org/article/Coastal_zone), the coastal zone accounts for at least 15% of oceanic primary production, 80% of organic matter burial, 90% of sedimentary mineralization, 75–90% of the oceanic sink of suspended river load and ca. 50% of the deposition of calcium carbonate. Additionally, it represents 90% of the world's fish catch and its overall economic value has been recently estimated to be at least 40% of the value of the world's ecosystem services and natural capital. As discussed above, the already complicated wind-wave-current interactions, coupled with that arising from coastal geometry and bottom topography, still pose a challenge for the foreseeable future. Wave studies are very much an integrated part of this grand scheme.

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