Comparison of Doppler Centroid Estimation Methods in SAR

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ABSTRACT

This paper compares five Doppler centroid estimation methods which are :energy balancing, matched-correlation , maximum likelihood, correlation Doppler estimator(CDE) and sign Doppler estimator (SDE). Their estimation performances in raw data domain and image domain are studied. The computer simulation results are presented. ERS-1 raw data are also used to test the performances of every method.

Introduction

Synthetic aperture radar (SAR) is a class of high resolution radar which obtains fine azimuth resolution by coherent processing of backscattered Doppler histories. An important parameter in relation to the azimuth processing is the Doppler centroid, which is used to generate the azimuth matched-filters, together with the Doppler frequency rate. In principle, it is possible to calculate Doppler centroid from orbit and attitude data. However, measurement uncertainties will limit the accuracy. This error leads to the degradation in signal-to-noise ratio and signal-to-azimuth ambiguity ratio.

This paper studies the Doppler centroid estimation methods from the received echo data, which is also referred to as "clutterlock". So far, there are about five methods proposed by people. They are: (1)Energy balancing[1], (2)Matched-correlation estimation[2], (3) Maximum-likelihood estimation[3], (4) Correlation Doppler estimator (CDE)[4], (5)Sign Doppler estimator (SDE)[4]. The first three methods are implemented with azimuth power spectrum. The last two ones directly use the complex raw data.

The every method mentioned above can also be carried out in complex image domain[5], so the bias arised from the strong scatterers passing part of the synthetic aperture will be avoided. However, estimation in image domain is always an iterating procedure.

R. Bamler[2] has pointed out that all the methods are equivalent to the correlation of the signal power spectra with a particular weighting function and finding the minimum value of the correlation results. The different weighting function achieves different estimation variance. Special case of a correlation-based estimator is the maximum-likelihood estimator that can achieve the Cramer-Rao bound. He has compared the first four methods and derived their variances in theory.

This paper reviews all these methods and compares their performance when they are implemented in image domain. The iterating coefficients used for estimation in image domain are derived.

The computer simulation results are presented. All the methods also have been tested by the data from ERS-1.

II. Estimating Doppler centroid in data domain

The principal of the estimation use the fact that the high azimuth bandwidth time product of a SAR locks Doppler frequency to the position along tracks. Thus, the components at any particular Doppler frequency originate from the targets in a specific part of the radar beam. As a consequence, the azimuth power spectrum S(f) should follow the shape of the two-way azimuth power pattern $G^2(f)$ of the antenna. If this pattern is symmetry, the center frequency can be found by finding the energy centroid of the azimuth power spectrum, as shown by the following equation

$$\int_{-B_d/2}^{f_{dc}} S(f) = \int_{f_{dc}}^{B_d/2} S(f) \qquad (1)$$

where B_d is Doppler band and f_{dc} is the Doppler centroid. This idea is often implemented by correlating the azimuth power spectrum with some odd reference function R(f) and finding the zero position of the correlation result D(f) as

$$D(f) = \int S(\mu) \cdot R(\mu - f) d\mu \qquad (2)$$

$$D(\hat{f}_{dc}) = 0 \qquad (3).$$

According to the reference function chosen, three methods can be constructed. They are *energy balancing* (EB) with

$$R(f) = \begin{cases} 1 & \frac{B_d}{2} \le f < 0 \\ -1 & 0 \le f \le \frac{B_d}{2} \end{cases} \qquad (4),$$

matched-correlation (MC) with

$$R(f) = W'(f) \tag{5}$$

where W(f) is the nominal antenna two-way power pattern as

$$W(f) = G^2(f) \tag{6}$$

and *maximum-likelihood* (ML) with

$$R(f) = -\frac{W'(f)}{W(f)} \tag{7}$$

Matched-correlation takes account of the antenna pattern and maximum-likelihood utilizes the statistic characteristic of the azimuth power spectrum which is a stochastic process with exponent distribution. So, MC and ML both have superior estimation performance than that of EB. The estimation variance achieved by these three methods is

$$\operatorname{var}\{\hat{f}_{dc}\} = \frac{PRF}{N} \frac{\int_{-B_d/2}^{B_d/2} [W(f)\cdot R(f)]^2}{\left[\int_{-B_d/2}^{B_d/2} W'(f)\cdot R(f) df\right]^2} \tag{8}$$

where PRF is the pulse repetition frequency and N is the spectrum sampling points. It is shown by (8) that the ML method arrived Cramer-Rao bound.

According to the Fourier relationship between the power spectrum S(f) and its autocorrelation function (ACF) $r(\tau)$, CDE estimates f_{dc} by calculating the autocorrelation coefficient $r(\Delta t)$ (Δt =1/PRF) and its argument (arg[.]) in the following way:

$$r(\Delta t) \approx \frac{1}{N-1}\sum_{i=0}^{N-2} s(i\cdot\Delta t)s((i+1)\cdot\Delta t) \tag{9}$$

$$\hat{f}_{dc} = \frac{\arg[r(\Delta t)]}{2\pi}\cdot PRF \tag{10}$$

where s(t) is the complex azimuth echo signal. Its has an up limit of estimation variance as

$$\operatorname{var}\{\hat{f}_{dc}\} \le \frac{\int_{-B_d/2}^{B_d/2} W^2(f) df}{NB_d(2\pi\Delta t)^2 |r(\Delta t)|^2} \tag{11}$$

CDE has an advantage of smaller computation and simpler hardware structure when being realized.

SDE also utilizes the ACF but using a different calculating procedure. Based on Van Vleck theorem, the ACF of a real stationary Gauss stochastic process X(t) can be derived from its sign and variance as

$$r_X(\tau) = \sigma^2 \cdot ar\sin[\frac{\pi}{2}\cdot r_Y(\tau)] \tag{12}$$

where $r_Y(\tau)$ is the ACF of a new process Y(t) generated by the following operation.

$$Y(t) = \begin{cases} 1 & X(t) \ge 0 \\ -1 & X(t) < 0 \end{cases} \tag{13}$$

Because Y(t) only take values of "+1" and "-1", we call it the sign of X(t). This means that the computation involved in the estimation will be reduced greatly. In fact, $r(\tau)$ is always complex, so it must be divided into four real parts when applying Van Vleck theory. This procedure can be shown as

$$s(t) = I(t) + j\cdot Q(t) \tag{14}$$

$$p(t) = p_I(t) + j\cdot p_Q(t) \tag{15}$$

$$p_X(t) = \begin{cases} 1 & X(t) \ge 0 \\ -1 & X(t) < 0 \end{cases} \quad X = I, Q \tag{16}$$

$$r_{XY}(\tau) = \sigma^2 \sin[\frac{\pi}{2}\rho_{XY}(\tau)] \quad X,Y = I,Q \tag{17}$$

$$\begin{aligned} r(\tau) = \sigma^2\{&\sin[\frac{\pi}{2}\rho_{II}(\tau)] + \sin[\frac{\pi}{2}\rho_{QQ}(\tau)] \\ &+ j\cdot\sin[\frac{p}{2}\rho_{IQ}(\tau)] - j\cdot\sin[\frac{\pi}{2}\rho_{QI}(\tau)]\} \end{aligned} \tag{18}$$

where $r_{XY}(\tau)$ (X,Y=I,Q) is the ACF when X=Y or cross ACF when X≠Y of X(t) and Y(t). So is $\rho_{XY}(\tau)$ (X,Y=I,Q) but for $p_X(t)$ and $p_Y(t)$. Reference [4] also said SDE is a more robust estimator because of its nonlinearity. However, the estimation variance of SDE can not be defined in theory.

At last of this section, we compare estimation performance of the five methods. Table 1 presents the standard estimation deviations of the simulation and in theory. Simulations are made under the following conditions :

- Homogeneous scene
- SNR=20dB
- Wavelength λ=0.057m
- Slant range R_0=847km
- Sensor velocity v=7500m/s
- PRF=1680Hz
- Antenna width L_a=10m
- Antenna two-way pattern:

  in frequency domain

$$G(f) = \operatorname{sinc}^2\left(\frac{\pi\cdot f\cdot L_a}{2v}\right) \quad -\frac{PRF}{2} \le f \le \frac{PRF}{2} \tag{19}$$

  in space (time) domain

$$G(x) = \operatorname{sinc}^4\left(\frac{\pi\cdot x\cdot L_a}{R_0\lambda}\right) \quad -\frac{\lambda R_0}{2L_a} \le f \le \frac{\lambda R_0}{2L_a} \tag{20}$$

- Azimuth spectrum sampling points N=2048
- Estimations are made in one range bin

Then five methods are tested by ERS-1 raw data.

Estimations have been obtained based on data blocks of 32 range bins by 2048 azimuth samples. These estimations have been fitted to a linear function (may be divided into two segments) in range direction. The deviation of each estimate from this function has been taken as the estimation error. The standard deviations of the five estimation methods are shown in Table 2.

Table 1 Comparison of standard deviations in theory and of simulations achieved by five estimation method in raw data domain(Hz)

	EB	MC	ML	CDE	SDE
Theory	12.08	10.35	4.5	≤12.0	/
Simulation	13.69	10.37	4.54	8.91	12.10

Note: Simulation results are the statistics of ten times.

Table 2 Standard estimation deviations using ERS-1 raw data (Hz)

	EB	MC	ML	CDE	SDE
Theory	9.52	8.21	3.57	≤9.52	/
Measured	6.337	5.64	5.8	5.63	7.26

Note: The image contrast is about 20.

III. Estimating Doppler centroid in image domain

As is well known, the point response of SAR in data domain is always dispersed. When a raw data segment is selected, part of echo signal backscattered by the targets at the edge of the synthetic aperture will be missed any way. If these targets are strong scatterers, they will defeat the symmetry of the azimuth spectrum and cause estimation bias. In order to avoid this bias, estimating Doppler centroid in image domain should be considered where all the targets' response has been compressed. So, any image segment must include the whole energy of targets imaged.

The image used can be real or complex. The former is always incorporated with the multilook processing and a special case of the later. Here, we just discuss the estimation methods in complex image domain.

Assuming we have got a complex image z(t) (along azimuth direction and just for one range bin) using an initial f_{dc}^0 and a correct Doppler rate. Now the power spectrum Z(f) of z(t) will be used, when EB,MC or ML are applied. Different from the correlation procedure in data domain, we calculate the normalized energy difference defined as

$$\Delta E = \frac{\int_{-B_d/2}^{B_d/2} Z(f)\cdot R(f-f_{dc}^0)df}{\int_{-B_d/2}^{B_d/2} Z(f)df} \qquad (21).$$

Then the $\hat{f}_{dc}$ can be derived by the following correction to f_{dc}^0.

$$\hat{f}_{dc} = f_{dc}^0 + \Delta f_{dc} \qquad (22)$$

$$\Delta f_{dc} = \frac{\Delta E}{c} \qquad (23)$$

where c is a coefficient related to the two-way antenna power pattern and the reference function chosen. If the image scene is homogeneous and f_{dc}^0 is close to the true f_{dc}, we have

$$c = \frac{\int_{-B_d/2}^{B_d/2} W(f)\cdot R'(f)df}{\int_{-B_d/2}^{B_d/2} W(f)df} \qquad (24).$$

The derivation of (24) is given in the appendix. The coefficients for EB,MC and ML are

$$c_{EB} = \frac{2[W(0)-W(B_d/2)]}{\int_{-B_d/2}^{B_d/2} W(f)df} \qquad (25)$$

$$c_{MC} = \frac{\int_{-B_d/2}^{B_d/2} [W'(f)]^2}{\int_{-B_d/2}^{B_d/2} W(f)df} \qquad (26)$$

$$c_{MLL} = \frac{\int_{-B_d/2}^{B_d/2} \left[\frac{W'(f)}{W(f)}\right]^2}{\int_{-B_d/2}^{B_d/2} W(f)df} \qquad (27)$$

respectively.

In fact, the correction procedure is always iterated using the value $\hat{f}_{dc}$ as the new f_{dc}^0. After several iterations, the more accurate $\hat{f}_{dc}$ can be achieved.

CDE and SDE can also be implemented in image domain. The estimation result f_p with initial value f_{dc}^0 will related to the true f_{dc} as

$$f_p = f_{dc} + c\cdot(f_{dc}^0 - f_{dc}) \qquad (28).$$

So the true f_{dc} will be

$$f_{dc} = \frac{f_p - c\cdot f_{dc}^0}{1-c} \qquad (29).$$

Like the estimation using EB,MC and ML, the iteration is also carried out here. However, it is difficult to give a formula to calculate the coefficient c here. This coefficient can always be found using experimental techniques.

Now, we compare estimation performance of the four methods in image. Table 3 presents the standard estimation deviations of the simulation and in theory (variance in theory still can be derived as that in data domain). Simulation conditions are as same as that of data domain with a difference that only half azimuth points are used in image domain compared with that of data domain. This is because we can only generate an image of N/2 points in azimuth from a raw data segment of N points if the synthetic aperture is N/2 points long.

Table 3 Comparison of standard deviations in theory and of simulations achieved by four estimation method in complex image domain(Hz)

	EB	MC	ML	CDE
Theory	17.08	14.63	6.36	≤16.97
Simulation	21.997	18.227	7.331	19.195

Note: Simulation results are the statistics of ten times.

The estimation using ERS-1 complex image has not been completed. So, the results can not be presented here.

Conclusion

Five Doppler centroid estimation methods are reviewed and compared. The estimation in image domain is discussed and the iteration coefficient c is given from a generalized view. If the scene is homogeneous, estimations in data domain and image domain are equivalent. ML is optimal because it achieves Cramer-Rao bound. CDE and SDE both have a smaller computation burden than the methods using azimuth power spectrum.

Appendix The derivation of c

Let

$$D(\phi) = \int Z(f) \cdot R(f - f_{dc}^{0} - \phi)df \quad (30)$$

We have

$$\Delta E = \frac{D(0)}{\int Z(f)df} \quad (31)$$

and

$$E\{Z(f)\} = W(f) \quad (32),$$

Then

$$E\{D(\phi)\} = \int W(f - f_{dc}^{0} - \Delta f_{dc}) \cdot R(.)df \quad (33)$$

Expanding $E\{D(\phi)\}$ at $\phi=0$ to linear term as

$$E\{D(\phi)\} = c' \cdot \phi + E\{D(0)\} \quad (34)$$

$$\begin{aligned} c' &= \left.\frac{dE\{D(\phi)\}}{d\phi}\right|_{\phi=0} \\ &= \int W(f - f_{dc}^{0} - \Delta f_{dc}) \cdot R'(f - f_{dc}^{0})df \quad (35) \\ &\approx \int W(f) \cdot R'(f)df \end{aligned}$$

so we get the coefficient c as

$$\begin{aligned} c &= E\left\{\frac{c'}{\int Z(f)df}\right\} \\ &= \frac{\int W(f) \cdot R'(f)df}{\int W(f)df} \end{aligned} \quad (36)$$

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Author Biographies

Weidong Yu was born in Henan Province China in 1969. He received B.S. and M.S. in 1991 and 1994, respectively, from the Dept. of Electr. Eng., NUAA. Presently he is a doctoral student in NUAA. His doctoral theme is "SAR Signal Processing".

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