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Discussion

On the vertical distribution of $\langle \tilde{u}\tilde{w} \rangle$ by F.J. Rivero and A.S. Arcilla: comments

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The wave Reynolds stress $-\rho\overline{\tilde{u}\tilde{w}}$ plays an important role in the deviation of current profiles from the standard logarithmic distributions in the presence of waves as studied first by Nielsen (1992) and recently by You et al. (1995) and You (1996). The study of Rivero and Arcilla (1995) $\overline{\tilde{u}\tilde{w}}$ is therefore of great importance in developing a new generation model of combined wave–current flows in coastal zones. The main part of the study conducted by the authors was to analytically determine the vertical distribution of $\overline{\tilde{u}\tilde{w}}$ for irrotational and rotational flows simply based on the concept of vorticity and the equation of continuity. The vertical distribution of $\overline{\tilde{u}\tilde{w}}$ in the irrotational flow was derived as

$$\frac{\partial}{\partial z}\left(\overline{\tilde{u}\tilde{w}}\right) = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\overline{\tilde{u}^2} - \overline{\tilde{w}^2}\right)\right] = -\frac{1}{2}\frac{\partial}{\partial x}\left(G\frac{E}{\rho h}\right) \tag{17}$$

which was also proven to be valid for the rotational flow. However, some of their statements and conclusions on the vertical distribution of $\overline{\tilde{u}\tilde{w}}$ need readjustments. The reasons for that are given as follows.

[1] $\overline{\tilde{u}\tilde{w}}$ should be zero in the irrotational flow ($\tilde{\omega} = 0$). For simplicity, a horizontal bottom is assumed in the present study. Similarly, multiplying $\tilde{\omega}$ (10b) by $\tilde{u}$ leads to

$$\tilde{u}\tilde{\omega} = \tilde{u}\frac{\partial \tilde{u}}{\partial z} - \tilde{u}\frac{\partial \tilde{w}}{\partial x} = \frac{1}{2}\frac{\partial \tilde{u}^2}{\partial z} - \frac{\partial \tilde{u}\tilde{w}}{\partial x} + \tilde{w}\frac{\partial \tilde{u}}{\partial x} \tag{49}$$

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and using the equation of continuity (14), we obtain

$$\frac{\partial \tilde{u}}{\partial x} = -\frac{\partial \tilde{w}}{\partial z} \tag{50}$$

Combining Eqs. (49) and (50) yields

$$\tilde{u}\tilde{\omega} = \frac{1}{2}\frac{\partial \tilde{u}^2}{\partial z} - \frac{\partial \tilde{u}\tilde{w}}{\partial x} - \tilde{w}\frac{\partial \tilde{w}}{\partial z} \tag{51}$$

which can be rewritten as

$$\frac{\partial\left(\overline{\tilde{u}\tilde{w}}\right)}{\partial x} = \frac{1}{2}\frac{\partial}{\partial z}\left(\overline{\tilde{u}^2} - \overline{\tilde{w}^2}\right) - \overline{\tilde{u}\tilde{w}} = \frac{1}{2}\frac{\partial}{\partial z}\left(G\frac{E}{ph}\right) - \overline{\tilde{u}\tilde{\omega}} \tag{52}$$

Since $\tilde{\omega} = 0$ in the irrotational flow, Eq. (52) becomes

$$\frac{\partial\left(\overline{\tilde{u}\tilde{w}}\right)}{\partial x} = 0 \quad \text{or} \quad \frac{\partial}{\partial z}\left[\frac{\partial}{\partial x}\left(\overline{\tilde{u}\tilde{w}}\right)\right] = 0 \tag{53}$$

On the other hand, Eq. (17) can be rewritten as

$$\frac{\partial}{\partial z}\left[\frac{\partial}{\partial x}\left(\overline{\tilde{u}\tilde{w}}\right)\right] = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\left[G\frac{E}{Ph}\right] \tag{54}$$

Since Eqs. (53) and (54) have to be satisfied at the same time, combining Eqs. (53) and (54) leads to

$$\frac{\partial^2}{\partial x^2}\left(G\frac{E}{\rho h}\right) = 0 \quad \text{or} \quad \frac{\partial}{\partial x}\left(G\frac{E}{\rho h}\right) = C \tag{55}$$

in which C is constant and independent of x. For simplicity, the wave height is assumed to decay exponentially, i.e., $H(x) = H_0\exp(-\alpha x)$, Eq. (55) can be rewritten as

$$-\alpha H^2(x) = \frac{4Ch}{Gg} \tag{55b}$$

which leads to $\alpha = C = 0$. This indicates that for $\tilde{\omega} = 0$, the wave height attenuation $\partial H/\partial x$ in Eq. (17) should be zero and $\overline{\tilde{u}\tilde{w}}$ equals zero too. Alternatively, the same result can also be derived from the time-averaged momentum equation

$$\frac{\partial\overline{\tilde{u}^2}}{\partial x} + \frac{\partial\left(\overline{\tilde{u}\tilde{w}}\right)}{\partial z} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x} \tag{56}$$

in which the flow is assumed to be irrotational. Since the time-averaged pressure distribution in Eq. (56) can be expressed as

$$\frac{\bar{p}}{\rho} = g\left(z - \bar{\xi}\right) - \overline{\tilde{w}^2} \tag{57}$$

in which $\overline{\xi}$ is the time-averaged mean water level. Combining Eqs. (56) and (57) yields

$$\frac{\partial \overline{\tilde{u}^2}}{\partial x} - \frac{\partial \overline{\tilde{w}^2}}{\partial x} + \frac{\partial\left(\overline{\tilde{u}\tilde{w}}\right)}{\partial z} = -g\frac{\partial \overline{\xi}}{\partial x} \tag{58}$$

which can be rewritten as

$$\frac{\partial}{\partial x}\left(G\frac{E}{\rho h}\right) + g\frac{\partial \overline{\xi}}{\partial x} = -\frac{\partial\left(\overline{\tilde{u}\tilde{w}}\right)}{\partial z} \tag{59}$$

Since the terms on the left side of Eq. (59) are dependent of x while $\overline{\tilde{u}\tilde{w}}$ is independent of x as shown by Eq. (53), the vertical distribution of $\overline{\tilde{u}\tilde{w}}$ in Eq. (59) must be

$$\frac{\partial}{\partial z}\left(\overline{\tilde{u}\tilde{w}}\right) = 0 \tag{60}$$

Consequently, $\overline{\tilde{u}\tilde{w}}$ is zero in the irrotational flow as proven previously.

[2] The assumption of $\tilde{\omega} = 0$ is not correct when $\partial H/\partial x \neq 0$. This can be seen from the following example.

The horizontal velocity over a flat bottom may be structured from linear wave theory as

$$\tilde{u}(x,z,t) = \hat{u}_0 \cosh kz \cos(kx - \sigma t) \tag{61}$$

in which $\hat{u}_0$ is the near bed velocity amplitude and calculated as

$$\hat{u}_0 = \frac{\sigma H(x)}{2 \sin hkh} \tag{62}$$

in which the wave height $H(x)$ varies with x. The vertical velocity component $\tilde{w}(x,z,t)$ can be calculated from the equation of continuity as

$$\tilde{w} = -\int_0^z \frac{\partial \tilde{u}}{\partial x} \mathrm{d}z = -\frac{1}{k}\frac{\partial \hat{u}_0}{\partial x} \sinh kz \cos(kx - \sigma t) + \hat{u}_0 \sinh kz \sin(kx - \sigma t) \tag{63}$$

Therefore, the vorticity $\tilde{\omega}$ defined by Eq. (10b) can be calculated from Eqs. (61) and (63) as

$$\tilde{\omega} = \frac{\partial \tilde{u}}{\partial z} - \frac{\partial \tilde{w}}{\partial x} = -2\frac{\partial \hat{u}_0}{\partial x} \sinh kz \sin(kx - \sigma t) \tag{64}$$

which indicates that the assumption of $\tilde{\omega} = 0$ is not correct when $\partial H/\partial x \neq 0$. From Eqs. (63) and (64), the term of $\overline{\tilde{w}\tilde{\omega}}$ in Eq. (17) can be calculated as

$$\overline{\tilde{w}\tilde{\omega}} = -\frac{1}{2}\frac{\partial \hat{u}_0^2}{\partial x} \sinh^2 kz \tag{65}$$

and then substituting Eq. (65) into Eq. 16 leads to

$$\frac{\partial}{\partial z}\left(\overline{\tilde{u}\tilde{w}}\right) = -\frac{1}{2}\frac{\partial \hat{u}_0^2}{\partial x} \sinh^2 kz - \frac{1}{4}\frac{\partial \hat{u}_0^2}{\partial x} \tag{66}$$

which reduces to Eq. (17) in shallow water. Alternatively, $\overline{\tilde{u}\tilde{w}}$ can be directly calculated from Eqs. (61) and (63) as

$$\overline{\tilde{u}\tilde{w}} = -\frac{1}{4k}\frac{\partial \hat{u}_0^2}{\partial x}\cosh kz \sinh kz \tag{67}$$

which is equal to Eq. (17) in shallow water. Therefore, the assumption of $\tilde{\omega} = 0$ applied in Eq. (17) is not correct when $\partial H/\partial x \neq 0$. It should be mentioned here that Eq. (17) may be considered to be the approximation of Eq. (66) in shallow water by ignoring the second-order term $\overline{\tilde{w}\tilde{\omega}}$ rather than by assuming $\tilde{\omega} = 0$.

[3] Although the derivation of Eq. (17) or Eq. (67) seems to be correct mathematically, the sign of $\overline{\tilde{u}\tilde{w}}$ given by Eq. (17) is not supported by experimental data. The time-averaged horizontal momentum equation given by Eq. (46) can be rewritten as

$$\epsilon_c \frac{\partial \bar{u}}{\partial z} = \overline{u_*}\left|\overline{u_*}\right|(1 - z/h) - C_1 z + \overline{\tilde{u}\tilde{w}} \tag{68}$$

in which the eddy viscosity ϵ_c and C_1 are expressed as

$$\frac{\bar{\tau}}{\rho} = \epsilon_c \frac{\partial \bar{u}}{\partial z} \quad \text{and} \quad C_1 = -\frac{\partial}{\partial x}\left(\overline{\tilde{u}^2} - \overline{\tilde{\omega}^2}\right) = -g\frac{\partial H}{\partial x}\frac{kH}{2\sinh kh} \tag{69}$$

The current eddy viscosity ϵ_c outside the wave boundary layer may be assumed to be of the form

$$\epsilon_c = \kappa\left|\overline{u_*}\right| z(1 - z/h) \tag{70}$$

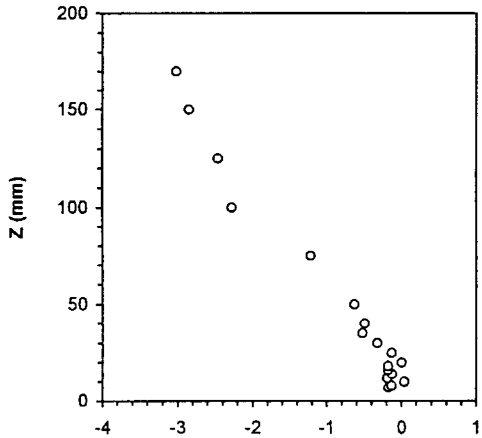
and then the current profile can be obtained from Eq. (68) as

$$\bar{u} = \frac{\overline{u_*}}{\kappa}\ln\frac{z}{z_1} + \frac{C_1 h}{\kappa\left|\overline{u_*}\right|}\ln\frac{(h-z)}{h - z_1} + \int_{z_1}^{z}\frac{\overline{\tilde{u}\tilde{w}}}{\kappa\left|\overline{u_*}\right| z(1 - z/h)}\mathrm{d}z \tag{71}$$

in which z_1 is the apparent roughness and $\overline{u_*}$ is the shear velocity. Since the parameters except $\overline{\tilde{u}\tilde{w}}$ in Eq. (71) can be experimentally measured, e.g., by Kemp and Simons (1983), the sign of $\overline{\tilde{u}\tilde{w}}$ will be determined from Eq. (71). It is shown in Fig. 1 of these comments that the sign of $\overline{\tilde{u}\tilde{w}}$ determined from Kemp and Simons' (1983) experimental data is negative rather than positive as shown by Eq. (17). The sign change may be due to the fact that the contribution of the wave boundary layer to $\overline{\tilde{u}\tilde{w}}$ in Eq. (17) has not been taken into account.

You et al. (1995) presented a semi-empirical model to study the derivation of current profiles from the standard logarithmic profiles. The vertical profile of $\overline{\tilde{u}\tilde{w}}$ was assumed to be the form of $\overline{\tilde{u}\tilde{w}} = -C_2 z$, where $C_2 > 0$. It is shown in Fig. 2 of these comments that the model of You et al. (1995) gives a good prediction of current profiles with the presence of waves opposing and following currents, respectively.

It can be concluded here that [a] the current profiles with the presence of waves especially near the mean water surface are affected by the wave Reynolds stress $\overline{\tilde{u}\tilde{w}}$ as clearly shown by Fig. 2; [b] the vertical distribution of $\overline{\tilde{u}\tilde{w}}$ due to the wave height attenuation should be zero in the irrotational flows, and may be expressed by Eq. (17) in



$$\int_0^z \frac{\overline{\tilde{u}\tilde{w}}}{\kappa|\bar{u}_*|z(1-z/h)}dz$$

Fig. 1. The sign of $\overline{\tilde{u}\tilde{w}}$ is negative as shown by Kemp and Simons' (1983) measurements of current velocities and wave parameters. The input parameters used in Eq. (71) are $H = 2.8$ cm, $T = 1.0$ s, $|\bar{u}_*| = 1.2$ cm/s, $L = 135$ cm, $h = 20$ cm, $\partial H/\partial x = 1.35 \times 10^{-3}$, $|\bar{u}_*| = 1.2$ cm/s, and $z_1 = 0.24$ cm.

the rotational flows, [c] the contribution of the wave boundary layer to $\overline{\tilde{u}\tilde{w}}$ may be significant and can not be ignored in Eq. (17) as indicated by Fig. 1, and [d] direct measurements of $\overline{\tilde{u}\tilde{w}}$ in the laboratory are definitely needed.

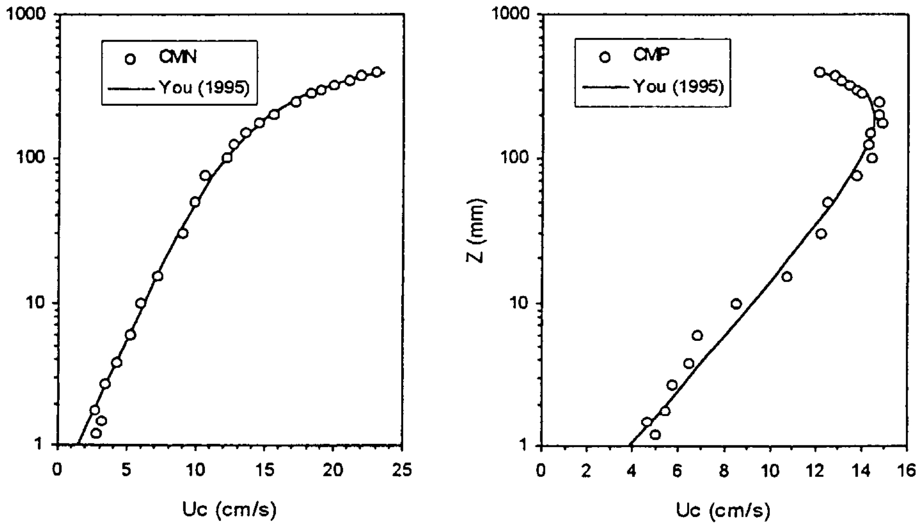


Fig. 2. Comparison of the semi-empirical model of You et al. (1995) with Klopman's (1994) experimental data: Currents opposing waves (CMN) and Currents following waves (CMP).

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