UDC 551.551:551.465.75 Ard.

V. V. YEFIMOV

ON THE STRUCTURE OF THE WIND VELOCITY FIELD IN THE ATMOSPHERIC NEAR-WATER LAYER AND THE TRANSFER OF WIND ENERGY TO SEA WAVES*

A theoretical model is proposed for the wind velocity field in the atmospheric near-water layer above waves. The turbulent character of the motion is taken into account by introducing a coefficient of turbulent viscosity. The equations of motion for air are considered within the framework of a plane problem. A stream function is introduced, and numerical solutions for it (obtained on a computer) are given. The transfer of wind energy to sea waves is discussed. Graphs showing the relationship between the amount of energy transferred to waves and the parameters of the mean wind velocity are presented.

Surface wind wave motion is one of the fundamental phenomena of atmosphere-ocean interaction. Wind waves lead to a feature that is characteristic of the turbulent structure of the boundary layers between atmosphere and ocean—the wave perturbations of the velocity field in the near-water layer of the atmosphere and upper layer of the ocean, which determine the energy and momentum transfer across the air-water boundary.

The investigation of energy transfer from wind to ocean waves is a basic task in the study of small-scale interaction between atmosphere and ocean and presupposes a detailed study of the velocity field in the atmospheric near-water layer. Although attempts to describe the mechanism of transfer of wind energy to ocean waves have been undertaken for a long time, it is only in the last few years that theoretical studies have produced notable results. Among such studies, prime mention should be made of the theoretical models of Phillips, Miles and Benjamin, where both the transfer of energy from wind to waves and related problems of the velocity field structure in the near-water atmospheric layer over waves have been discussed. These models have been analyzed in detail in [1]. There, in addition, the Phillips-Miles model, which has been accepted at the present time by a number of authors as a working theory, is formulated and generalized.

However, the assumption as to the decisive role of the turbulent fluctuations for the more developed wave components is rather more hypothetical. Thus Phillips [11] postulates that in a turbulent flow of air the correlation between vorticity and vertical velocity differs from zero not only in the critical area, but outside it as well. In order to allow for turbulence, a formula is introduced, by analogy with Miles' expression, which contains a certain coefficient determined by Miles from empirical data. The data used were the results of Motzfeld [2] on airflow over solid wave models in wind tunnels.

In this paper, perturbations introduced in the velocity field of the air flow in the near-water atmospheric layer by surface waves are calculated and the results obtained are applied to a calculation of the energy transferred by the wind to the waves. Here our principal attention will be focussed on the particular features to which allowing for the wind's turbulent structure leads.

FORMULATION OF THE PROBLEM

Consider the two-dimensional problem in which a wave of amplitude a, wave number k and frequency ω is propagated along the water surface in the direction of the x-axis. We assume that the flow of air above the waves is turbulent. Let us write out the initial Reymolds equations for air:

$$\frac{\partial \mathcal{U}}{\partial t} + \mathcal{U}\frac{\partial \mathcal{U}}{\partial x} + W\frac{\partial \mathcal{U}}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_m \Delta \mathcal{U} + \frac{\partial}{\partial x}(-\overline{u'^2}) + \frac{\partial}{\partial z}(-\overline{u'w'}) \qquad (1)$$

$$\frac{\partial W}{\partial t} + \mathcal{U}\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu_m \Delta W + \frac{\partial}{\partial x}(-\overline{u'w'}) + \frac{\partial}{\partial z}(-\overline{w'^2}). \qquad (2)$$

Here ρ is the density of air; z is the axis directed vertically upwards from the mean surface level; p is the pressure, without taking hydrostatic pressure into account; t is the time; U and W are the horizontal and vertical components of velocity; ν_m is the coefficient of molecular viscosity; u' and w' are the turbulent velocity fluctuations, having a time scale much less than the period of a surface wave.

Further, let us introduce the following assumptions:

a) Assume that the turbulent fluctuations u' and w' have a dual nature. First, they are generated by the gradient of the mean velocity U(z) in the near-water layer and second, by the wave-caused perturbations of the velocity. Then the tangential Reynolds wind stress which enters into the right side in the equations of motion can be represented in the form of two terms which, in accordance with the generally accepted method of introducing coefficients of turbulent exchange, we shall express by means of the corresponding deformation rate tensors

$$-\overline{u'w'} = K\frac{\partial U}{\partial z} + \nu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad (3)$$

where u and w are the horizontal and vertical components of wave caused velocity perturbations.

As is customary, assume for the boundary layer that K, the coefficient of turbulent exchange, which is related to the generation of turbulent fluctuations by the mean velocity gradient, is

$$K = \varkappa U_* z,$$

which increases linearly with height and corresponds to the undisturbed mean velocity profile

$$U = \frac{U_*}{\varkappa}\ln\frac{z}{z_0},$$

*Izv., Atmospheric and Oceanic Physics, Vol. 6, No. 10, 1970, pp. 1043-1058, translated by J. D. L. McIntosh.

where U_* is the dynamic velocity and $\varkappa \approx 0.4$ is von Kármán's constant. Assume that ν, the coefficient of turbulent exchange, which is related to the generation of turbulence by the wave-caused velocity fluctuations, is constant, although further numerical solutions may be calculated with ν designated as a certain function of the coordinate z. Assume that $\nu \gg \nu_m$ everywhere except in the thin laminar sublayer in which we shall neglect the effect of turbulent viscosity.

b) In relation to normal Reynolds stresses, let us assume that

$$-\overline{u'^2} = -\overline{w'^2}. \qquad (4)$$

c) Assume that the slope of the waves is small, so that the expressions for the velocities on the boundary predicted by the theory of waves of small slope can be applied.

Let us formulate the boundary conditions. At infinity we require an attenuation of the periodic wave-caused velocity perturbations. On the air-water boundary we specify continuity of the horizontal and vertical components, which in accordance with the assumed theory of small waves are

$$u_\eta = a\omega\cos(kx - \omega t), \qquad w_\eta = a\omega\sin(kx - \omega t) \qquad (5)$$

where $z = \eta$ for a wave which has a surface of the form

$$\eta = a\cos(kx - \omega t).$$

Thus, boundary conditions have been specified at infinity and on the surface of the wave. Earlier we presupposed a two-layer model of the atmospheric near-water layer, i.e., a thin laminar sublayer in which turbulent effects may be neglected and an external main turbulent layer. Generally speaking, this requires that a solution be found for each sublayer and each transition. In the analysis carried out by Benjamin [3], in which these two layers differed only in the appearance of the mean velocity profile (the coefficient of viscosity was taken to be the molecular viscosity everywhere), the complete solution was found in the form of the sum of two functions, one of which described the velocity within the viscous frictional boundary layer, and the other outside it. Let us make use of his conclusion that, firstly, the "viscous" solution plays a part only within the boundaries of a very thin layer, beyond which it is negligibly small, and secondly, that viscous molecular effects in this layer make a small contribution to the total energy relations.

This will be all the more correct in our case, for which the main features are related to the turbulent nature of the motion. According to [3], it can be assumed that within the limits of the viscous frictional layer, there occurs a change in phase of the horizontal component of perturbation velocity by π relative to the phase on the surface. It follows from the solution of Miles [4] that there is an analogous condition on the lower boundary of the atmospheric near-water layer, in which the phase of the horizontal component differs by π from the phase of the horizontal component on the surface of the water. This conclusion also follows from general considerations relating to the fact that molecular viscosity plays a part only in a thin viscous frictional sublayer of thickness $\delta_1 = (2\nu_m/\omega)^{1/2}$, while outside this sublayer the velocity field approximates a nonviscous field [5]. For air, δ_1 is of the order of 1 mm. Thus, assuming the thickness of the laminar sublayer to be much greater than δ_1, let us write the boundary conditions for the lower boundary of the main turbulent boundary layer:

$$u = -a\omega\cos(kx - \omega t), \qquad w = a\omega\sin(kx - \omega t). \qquad (6)$$

In view of the thinness of the laminar sublayer and the assumption that the wave slope is small, we apply these conditions to the level $z = 0$ [6], by doing so we introduce an error of the same order of magnitude as the error committed in putting u_η and w_η in the form of (5).

Making use of the continuity equation and the assumed two-dimensionality of the motion, we change, in the equations of motion (1) and (2), from the velocity components $\mathcal{U} = U + u$, $W = w$ to the stream function of the perturbations, where u and w are wave perturbations of the velocity of air flow with a mean undisturbed horizontal velocity component U(z) and a vertical component equal to zero. Let us assume that

$$u = \frac{\partial\Phi}{\partial z}, \qquad w = -\frac{\partial\Phi}{\partial x}.$$

Then, after eliminating the pressure and taking into account assumptions "a" and "b", we obtain an equation for the stream function which, after separation into stationary and nonstationary parts [6, 10], breaks down into two functions,

$$\tilde{\Omega}_t + \bar{\Phi}_z\tilde{\Omega}_x - \tilde{\Phi}_x\bar{\Omega}_z + (\tilde{\Phi}_z\tilde{\Omega}_x - \tilde{\Phi}_x\tilde{\Omega}_z)_\sim + U\tilde{\Omega}_x + U_{zz}\tilde{\Phi}_x = \nu\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right)(\tilde{\Phi}_{zz} - \tilde{\Phi}_{xx}), \qquad (7)$$

$$(\tilde{\Phi}_z\tilde{\Omega}_x - \tilde{\Phi}_x\tilde{\Omega}_z)_s = \nu\bar{\Phi}_{zzzz}. \qquad (8)$$

Here the following notation is introduced. The vorticity is $\Omega = \Phi_{zz} + \Phi_{xx}$; the stationary part of each variable is indicated by a straight line above it and the nonstationary part by a wavy line; $(\ )_s$ and $(\ )_\sim$ are respectively the stationary and nonstationary quantities in parentheses, the subscript indicating differentiation with respect to the given variable.

Then we change to dimensionless variables, choosing as a scale the length k^{-1}, the time ω^{-1}, the nonstationary velocity $a\omega$ and the stationary velocity of perturbations $kaa\omega$ [6]. For convenience in solving for the dimensionless variables we shall keep to the previous notations. As a result,

$$\tilde{\Phi}_{txx} + \tilde{\Phi}_{tzz} - \frac{1}{R}\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right)(\tilde{\Phi}_{zz} - \tilde{\Phi}_{xx}) + \varepsilon[U(\tilde{\Phi}_{xxx} + \tilde{\Phi}_{xzz}) - U_{zz}\tilde{\Phi}_x] = \varepsilon(\tilde{\Phi}_z\tilde{\Phi}_{xxx} + \tilde{\Phi}_z\tilde{\Phi}_{xzz} - \tilde{\Phi}_x\tilde{\Phi}_{zxx} - \tilde{\Phi}_x\tilde{\Phi}_{zzz}) - \varepsilon^2(\bar{\Phi}_z\tilde{\Phi}_{xxx} - \bar{\Phi}_z\tilde{\Phi}_{xzz} - \tilde{\Phi}_x\bar{\Phi}_{zzz}), \qquad (9)$$

$$(\tilde{\Phi}_z\tilde{\Phi}_{xxx} + \tilde{\Phi}_z\tilde{\Phi}_{xzz} - \tilde{\Phi}_x\tilde{\Phi}_{zxx} - \tilde{\Phi}_x\tilde{\Phi}_{zzz})_s = R^{-1}\bar{\Phi}_{zzzz}, \qquad (10)$$

where the slope $\epsilon = ka$ and the parameter $R = \omega/\nu k^2$.

The characteristic feature of Eq. (9) is the term in square brackets on the left side, which characterizes the interaction of the waves with the average undisturbed air flow having a mean velocity profile given by U.

In conformity with the chosen boundary conditions, the boundary conditions for the stream function will be written in the following form:

$$\tilde{\Phi}_x(0) = \sin S, \quad \tilde{\Phi}_z(0) = -\cos S, \quad \bar{\Phi}_z(0) = 0, \quad S = x - t, \qquad (11)$$

$$\tilde{\Phi}_x(\infty) = \tilde{\Phi}_z(\infty) = 0, \qquad |\bar{\Phi}_z(\infty)| < \infty. \qquad (12)$$

SOLUTION OF THE PROBLEM

An analytic solution to the Eqs. (9) and (10) by the method of successive approximations for the particular case $U = 0$ has been considered in [6]. In this case the series of successive approximations converged if the relation $\epsilon^2 R^{3/2} \leq 0$ (1) was satisfied. There is no apparent reason to expect this limitation not to apply also to Eqs. (9) and (10), which differ in our case only by terms containing the function U. Here the main features were already obtained in the zeroth approximation. Subsequent approximations contributed only to the magnitude of the mean velocity.

As the zeroth approximation, let us use Eq. (9) with the right side equal to zero. Note that, although on the left side of the equation terms containing U have the small quantity ϵ as coefficient, the mean velocity U for the wind waves is of a higher order of magnitude than the wave perturbations u and w, i.e. in the dimensionless case, of order greater than one. These same terms are of most interest to us. As all functions which enter into the right-hand side of the equation are of order 1 and are multiplied by ϵ or ϵ^2, this enables us to neglect them when finding the zeroth approximation.

Let us look for a solution for the stream function for the perturbations in the following form:

$$\Phi(z, S) = \varphi_0(z) + \varphi_1(z)\cos S + \varphi_2(z)\sin S, \qquad (13)$$

where $\varphi_0(z)$ specifies the stationary velocity, which is independent of the time t and the x coordinate; $\varphi_1(z)$ and $\varphi_2(z)$ are wave velocity fluctuations in the near-water layer. Then for the stream function from (9) and (10) we obtain the following equations:

$$\left(U - \frac{1}{\varepsilon}\right)(\varphi_1'' - \varphi_1) - U''\varphi_1 = -(\mathrm{R}\varepsilon)^{-1}(\varphi_2^{\mathrm{IV}} + 2\varphi_2'' + \varphi_2), \qquad (14)$$

$$\left(U - \frac{1}{\varepsilon}\right)(\varphi_2'' - \varphi_2) - U''\varphi_2 = (\mathrm{R}\varepsilon)^{-1}(\varphi_1^{\mathrm{IV}} + 2\varphi_1'' + \varphi_1), \qquad (15)$$

$$-\frac{1}{2}(\varphi_1'\varphi_2 - \varphi_2'\varphi_1) = \mathrm{R}^{-1}\varphi_0^{\mathrm{IV}}. \qquad (16)$$

Equation (16) serves for the calculation of the stationary perturbation velocity $\bar{u} = \partial\varphi_0/\partial z$. For the sake of clarity, it can be rewritten

$$(\overline{uw})'' = \mathrm{R}^{-1}\bar{u}''', \qquad (17)$$

whence it is obvious that the reasons for the appearance of a stationary addition to the mean undisturbed velocity U are the nonzero Reynolds wave stresses $\overline{uw}$ (for convenience we shall not include ρ, the density of air, when determining the Reynolds wave stress).

Equations (14) and (15) determine the wave-caused velocity perturbations. In essence they amount to a form of a well-known equation in the theory of hydrodynamic instability, the Orr-Sommerfeld equation. In fact, by introducing the complex function $\varphi_k = \varphi_2 + i\varphi_1$, we obtain an equation which differs from the complete Orr-Sommerfeld equation only in the sign preceding the term $2\varphi_k''$ on the right-hand side. Clearly, this difference, which is related to the introduction of the coefficient of turbulent viscosity ν, is very important. In addition, it is also significant that in our case the parameter R is substantially less than the Reynolds number which enters into the Orr-Sommerfeld equation and which is determined by means of the coefficient of molecular viscosity.

These equations, according to Miles and Benjamin, for example, could have been obtained somewhat more easily from the equations of motion for the boundary layer, (1) and (2), using the method of small perturbations and neglecting small quadratic terms; however, at the same time, Eq. (16) for the stationary velocity $\bar{u}$ is left out.

Equations (14) and (15) are the basic equations for solving the problem. The presence of terms which include the mean undisturbed velocity U, described by the logarithmic law, prevents one from finding an analytical solution which has to be found numerically. The function of mean undisturbed velocity has been assumed in the following form:

$$U = \frac{U_*}{\varkappa}\ln\left(\frac{z}{z_0} + 1\right) = U_0 \ln\left(\frac{z}{z_0} + 1\right) \qquad (18)$$

everywhere except in the thin layer δ of the lower boundary, z = 0. In the layer δ, the quantity U(z) is approximated by a linear function of height so as to exclude physically unreally large values of the slope for the mean velocity profile U'' in direct proximity to z = 0. Actually, the thickness of δ in the calculations was of the order of 1 cm. Generally speaking, it was only essential to set U'' = 0 at the boundary itself; a further increase in the thickness of δ to 10^{-1} (in terms of nondimensional height, i.e., normalized to k^{-1}) had practically no effect on the results of the calculations.

Numerically, the problem was solved with the following boundary conditions, which result from (11) and (12):

$$\varphi_1(0) = 1, \quad \varphi_1'(0) = -1, \quad \varphi_2(0) = \varphi_2'(0) = 0, \qquad (19)$$

$$\varphi_1(H) = \varphi_1'(H) = \varphi_2(H) = \varphi_2'(H) = 0. \qquad (20)$$

Thus in the calculations, the boundary layer was limited to a certain height H, where the wave-caused velocity perturbations were set equal to zero. The upper limit of the boundary layer H was chosen sufficiently high so that it was possible to assume that the wave-caused perturbations at the upper boundary had fully disappeared. For practical reasons H was chosen to be greater than 10, i.e., greater than one and a half wave lengths. Analysis showed that starting with H = 10 and higher, a stationary solution was obtained. (For a wavelength of 25 m, for example, a height of H = 0 corresponds to 40 m. Obviously, starting from such a height, wave-caused perturbations of the air velocity are negligible.) The matrix distillation method was used to obtain a numerical solution.

As a result, the functions φ_1 and φ_2, which satisfy Eqs. (14) and (15) and the boundary conditions (19) and (20) and which characterize the periodic perturbations of air flow velocity, were found. Then by means of φ_1 and φ_2, those characteristics of the velocity field in a boundary layer which have a clear physical meaning were calculated. Among them are the dispersions of the horizontal (σ_u) and vertical (σ_w) components of the fluctuation velocity, the stationary addition $\bar{u}$ to the undisturbed velocity, and the Reynolds wave stresses $\tau = -\overline{uw}$. From expressions (13) and (17), taking boundary conditions (11) and (12) into account, it follows that

$$\sigma_u = \sqrt{\overline{u^2}} = 2^{-1/2}(\varphi_1'^2 + \varphi_2'^2)^{1/2}, \quad \sigma_w = \sqrt{\overline{w^2}} = 2^{-1/2}(\varphi_1^2 + \varphi_2^2)^{1/2}, \qquad (21)$$

$$\tau = -\overline{uw} = -{}^1/_2(\varphi_2'\varphi_1 - \varphi_1'\varphi_2), \qquad (22)$$

$$\bar{u} = \mathrm{R}\int \overline{uw}\, dz. \qquad (23)$$

Note that the quantities determined by (21)-(23) are written in dimensionless variables. To find the true values of the dispersions, they should be multiplied by $a\omega$, of the wave stresses, by $(a\omega)^2$, and of the velocities, by $(aka\omega)$.

ANALYSIS OF RESULTS

a) Field of perturbation velocity in the atmospheric boundary layer. Equations (14) and (15), which determine the functions φ_1 and φ_2, contain dimensionless parameters: the wave slope ϵ and the wave number R. Clearly they are determined by the characteristics of the wave motion a, ω, and k and by ν, the coefficient of turbulent viscosity. Allowing for the fact that the wave motion is assumed to be subject to the consequences of the theory of waves of small slope, it is possible, on the basis of the dispersion relation $\omega^2 = gk$ (where g is the acceleration due to gravity), to reduce the number of independent parameters to a, ω and ν. In addition, the mean undisturbed velocity U is determined by two more parameters: the velocity U_0 and the roughness parameter z_0. As a result, the solution of the problem depends on the values a, ω, ν, U_0 and z_0, with the only unknown parameter among them being ν, the coefficient of turbulent exchange.

In this connection it is natural to adopt the following scheme for analyzing the solutions obtained for various interrelations of the indicated parameters: for the given parameters of wave motion, i.e., constant values of amplitude a and frequency ω, we investigate the solution for various profiles of mean velocity $U(z)$. One can vary the mean velocity profile by changing, for example, U_0 or z_0. These calculations were carried out for several values of ν which exceeded the coefficient of molecular viscosity by three to five orders of magnitude. A second method of analysis is possible: for the given mean velocity profile, we vary the parameters of wave motion, for example the wave slope of the wave frequency. However, in using this method of systematization of solutions, a difficulty arises relating to the coefficient ν. It is known that the coefficient of turbulent viscosity characterizes not the physical properties of a liquid, but the statistical properties of the fluctuations. Therefore, generally speaking, it does not have to be constant, and in our case can vary depending on the wave parameters. In the first case it can be assumed to be a constant since the parameters are constant: in the latter case, with a given profile and varying wave parameters, additional assumptions have to be made concerning the coefficient of turbulent viscosity.

Before proceeding with a detailed analysis of the wind velocity field in the boundary layer, let us note some characteristic features of the solution of the problem which are evident in the particular case $U_0 = 0$. This case corresponds to swell waves propagating in initially calm air. For this case, Eqs. (14) and (15) can be solved analytically, and the solution is considered in detail in [6]. Numerical calculation of the same equations yields results which coincide with the analytical solution, which, in particular, enables us to test the method for solving the equation numerically and to choose an optimal upper limit H for the calculations. An analysis of the solutions shows that, beginning with H = 10 and higher, the numerical treatment gives values of σ_u, σ_w, and τ which differ very little from their accurate values obtained analytically.

The solution shows that for values of the turbulent viscosity coefficient ν which exceed the magnitude of the turbulent viscosity coefficient for air by no more than five orders of magnitude, $\varphi_2 \ll \varphi_1$. When this coefficient decreases, $\varphi_2 \to 0$, $\varphi_1 \to e^{-z}$, and $\overline{uw} \to 0$. This means that the motion of the air in the boundary layer approximates the simple potential wave motion for which the velocity perturbations are described by the formulas

$$u = -e^{-z}\cos(x-t), \qquad w = e^{-z}\sin(x-t). \tag{24}$$

For this case ($U_0 = 0$) it is interesting to trace the difference in the effect of turbulent viscosity as compared to molecular viscosity. Neglecting turbulent velocity fluctuations in the equations of motion (1) and (2), we obtain, instead of Eqs. (14) and (15), the following system of equations, which describes wave-caused perturbations of the velocity for laminar motion:

$$\begin{aligned}\left(U-\frac{1}{\varepsilon}\right)(\varphi_1''-\varphi_1)-U''\varphi_1 &= (R_m\varepsilon)^{-1}(\varphi_2^{IV}-2\varphi_2''+\varphi_2),\\ \left(U-\frac{1}{\varepsilon}\right)(\varphi_2''-\varphi_2)-U''\varphi_2 &= (R_m\varepsilon)^{-1}(\varphi_1^{IV}-2\varphi_1''+\varphi_1),\end{aligned} \tag{25}$$

where the number R_m is expressed by means of the molecular viscosity coefficient in the form $R_m = \omega/\nu_m k^2$.

As we recalled earlier, these equations are in the exact form of the Orr-Sommerfeld equation, and they differ from (14) and (15) in the sign before the terms $2\varphi_2''$ and $2\varphi_1''$ on the right side of these equations. This difference turns out to be significant. For example, if we formally set $\nu = \nu_m$, so that for the given wave motion $R = R_m$, then the solution of Eqs. (14), (15) and (25) is of a different form. This difference is not great for the velocities u and w themselves, which in both cases are close to the form of (24); however, the difference is substantial for quantities of a higher order of smallness, such as the Reynolds wave stress τ.

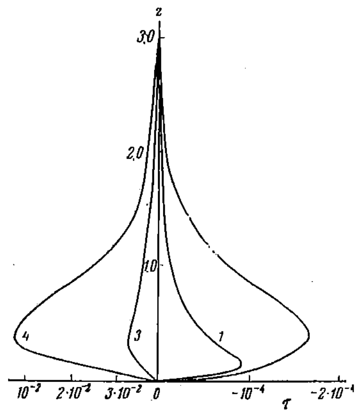

Fig. 1. Distribution of wave-caused stresses with height for $U_0 = 0$ and R and R_m equal to, respectively:

1) R = 100, 2) R = 10, 3) R_m = 100, 4) R_m = 10.

Figure 1 shows the behavior of the stress τ with respect to height for two values of the parameters R. There also are shown the values of τ, for the same values of the parameter R_m, obtained as a result of solving system (25). Obviously, both the magnitudes and the behavior of τ are completely different for these two cases; i.e., a formal increase in the molecular viscosity coefficient by several orders of magnitude does not lead to the results which are obtained for the same values of the turbulent velocity coefficient;—the mechanisms for the action of molecular and turbulent viscosity are different.

Consider the structure of wave-caused perturbations of the near-water atmospheric layer in the case $U_0 = 0$, i.e., in the case of wind waves. Let us assume a logarithmic form of the profile of mean undisturbed air velocity, where U_0 and z_0 are expressed in dimensionless form. Let a and ω, the parameters of surface wave motion, be given. Consider the behavior of the solution as the velocity gradually increases from zero while z_0, the roughness parameter, remains constant. It is known that the logarithmic function describing the mean velocity profile increases rapidly at first and then changes insignificantly. When the values of U_0 are not large, the mean wind velocity U is less than c, the phase velocity of the wave, over the whole atmospheric boundary layer. When U_0 is gradually increased, the mean velocity U increases, so that at a certain height it becomes equal to the wave velocity c, and for still higher values of U_0, the wind velocity U exceeds the wave velocity c over practically the whole boundary layer with the exception of the thin layer near the lower boundary where, as before, $U < c$.

It is obvious from physical considerations that for small velocities U_0 the structure of the near-water layer must differ from the case when the velocities are high. In the former case, waves attenuate, and in the latter,

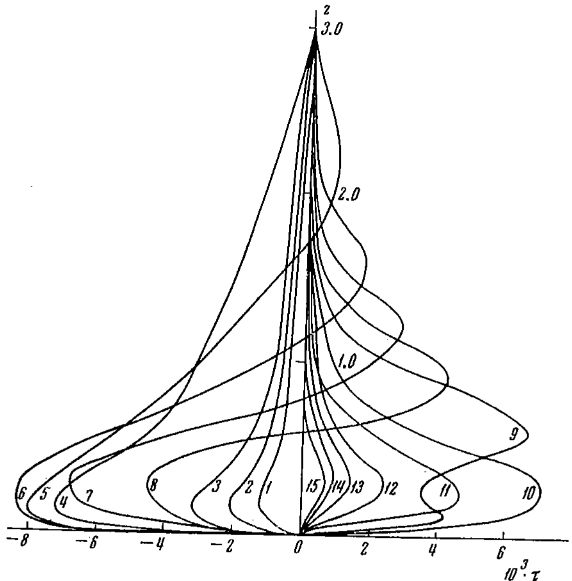

Fig. 2. Distribution of wave-caused stresses with height for $R = 10^3$, $\epsilon = 0.1$, $z_0 = 10^{-5}$ and U_0 equal, respectively, to:

1) 0.4, 2) 0.6, 3) 0.7, 4) 0.8, 5) 0.82, 6) 0.84, 7) 0.86, 8) 0.88, 9) 0.9, 10) 0.95, 11) 1.0, 12) 1.1, 13) 1.2, 14) 1.6, and 15) 1.8.

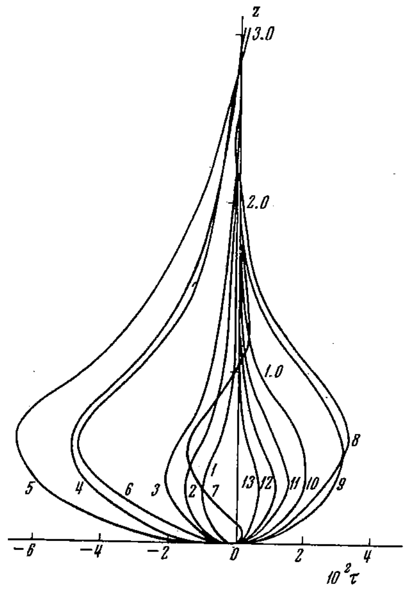

Fig. 3. Distribution of wave-caused stresses with height for $R = 10^2$, $\epsilon = 0.1$, $z_0 = 10^{-5}$ and U_0 equal, respectively, to:

1) 0.4, 2) 0.6, 3) 0.7, 4) 0.8, 5) 0.82, 6) 0.83, 7) 0.84, 8) 0.88, 9) 0.9, 10) 1.0, 11) 1.1, 12) 1.2, and 13) 1.6.

they increase in size owing to the effect of the wind. A graphic example of the first case is shown by swell waves; of the second, by the initial period of wave formation under the action of wind. However, for intermediate values of U_0, when the mean wind velocity in the main part of the boundary layer is close to the wave velocity c, it is difficult to come to any definite conclusions in advance.

Let us cite the results of a numerical solution according to this scheme of analysis for several values of the coefficient of turbulent viscosity. For convenience in the following description, we select characteristic parameters for the wind waves and for the mean wind velocity profile. We take as a typical case of a gravitational wind wave a wave with a wave number $k = 0.25\ m^{-1}$ and amplitude $a = 0.4$ m, so that for this wave, ϵ, the small slope parameter equals 0.1. For the mean velocity profile, let us take the roughness parameter z_0 to be equal to $3 \cdot 10^{-4}$ m and the friction velocity U_* to 0.25 m/sec. Assume that the values of R are 10^3, 10^2 and 10, which, for the characteristic parameters of wave motion chosen, correspond to coefficients of turbulent viscosity equal to 0.024, 0.24 and 2.4 m^2/sec, respectively.

Figures 2-4 show the distribution of wave stresses τ with height for three values of R. The individual curves of the family correspond to different values of U_0 indicated in the caption beneath the figures. The following features of the behavior of τ are noteworthy:

a) the increase in absolute magnitude of τ as R decreases;

b) the presence of a maximum absolute value of τ at a certain height above the wave surface. This height is, on the average, around 0.5, which in dimensional terms corresponds to a height of $0.5\ k^{-1}$ m;

c) the change in sign of τ as U_0 gradually increases. For U_0 greater than zero, but less than a certain fixed quantity that depends on the rest of the solution's parameters, $\tau < 0$; when U_0 increases still further, $\tau > 0$. In the case where the mean wind velocity U is less than the wave phase velocity c over the whole boundary layer, $\tau < 0$. In the case where the mean wind velocity in the upper part of the boundary layer, (i.e., where it does not change substantially) is much greater than the wave velocity c, $\tau > 0$ everywhere;

d) in the intermediate case where the mean wind velocity in the upper part of the boundary layer approaches c, the behavior of τ is characteristically complex. This case is the most interesting one from our point of view, as it

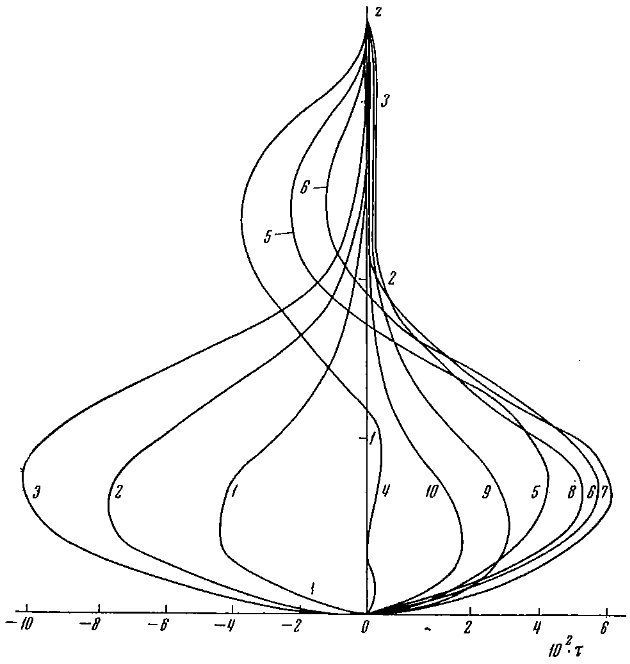

Fig. 4. Distribution of wave-caused stresses with height for R = 10, $\epsilon = 0.1$, $z_0 = 10^{-5}$, and U_0 equal, respectively, to:

1) 0.4, 2) 0.7, 3) 0.78, 4) 0.82, 5) 0.85, 6) 0.88, 7) 0.91, 8) 1.0, 9) 1.3 and 10) 1.8.

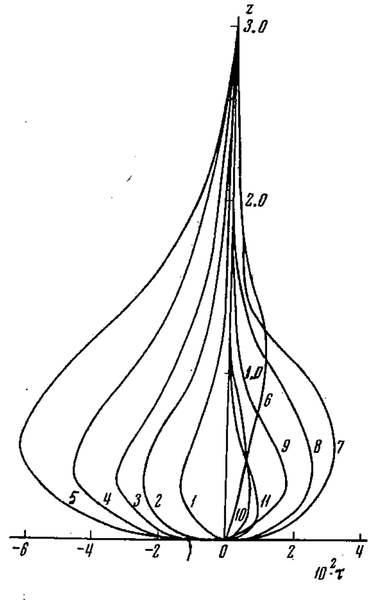

Fig. 5. Distribution of wave-caused stresses over height for $R = 10^2$, $\epsilon = 0.1$, $U_0 = 0.9$ and z_0 equal, respectively, to:

1) 10^{-3}, 2) 10^{-4}, 3) $6 \cdot 10^{-5}$, 4) $4 \cdot 10^{-5}$, 5) $3 \cdot 10^{-5}$, 6) $2 \cdot 10^{-5}$, 7) 10^{-5}, 8) $5 \cdot 10^{-6}$, 9) 10^{-6}, 10) 10^{-7} and 11) 10^{-8}.

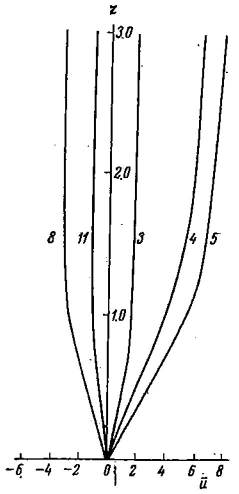

Fig. 6. Distribution of mean perturbation velocity $\bar{u}$ with height for $R = 10^2$, $\epsilon = 0.1$, $z_0 = 10^{-5}$ and U_0 as in Fig. 3.

concerns fully developed wind waves for which the critical height is not too near the surface. For example, it is known from actual experience that the mean velocity of waves bearing maximum energy in the spectrum of fully developed wave motion is close to the average wind velocity measured at a height of 10 m.

It is obvious from the diagrams that an important result of the solution is that the change of sign of τ as the velocity U_0 increases is not accompanied by a monotonic increase in τ from negative to positive values. Near the transition of τ through zero, its absolute value increases on both sides, i. e., the dependence of the stresses on the

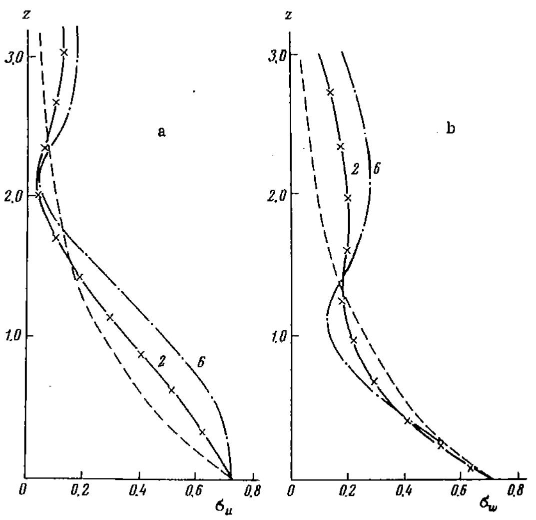

Fig. 7. Attenuation with respect to height:

a) dispersions σ_u and b) dispersions σ_w for $R = 10$, $\epsilon = 0.1$. For curves 2 and 6, see Fig. 4.

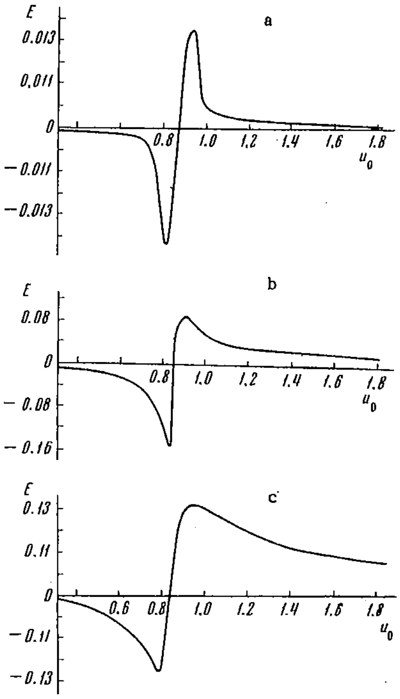

Fig. 8. Dependence of energy on the velocity U_0 for $\epsilon = 0.1$, $z_0 = 10^{-5}$: a) $R = 10^3$, b) $R = 10^2$, c) $R = 10$.

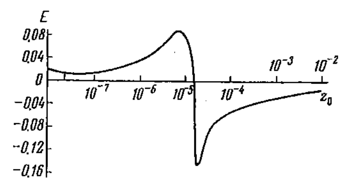

Fig. 9. Dependence of energy on the roughness parameter for $R = 10^2$, $\epsilon = 0.1$, $U_0 = 0.9$.

velocity U_0 is resonant and selective in character. As will be obvious from what follows, this leads directly to selectivity in the transmission and absorption of wind energy by the waves.

An analogous picture is observed when an analysis is made of the dependence of the stress τ on the roughness parameter z_0 for constant values of U_0. In this case the increased wind velocity is achieved by a decrease in z_0. Figure 5 shows the family of functions $\tau(z)$ for various values of z_0 when $R = 10^2$. The wave parameters remain the same as for Figs. 2-4, and the velocity $U_0 = 0.9$. As can be seen from Fig. 5, the above-mentioned features of the behavior of τ as the mean wind velocity increases remain as before. This applies also to the cases $R = 10^3$ and $R = 10$. As the graphs of the functions $\tau(z)$ do not differ in principle from Figs. 2 and 4, we do not show them.

According to Eq. (17), nonzero values of τ give rise to the appearance of a mean horizontal velocity of the perturbations $\bar{u}(z)$. It is clear from (23) and the values of τ shown in Figs. 2-5 that this addition to the mean undisturbed velocity $U(z)$ is on the whole not large, except in cases where τ reaches maximum values near the point of transition through zero. The dependence of $\bar{u}$ on height can be obtained directly by using the graphs of $\tau(z)$ (Figs. 2-5) and formula (23). For example, Fig. 6 shows profiles of the mean velocity of perturbations $\bar{u}$ for various values of U_0 when $R = 10^2$ (for this case, $\tau(z)$ is shown in Fig. 3).

Clearly, the velocity $\bar{u}$ has different signs for different U_0. In the case of U_0 being small enough so that the wave stress τ is negative, $\bar{u}(z) > 0$. In the case of larger U_0, when $\tau > 0$, $\bar{u}(z) < 0$. In the former case the total mean wind velocity, which is equal to the sum of the given undisturbed wind velocity U and the stationary addition $\bar{u}$, becomes greater than U, and in the latter case, less than U. As a result, the mean wind velocity profile in the near-water layer will differ from the logarithmic form. This difference becomes greater when $U \approx c$, and is especially marked in the lower part of the boundary layer at a height of the order of 1, i.e., where the magnitude of the stress τ is maximal. Note that if the stress τ depends significantly on the value of the turbulent viscosity coefficient, $\bar{u}$ varies insignificantly with a change in ν. This is qualitatively evident, particularly from expression (23), if we allow for the fact that lower values of τ correspond to higher values of R.

Two characteristics of the field of perturbations introduced into the air boundary layer by interfacial wave motion have been considered above: Reynolds wave stresses and the horizontal component of the mean perturbation velocity. As indicated, when $U_0 = 0$, the horizontal and vertical perturbation velocity components themselves are close to the form of (24). This applies also when $U_0 \neq 0$, with the exception, however, of cases which correspond to resonance values of τ. For such cases, the absence of components of the fluctuation velocity from the simple form of expression (24) becomes important.

Figures 7a and 7b show the dispersions of the horizontal and vertical velocity components σ_u and σ_w, calculated according to (21) for $R = 10$. The remaining parameters and the numbering of curves corresponds to Fig. 4. The case $R = 10$ is chosen because the characteristic features of the behavior of σ_u and σ_w are more marked. For higher values of R these features remain, but are not so well defined. The dashed line in Fig. 7 represents the function $2^{-1/2}\exp(-z)$, which describes the attenuation of the dispersions σ_u and σ_w with respect to height for the velocity field described by the expressions (24). Here, only dispersions which correspond to values of U_0 for which the stress τ increases by resonance are mentioned. For these dispersions, there is an appreciably slower attenuation of σ_u with respect to height and a more rapid attenuation of σ_W compared to the exponent in the lower part of the boundary layer.

Note that σ_u and σ_W are quantities which change easily; for that reason it is interesting to compare the results obtained with the available experimental data. Thus, in [8], spectral density functions of the fluctuations of the horizontal velocity component, measured at various distances from the interface, are cited. Obviously, the spectral density does not attenuate very much in the layer extending to 4.5 m in thickness. In dimensionless coordinates a height of 4.5 m is ~1. As can be seen from Fig. 7, the attenuation of σ_u in this layer is in fact not great. Thus there has been some experimental confirmation of a slower attenuation of the horizontal component of wind velocity over waves in relation to the exponent.

The resonance behavior of the solution of Eqs. (14) and (15) is clearly manifested in the analysis of integral relations such as the amount of energy transferred through the interface from wind to waves.

b) Transfer of wind energy to the waves. Wave perturbations of the velocity in the near-water atmospheric layer interact with the average motion of the air. From the equation for the turbulent energy balance, it is known that if the tangential Reynolds wind stress and the mean velocity gradient differs from zero, there occurs an exchange of energy between the average and fluctuation-motions. The direction of the energy transfer is determined by the signs of the mean velocity gradient and the Reynolds stress. Here the rate of energy transfer per unit volume is [7]:

$$e = -\rho\, \overline{uw} \frac{\partial U}{\partial z}. \tag{26}$$

The energy transferred by the average motion of the air to the wave fluctuations (or vice versa) calculated per unit area of the interface, can be written as follows:

$$E = -\rho \int_0^\infty \overline{uw} \frac{\partial \overline{U}}{\partial z}\, dz. \tag{27}$$

When $E > 0$, wave perturbations in the air are maintained due to the average motion. $E < 0$ means that the wave perturbation energy is being transferred to the average motion. From the equation for the balance of fluctuation energy, integrated with respect to z from 0 to ∞, it also follows that if we neglect viscous dissipation of fluctuation energy in the boundary layer, Eq. (27) will represent the energy transferred across the interface, i.e., the energy transferred from the wind to the waves. Note that this expression for the energy has been used in a number of papers (see [9], for example).

The energy E was calculated using (27). Calculations of the mean velocity were made, allowance being made for the fact that the magnitude of the stationary addition $\bar{u}(z)$ to the mean undisturbed velocity U(z) is small, by using the undisturbed velocity U(z) from (18).

Figure 8 shows the values of E as they depend on the velocity u_0, and Fig. 9, as they depend on the roughness parameter z_0. All the other parameters apply to cases for which the behavior of the stress τ as the height changes was shown in Figs. 2-5, respectively. In order to obtain the dimensional values of the energy, these values have to be multiplied by $\rho\,(a\omega)^3$. As is clear from the graphs, $E < 0$ for low mean wind velocities, i.e., the wave energy is transferred to the averaged motion in the near-water layer. For large mean wind velocities one has $E > 0$, and energy is transferred from the wind to the waves.

The transition from negative to positive energy values is accompanied by two resonance maxima. At these maxima, the amount of energy transferred by the wind to the waves or vice versa reaches maximum values. This result, generally speaking, is not physically evident, as it may seem that the larger the difference between the mean wind velocity and the velocity of wave propagation, the greater is the amount of energy transferred. From Figs. (8) and (9), it follows that the energy is maximal when $U \approx c$ (U is the mean wind velocity in the upper region of the boundary layer), i.e., where it increases insignificantly.

For the time being it is difficult to propose a quantitative formula for an approximate calculation of the energy transferred from the wind to the waves on the basis of the numerical solution obtained. Firstly, the magnitude of the coefficient of turbulent viscosity ν is unknown. Secondly, there exists, in principle, no method for solving the equations numerically on a computer with accuracy comparable to the accuracy of analytical solutions. In spite of the introduction of dimensionless variables, the number of parameters which determines the solution is rather large: U_0, z_0, R, ω, and ϵ. Therefore the investigation of the way the solution depends on the change of all parameters is an extremely laborious process. Above, we traced the behavior of the solution for various mean wind velocities and three values of R under the condition that the other parameters remained constant. The relations obtained apply to the given parameters. In the case of other values, the solution will be different. However the general features, such as the change in sign of E in the case of low and high wind velocities or the resonance behavior of E, apparently remain. Note that those energies transferred by the wind to the waves, as can be easily seen from Figs. 8 and 9, have reasonable values. At least for R = 10 and R = 100 they coincide in order of magnitude with values that are recorded in the literature.

CONCLUSION

To a first approximation, the basic characteristics of the perturbation-velocity field in the atmospheric near-water layer above waves have been obtained as a result of a numerical solution of the equations of motion, taking into account the effect of turbulent viscosity. Among these characteristics are the dispersions of the horizontal and vertical components of the velocity fluctuations, the Reynolds wave stress, and the stationary horizontal component of the perturbation velocity. From these was calculated the energy transferred from wind to waves. An analysis of the solution showed that allowing for the turbulence structure accounts for a number of particular features of the wind velocity field in the atmospheric near-water layer. These features already become important for turbulent viscosity coefficients that exceed the molecular viscosity coefficient by three orders of the magnitude or more.

As a result, the wave-caused motion of the air in the near-water layer is essentially nonpotential; a nonzero vorticity of the velocity and related Reynolds wave stresses appear over the whole boundary layer. The horizontal and vertical perturbation-wave velocity components have a phase shift which differs from 90°. Attenuation of these components with respect to height differs from an exponential decay.

Interacting with the average air motion, the Reynolds wave stress gives rise to an exchange of energy between the wave-caused fluctuation—motion and the average motion. For low mean wind velocities, the energy of the wave-caused fluctuations becomes the energy of the average motion; in the case of higher velocities, the wind energy maintains the wave fluctuations, signifying a transfer of energy from the wind to the waves.

Other parameters being equal, the difference between the motion of the air and the potential motion increases as the mean wind velocity in the upper part of the boundary layer, where it changes insignificantly, approaches the velocity of wave propagation. As a result, the transition from negative to positive energies as the mean wind velocity increases, is not monotonic. Near the point of transition, the absolute energy increases on either side.

Thus, for example, the energy will be greatest when the wave velocity is close to the mean wind velocity. This result is important, as it is well known, that in the case of developed wave motion, for example, the velocity of waves bearing maximum energy in the energy spectrum is close to the mean wind velocity. In order to explain this, it is assumed that such developed waves are maintained due to the direct effect of turbulent fluctuations of wind pressure on the surface. The solution enables us to explain this by taking into account the turbulent nature of the motion of air in the near-water layer of the atmosphere above the ocean.

Academy of Sciences of
the UkrSSR Marine
Hydrophysical Institute

Received
November 19, 1969

REFERENCES

1. Phillips, O.M. The Dynamics of the Upper Ocean. Cambridge University Press, 1966.

2. Motzfeld, H. Die turbulente Strömung an welligen Wänden. Z. angew Math. Mech., 17, 193-212, 1937.

3. Benjamin, T.B. Shearing Flow over a Wavy Boundary. J. Fluid Mech., 6, No. 2, 1959.

4. Miles, J.W. On the Generation of Surface Waves by Shear Flows. J. Fluid Mech., 3, No. 2, 1957.

5. Schlichting, G. Boundary Layer Theory. (Russian Translation). IL, 1956

6. Yefimov, V.V. and V.L. Pososhkov. Dynamics of Wave Perturbations in the Near-Water Atmospheric Layer by Swell-Waves. Bull. (Izv.) Acad. Sci. USSR, Atmos. and Ocean. Phys. 6, No. 6, 1970.

7. Monin, A.S. and A.M. Yaglom. Statistical Hydromechanics, Part 1, Nauka, 1965.

8. Yefimov, V.V. and A.A. Sizov. Experimental Study of the Wind Velocity Field Above Waves. Bull. (Izv.) Acad. Sci. USSR, Atmos. and Ocean. Phys. 5, No. 9, 1969.

9. Kinsman, B. Wind Waves. Englewood Cliffs, New Jersey, Prentice-Hall, 1965.

10. Chang-Yi, W. On High-Frequency Oscillatory Viscous Flows. J. Fluid Mech., 32, No. 1, 1968.