Breaking of Wind-Generated Waves

DELUN XU*, PAUL A. HWANG** AND JIN WU

Air–Sea Interaction Laboratory, College of Marine Studies, University of Delaware, Lewes, DE 19958

(Manuscript received 18 February 1986, in final form 21 July 1986)

ABSTRACT

Breaking of wind-generated waves was studied in a laboratory tank. The critical surface slope and global wave steepness for inception of breaking were evaluated. Besides the frequency of occurrence, two other characteristic quantities, height and duration of breaking, were measured. The frequency of breaking was found to increase rapidly with wind velocity, following a power law $U^{2.2}$. The period of breaking remained about 7% of the wave period at all wind velocities. The height of breaking was about 30% of the wave height. Portions of these results compare favorably with other available measurements.

1. Introduction

Breaking of ocean waves is a very important, but scarcely studied, phenomenon. Breaking waves are suggested to play an important role in all aspects of air–sea exchange processes, including momentum, heat, and mass (Blanchard and Woodcock, 1957; Banner and Melville, 1976); breakers also initiate a number of oceanographic phenomena, including whitecaps on the sea surface, bubbles in the near-surface ocean, and spray in the atmospheric surface layer (Monahan, 1971; Wu, 1981).

As a start toward quantifying wave breaking, Longuet-Higgins and Smith (1983) used a surface jump meter in the field to determine the frequency of occurrences of breaking waves. Their method was extended to include measurements of duration and intensity of breaking. Two criteria for breaking of wind-waves, local surface slope and global wave steepness, were evaluated; quantitative results on occurrences, durations, and intensities of the breaking under various wind velocities were provided. These results were compared with laboratory simulations reported by Duncan (1981) and Ochi and Tsai (1983) and oceanic results by Longuet-Higgins and Smith (1983), Snyder et al. (1983), and Weissman et al. (1984).

2. Measurements of breaking waves

a. Principle

Breaking criterion. If $\eta(t)$ is a time series of sea-surface displacements measured at a fixed point, two parameters—the temporal rate of variation $R(t)$ and the slope of water surface $s(t)$—can be expressed (Longuet-Higgins and Smith, 1983) as

$$R(t) = d\eta(t)/dt, \quad s(t) = R(t)/c \qquad (1)$$

where c is the phase velocity of waves. A drastic increase of $R(t)$, the so-called jump (Longuet-Higgins and Smith, 1983), occurs when the wave breaks. Note that absolute magnitudes of R and s are dealt with in this article.

Numerical calculations of Longuet-Higgins and Fox (1977) indicated that the maximum slope for a regular, progressive gravity wave is 0.586, or

$$s_{max} = 0.586, \quad R_{max} = 0.586c. \qquad (2)$$

Inasmuch as this is a local parameter, it should also be applicable to wind-generated waves.

In summary, for our measurements, a wave is first identified from the record by its zero up-crossings; subsequently, the phase velocity of this wave and the value of $R(t)$ along its profile are calculated. The wave is classified as breaking if $R(t) > R_{max}$ anywhere between the zero up-crossings. In some cases, the value of $R(t)$ may exceed R_{max} at more than one place along the profile of the same wave, but still only one breaking wave is counted. The breaking wave is subsequently characterized by the following three parameters.

Frequency of occurrence, breaking height, and breaking duration. The frequency of occurrence of breaking, P, is defined as the ratio between the number of breaking waves, n, and the total number of waves examined, N,

$$P = n/N. \qquad (3)$$

The jump height, J_h, is defined, as in Longuet-Higgins and Smith (1983), as the difference in surface elevations where the surface slope rises above and falls below the critical value s_{max},

$$J_h = \int_{t_r}^{t_f} \frac{d\eta(t)}{dt} dt = \eta(t_f) - \eta(t_r) \qquad (4)$$

* Present address: Shandong College of Oceanography, Qingdao, Shandong, China.

** Present address: Ocean Research & Engineering, La Canada, CA 91011.

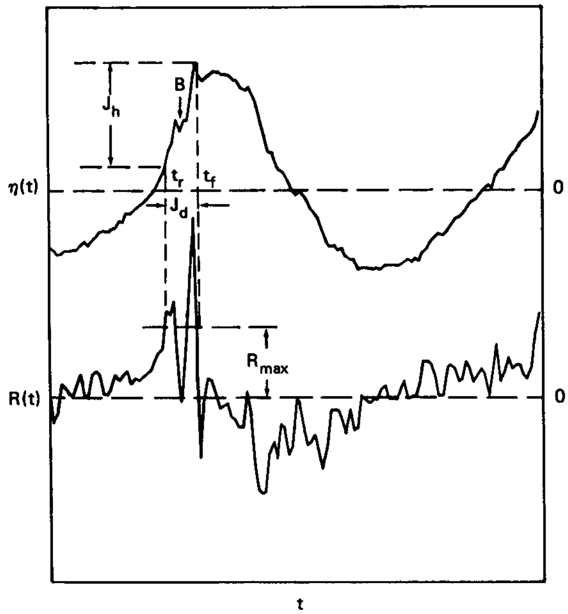

FIG. 1. Definition sketch.

where t_r and t_f are, respectively, the instances when $R(t)$ rises above and drops below R_{max}.

The jump duration, J_d, is the period during which the surface slope exceeds the critical value and is defined as

$$J_d = t_f - t_r. \tag{5}$$

Both definitions, J_h and J_d, are illustrated in Fig. 1. Signals such as that marked as B in the figure, believed to be associated with capillary waves, are ignored in the data analysis; more will be discussed in the following section.

b. Experiments

Experiments were conducted in the Wind-Wave-Current Research Facility. The tank is 40 m long, and 1 m wide; the water was 75 cm deep, and the wind tunnel 60 cm high. A capacitance-type probe was used to measure the surface displacement at fetches of 10.0, 15.1 and 20.4 m under wind velocities from 5 to 16 m s^{-1}. A typical wave record at the wind velocity of $U = 14$ m s^{-1} and fetch of $L = 20.4$ m is shown in Fig. 2a, where U is the maximum wind velocity in the tunnel. The temporal derivative of surface displacement, computed by finite differencing, is shown in Fig. 2b; all jumps characterizing the wave breaking are seen downwind from the wave crest. In order to preserve features of the "jump," a sampling rate of 128 Hz was used for digitizing.

The length of the wave record analyzed was 128 seconds for each sequence, containing 170 to 377 waves depending on the wind velocity. Individual waves, as discussed earlier, were first identified from their zero up-crossings. Owing to the randomness of wind-generated waves and the high sampling rate in digitizing, some very short waves were included in the data. These waves were mostly capillary waves, to which the criterion shown in Eq. (2) was obviously not applicable. Since the influence of surface tension is negligible for waves with period longer than 0.15 seconds (Kinsman, 1965), the wave with its time interval between the zero up-crossings less than this period was excluded from the computation.

The phase velocity, c, was then computed for each identified wave from the dispersion relation to the second order,

$$c = \frac{gT}{2\pi}[1 + k^2(H/2)^2], \quad k = 4\pi^2/gT^2 \tag{6}$$

where g is the gravitational acceleration, T the wave period, k the wavenumber, and H the wave height defined as the maximum difference of surface elevations between the zero up-crossings. Since we are dealing here with relatively long waves, effects of drift currents on their propagation are negligible. Using the actual phase velocity, the value of the breaking criterion

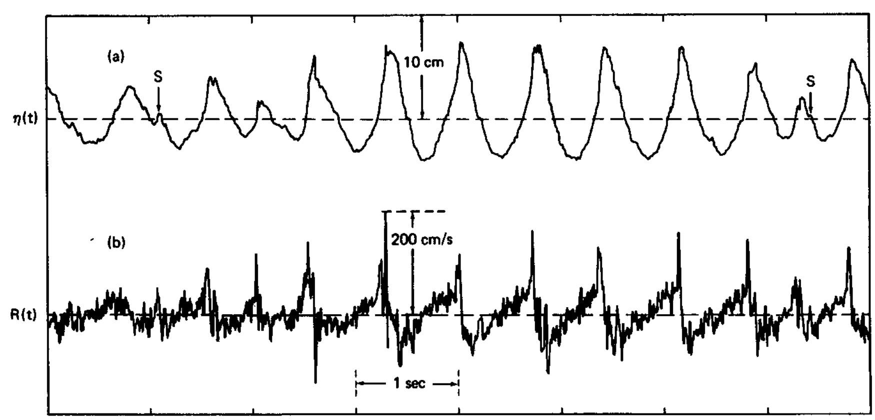

FIG. 2. Sample record and analysis. The wave record $\eta(t)$ at $U = 14$ m s^{-1} and $L = 20.4$ m is shown in (a); the time runs from right to left. The temporal derivative $R(t) = d\eta/dt$ is shown in (b).

R_{max} varies from wave to wave in our analysis. This differs from Longuet-Higgins and Smith's analysis (1983), which used a fixed value calculated from the average phase velocity of all waves in the record.

Tests were first conducted with mechanically generated waves. Three trains of waves with the generator operated at the same stroke length but different frequencies were produced. Waves at two lower frequencies, 1.25 and 1.43 Hz, were smooth and regular and did not break. When the generator frequency was increased to 1.67 Hz, the waves became irregular and distinct wave groups were observed. In this case, breaking waves observed visually near the test section, where the wave probe was located, were counted and were found to be in good agreement with the number determined by applying our breaking criterion to the wave record. With this calibration, we then proceeded to conduct experiments with wind-generated waves.

3. Results

a. Probability of occurrence

The measured frequencies of occurrence of breaking, P, at various fetches were plotted against the wind velocities, U, in Fig. 3. It is seen clearly that at all fetches the increase of P with wind velocity follows a similar power-law relationship and that the frequency of occurrence is greater at a longer fetch. Overall, the data can be represented by

$$P \sim U^{2.2}. \tag{7}$$

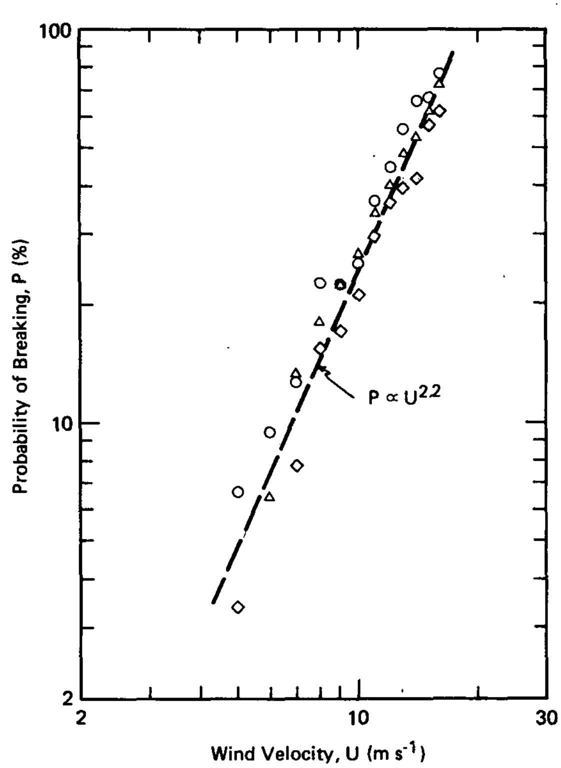

FIG. 3. Frequencies of occurrence of breaking. The data were obtained at fetches of 10.0 (◇), 15.1 (△) and 20.4 (○) m.

The line shown in Fig. 3 was fitted to the data obtained at the intermediate fetch.

b. Breaking height

The mean jump height $\bar{J}_h$ averaged over breaking waves is defined as

$$\bar{J}_h = \frac{1}{n}\sum_{i=1}^{n} J_{hi}. \tag{8}$$

The results obtained under various wind velocities are presented in Fig. 4a. The heights of breaking waves are also averaged, and the average height is designated as $\bar{H}_b$. The nondimensional jump heights, $\bar{J}_h/\bar{H}_b$, are presented in Fig. 4b. It is interesting to see that the nondimensional jump height has a constant value of approximately 0.26 for a wide range of wind velocities and increases slightly with wind velocity at high winds.

c. Breaking duration

The mean jump period, $\bar{J}_d$, averaged over breaking waves is defined as

$$\bar{J}_d = \frac{1}{n}\sum_{i=1}^{n} J_{di}. \tag{9}$$

The mean jump periods were obtained along with the average period of breaking waves, $\bar{T}_b$. The results are also shown in Fig. 4a, b. The nondimensional breaking duration is seen in Fig. 4b to have a constant value of about 5.5%.

4. Discussion

a. Breaking of wind-generated waves

Many studies have been conducted to investigate the breaking criterion of mechanically generated waves; for a review see Ochi and Tsai (1983). The present study, however, concentrates on determining the breaking criterion of wind-generated waves. The wind introduces various effects to waves; the wind stress induces drift currents in the upper layer, and the profile of wind-generated waves is skewed with steeper front and flatter rear faces. Both of these effects should promote the wave breaking.

The surface slope used to identify breaking is a local parameter, which should be valid for all waves regardless of whether they are mechanically produced or wind generated. However, such a parameter was not measured in previous studies; a global type parameter, $\bar{H}_b/g\bar{T}_b^2$, was often used. The present results are shown in this form in Fig. 5, where the data were grouped into a number of bands in $g\bar{T}_b^2$, with the full range being divided into 20 bands. It appears that the dataset can be separated into two distinct groups, one with the ratio of $\bar{H}_b/g\bar{T}_b^2$ much larger, and the other much less, than 0.01. The former group is seen in Fig. 5 to be well

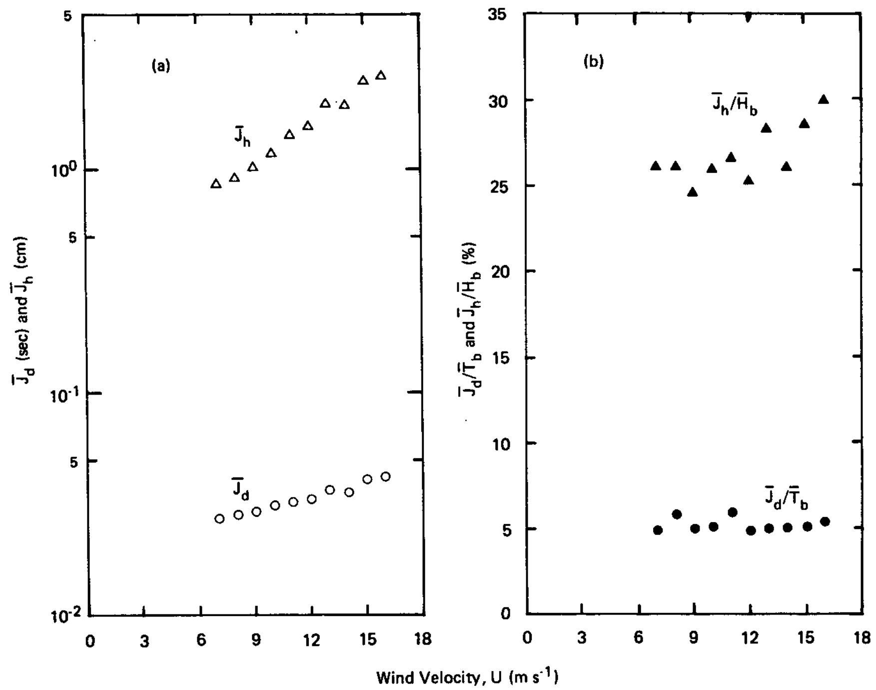

FIG. 4. Periods and heights of breaking waves at various wind velocities.

confined between the ratios of 0.015 and 0.025 with the most probable value of 0.019,

$$\bar{H}_b = 0.019g\bar{T}_b^2. \tag{10}$$

This group is believed to be associated with the breaking of "dominant" waves. The second group, consisting of waves of much smaller "apparent" global steepnesses, is probably associated with either the breaking of small waves riding on long waves or other profile irregularities. Since T_b was obtained in the present method from zero up-crossings of the surface elevation, small waves were not counted as individual waves. In other words, in this case the local steepness of small waves might exceed the breaking criterion, but their long carrier

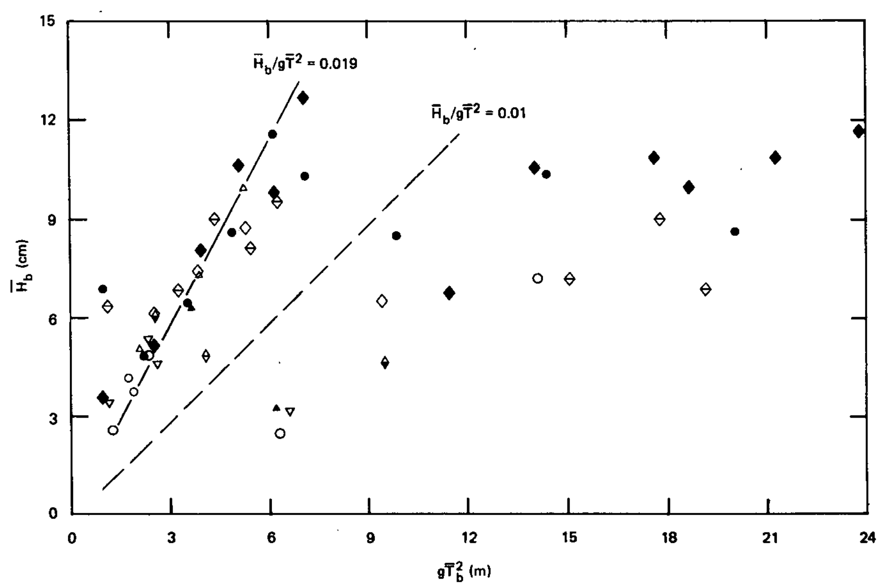

FIG. 5. Global parameterization of incipient wave breaking. The results were obtained at wind velocities of 7 (○), 8 (▽), 9 (▲), 10 (♦), 11 (●), 12 (◇), 13 (△), 14 (⊖), 15 (♦) and 16 (○) m s^{-1}.

wave was still not steep enough to reach the breaking state. The period of the long carrier wave, however, was used in the analysis, resulting in a much smaller global steepness. This shows a definite advantage of using a local parameter to define the rather localized breaking phenomenon; the global steepness becomes meaningless, especially for the small waves.

Accepting $\bar{H}_b = 0.01g\bar{T}_b^2$ as the dividing criterion of the two breaking groups, results of the mean height and duration of breaking for two groups are presented separately in Fig. 6. The dimensional magnitudes of $\bar{J}_h$ and $\bar{J}_d$ from two groups are similar; both increase with the wind velocity, but the group with $\bar{H}_b/g\bar{T}_b^2 \gg 0.01$, as expected, has a larger magnitude. The nondimensional jump duration for the first group now reaches about 7%, while that for the second group goes down to about 3%. The results of $\bar{J}_h/\bar{H}_b$ are rather scattered with the value of the first group falling in the range of 25%–34% and the second in the range of 19%–32%. The still large ratio of $\bar{J}_h/\bar{H}_b$ for the second group is due to H_b being much smaller in this group (see Fig. 5). For example, a typical jump in the first group at $g\bar{T}_b^2 = 6$ m is about 3.4 cm, but is less than 1 cm for the second group. In any event, the exclusion of the second group seems to render the following ratios of $\bar{J}_h/\bar{H}_b = 0.3$ and $\bar{J}_d/\bar{T}_b = 0.07$.

b. Comparison with other results

Rather limited measurements on wind-wave breaking have been reported; those available are compared with our results in the following.

Frequency of occurrences. In the open sea under a wind velocity of about 6 m s^{-1}, Longuet-Higgins and Smith (1983) observed roughly two breaking waves in a 10-minute period, estimated roughly to be $P = 1.3\%$. On the other hand, Weissman et al. (1984) found $P = 8.6\%$ under a similar wind velocity but at a fetch of 8 km. The latter results were obtained from the filtered wave record band passed between 18 and 32 Hz. The present laboratory results, about 6.5% under a wind velocity of 6 m s^{-1} at a fetch of 20.4 m, are in order-of-magnitude agreement with these measurements.

The method of Weissman et al. is too sensitive to small-scale structures. The method is also unable to distinguish whether there is more than one breaking occurrence in the same wave period defined by zero up-crossings; consequently, a breaking wave may be counted more than once. These deficiencies tend to give a higher percentage of breaking. On the other hand, the results obtained by Longuet-Higgins and Smith were for the open sea, where the waves are generally much less choppy than those in laboratory tanks. Field

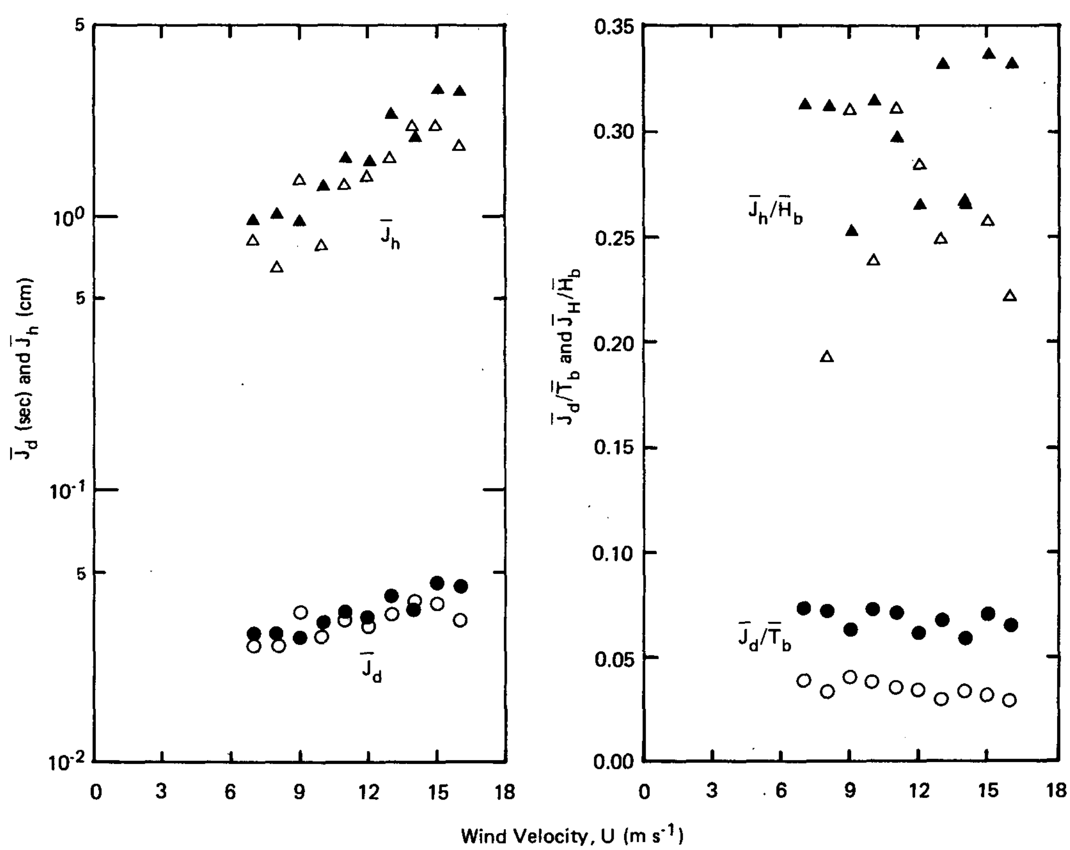

FIG. 6. Periods and heights of separately grouped breaking waves. The solid symbols are for the case with $\bar{H}_b/g\bar{T}_b^2 \gg 0.01$ and the open symbols for the case with $\bar{H}_b/g\bar{T}_b^2 \ll 0.01$.

measurements of breaking waves were also made by Thorpe and Humphries (1980); waves having more than one breaking event were counted only once by them. The percentage of breaking was found in the range of 2.5%–6.5% under a similar wind velocity. Our results are just the upper limit of theirs.

Breaking period. A hydrofoil was towed by Duncan (1981) at a shallow depth below the water surface in a tank to simulate breaking waves. The breaker had a small zone of turbulent water riding on its forward slope; this zone was considered as the breaking region. Measurements were made of the area of this region (A) and the wave length of the breaker (λ_b). The characteristic length of this region can therefore be found from $(A)^{1/2}$ and be compared with λ_b; see Fig. 7. The ratio $A^{1/2}/\lambda_b$ is seen to remain roughly constant, about 9.5%. The constancy of the ratio shown in Fig. 7 is consistent with our results shown in Fig. 6b. It is also understandable that the ratio $A^{1/2}/\lambda_b$ is greater than that of $\bar{J}_d/\bar{T}_b$. The former, related to the turbulent zone which persists after the active breaking, should therefore be greater than the latter, associated only with the active breaking.

The experiments of Synder et al. (1983) were conducted with a photographic method. Their field results indicated that the breaking event lasted about a photographing interval (0.125 s). The period of typical waves in their experiments was about 2–3 seconds, which places $\bar{J}_d/\bar{T}_b$ to be about 4%–6%; the latter is in fair agreement with our measurements.

Whitecap coverage. Photographs of whitecaps on the sea surface are the only spatial data available. Whitecap coverage is actually a measurement of spatial proportion of the foam produced by wave breaking, not the intermittency of wave breaking itself. Whitecap coverages were measured on lakes (Monahan and Zietlow, 1969) and ocean (Monahan, 1971). Their results for a wind velocity of about 6 m s^{-1} and freshwater is roughly 0.12%. The temporal intermittency of wave breaking in our tank can be estimated from $P\bar{J}_d/\bar{T}$, where $\bar{T}$ is the average wave period; our result for the same wind velocity at a fetch of 20.4 m and freshwater is about 0.26%, the same order as field results.

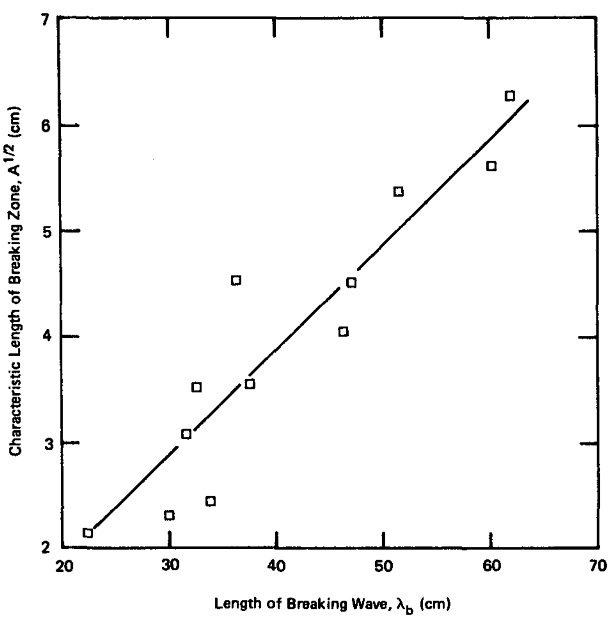

FIG. 7. Variation of characteristic lengths of breaking zone. The data were obtained by Duncan (1981) and the line corresponds to $A^{1/2}/X_b = 0.095$.

Weissman et al. (1984) estimated, from their measurement of a similar wind velocity, the spatial intermittency of wave breaking, corresponding to P, to be 1.2%, an order higher than Monahan's, indicating that the breaking measurements at this stage are only in order-of-magnitude accuracy.

Breaking criteria. Ochi and Tsai (1983) studied the breaking criterion of irregular waves, which were generated by a random wave generator with a preprogrammed spectrum. Heights and periods of 40 waves at the point of breaking were measured. The functional relationship between height and period of irregular waves was found to differ from that of regular, monochromatic waves. Based on their data, Ochi and Tsai suggested $\bar{H}_b = 0.020g\bar{T}_b^2$ as the breaking criterion for irregular waves. The coefficient obtained by Ochi and Tsai for irregular waves falls between those for regular waves (0.027) and wind-generated waves reported here (0.019). In other words, the irregular waves were first shown to be easier to break than regular waves. The wind-generated waves as discussed earlier are not only irregular, but also influenced by the effects of wind. Therefore, the wind-generated waves should be even easier to break, as indicated by the results.

Snyder and Kennedy (1983), Kennedy and Snyder (1983), and Snyder et al. (1983) used the vertical acceleration as the breaking criterion. We feel that the acceleration is a criterion equivalent to the surface slope, since the fourth moment of wave frequency spectrum, from which they derived the vertical acceleration, can be interpreted as the slope in terms of the linear theory. Currently, we are conducting experiments to evaluate these two criteria; our preliminary results indicate that the acceleration, being a second derivative of the measured wave profile, appears to be much more noisy than the surface slope, the first derivative. Consequently, the results depend strongly on the assigned cutoff frequency as experienced by Snyder and his co-workers.

5. Concluding remarks

Although the present technique has already been improved over that reported by Longuet-Higgins and Smith (1983), it still encounters the inherent problem of performing measurements with a single probe: the breaking may not occur at the probe location. Much still needs to be done with a complete mapping of this highly transient phenomenon and to correlate spatial and temporal descriptions of wave breaking.

Acknowledgments. We are very grateful for the sponsorship of our work provided by Fluid Dynamics Program, Office of Naval Research, under Contract N00014-83-K-0316, and Physical Oceanography Program, National Science Foundation, under Grant OCE-8214998.

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