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# Probability of wave breaking and whitecap coverage in a fetch-limited sea

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**Abstract.** The analytical model of *Snyder and Kennedy* [1983] for the whitecap coverage $W$ and that of *Srokosz* [1985] for the probability of wave breaking $B$ are developed into practically applicable forms by closely combining theoretical analysis and data fitting. The database consists of two parts: the data collected from published literature and those recently measured in the Bohai Bay. A time-averaging technique is used to estimate the fourth spectral moment of the mean Joint North Sea Wave Project spectrum. The resulting expressions are simple, in which $W$ and $B$ depend only on the nondimensional fetch for a fetch-limited sea.

## 1. Introduction

Breaking of ocean waves is an important phenomenon in close relation to many fields of ocean study, especially to air-sea interaction, remote sensing, and ocean engineering. Many investigations have been performed to detect breaking waves and estimate their probability [*Thorpe and Humphries*, 1980; *Longuet-Higgins and Smith*, 1983; *Weissman et al*., 1984; *Holthuijsen and Herbers*, 1985; *Xu et al*., 1986]. Whitecapping is a consequence of wave breaking. Many field studies of whitecaps have focused on the experimental determination of whitecap coverage as a function of wind speed [*Monahan*, 1971; *Monahan and O'Muircheartaigh*, 1980; *Toba and Chaen*, 1973; *Wu*, 1979]. Effort has also been made to relate whitecap coverage empirically to both wind speed and fetch [*Ross and Cardone*, 1974]; however, the weak database available did not allow for an integrated description.

*Snyder and Kennedy* [1983] were the first to estimate analytically, in terms of a threshold mechanism, the fraction of sea surface covered by breaking water (referred to as whitecap coverage $W$ in this paper) and found that

$$W = 1 - \Phi\left(\frac{\beta g}{m_4^{1/2}}\right) \tag{1}$$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-y^2/2}\,dy, \tag{2}$$

where $\beta g$ is the critical downward acceleration ($\beta$ is a positive numerical coefficient) and $m_4$ is the variance of the vertical acceleration of the sea surface, i.e., the fourth moment of wave spectrum $S(\omega)$ defined by

$$m_n = \int_0^{\infty} \omega^n S(\omega)\,d\omega. \tag{3}$$

Also, in terms of the threshold mechanism, *Srokosz* [1985] analytically estimated the probability of wave breaking and found that

$$B = \exp\left\{-\frac{\beta^2 g^2}{2m_4}\right\}. \tag{4}$$

Although (1) and (4) are based on linear wave theory and Gaussian statistics and cannot therefore be representative of nonlinear wave effects, they are simple, depending only on the nondimensional variance $m_4 g^{-2}$ and the parameter $\beta$, and therefore relatively easy to be developed into practically applicable forms. Moreover, as noted by *Srokosz* [1985], choosing the parameter $\beta$ to make some allowance for nonlinear effects might be possible.

The fundamental difficulties in applying (1) and (4) are estimating $m_4$ and determining $\beta$ because $m_4$ is theoretically indeterminable by the wave spectra characterized by the Phillips equilibrium range and the values of $\beta$ are quite variable in both theory and experiment. Moreover, $W$ and $B$ defined in (1) and (4) depend not only on $m_4$ and $\beta$ themselves but also on the ratio $\beta/m_4^{1/2}$; this means that if $m_4$ is estimated with some deviation, a deviated $\beta$ must be used to balance it in order to keep the ratio suitable. *Snyder and Kennedy* [1983] employed the spectral cutoff method to estimate the $m_4$ of the mean Joint North Sea Wave Project (JONSWAP) spectrum and then related $W$ to the nondimensional fetch through *Snyder*'s [1973] empirical formula

$$\alpha = 0.57\tilde{X}^{-0.5}, \tag{5}$$

where $\alpha$ is the scale coefficient of the JONSWAP spectrum and

$$\tilde{X} = gX/U_{10}^2, \tag{6}$$

where $X$ is the fetch and $U_{10}$ is the wind speed at 10 m height above the sea level. Owing to, perhaps, the difficulties mentioned above, they did not give a definite expression of $W$ as a function of $\tilde{X}$ but gave a range of such functions for different combinations of $\beta$ and the cutoff frequency.

The goal of the present study is to develop the models (1) and (4) into definite expressions for $W$ and $B$ as functions of $\tilde{X}$ by closely combining theoretical analysis with data fitting. In estimating the $m_4$ of the mean JONSWAP spectrum the time-averaging method [*Glazman*, 1986] will be employed. In relating $m_4$ to $\tilde{X}$ we will follow the path provided by *Snyder and Kennedy* [1983].



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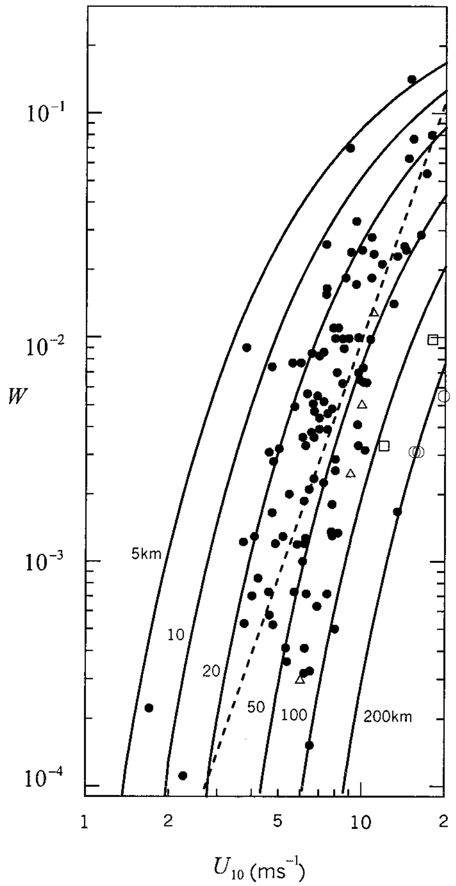


**Figure 1.** Dependence of the whitecap coverage $W$ on the wind speed $U_{10}$. Solid lines are $W$ versus $U_{10}$ for different fetches given by (27). Dashed lines are the power law (7). Solid circles are the data collected by *Monahan and O'Muircheartaigh* [1980]. The open symbols are the Bohai data measured at the fetches of 60 (triangles), 120 (squares), 170 km (circles), respectively.

## 2. Database

The database of the present analysis consists of two parts: the data collected from published literature and those we recently measured in Bohai Bay.

### 2.1. Data Collected From Published Literature

The data of $W$ versus $U_{10}$ collected by *Monahan and O'Muircheartaigh* [1980] are shown in Figure 1. The variance of these data points is so large that the difference of $W$ at the same $U_{10}$ is up to two orders of magnitude. Regardless of the large variance, *Monahan and O'Muircheartaigh* [1980] fitted these data to the power law

$$W = 2.95 \times 10^{-5} U_{10}^{3.52} \tag{7}$$

as shown in Figure 1 (dashed line). The large variance of these data should not be mainly attributed to the errors of measurements. A possible interpretation is that it is mostly due to the fetch (or duration) effect. In fact, the laboratory experiment of *Xu et al.* [1986] has shown that the probability of wind-wave breaking, which is in close relation to $W$, depends sensitively on the fetch. Unfortunately, the data collected by *Monahan and O'Muircheartaigh* [1980] cannot be directly used as the database of the present analysis because they lack fetch information. We will, however, use these data as indirect evidence to support the developed expression of $W$ as a function of $\tilde{X}$ given in (27).

*Thorpe and Humphries* [1980] made spatial and temporal measurements and calculated the number of braking waves as a fraction of total crests. They noted that their measurements were at a fetch of "some 10 km." Taking $X = 20$ km, we calculated $B$ of their spatial measurements as a function of $\tilde{X}$ and plotted them in Figure 2 (solid circles). Note that we did not plot their temporal measurement data in Figure 2 because they did not count individual crests and the fractions of breaking waves of the temporal measurement are significantly smaller than those of the spatial measurement, as they noted. We speculate that this discrepancy arises from the fact that the critical value of time derivative used by them is significantly greater than those commonly used later for the detection of breaking events in the field.

*Longuet-Higgins and Smith* [1983] detected wave breaking with a jump meter at the Nordwijk Observation Tower 10 km from the Dutch coast. Under relatively steady winds with speeds ~7 and 14 m s$^{-1}$ they found the fractions of breaking waves to be 1.3 and 4.0%, respectively. They did not explicate the fetches but gave the wind directions (220° and 300°). From a large-scale geographical chart we estimated the fetches to be roughly 100 and 200 km, respectively. We calculated $B$ as a function of $\tilde{X}$ for the two cases and plotted them in Figure 2 (open diamonds). Their data measured under unsteady winds were not used for the present analysis, as the fetch is indeterminable under unsteady winds.

*Weissmann et al.* [1984] measured wave breaking on Lake Washington during northerly winds when the fetch was ~7 km. They used the variation of high-frequency energy of the wave

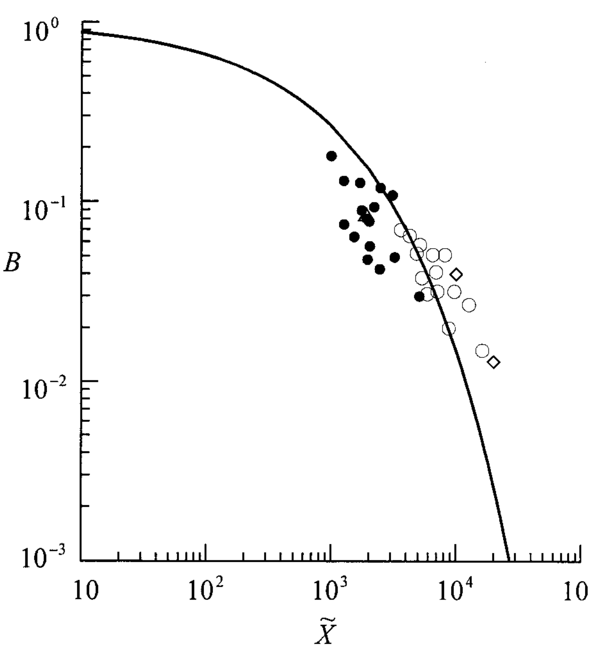


**Figure 2.** Dependence of the probability of wave breaking, $B$, on the nondimensional fetch $\tilde{X}$ given by (28). Solid circles are the data measured by *Thorpe and Humphries* [1980]. Open diamonds are the data measured by *Longuet-Higgins and Smith* [1983]. Open triangles are the data measured by *Weissmann et al.* [1984]. The open circles are the data measured in the Bohai Bay in 1995.

spectrum as a discriminator to detect breaking events and found that $B = 8.6\%$ for a wind speed of ~6 m s$^{-1}$, which was also plotted in Figure 2 (open triangles). Other related data of $W$ and $B$ have appeared in published literature, but they could not be incorporated into the database because of the lack of pertinent information for estimating their corresponding fetches.

### 2.2. Data Measured in the Bohai Bay

Field measurements were conducted from October 18 to November 2, 1995, at an oil platform in the Bohai Bay. The measured data are listed in Table 1 and Table 2 and plotted in Figure 2 and Figure 3. The platform is located at 39°09′N, 119°49′E (Figure 4a) where the water is 27 m deep. During the measurements the frequent wind directions were NNE, N, and NW, and their corresponding fetches were roughly 170, 120, and 60 km, respectively, as can be estimated from the chart shown in Figure 4a. The environmental conditions during the measurements are listed in Table 1.

The whitecap coverage was observed by photography. A camera equipped with a shutter remote controller and a wide angle lens (90°) was used to take pictures of the sea surface for the measurement of whitecap coverage. With its lens vertically downward the camera was fixed at the top of the platform arm (see Figure 4b), where it is ~46 m above the sea level and 46 m away from the platform body. The picture taken in this way covers 80 × 80 m$^2$ of sea surface with uniform scale and high resolution. From such a picture, distinguishing between breaking and unbroken surfaces and hence measuring the fraction of breaking surface is relatively easy. Under each wind condition (the wind speed and direction were relatively steady), 10 pictures were taken at intervals of 30 s. The fraction averaged over the 10 pictures was taken to be a value, and the value averaged over those measured under close winds (with difference ±0.2 m s$^{-1}$ in speed and 10° in direction) was taken to be the value of $W$ at that wind condition. The data obtained thus are listed in Table 2 and plotted in Figures 1 and 3.

The probabilities of wave breaking under different wind speeds and fetches were determined from surface elevation records, which were measured with a capacitance wave meter whose sensing wire is 10 m in length and 5 mm in diameter. The wind speed was measured by a cup-anemometer. The setting of these apparatuses is shown in Figure 4b. Records were taken every 1–2 hours, each lasting 20 min. During the measurements, 145 surface elevation records were obtained in all, from which 78 measured under relatively steady winds were chosen for the detection of breaking waves. In order to check the validity of the detecting method, which will be addressed below, visual counts of breaking events were simultaneously made for several typical records by daylight, showing an encouraging agreement with the detected results.

Using the acceleration criterion for detecting wave breaking from the wave records is difficult because the second derivatives $d^2\zeta(t)/dt^2$ calculated from the records are quite erratic. Instead, we used the surface slope criterion. The surface slope at a fixed point can be expressed as $d\zeta(t)/c(t)dt$ [*Longuet-Higgins and Smith*, 1983], where $c(t)$ is the local phase speed. The critical value of surface slope derived by *Longuet-Higgins and Fox* [1977] for regular progressive gravity waves is tan 30.37° ≈ 0.586. *Longuet-Higgins and Smith* [1983] have noted that for their results of breaking waves detected by a surface meter, all the ratios $d\zeta/c_0dt$ ($c_0$ was taken by *Longuet-Higgins and Smith* [1983] to be the averaged wave speed) lie somewhat above this value. Therefore the criterion

$$\frac{d\zeta(t)}{c(t)dt} \geq 0.586 \tag{8}$$

was employed for detecting wave breaking in the Bohai experiments. *Hwang et al.* [1989] have used this criterion for detect-

**Table 1.** Range of Environmental Factors During the Measurements

| | Values |
|---|---|
| Wind speed, m s$^{-1}$ | 3.0–20.1 |
| Averaged wave height, m | 0.22–3.31 |
| Averaged wave period, s | 2.61–7.89 |
| Air temperature, °C | 9.4–16.2 |
| Temperature of surface layer, °C | 15.5–16.9 |

**Table 2.** Results of Measurements in Bohai Bay

| Wind Direction | $X$, km | $U_{10}$, m s$^{-1}$ | $\tilde{X}$ | $W$, % | $B$, % |
|---|---|---|---|---|---|
| Northwest | 60 | 11.0 | 4860 | 1.3 | 5.2 |
| | 60 | 9.1 | 7100 | 0.25 | 3.2 |
| | 60 | 10.5 | 5333 | (night) | 3.8 |
| | 60 | 10.0 | 5880 | 0.51 | 3.1 |
| | 60 | 6.0 | 16333 | 0.030 | 1.5 |
| | 60 | 8.2 | 8745 | (night) | 2.0 |
| North | 120 | 18.0 | 3630 | 0.98 | 7.0 |
| | 120 | 12.0 | 8167 | 0.33 | 5.1 |
| | 120 | 11.0 | 9719 | (night) | 3.2 |
| | 120 | 9.6 | 12760 | (night) | 2.7 |
| North–Northeast | 170 | 19.8 | 4250 | 0.55 | 6.5 |
| | 170 | 18.0 | 5142 | (night) | 5.8 |
| | 170 | 16.0 | 6508 | 0.31 | 5.1 |
| | 170 | 15.5 | 6934 | 0.31 | 4.1 |

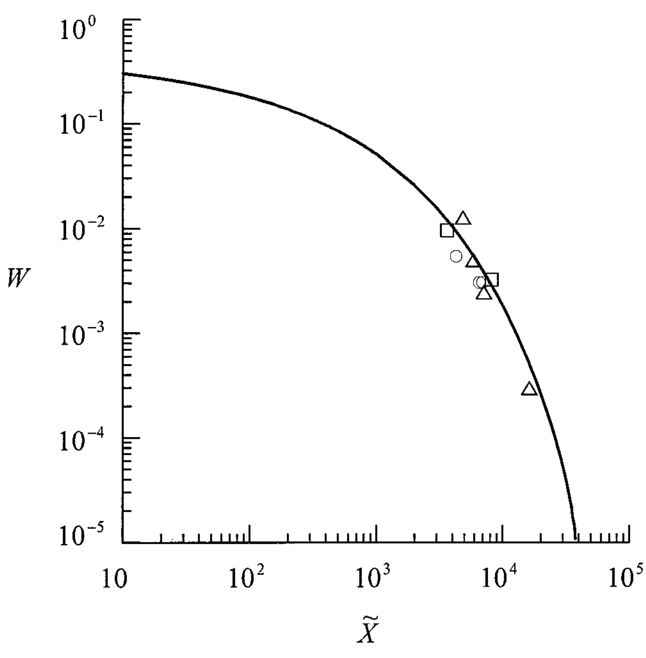


**Figure 3.** Dependence of the whitecap coverage $W$ on the nondimensional fetch $\tilde{X}$ given by (27). The open symbols are the same as in Figure 1.

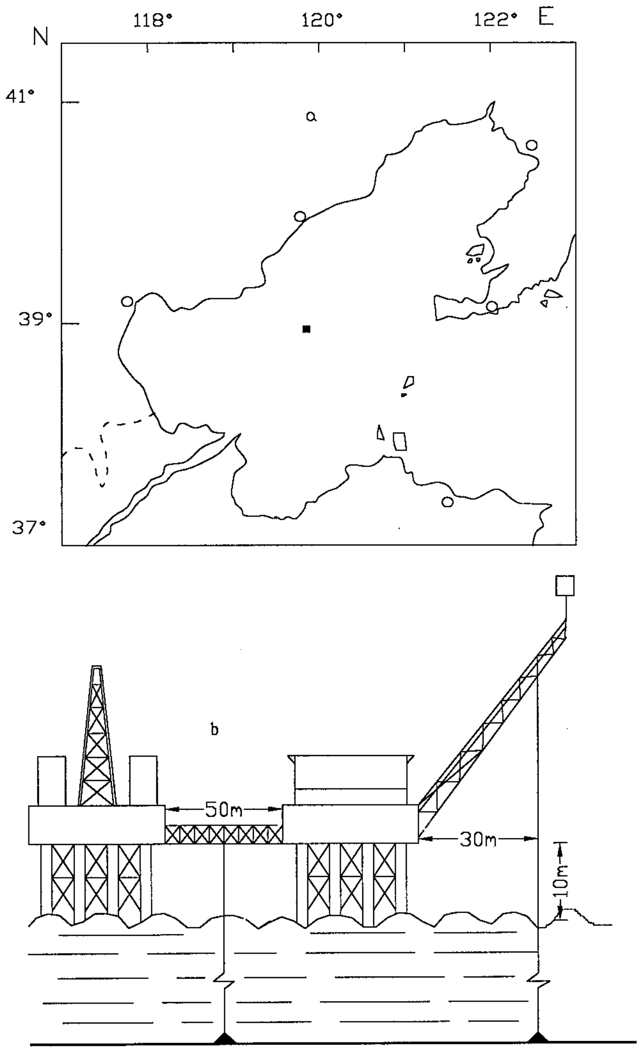


**Figure 4.** (a) The Bohai Bay and measurement site (solid squares). (b) The oil platform and instrument setting.

ing wave breaking from laboratory wind-wave records and obtained a series of reasonable results on the statistical characteristics of wind-wave breaking.

The local phase speed $c(t)$ was computed through the Hilbert transform technique [*Melville*, 1983]: The Hilbert transform of $\zeta(t)$ is defined as

$$\hat{\zeta}(t) = \frac{1}{\pi t} * \zeta(t), \tag{9}$$

where the asterisk denotes the convolution. In narrow-band cases, if

$$\zeta(t) = a(t) \cos \phi(t), \tag{10}$$

then

$$\hat{\zeta}(t) = a(t) \sin \phi(t); \tag{11}$$

thus

$$\phi(t) = \arctan [\hat{\zeta}(t)/\zeta(t)]. \tag{12}$$

The local phase speed $c(t)$ is given, to the first-order approximation, by

$$c(t) = g/\omega(t), \tag{13}$$

where

$$\omega(t) = d\phi(t)/dt. \tag{14}$$

From (9), that

$$F\{h(t)\} = \begin{cases} 2F\{\zeta(t)\} & \omega \geq 0 \\ 0 & \omega < 0 \end{cases} \tag{15}$$

where $F\{\quad\}$ denotes the Fourier transform and

$$h(t) = \zeta(t) + i\hat{\zeta}(t) \tag{16}$$

can be shown. Equation (15) makes computing $\hat{\zeta}(t)$ and hence $\phi(t)$ from $\zeta(t)$ fast and easy.

Note that in order to fit the Hilbert transform technique and be consistent with the time-averaging addressed below, the surface elevation records were running-averaged with the time-averaging scale $0.25\bar{\tau}/(2\pi)$ prior to detecting wave breaking. The data of $B$ measured under wind conditions identical to those of $W$ are given in Table 2 and Figures 2 and 5.

## 3. On $m_4$ and $\beta$

Starting from some frequency, any wave spectrum, either theoretical or experimental, becomes invalid. *Glazman* [1986] has shown that the high-frequency tail of the JONSWAP (or Pierson-Moscowitz (P-M)) spectrum is so ill posed as to make the surface slope discontinuous in the mean square sense, i.e., to have the surface possessed with sharp crests whose local curvature radii tend to zero. The discontinuity gives rise to the divergence of the integral in (3) for $i \geq 4$. In fact, infinitely small radii of curvature at sea surface are unrealistic because they require an infinitely large capillary pressure. In order to discard the inadequate information concerning the spectrum

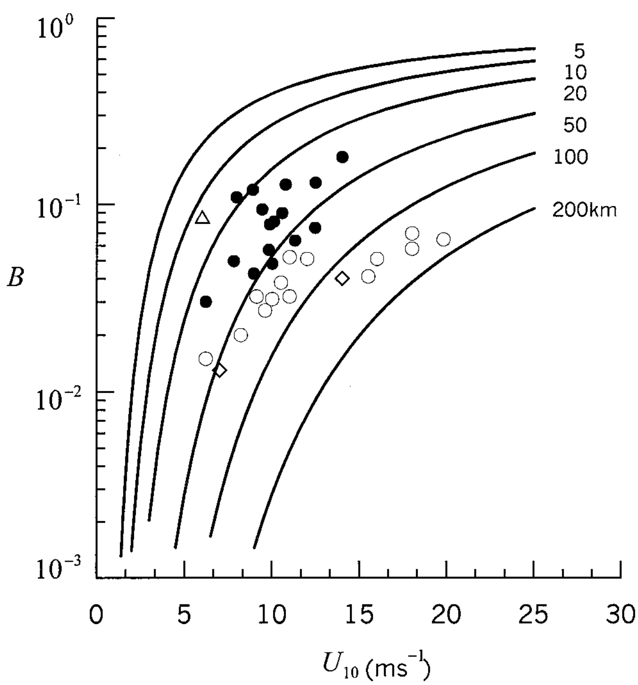


**Figure 5.** The probability of wave breaking $B$ versus the wind speed $U_{10}$ for different fetches given by (28).

and make the spectral moments of $i \geq 4$ determinate, Glazman used time averaging (smoothing):

$$\tilde{\zeta}(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} \zeta(t')\, dt', \qquad (17)$$

where $\zeta(t)$ denotes the surface elevation represented by the JONSWAP (or P-M) spectrum and $T$ is the averaging scale. The spectral version of the averaging is

$$\tilde{S}(\omega) = S(\omega)V^2(\omega T), \qquad i \leq 3, \qquad (18a)$$

$$\tilde{S}(\omega) = S(\omega)V^i(\omega T), \qquad i = 4, \qquad (18b)$$

$$V(\omega T) = \sin(\omega T/2)/(\omega T/2), \qquad (19)$$

where $S(\omega)$ is the spectrum of $\zeta(t)$ (called the raw spectrum) and $\tilde{S}(\omega)$ is that of $\tilde{\zeta}(t)$ (called the smoothed spectrum). With the definitions

$$\tilde{m}_i = \int_0^\infty \omega^i \tilde{S}(\omega)\, d\omega \qquad (20)$$

$$\tilde{M}_i = \int_0^\infty \Omega^i \tilde{A}(\Omega)\, d\Omega \qquad (21)$$

$\tilde{M}_i$ can be shown to be related to $\tilde{M}_i$ by

$$\tilde{m}_i = \alpha g^2 \omega_0^{i-4} \tilde{M}_i, \qquad (22)$$

where $\omega_0$ is the peak frequency, $\Omega = \omega/\omega_0$, and $\tilde{A}(\Omega)$ is the nondimensional counterpart of $\tilde{S}(\omega)$, for the JONSWAP (or P-M) spectrum.

As can be seen from the above descriptions, the time-averaging method is theoretically more reasonable than the spectral cutoff one in estimating $m_4$. We will employ the former to estimate $m_4$ from the mean JONSWAP spectrum. However, the value of $m_4$ estimated by this method still depends on the time-averaging scale $T$, whose selection is difficult for a specific problem.

The value of $\beta$ derived by *Longuet-Higgins* [1963] in water wave theory for the highest wave is 0.5; *Ochi and Tsai* [1983] suggested $\beta = 0.4$, on the basis of experiments for machine-generated irregular waves; and *Kennedy and Snyder* [1983] found $\beta$ ranging from 0.25 to 0.4 for trial 3 in their Monte Carto–type experiments. *Kennedy and Snyder* [1983] also estimated $\beta$ from Barbado Oceanographic and Meteorological Experiment (BOMEX) data and found a range of values, 0.52–0.4, for a range of wind speeds, 5–10 m s$^{-1}$; they noted that these values appear to decrease with increasing wind speed. *Snyder et al*. [1983] found that $\beta = 0.5$ gives a good agreement between their experimental result under low wind speeds (<8 m s$^{-1}$) and the theory of Snyder and Kennedy, but they noted the difficulty of measuring the high-frequency wave components, which can significantly affect the value of $\beta$ estimated from the data. They speculated that the appropriate threshold for the linear part of the downward acceleration might be <0.5 g. In estimating the rate of steep wave occurrence, *Glazman* [1986] selected $\gamma$ (equivalent to $\beta$) = 0.3 as a guess for a "criterion" of wave breaking; he noted that the guessing was based largely on *Longuet-Higgins*'s [1985] analysis of the distribution of the vertical acceleration along the wave profile and also on the fact that the time-averaging procedure employed leads to a reduction in the slope (and vertical acceleration) of the smoothed field. The above mentioned discrepancy of $\beta$ makes determining a suitable value of $\beta$ for the present problem difficult. Even if the value of $\beta$ were consistent in theory and experiment, it might be unfit to the models (1) and (4), which are based on linear theory and Gaussian statistics.

## 4. Results and Comparisons

As can be seen, both the whitecap coverage defined by (1) and the probability of wave breaking by (4) are sensitive to the ratio $\beta/m_4^{1/2}$. Owing to the difficulties mentioned in the proceeding sections, determining the ratio by exactly estimating $m_4$ and definitely selecting $\beta$ is unrealistic. In the present study the ratio is determined according to the following principles: (1) The value selected for $\beta$ should be acceptable both in theory and experiment. (2) The time-averaging scale selected for estimating $m_4$ of the mean JONSWAP spectrum should be theoretically acceptable and should experimentally fit the Bohai observation instrumentation. (3) The resulting value of the ratio should be such as to make both (1) and (4) fit well to the data addressed in section 2. Following these three principles, our ultimate selections are

$$\beta = 0.3 \qquad (23)$$

$$T = 0.25\left(\frac{m_0}{m_2}\right)^{1/2} = 0.25\,\frac{\bar{\tau}}{2\pi} = 0.040\bar{\tau}. \qquad (24)$$

Considering the wind effect and the reduction in vertical acceleration at the crests due to the time-averaging procedure, as noted by *Glazman* [1986], the selection of $\beta = 0.3$ is theoretically acceptable. Moreover, as (1) and (4) are based on linear theory and Gaussian statistics and cannot therefore be representative of the nonlinear wave effects on wave breaking, the selection of a smaller $\beta$ is advantageous to making some allowance for the nonlinear effects. Experimentally, the value of $\beta = 0.3$ is somewhat less than the 0.4 suggested by *Ochi and Tsai* [1983] for machine-generated irregular waves and considerably less than the 0.5 suggested by *Snyder et al*. [1983] for the linear (negative) part of the vertical acceleration field, but it falls within the range 0.25–0.4 employed by *Snyder and Kennedy* [1983] for $\beta$ in their trial 3.

As can be seen from (18b) and (20), no matter how small the averaging scale is, it can ensure the existence of $m_4$ and is therefore theoretically acceptable. The point is whether the observation instrumentation, by which the Bohai data were obtained, could discriminate the small $T$ defined by (24). The Bohai data were measured under wind speeds mostly over 9 m s$^{-1}$; the characteristic scale of $\bar{\tau}$ is ~7 s. With $\bar{\tau} = 7$ s, (24) gives $T = 0.24$ s, which corresponds to a length scale roughly of 9 cm. The Bohai observation instrumentation (including the wave recorder and the photographic system) was nominally possessed of such a resolution. Moreover, the time-averaging scale $T$ defined in (24) depends on $m_0$ and $m_2$ and hence is an intrinsic scale for a given spectrum, and its remarkable feature is that the smoothed spectra preserve self-similarity, as noted by *Glazman* [1986].

With $T$ defined by (24), $\tilde{M}_i$ are estimated from the mean JONSWAP spectrum and listed in Table 3. For comparison, the nondimensional spectral moments, $M_i$ for $i \leq 3$, defined by

$$M_i = \int_0^\infty \Omega^i A(\Omega)\, d\Omega \qquad (25)$$

**Table 3.** Values of Nondimensional Moments of the Mean JONSWAP Spectrum

| Order $i$ | $M_i$ | $\tilde{M}_i$ $[T = 0.25(m_0/m_2)^{1/2}]$ |
|---|---|---|
| 0 | 0.3050 | 0.3034 |
| 1 | 0.3656 | 0.3627 |
| 2 | 0.5046 | 0.4950 |
| 3 | 0.9679 | 0.8699 |
| 4 | | 2.1386 |

are also listed in this Table 3 where $A(\Omega)$ is the nondimensional counterpart of the mean JONSWAP spectrum,

$$A(\Omega) = \Omega^{-5} \exp\{-1.25\Omega^{-4}\} p^{\exp\{-(\Omega-1)^2/2q^2\}},$$

where $p = 3.3$, $q = 0.07$ for $\Omega \leq 1$, and $q = 0.09$ for $\Omega > 1$.

From (22) and $\tilde{M}_4 = 2.14$ (Table 3),

$$\tilde{m}_4 = 2.14\alpha g^2. \tag{26}$$

Substitutions of $\beta$ and $m_4$ in (1) and (4) with 0.3 and $\tilde{m}_4$, respectively, and the use of (5) yield

$$W = 1 - \Phi(0.29\tilde{X}^{0.25}) \tag{27}$$

$$B = \exp\{-0.042\tilde{X}^{0.5}\}. \tag{28}$$

The expressions (27) and (28) are graphically presented in Figures 1, 2, 3, and 5. Figures 2 and 3 show $B$ and $W$ versus $\tilde{X}$, respectively. In Figures 1 and 5, $W$ and $B$ are drawn versus $U_{10}$ for different $X$.

As can be seen in Figure 3, the agreement between the Bohai data points of $W$ and the curve given by (27) is quite good. In Figure 2 we see that overall, the agreement between the data points of $B$ and the curve given by (28) is satisfactory, although a part of the data points deviate somewhat from the curve.

In Figure 1 the set of curves (solid lines) given by (27) reasonably covers almost all the data points. This set appears able to interpret the fetch effect on $W$ and therefore provides, to some extent, evidence to support (27). The Bohai data of $W$ plotted in Figure 1 really support the expression.

In Figure 5 we can also see that the data points of $B$ are consistent with the set of curves. The Bohai data points (open circles) are located around the curve marked by 100 km that is roughly the weighted average of the three fetches listed in Table 2; the data points (open diamond) of *Longuet-Higgins and Smith* [1983] are located near the 50 and 100 km curves (deviating somewhat from the chart-estimated fetches of 100 and 200 km); the data point (open triangle) of *Weissman et al.* [1984] is located a little below the 10 km curve, while the fetch they reported is 7 km; although the data points (solid circles) of *Thorpe and Humphries* [1980] are scattered as they originally were, most of them are located between the 10 and 50 km curves, in agreement with the fetch of some 10 km.

## 5. Discussions and Conclusions

The models (1) and (4) have been developed into definite forms by closely combining theoretical analysis and a database. The resulting expressions, (27) and (28), are simple, with $W$ and $B$ depending only on the nondimensional fetch, and therefore applicable and convenient in practice.

So far as our database, addressed in section 2, is concerned, the choices $\beta = 0.3$ and $T = 0.040\bar{\tau}$ are suitable, as can be seen in the comparisons of (27) and (28) and the data. While developing (1) and (4), several attempts were made to choose appropriate $\beta$ and $T$. During one attempt, $\beta = 0.4$ was chosen. This choice needed to be balanced with a $T$ much less than $0.040\bar{\tau}$ to make both (1) and (4) fit the data; such a small averaging scale is unfit for the Bohai experiments. Another attempt employed *Glazman*'s [1986] intrinsic scale, $T = \bar{\tau}/2\pi$. Use of this scale needs to be balanced with a value of $\beta \ll 0.3$; such a mall $\beta$ is hard to accept in both theory and experiment.

The data used for our analysis show considerable scatter. This is because they were measured with different instrumentation and discriminations and, particularly, the corresponding fetches were estimated from charts. As is well known, only for a steady wind blowing for a sufficiently long duration is the fetch definite. In fact, rarely is such a steady wind encountered in field experiments, especially when the wind speed is low and the experimental site is relatively far from the coast from which the wind originates. Therefore the data points shown in Figures 2 and 3 are expected to be somewhat scattered, with some of them deviating considerably from the curves given by (27) and (28). To refine the two expressions, more comprehensive and pertinent data are needed.

Deductively, the critical acceleration level $\beta$ for wind waves should decrease with increasing wind speed as both wind force and drift current due to wind might be promoting wave breaking. If this inference is true, selecting $\beta \geq 0.3$, $T \leq 0.040\bar{\tau}$ and $\beta \leq 0.3$, $T \geq 0.040\bar{\tau}$ for lower and higher wind speeds, respectively, is reasonable.

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