NONLINEARITY IN IRREGULAR WAVES
FROM LINEAR LAGRANGEIAN SUPERPOSITION

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The linear superposition model is a common and useful tool to deal with irregular wave problems. However, time-series of surface and other wave related quantities created by this method do not contain higher order components. To enhance the capability of the method with respect to nonlinearity, a superposition method based on a LAGRANGEian view of the linear wave theory (superposition of orbital positions rather than EULERian components) is performed. The method is demonstrated on examples of regular waves and irregular wave trains in deep water, but can be applied in shallower water, too.

1. Introduction

To deal with irregular sea waves (e.g create time-series of surface and kinematics for theoretical investigations, hydraulic model tests, and numerical calculations, or simulate zero-crossing wave height and period statistics), the linear superposition model is a most common and useful tool. But time-series generated by this method do not contain higher order components and bound long waves, when superposition is carried out in the usual EULERian way.

One step towards inclusion of higher or lower order components is what e.g. Sand (1982) and Sand and Mansard (1986) have introduced with 2^{nd} order higher harmonics and bound long waves.

Woltering (1996) followed an other way during investigations of wave kinematics in regular waves. It came out, that a LAGRANGEian approach gives information on higher harmonics (usually named as nonlinearities) straightforward from elements of linear wave theory. This approach will be used in the following to include nonlinearities in irregular wave trains.

2. LAGRANGEian Treatment of Regular Waves in Deep Water

The LAGRANGEian approach is demonstrated by the example of the surface of a regular wave in deep water. The following Figure 1 is well known as the principle, to explain how a wave surface propagates, related to the orbital movement of water particles.

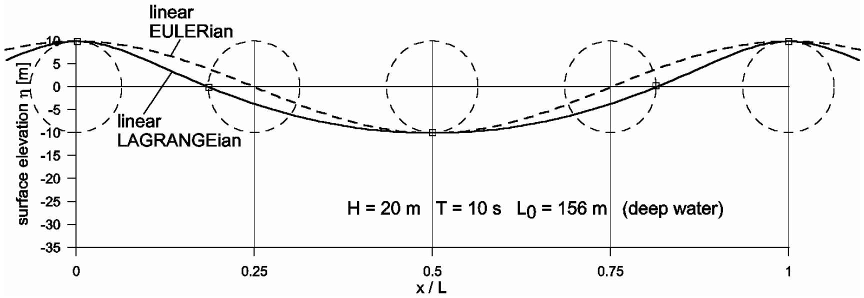

Figure 1: Surface of a regular deepwater wave (H = 20 m, T = 10 s) from LAGRANGEian treatment of linear wave theory

A precise construction of the surface from the (circular) orbital path obviously does not result in the sinusoid of the linear theory (but in a highly nonlinear surface) and corresponds to the results of the first wave theory published by Gerstner (1804), which in the literature often is mentioned as an exact solution (e.g. Wiegel 1964).

Such a surface in space can be calculated with the formulae given by the theory of Gerstner. However, we get the same results by using the equations of the orbital path from the linear wave theory and calculate the surface as the sequence of succeeding surface points from equally spaced orbital paths along the known wave length (this is what the figure shows and what is the principle of the LAGRANGEian approach).

The result, however, is a number of surface points, which are nor equally spaced. The disadvantage of not equally spaced results (neither in time nor in space), can easily be overcome by letting the computer find by iteration the positions of orbital centres, which result in surface points equally spaced. Of course one is not restricted to calculate in space domain, as was used for easy presentation here, it is as straightforward to do this calculations in time domain. Such surfaces or time-series can be analyzed then by Fourier transformation to get the higher order components.

Based on the deep water equations of linear wave theory, a number of wave surfaces where calculated by the LAGRANGEian approach and analyzed by Fourier technique with respect to the higher order components

In Figure 2 the magnitude of the higher harmonics of a wave surface, calculated from the orbital paths according to linear wave theory for a 5 m wave with 10 s period, is compared with results according to Fenton's Fourier Series Wave Theory.

The results are plotted as relative amplitudes, which are obtained by dividing all components, inclusive the first, by H/2 (the half wave height). Besides a linear vertical axis (left diagram in the figure), a logarithmic vertical axis has been used, to highlight the tendency of differences. The crosses are results according to Fenton (ACES 1992), the line results from the LAGRANGEian approach.

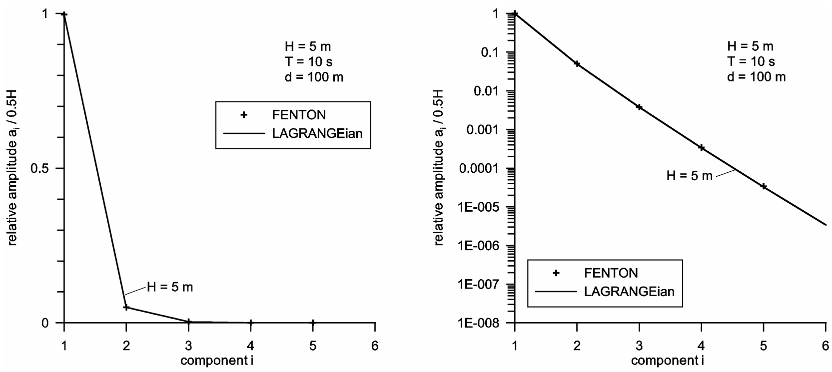

Figure 2: Comparison of (relative) higher order components after FENTON (ACES 1992) with results from LAGRANGEian treatment of linear wave theory (deepwater) (H = 5 m, T = 10 s)

Using the linear wave theory in a LAGRANGEian way gives excellent results (higher harmonics up to any order).

In Figure 3 results from steeper waves, 5 m, 10 m and 15 m wave heights are given.

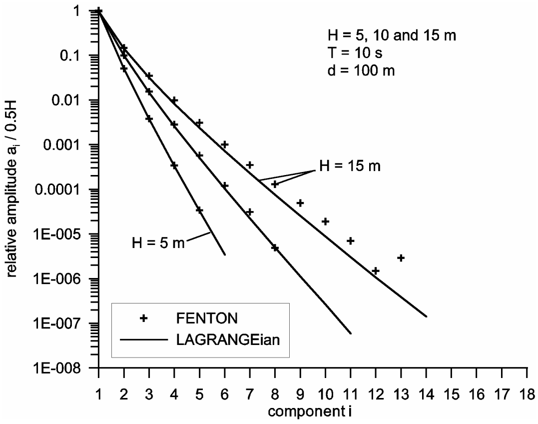

Figure 3: Comparison of (relative) higher order components after Fenton (Aces 1992) with results from LAGRANGEian treatment of linear wave theory (deepwater, H = 5 m, 10 m and 15 m, T = 10 s)

The results are good for steep waves, too. Some scatter in the very high orders might be due to the limited exactness of the input data, which were Fourier analyzed just from the output of the ACES program, which produces two decimals only.

Theses examples are all related to the surface, but can be applied on orbital velocities as well.

In earlier publications (Woltering 1996, Woltering and Daemrich 1994a,b,c,d and 1995a,b) the behaviour of the LAGRANGEian approach is described with respect to mean water level, mass transport, wave length and vorticity. Thus in the following, only a short summery is given.

A nonlinear surface calculated by the LAGRANGEian approach has a mean water level which is Δh lower than the initial one (Δh is the horizontal asymmetry of the wave, the difference between crest height and half of the wave height). According to Figure 4 it is obvious that a volume $\Delta h \cdot L$ is lost.

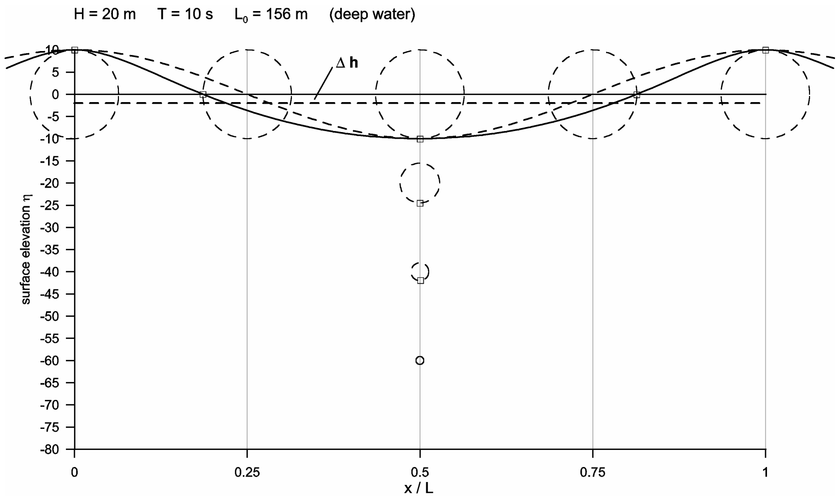

Figure 4: Position of mean water level

It can be shown, that this corresponds to the volume of water which is transported via mass transport during a single wave period. Thus one can argue, that in reality the mean water level will not go down, as the following wave will contribute with the same quantity, and balance the “loss”.

This mass transport, on the other hand, can be derived immediately from the linear theory orbital paths. Assuming for a moment no mass transport, and taking the centres of the orbital paths fixed: in the same way as a “non-linear surface” can be constructed, a “non-linear” velocity time-series can be constructed in a certain fixed location, which can be seen as a velocity probe location (Figure 5).

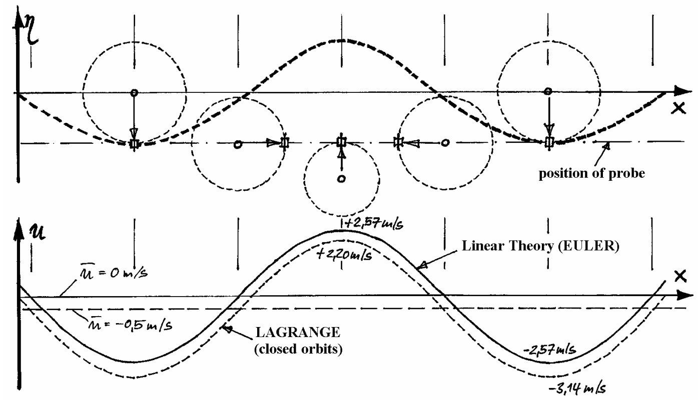

Figure 5: EULERian horizontal velocity from LAGRANGEian orbits

This EULERian time series has obviously a negative mean, like the surface has. This can be estimated roughly already from the velocities under the crest and under the trough. The velocity under the crest is from a smaller orbit and lower, compared to the higher velocity from the bigger orbit during passage of the trough. So, assuming fixed centres of the orbital path or “closed orbits”, would result in a “negative“ EULERian mean velocity. Again it is easy to show, that the profile of this negative mean current is the mass transport profile, with a negative direction (Figure 6).

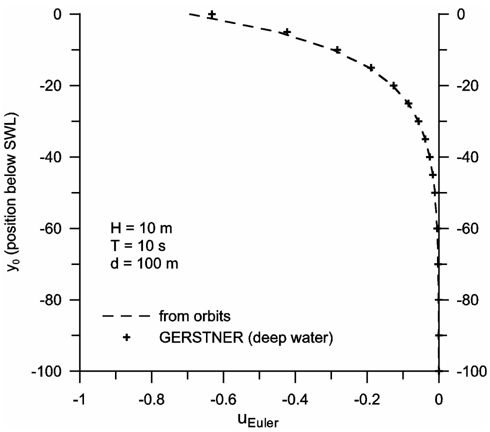

Figure 6: Negative EULERian current from closed orbits and mass transport according to the theory of Gerstner

Thus the information on the mass transport profile can be derived from the orbital path, too. Superposing this to the LAGRANGEian kinematics from the closed orbits, the correct orbital velocity information is available.

Adding the mass transport velocity at the still water level to the linear wave theory propagation velocity, propagation velocity and wave length become non-linear, dependent on the wave height Figure 7.

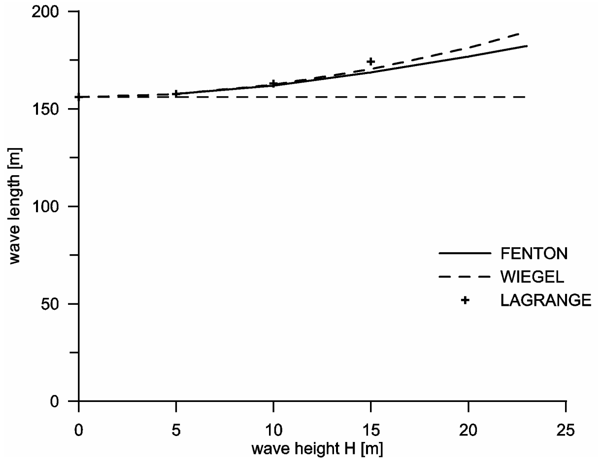

Figure 7: Wave length variation with wave height

It can be shown that the theory of Gerstner is not irrotational, which is a stigma for a respectable wave theory. However, the amount of rotation in the theory of Gerstner is exactly due to the "negative current", when closed orbits are assumed. So, adding the mass transport velocity profile, the theory of Gerstner and the LAGRANGEian approach are irrotational.

3. LAGRANGEian Treatment of Irregular Wave Trains in Deep Water

The superposition of a simple wave group from two sinusoidal wave components, which is the standard situation for irregular waves, is treated first. Again deepwater is assumed for simplicity. Wave heights in both components are H = 5 m, wave periods are 10 s and 8.33 s (0.1 Hz an 0.12 Hz). The results from the LAGRANGEian superposition are shown in Figure 8.

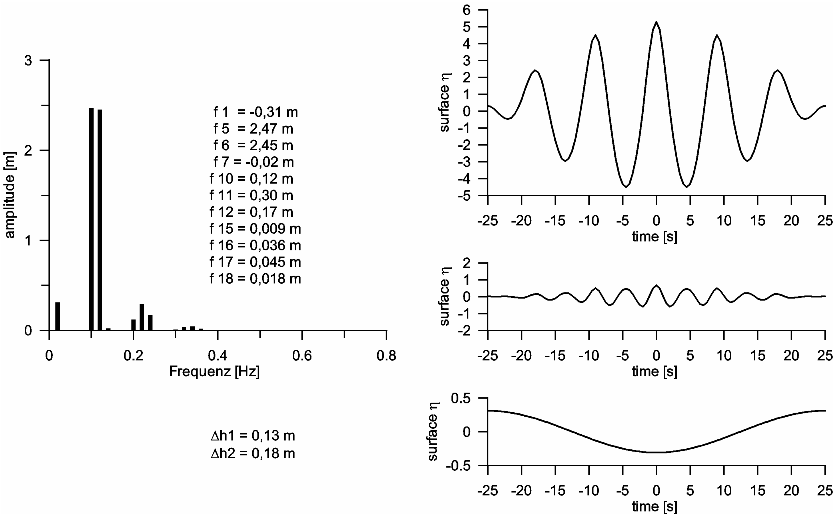

Figure 8: LAGRANGEian superposition of two linear waves

First, on the right of Figure 8 (time-domain), in the upper part the composed wave group with peaked crests and flatted troughs is to be seen, in the middle the isolated sum of the higher harmonics, and in the lower part, the isolated bound long wave.

On the left, the results of the Fourier analysis of the time-series are shown. The two initial main components are to be seen at frequency positions 5 and 6, components at the double frequencies (positions 10 and 12), and even more important, the interaction component 11 between. This is already well known from the 2nd order theory. But there are also components of 3rd order and higher, and components immediately adjacent the second main component. Additionally, there is a bound long wave.

For comparison, 2nd order results according to Sand and Mansard are shown in Figure 9.

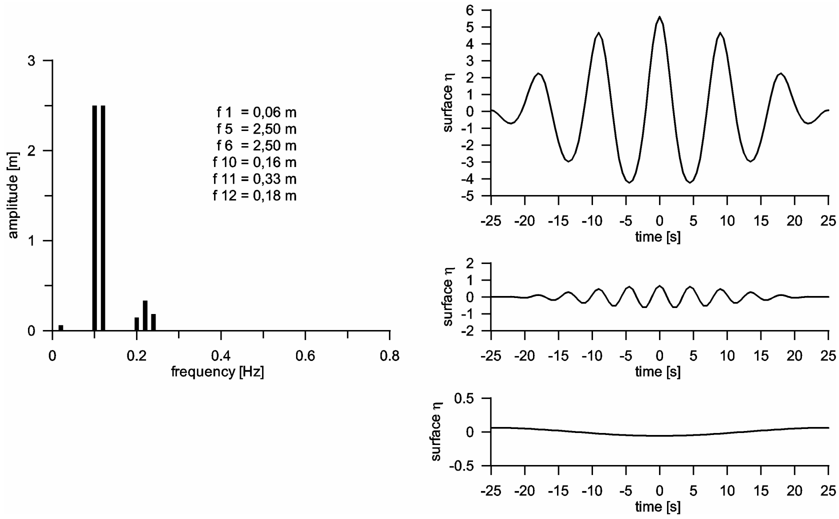

Figure 9: Second order results according to Sand and Mansard, 1986 and Sand, 1982

Results look similar. The magnitudes of the higher harmonics in fact are close, the bound long wave component, which is calculated here according to Ottesen Hansen (1978), is clearly lower.

In this point, we do not have an explanation, yet. We are still investigating whether this “depression” represents the driving quantity of the mass transport only, which should, however, be partly compensated by opposite transport (back flow) due the local variations of drop gradients (in regular waves the “Δh” requires a contribution from the following wave, see Chapter 2). It shall not be speculated here further, as the proof e.g. in a physical model is not really without problems, due to the closed channel situation in hydraulic models.

After this academic example, results from a real spectrum are presented. As an example, a typical linear spectrum for model testing (for simplicity and better figures with a relative short time series of about 20 waves) will be treated.

The significant wave height is H_{m0} = 0,15 m, the peak period is T_p = 1,25 sec. The spectrum has JONSWAP-shape for deep water. With a certain phase setting, this results in the time-series shown in Figure 10, when linear superposition is used.

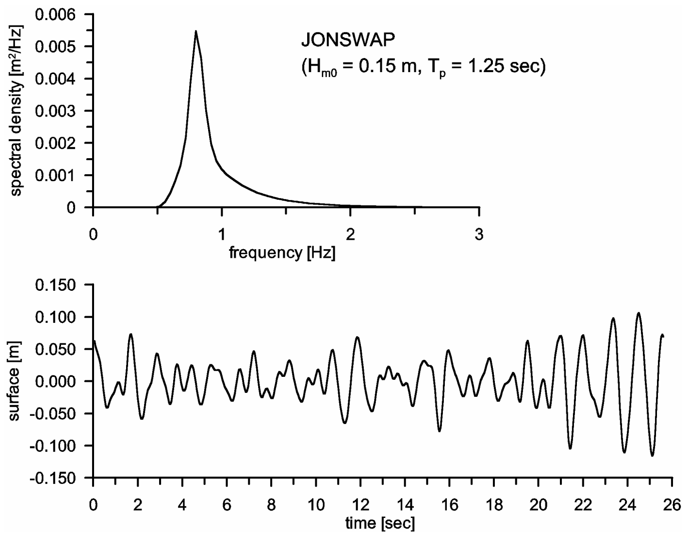

Figure 10: Linear spectrum and related time-series

Applying the LAGRANGEian superposition, the non-linear spectrum shows up as follows (Figure 11):

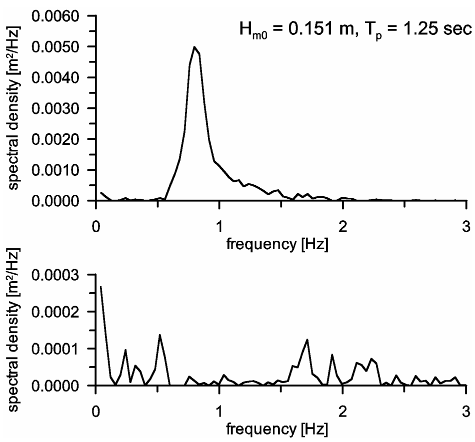

Figure 11: Non-linear spectrum from LAGRANGEian superposition

The bound harmonics are isolated by complex subtraction. In the figure only the spectral densities are plotted, but of course there is also a phase information, which has to be considered.

To get a better impression of the dimensions, amplitudes are plotted rather than energy densities in the following figures.

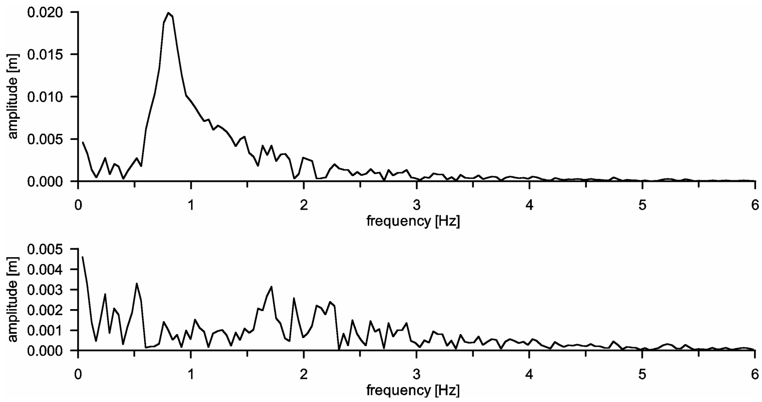

Figure 12: Amplitude spectrum from LAGRANGEian superposition

The time-series related to the LAGRANGEian spectrum in comparison to the linear spectrum is shown in Figure 13:

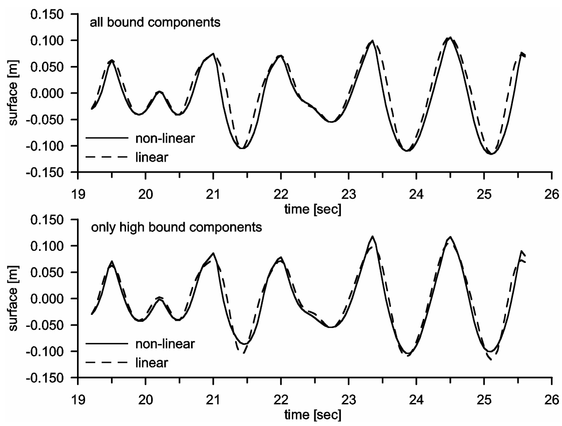

Fig. 13: Time-series from LAGRANGEian and linear superpositiontop: all bound components bottom: bound higher components only

Applying the LAGRANGEian approach results in the higher order components and also gives information about bound long waves. This allows to use the valuable method of linear superposition with the linear part of a spectrum and to add via LAGRANGEian approach the bound harmonics. In the same way it is also possible to calculate the non-linear wave kinematics from the linear components.

However, this requires the knowledge of the linear part of the spectrum. So, if one has to deal with a measured (real) wave train (which contains bound harmonics), e.g. for calculating the pertinent orbital velocities or for calculating the variation of the wave train in space, the bound harmonics have to be eliminated before starting the process.

Fortunately it came out that the whole process is good-natured. If the LAGRANGEian approach is applied to the non-linear spectrum and the bound components are determined from this, this is a good first estimate. Subtracting this first estimate from the non-linear spectrum, one gets a first estimate of the linear part. Now, applying the LAGRANGEian approach to this "first" estimate, results in an improved estimate of the true bound harmonics. This again is subtracted from the non-linear spectrum and the whole process is repeated. It has to be repeated several times. It depends on the width of the linear spectrum, how fast the process converges.

In Figure 14 some steps of the "growing" of the linear spectrum are shown.

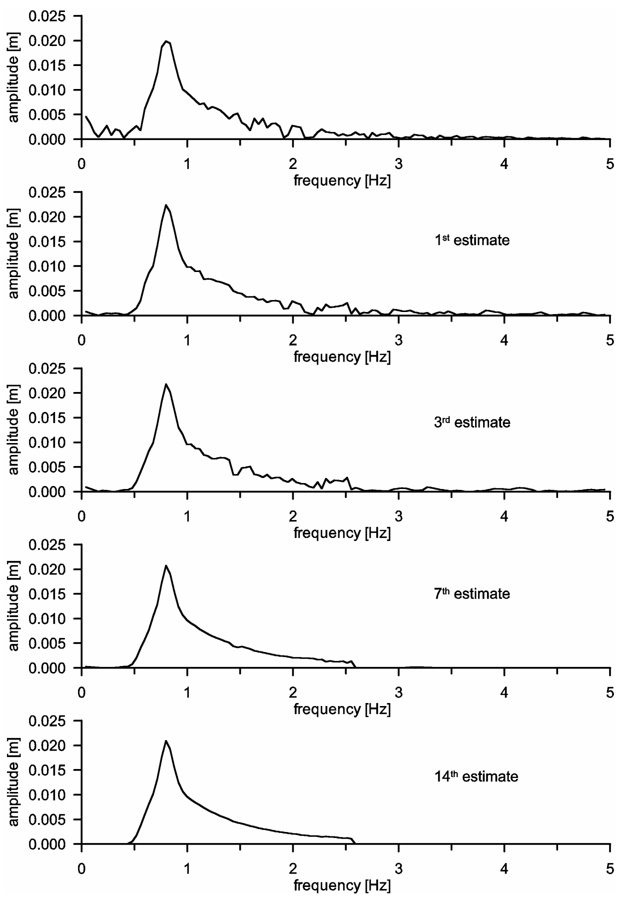

Figure 14: Estimation of linear spectrum from non-linear spectrum

It is important to estimate the significant frequency range of the linear spectrum and to apply the LAGRANGEian approach only to this. In model tests it might be quite easy. In real measured wave trains from nature there might be more problems, besides the up to now not tackled problem of directionality.

4. Summary and concluding remarks

The LAGRANGEian approach allows to calculate the non-linear surface and spectrum from linear components of any spectral shape. In the same way orbital velocities can be treated.

For deep water conditions the good agreement with the best wave theories has been shown for regular waves. From this and from comparison with 2nd order calculations of higher harmonics according to Sand and Mansard (1986) the results are absolutely trustworthy for the higher harmonics.

The results are reasonable in shallower water, too. However up to now with the input from present linear wave theory, the higher harmonics tend to be underestimated. This subject is under progress at present.

By iterative calculations with the LAGRANGEian approach, the linear part of non-linear wave trains or spectra can be extracted. This allows an improved use of the superposition method for simulation in frequency domain.

The method itself gives a good insight in the nature of non-linearity and mass transport.

It has to be solved the problem of the bound long wave components, where the LAGRANGEian approach determines higher amplitudes than resulting from the method of Ottesen Hansen (1978).

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KEYWORDS – ICCE 2004

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Abstract number 491

Wave theory
Nonlinearity
Bound components
Irregular waves
LAGRANGEian superposition