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IMPLEMENTATION OF LOCAL SATURATION-BASED DISSIPATION IN SWAN

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Abstract: This study investigates the implementation in SWAN of a whitecapping source term expression that links whitecapping dissipation to nonlinear hydrodynamics within wave groups. It is investigated whether this alternative whitecapping expression is able to correct the tendency towards underprediction of period measures that has been identified in SWAN. This whitecapping expression was combined with an alternative wind input source term that is more accurate for young waves than the default expression. The shallow water source terms were left unaltered. It is shown that this alternative source term combination yields improved results of idealised fetch- and depth-limited growth curves, and of spectra in a shallow water field case. The improvement is most notable in the prediction of period measures. The investigated deep water source term combination also corrects the erroneous behaviour that the default model displays in the presence of ambient swell, and results in faster model convergence.

INTRODUCTION

The spectral wind wave model SWAN (Booij *et al.* 1999) is a well-established tool for the prediction of wave fields in coastal waters. The model simulates wave spectra by means of the action balance equation, which features a range of source terms that describe physical processes in deep and shallow water. Experience with SWAN in a range of situations, including shelf seas, deep- and shallow water lakes, estuaries and inter-tidal areas, has shown that significant wave height tends to be well predicted, but that period measures are typically somewhat underestimated. This tendency towards underprediction of period measures is related to the following underlying problems: firstly, the energy density at lower frequencies is typically underpredicted, resulting in an overestimation of the peak frequency; secondly, energy levels in the tail are generally overpredicted; thirdly, erroneous results of wind wave growth are obtained in the presence of swell—swell energy experiences enhanced dissipation in the presence of wind sea, whereas the wind sea part of the spectrum experiences reduced dissipation in the model due to the swell, leading to accelerated wind sea growth.

Analysis of the model's performance suggests that these inaccuracies may be corrected in part by altering its deep water source terms, in particular its whitecapping dissipation

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term. The default setup of SWAN features the deep water source terms of wind input and whitecapping of Komen *et al.* (1984), together with the Discrete Interaction Approximation (DIA) for computing quadruplet interaction (Hasselmann *et al.* 1985). The whitecapping term used in Komen *et al.* (1984) is based on the quasi-linear model of Hasselmann (1974). This model represents the negative work done by a whitecap on the forward face of a broken wave, and is based on the assumption that dissipation is local in geographical space and thus distributed in spectral space (Komen *et al.* 1994). The model inaccuracies reviewed above can all be traced back to this whitecapping expression. Primarily, too little energy appears to be dissipated at high frequencies and too much at low frequencies (Rogers *et al.* 2003). Furthermore, the source term's dependency on a spectral mean steepness that is computed with the mean wavenumber has been identified as the root cause of the model's overprediction of wind sea growth in the presence of swell (Hurdle 1998).

A number of alternative whitecapping expressions have been proposed to improve the accuracy of SWAN. These range from alternative calibrations of the Komen *et al.* (1984) expression, e.g. Rogers *et al.* (2003), to alternative ways of calculating mean spectral steepness, e.g. Van Vledder and Hurdle (2002). However, none of these alternatives has comprehensively solved the above-mentioned accuracy problems. In this study, we address the identified inaccuracies by investigating the performance of a whitecapping expression based on that of Alves and Banner (2003) in SWAN. This expression is based on experimental findings that whitecapping dissipation appears to be related to the nonlinear hydrodynamics within wave groups. This yields a dissipation term that primarily depends on quantities that are local in the frequency spectrum, as opposed to ones that are distributed over the spectrum, as in the expression of Komen *et al.* (1984). However, the final whitecapping expression proposed by Alves and Banner (2003) features additional dependencies on the spectral mean wavenumber and steepness, which, as discussed above, is problematic in situations of mixed sea and swell often encountered in the nearshore. Therefore, their whitecapping expression is applied here without these mean spectral dependencies. This adapted whitecapping expression is used together with a wind input term that is based on that of Yan (1987) and respectively an exact method and the approximate DIA method for computing quadruplet interactions. (For the sake of brevity, only the results obtained with the computationally more efficient DIA are presented here.) Avoiding the use of spectrally averaged quantities, the parameter choice of the investigated whitecapping expression is made on the basis of scaling arguments for the deep water source terms, so that this whitecapping term has the same frequency scaling as the wind input term. This yields a whitecapping source term that has a secondary dependency on wave age. The resulting source term combination is calibrated for fetch- and depth-limited cases and subsequently evaluated for a shallow water field case.

The paper is structured as follows: First, the basic model background and the default- and new source term combinations investigated in this study are presented. Thereafter, the calibration of the source term combination featuring saturation-based dissipation is discussed, and the calibrated model is applied to a shallow water field case. Subsequently, two characteristics of the new balance, namely its performance under ambient swell and its convergence speed, are presented. The paper closes with conclusions.

MODELLING APPROACH

In this section, the action balance equation and the current default combination of wind input and whitecapping source terms used in SWAN are presented. This is followed by a description of the new source term combination investigated in this study, which features saturation-based whitecapping dissipation based on Alves and Banner (2003). The section closes with a description of the parameter choice for the new whitecapping expression.

Action Balance Equation

In stationary simulations, as are considered in this study, SWAN computes the evolution of wave action density N (equal to E/σ, where E is the variance density and σ the radian frequency) using the time-independent action balance equation (Booij *et al.* 1999):

$$\frac{\partial}{\partial x}(c_{g,x}N) + \frac{\partial}{\partial y}(c_{g,y}N) + \frac{\partial}{\partial \theta}(c_{\theta}N) + \frac{\partial}{\partial \sigma}(c_{\sigma}N) = \frac{S_{tot}}{\sigma} \, . \tag{1}$$

The first two terms on the left-hand side represent the propagation of wave action in two-dimensional (x, y) geographical space, where $c_{g,x}$ and $c_{g,y}$ are the wave group velocities, including ambient current. The third term describes depth- and current-induced refraction, with c_θ the propagation velocity in directional space θ, and the fourth term represents the effect of shifting of the radian frequency due to variations in mean current, with c_σ the propagation velocity in frequency space. The right-hand side contains the total source term S_{tot} which represents physical processes that generate, dissipate or redistribute wave energy:

$$S_{tot} = S_{in} + S_{dis} + S_{nl4} + S_{bot} + S_{nl3} + S_{brk} \, . \tag{2}$$

We distinguish between source terms that are active primarily in deep water, namely energy transfer from wind to waves (S_{in}), dissipation due to whitecapping (S_{dis}) and nonlinear quadruplet interactions (S_{nl4}), and source terms that are active exclusively in shallow water, namely bottom friction (S_{bot}), nonlinear triad interactions (S_{nl3}) and depth-induced breaking (S_{brk}). The two alternative formulations for S_{in} and S_{dis} considered in this study are presented in detail below. The three shallow water source terms have been applied with their default formulations (see Booij *et al.* 1999).

Default Source Terms

The process of whitecapping dissipation is currently represented in SWAN by the pulse-based, quasi-linear model of Hasselmann (1974). The formulation used in the model is based on the expression proposed by Komen *et al.* (1984), as modified by Janssen (1991):

$$S_{dis}(\sigma,\theta) = -C_{ds}\left[(1-\delta) + \delta\left(\frac{k}{\tilde{k}}\right)\right]\frac{k}{\tilde{k}}\left(\frac{\tilde{s}}{\tilde{s}_{PM}}\right)^{q}\tilde{\sigma}\,E(\sigma,\theta) \tag{3}$$

where k is the wavenumber and $\tilde{\sigma}$ and $\tilde{k}$ the mean spectral wavenumber and radian frequency. Quantity $\tilde{s}$ is the mean spectral steepness, defined as $\tilde{k}\sqrt{E_{tot}}$, with E_{tot} the total variance, and $\tilde{s}_{PM}$ is the mean steepness of the Pierson-Moskowitz spectrum. The tuning parameters of this expression are C_{ds}, q and δ. The default setting in SWAN is $C_{ds} = 2.36 \times 10^{-5}$, $q = 4$ and $\delta = 0$, which reduces Eq. 3 to the expression of Komen *et al.* (1984). Note the dependency of this expression on the mean spectral steepness $\tilde{s}$ and mean wavenumber $\tilde{k}$.

The default expression for exponential wave growth due to wind is the formulation of Komen *et al.* (1984), which is based on the empirical results of Snyder *et al.* (1981):

$$\beta_{Snyder}(\sigma,\theta) = \frac{1}{\sigma E} S_{in}(\sigma,\theta) = \max\left[0\;,\;0.25\frac{\rho_a}{\rho_w}\left(28\frac{u_*}{c}\cos(\theta-\alpha)-1\right)\right] \tag{4}$$

where ρ_a and ρ_w are the densities of air and water respectively, u_* is the friction velocity of the wind, c the wave phase velocity and α the wind direction. It is important to note that Eq. 4 is based on measurements of waves for which $1 < U_5/c < 3$, where U_5 is the wind speed at 5 m height. The proven validity of this expression is therefore restricted to relatively fast, mature waves.

Saturation-Based Model

The whitecapping formulation investigated in this study is an adapted form of the expression of Alves and Banner (2003), which is based on the apparent relationship between wave groups and whitecapping dissipation. For use in SWAN, their expression was adapted for application to mixed sea-swell conditions and in shallow water. This was done by removing the dependencies on mean spectral steepness and wavenumber in the original expression, and by applying source term scaling arguments for its calibration (see below). This led to the following expression for whitecapping dissipation:

$$S_{dis}(\sigma,\theta) = -C_{ds}\left[\frac{B(k)}{B_r}\right]^{p/2}\left[\tanh(kh)\right]^{\frac{2-p_0}{4}} g^{\frac{1}{2}}\, k^{\frac{1}{2}}\, E(\sigma,\theta) \tag{5}$$

in which the density function $B(k)$ is the azimuthal-integrated spectral saturation, which is positively correlated with the probability of wave group-induced breaking. It is calculated from frequency space variables as follows

$$B(k) = \int_0^{2\pi} c_g k^3 E(\sigma,\theta)\, d\theta\;, \tag{6}$$

B_r is a threshold saturation level. When $B(k) > B_r$, waves break, and the exponent p is set equal to a calibration parameter p_0. For $B(k) < B_r$ there is no breaking, but some residual dissipation proved necessary. This is obtained by setting $p = 0$. A smooth transition between these two situations is achieved by (Alves and Banner 2003):

$$p = \frac{p_0}{2} + \frac{p_0}{2}\tanh\left\langle 10\left(\left[\frac{B(k)}{B_r}\right]^{1/2} - 1\right)\right\rangle. \tag{7}$$

The wind input expression used here is based on that by Yan (1987). This expression embodies experimental findings that for strong wind forcing, $u_*/c > 0.1$ say, the wind-induced growth rate of waves depends quadratically on u_*/c (e.g. Plant 1982), whereas for weaker forcing, $u_*/c < 0.1$ say, the growth rate depends linearly on u_*/c (Snyder *et al.* 1981). Yan (1987) proposes an analytical fit through these two ranges of the form:

$$\beta_{fit} = D\left(\frac{u_*}{c}\right)^2 \cos(\theta - \alpha) + E\left(\frac{u_*}{c}\right)\cos(\theta - \alpha) + F\cos(\theta - \alpha) + H \tag{8}$$

where D, E, F and H are coefficients of the fit. Yan imposed two constraints:

$$\beta_{fit} \approx \beta_{Snyder} \qquad \text{for } \frac{U_5}{c} \text{ near } 1 \quad \left(\text{or } \frac{u_*}{c} \approx 0.036\right) \tag{9}$$

and

$$\lim_{\frac{u_*}{c} \to \infty} \beta_{fit} = \beta_{Plant} \tag{10}$$

in which β_{Snyder} and β_{Plant} are the growth rates proposed by Snyder *et al.* (1981) and Plant (1982) respectively. Application of Eqs. 9 and 10 led us to parameter values of $D = 4.0 \times 10^{-2}$, $E = 5.52 \times 10^{-3}$, $F = 5.2 \times 10^{-5}$ and $H = -3.02 \times 10^{-4}$, which are somewhat different from those proposed by Yan (1987). We found that our parameter values produce better fetch-limited simulation results in the Pierson and Moskowitz (1964) fetch range than the original values of Yan (1987) (results not shown).

Finally, the choice of the exponent p_0 in Eqs. 5 and 7 is made by requiring that the source terms of whitecapping (Eq. 5) and wind input (Eq. 8) have equal scaling in frequency, after Resio *et al.* (2004). This leads to a value of $p_0 = 4$ for strong wind forcing ($u_*/c > 0.1$) and $p_0 = 2$ for weaker forcing ($u_*/c < 0.1$). A smooth transition between these two limits, centred around u_*/c = 0.1, is achieved by the expression

$$p_0(\sigma) = 3 + \tanh\left[w\left(\frac{u_*}{c} - 0.1\right)\right] \; . \tag{11}$$

where w is a scaling parameter for which a value of $w = 26$ is used here. In shallow water, under strong wind forcing ($p_0 = 4$), this scaling condition requires the additional dimensionless factor $\tanh(kh)^{-1/2}$ in Eq. 5, where h is the water depth.

CALIBRATION AND VALIDATION

This section presents the calibration of the source term balance described above, that is, the whitecapping expression of Eq. 5, the wind input expression of Eq. 8, with coefficients as adapted, and the DIA method for quadruplet interaction. Figure 1 presents the fetch-limited growth curves as a function of $X^* = gX/u_*^2$ produced by this set of source terms when the parameters of Eq. 5 are calibrated to $C_{ds} = 5.0 \times 10^{-5}$ and $B_r = 1.8 \times 10^{-3}$ and where p_0 varies according to Eq. 11. In the Kahma and Calkoen (1992) fetch range, the fit of dimensionless energy ($E^* = g^2 E_{tot}/u_*^4$) is of similar quality to that of the default

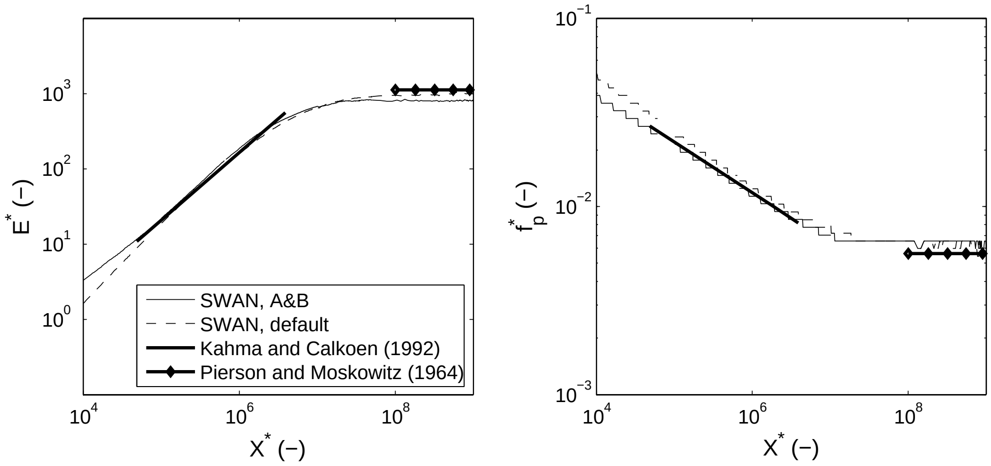

Fig. 1. Deep water, fetch-limited growth curves produced by the source term combination of Eqs. 5 and 8 (A&B), as calibrated, and the default model (Eqs. 3 and 4), both in combination with the DIA. Results for U_{10} = 10 m/s.

model, but in terms of dimensionless peak frequency ($f_p^* = f_p u_*/g$) the overprediction of the default model is corrected by the new source term combination. The source term combination of Eqs. 5 and 8 produces higher dissipation at higher frequencies and lower dissipation at lower frequencies than the default combination of Eqs. 3 and 4. The result is a spectrum with higher peak and mean periods. Quantitatively, the deep water spectrum produced using the saturation-based balance has about a 10% higher T_p and a 16% higher T_{m01} than those of the default model version within the Kahma and Calkoen fetch range. In the Pierson and Moskowitz (1964) equilibrium range, which is typically of less interest to coastal applications, the saturation-based model yields a poorer fit to observations than the default model.

Figure 2 presents the depth-limited growth results of the source term combination of Eqs. 5 and 8 as a function of dimensionless depth $\tilde{d} = gd/U_{10}^2$. The shallow water source terms have been applied with their default formulations and parameter values. These results are compared in terms of dimensionless energy and peak frequency, $\tilde{E} = g^2 E_{tot}/U_{10}^4$ and $\tilde{f}_p = f_p U_{10}/g$, with those of the default model, as well as with observations by Bretschneider (1973), Holthuijsen (1980) and Young and Verhagen (1996). The saturation-based model, using the default settings for shallow water source terms, agrees well with the observations. By contrast, the results of the default model appear to underestimate dimensionless wave energy at small dimensionless depths, while dimensionless peak frequencies are overestimated. Figure 3 presents the one-dimensional spectra of a field case from the Young and Verhagen (1996) data set (wind speed U_{10} = 10.8 m, direction N and mean depth 2 m), to investigate the spectra obtained in shallow water. The saturation-based model, as calibrated, yields more accurate predictions of the spectral peak than the default model at all stations. Also, the predictions of total energy are greater than in the default model at all locations, improving the correlation with the observations at all locations except at Stations 7 and 8 (results not shown). This while the energy levels in the spectral tail are consistently lower than in the default model, again improving the agreement with observations (Figure 3). On the basis of the results presented here, it is concluded that the saturation-based source term balance, as calibrated, reproduces both fetch- and depth-limited growth curves adequately.

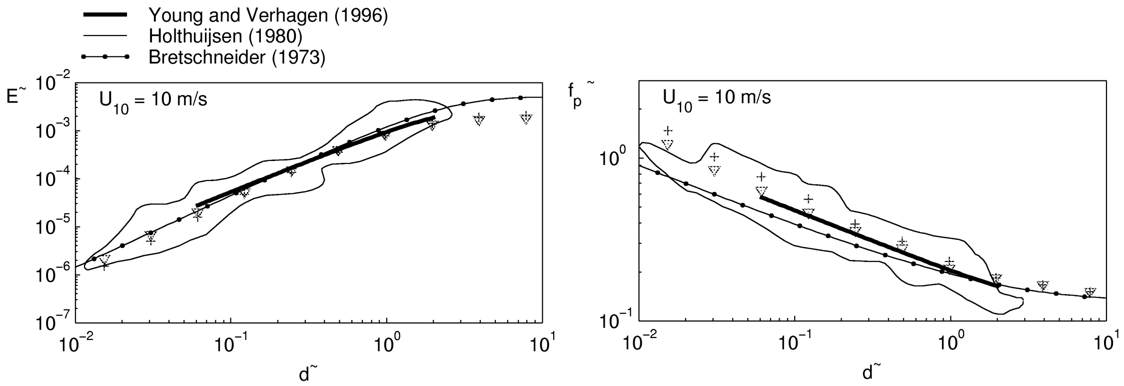

Fig. 2. Depth-limited growth curves produced by the source term combination of Eqs. 5 and 8 (inverted triangles) versus those by the default model (Eqs. 3 and 4, plusses). Observations as indicated. Results for U_{10} = 10 m/s.

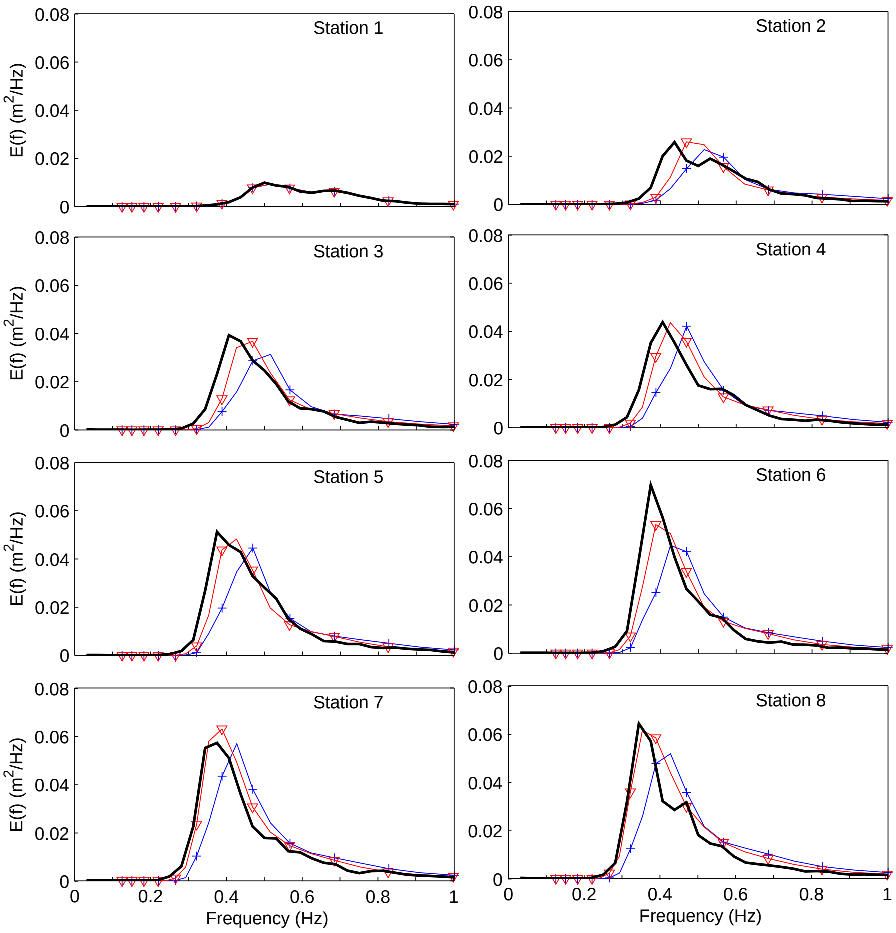

Fig. 3. Lake George observations of near-idealised, depth-limited wave growth. Spectra produced by the source term combination of Eqs. 5 and 8 (inverted triangles) and the default model (plusses). Thick line indicates the observations by the linear array (Stations 1–8) of Young and Verhagen (1996).

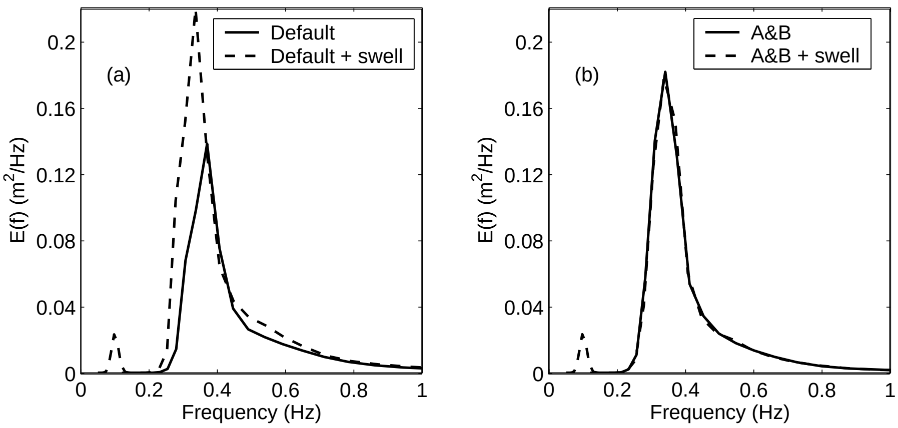

Fig. 4. Influence that background swell has on the wind sea part of the fetch-limited spectra produced by respectively (a) the default source balance and (b) the combination of Eqs. 5 and 8. Results for U_{10} = 10 m/s.

EVALUATION

Two characteristics of the saturation-based model, namely its performance under ambient swell and its convergence speed, are evaluated here. As discussed in the introduction, the default source terms of SWAN produce erroneous results of wind sea growth in the presence of background swell. Hurdle (1998) concluded that this faulty model behaviour is the result of the dependency of the whitecapping term of Eq. 3 on mean spectral steepness. Figure 4 presents the influence of a small amount of swell energy ($H_{m0,swell}$ = 0.1 m, $T_{p,swell}$ = 10 s and steepness $k\sqrt{E_{tot}}$ = 1×10^{-3}) on the growth of the wind sea part of the spectrum in a deep water fetch-limited simulation, at a dimensionless fetch of $X^* = 6 \times 10^5$. Figure 4(a) shows that with the default whitecapping expression, the addition of background swell yields accelerated growth of the wind sea peak (overestimation of energy and period with respect to the case without swell), as well as a further overestimation of energy levels in the tail region. With the whitecapping expression of Eq. 5, however, the background swell results in neither a change in the shape nor magnitude of the spectrum (Figure 4(b)). Therefore, whereas the results of the default model can be significantly affected by the presence of background swell, the local saturation-based model fully decouples the dissipation of swell and wind sea, leaving the latter unaffected by the presence of the swell.

Due to refraction and nonlinear wave energy transfer, the solution of the action balance equation (Eq. 1) needs to be repeated until the solution converges. The convergence speed, the number of iterations required by the stationary model to reach convergence, is an important aspect of model performance. Figure 5 presents the iteration behaviour of the default model and the combination of Eqs. 5 and 8 for a deep water, fetch-limited simulation at a dimensionless fetch of $X^* = 6 \times 10^5$. The convergence of the new source term combination is monotonic and more stable than that of the default model. As a result, the new model requires only about 20 iterations to converge, where the default model requires about 60. Investigation of the simulated wave spectra reveals the reason behind the differing convergence speeds: To speed up convergence, SWAN starts the iteration process with a so-called ‘first guess’ of the final solution. In the default model, both the position of the spectral peak and the energy levels in the tail of the converged spectrum differ from those of the first guess. Whereas the spectral peak quickly reaches its converged position, under weak whitecapping

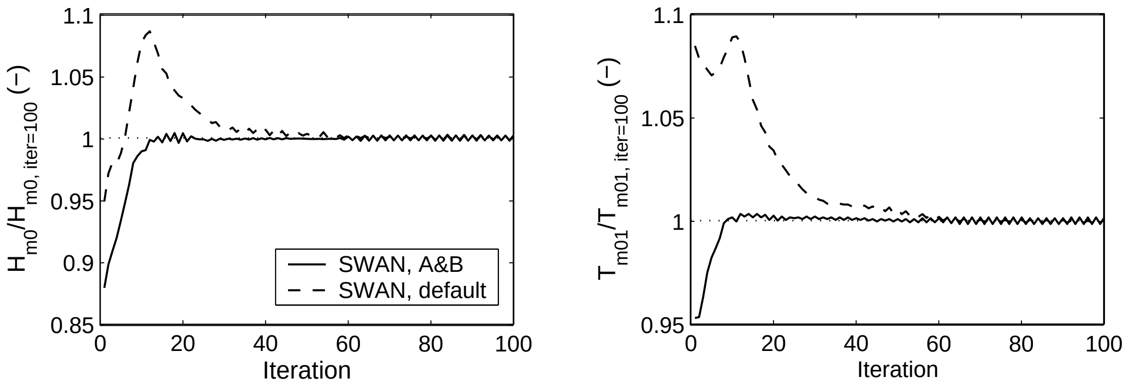

Fig. 5. Comparison of the iteration behaviour of the default model and the combination of Eqs. 5 and 8, for deep water, fetch-limited growth.

dissipation the tail requires many iterations to reach its final (overestimated) value. Therefore, the fact that the default model reproduces observed spectral shapes inaccurately also hampers its convergence speed. By contrast, the converged spectra of the new model agree better with the first guess, not only in terms of the position of the peak, but more importantly, also in the energy levels in the spectral tail. The new source term combination thus also improves on the default model in this respect.

CONCLUSIONS

This study investigated whether the accuracy of SWAN could be improved by application of a whitecapping dissipation expression based on that of Alves and Banner (2003), which relates whitecapping to nonlinear wave group hydrodynamics. The dependency on mean spectral quantities of the original Alves and Banner (2003) expression was removed, and the resulting expression was calibrated on the basis of existing scaling arguments for deep and shallow water. This led to a whitecapping expression that has a primary dependency on frequency-local spectral saturation and a secondary dependency on wave age. From the results presented here, it can be concluded that the investigated source term combination, featuring this new whitecapping expression, yields calibrated fetch- and depth-limited growth curves of closer correspondence to observations than those of the default model. The most notable difference is the general increase in peak and mean period measures, by which a general inaccuracy of the default model is corrected. It was also shown that, due to the alteration in whitecapping dissipation, the investigated source term combination converges faster and more smoothly than the default model. Finally, it was demonstrated that with the investigated whitecapping expression the spurious overprediction of wind sea growth in ambient swell, experienced by the default model, does not occur.

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