Observations of Inertial-Period Motions in the Deep Sea[1]

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Abstract. Inertial-period oscillations have been observed by numerous investigators at deep-sea locations ranging from subtropical to polar latitudes. Although observational techniques have favored surface-layer measurement, there is evidence for the existence of inertial motions at all depths. There is, however, no strong evidence that the amplitude of inertial motions is strongest near the surface. The character of inertial motions has been described more fully by recent observations with moored current meters. Inertial motions have a transient nature, with generation and decay times of a few days. An analysis of the data from a single simple experiment shows that the inertial motions are coherent horizontally over much greater scales than they are coherent vertically. Thus the picture that emerges is one of transient phenomena, of thin vertical extent, and of apparent possible occurrence anywhere in the oceans.

INTRODUCTION

Inertial-period motions are commonly observed by continuous long-period measurements of horizontal ocean velocities at fixed points in the deep sea. Such motions occur in free oceanic flow where no forces are acting on the fluid. In the ideal sense, the water will undergo uniform horizontal circular motion in the cum sole direction. At a latitude ϕ, inertial motion will have a period of rotation of (12 hours/sin ϕ). This period, called the 'inertial period,' is 12 hours at the poles, 24 hours at 30° latitude, and infinite at the equator. (Here, 'hours' are sidereal hours; in the remainder of this paper, 'hours' are mean solar hours.)

Moored current meters are well suited for collecting measurements by which inertial-period processes can be identified. Recently, the increased use of current meters on deep-sea moorings has provided numerous examples of the occurrence of inertial-period motions and has stimulated some speculation on their dynamic role. It is the purpose of this paper to discuss some of the properties of inertial-period motions inferred from recent deep-sea measurements.

Before describing the results of recent observations, a review of published reports of inertial-period processes will be presented. We review the older evidence because the interpretations of this evidence are confused and sometimes misleading, because the newer observations help put the older evidence in better perspective, and because the older evidence in turn complements the

[1] Woods Hole Oceanographic Institution Contribution 2081.

knowledge we have gained from newer time-series observations which do not yet have a global extent.

The description of recent current meter observations, which follows the review of published reports, considers some aspects of the following properties of inertial-period motions: (1) their possible resonance at 30° latitude, (2) their occurrence at great depth, (3) their transient nature, and (4) their scales of spatial coherence.

REVIEW OF PUBLISHED REPORTS OF INERTIAL-PERIOD MOTIONS

Inertial-period motion has been observed over an extensive range of latitudes and depths in the sea. Published reports of such motions have been summarized in Table 1, and the locations of the corresponding measurements are shown on the northern hemisphere chart of Figure 1. In many cases, entries in Table 1 were chosen as being representative of extensive experiments. The data given do not, however, adequately represent the full scope of the work summarized.

TABLE 1. Observations of Inertial-Period Motions

Map References (Figure 1)	Reported By	Publication Date	Date Observations Began	Duration, hours	Latitude	Longitude	Depth, m	Method	Water Depth, m
1	Ekman	1953	July 8, 1930	141	30°13	13°57′W	5	CM	3015
2	Gufstafson and Otterstedt	1932	Aug. 28, 1931	612	56°44′	19°42′E	25	CM	160
3	Gufstafson and Kullenberg	1936	Aug. 17, 1933	162	57°49′	17°49′E	14	CM	100
4	Defant	1940*b*	June 16, 1938	90	44°33′	38°58′W	5–800	CM	3500
2	Kullenberg and Hela	1942	July 23, 1939	279	56°19′	19°58′E	15–30	CM	31–136
5	Sanford	1967	Oct. 20, 1950	140	~38°	68°30′W	See text		4500
6	Reid	1962	May 11, 1953	360	30°06′	124°48′W	Surface	GEK	4400
7	Stommel	1954	Oct. 29, 1953	Various	32°20′	64°20′W	Surface	Drifting CM	~3500
8	Swallow	1957	Oct. 25, 1955	72	35°30′	11°26′W	1200	Float	4865
9	Rattray	1962	June 17, 1954	135	48°35′	127°17′W	Surface	GEK	2200
10	Saelen	1963	July 10, 1957	36	62°29′	1°41′E	10	CM	600
11	Knauss	1962	May 25, 1960	215	28°12′	139°07′W	1350	Float	~4500
12	Webster	1963	July 22, 1961	1046	39°29′	69°38′W	500	CM	2600
13	Webster	1963	July 29, 1961	913	34°04′	65°57′W	3000	CM	5500
14	Day and Webster	1965	Oct. 10, 1962	2616	28°07′	65°02′W	100	CM	5065
24	Verber	1964	Dec. 2, 1962	3340	43°10′	87°27′W	10	CM	75
15	Reed	1966	May 30, 1963	35	50°10′	179°57′W	10	Drogue	6500
6	Hendershott	1964	June 9, 1963	432	29°36′	124°48′W	Surface	GEK	4300
16	Nan-niti et al.	1964*b*	July 11, 1963	85	29°47′	141°41′E	1000	Float	4250
17	Hunkins	1967	Aug. 8, 1963	1080	83°	156°W	0	Ice island	
18	Lacombe and Gonella	1964	July 12, 1964	109	42°47′	7°29′E	20	CM	2680
19	Nan-niti et al.	1964*a*	July 21, 1964	74	31°38′	143°09′E	1000	Float	5500
20	Webster and Fofonoff	1967	July 21, 1964	108	23°51′	67°49′W	692	CM	5790
19	Nan-niti et al.	1964*a*	July 24, 1964	78	31°38′	143°09′E	2000	Float	5500
21	Pochapsky	1966	Nov. 10, 1964	100	27°56′	55°22′W	2340	Float	5000
18	Gonella et al.	1967	Dec. 6, 1964	206	42°47′	7°29′E	258 100	CM	2680
22	Nan-niti et al.	1965	May 19, 1965	64	28°03′	138°3′E	1000	Float	4300
23	Nan-niti et al.	1966	May 30, 1966	96	39°49′	131°44′E	800	Float	3160

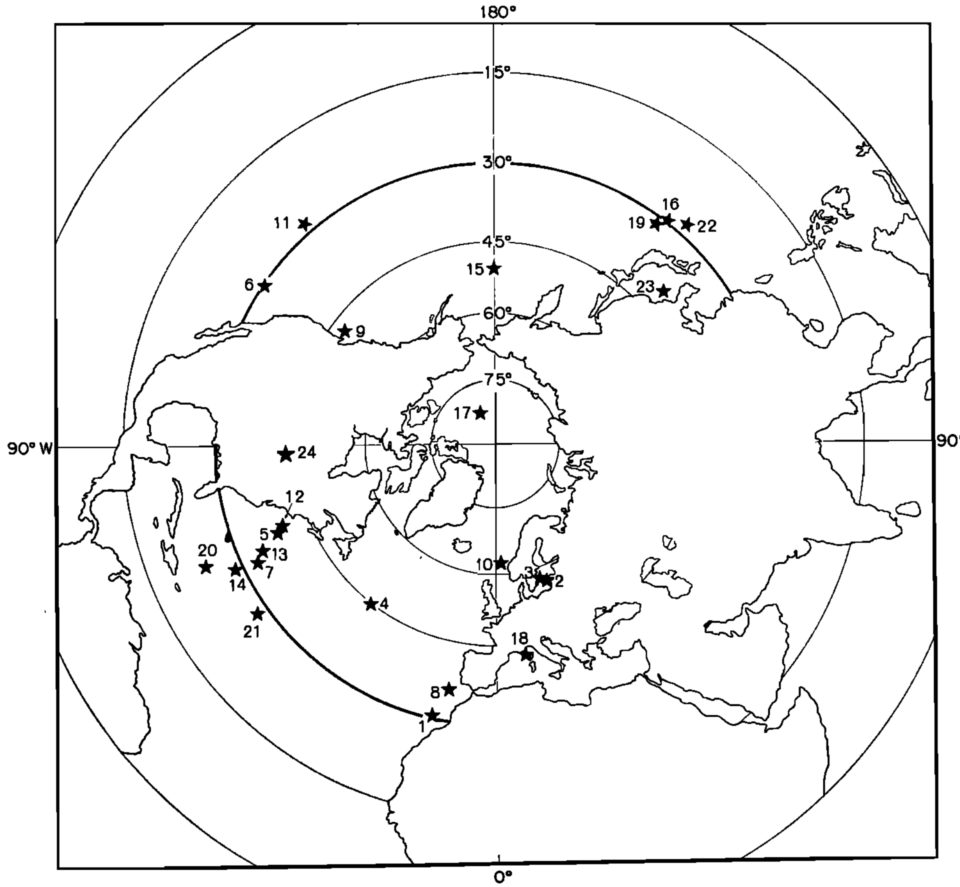

Fig. 1. Location of reported observations of inertial motions. The range of latitudes extends from 23°N to 83°N. No southern hemisphere observations of inertial motions have been reported as such.

The identification of inertial-period motions in time series of current measurements is often difficult. It is necessary to have a record that is long enough to contain several periods of the motion over which the estimates of amplitude and period can be averaged. Because inertial periods are of the order of a day in length, it is difficult to collect series of measurements, using ship-borne instruments, long enough for clear indications of inertial-period motions. The problem is aggravated because inertial-period motions have a transient behavior which can make identification, even in a long time series, difficult. Finally, inertial-period motions and tidal-period motions can be distinguished from each other only with difficulty near the latitudes at which their periods are the same. At the exact latitude where their periods coincide, there may be no way, in principle, to differentiate them

It is difficult in many of the older reports to determine clearly whether or not inertial-period motions have been truly observed. In the review that follows, results of current measurements are generally identified as showing inertial motions if they were so identified by those originally reporting the observations. This does not always give a consistent standard of identification. There are examples, particularly near 30° latitude, where it is difficult, if not impossible in principle, to distinguish between tidal and inertial motions. In other instances, cautious investigators may have been reluctant to interpret ambiguous results as inertial motions; their work may not be included here.

Pre-war observations. Ekman made a series of current measurements in 1930, using his current meter at an anchor station near 30°N, and found diurnal fluctuations in the signal. The first interpretation of these fluctuations [*Ekman and Helland-Hansen*, 1931] was that they were diurnal tidal motions. However, random variations in the amplitude and phase of the measurements were not consistent with the assumption of barotropic tidal motions. After further consideration, *Ekman* [1931, 1941, 1953] concluded that the diurnal variations were in fact inertial motions.

Perhaps Ekman was encouraged in this conclusion by the clear indications of inertial-period motions obtained during a series of experiments in the Baltic. These studies showed that inertial-period oscillations could exist, at least in the shallow Baltic, with relatively large amplitudes.

Gustafson and Otterstedt [1932] first reported finding inertial motions in the Baltic. Later work by *Gustafson and Kullenberg* [1933, 1936] showed that the inertial motions could have a dominant but time-varying amplitude. Further work was performed by *Kullenberg and Hela* [1942] with an array of simultaneous measurements in the eastern Baltic. Their array consisted of a cross of current meter stations, suspended from ships and buoys, having a long axis extending about 200 km along the 56°20′N parallel. Measurements were generally taken at 15- and 30-meter depths at nine stations along the array. At the 15-meter depth, inertial motions were found at all stations except the most easterly, 10 nautical miles off the coast of Latvia. The motions so observed were in phase over the area of the array, with phases within ±31° from their average value. In contrast, the measurements at the 30-meter depth showed no clear evidence of inertial motion or of coherence with the inertial motion at the 15-meter depth. It should be noted that the thermocline at the time of their experiments was at a depth of 23 meters.

The Baltic studies were well executed and inspired other oceanographers to look more carefully for evidence of such motions elsewhere. Because the Baltic is a shallow sea, however, there was doubt about the general applications of the results to deep-sea conditions.

A. Defant [1940*a*, *b*] measured currents for 90 hours at eight levels down to an 800-meter depth and found evidence of inertial-period motions. From the evident reversal of phase with depth, he inferred that the motions were of internal character. With Defant's measurements there was again a question of the general applicability of the results, since the measurements were made over the Altair Cone seamount and the possible effects of bottom topography were not understood.

Post-war observations. Inertial-period motions have been detected in the course of experiments intended to study more general properties of near-surface wind-drift currents and of deep currents. *Stommel* [1954] found inertial-period motions on many occasions during a 3-month-long study using drifting telemetering current meters near Bermuda. *Swallow* [1957] also found evidence of such motions at 1200-meter depth in the eastern Atlantic.

Three studies of inertial-period motions in the Pacific were reported in 1962. Rattray [1962] found such motions at 48°35′ latitude using geomagnetic electrokinetograph (GEK) measurements. *Knauss* [1962] used Swallow floats between 500- and 2500-meter depth at 28° latitude. He was plagued by the uncertainty of determining whether the 24-hour-period motions he observed were inertial motions or diurnal tidal motions. He concluded that the motions arose under a special set of circumstances wherein the diurnal tide was in resonance at the diurnal inertial latitude. Such motions, he reasoned, should be found only at latitudes in tune with the tide-producing forces and should not be expected elsewhere.

Knauss' view was re-enforced by *Reid* [1962], who reported on GEK measurements taken at 30°N. Reid concluded that, although observations of inertial motions with period different from the tidal constituents had been made by others, the observations '. . . do not imply that such motion is typical of the deep open ocean.'

The position held by Knauss and Reid that inertial motions occur primarily only in resonance with tides was an attractive theory and became widely accepted at that time; it has only slowly given way in the face of abundant evidence to the contrary.

Later work near latitude 30°N gave further evidence of the ubiquitous occurrence of inertial motion at all depths. These experiments did little to clarify the misconception that they could occur only at 30°. Among such investigations were those of *Day and Webster* [1965], *Hendershott* [1964], *Pochapsky* [1966], and *Nan-niti et al.* [1964*a, b,* 1965].

However, experiments at latitudes other than 30° have shown that inertial motions probably can occur at any latitude. *Saelen* [1963] reported inertial oscillations near the surface off the coast of Norway at latitude 62°. *Webster* [1963] found inertial-period peaks in the spectrum of ocean current speeds measured with current meters at 34° and 39°30′ latitude. *Reed* [1966] used drogues to find inertial motions at latitude 50° in the Pacific. Work by *Nan-niti et al.* [1966] in the Sea of Japan showed motions that could clearly be identified as inertial at latitude 40°. Current meter measurements in Lake Michigan show that inertial-period motions there are a dominant component of the flow. *Verber* [1964*a, b*] gives examples at three stations that are similar to results reported from a score of stations at other points in the lake.

Observations of inertial motions in the Mediterranean were first reported by *Lacombe and Gonella* [1964]. The work was extended by *Gonella et al.* [1967]. The Mediterranean observations, as well as those in the Sea of Japan, the Baltic, and Lake Michigan, show that inertial oscillations can occur in enclosed seas.

This occurrence, in both shallow- and deep-water sites, may be significant in considering the means of generation of inertial motions. In an enclosed sea, inertial motions are probably due to local generating forces. There is less possibility that the occurrence is due to local amplification at a critical latitude of motions generated at lower latitudes.

More recent investigations using current meters moored on deep-sea buoys have provided long time series in which inertial motions can be clearly resolved. Moored current meter measurements have been made over a wide range of latitudes in the western North Atlantic. The lowest-latitude observations have been made at 24°N [*Webster and Fofonoff*, 1967]. Moored-current-meter measurements are the basis for the studies of properties of inertial motions discussed in the latter part of this paper.

Indirect observations. Two investigations of a special nature furnish additional evidence of the universality of inertial motions. In a recent dissertation, *Sanford* [1967] used induced electromagnetic measurements taken by *von Arx* [1952] to estimate the difference between surface and vertically averaged currents in the Gulf Stream. Sanford compared these values with similar estimates derived from hydrographic stations taken by *Worthington* [1954]. When the difference in the results of the two methods was plotted against time, a variation of inertial period was found. On the basis of his results, Sanford speculated that a large horizontal region of the surface was undergoing coherent inertial motion.

Hunkins [1967] is another study of interest. Current meters suspended below a floating ice island showed inertial-period variations. Hunkins concluded that the ice island was itself undergoing inertial oscillations. He further showed that the amplitude of these oscillations was correlated with the strength of the observed winds.

RECENT STUDIES

Resonance near latitude 30°. As described above, the distribution of observations of inertial-period motions has encouraged the conclusion by some investigators that such motions could characteristically be found in the deep sea only at the 'inertial latitude,' that is, where inertial and tidal periods coincide. Given this hypothesis, *Hendershott* [1964] examined the character of inertial oscillations of diurnal tidal period. He studied a model in which uniform diurnal tidal currents were scattered into internal waves by bottom topography. The diurnal tidal period coincides with the local inertial period near 30° latitude. Hendershott's results predict that diurnal waves should have a maximum amplitude just south of 30° latitude. The amplitude of the waves has a fine structure at latitudes near 30°: to the north, the amplitude of inertial oscillations drops off sharply with latitude; to the south, there is an oscillating amplitude falling off more slowly as a function of latitude.

To examine Hendershott's theory further, an experiment was made in July 1964 using an array of moored current meters in the Sargasso Sea. The positions of the instruments were chosen to fit the expected latitude of resonance as predicted by Hendershott. The measurements collected during this experiment are summarized in Table 2. Summaries of the data have been presented by *Webster and Fofonoff* [1967].

The results of these measurements were surprising. It was expected that large-amplitude diurnal motions would be found in most measurements. Instead, inertial-period motions were evident only from data number 1663. These data are shown in the form of a progressive vector diagram in Figure 2. At

TABLE 2. Summary of Measurements Made to Examine Possible Resonance of Inertial-Period Motions near 30° Latitude

Station No.	Latitude	Longitude	Water Depth, m	Data No.	Depth, m	Duration of Data		
						Days	Hours	Min
165	28°50.0′N	68°49′W	5290	1651	55	7	6	40
				1652	56	2	3	32
				1653	620	6	10	20
				1654	3240	4	5	20
166	29°11.3′N	68°21′W	5200	1661	55	3	19	40
				1662	56	2	2	20
				1663	617	4	13	20
167*	29°39.5′N	67°54′W	5200	1671	55	3	22	40
				1672	56	1	23	28

* Mooring 167 broke and went adrift a few hours after setting. The data from this mooring are thus of little value as indicators of inertial-period motions.

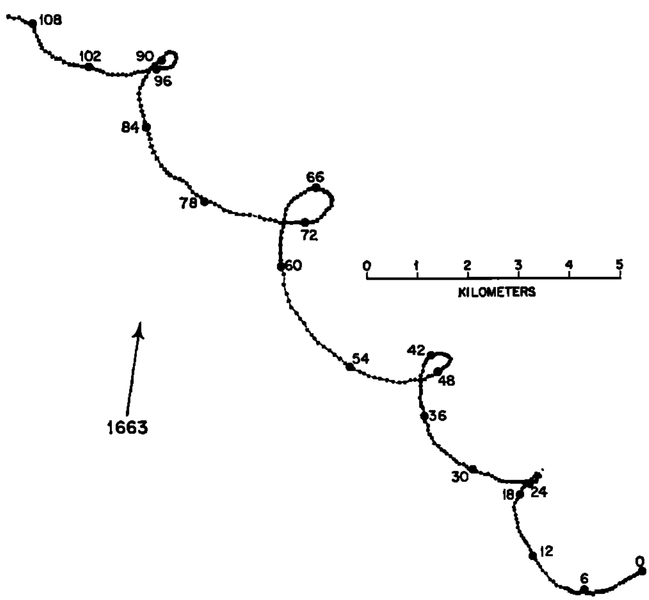

Fig. 2. Progressive vector diagram for data from film 1663. Depth of observations: 617 meter. Location: 29° 11.3′N, 68° 21.0′W. Time origin: 0152 UT, July 29, 1964. The heavy points are numbered in hours from time origin. The dominant motion has the inertial period of about 24 hours.

other stations, and even at the same station at other depths, there were no clear indications of inertial-period motions. Figure 3, for example, shows a progressive vector diagram for data collected at 55-meter depth. In contrast to the measurements at 617 meters, there is no indication here of inertial-period motion.

These results do not appear to agree with Hendershott's predictions. Probably Hendershott's assumption that inertial motions are generated primarily by tidal processes is invalid.

Occurrence at great depths. The mechanism of generation of inertial motions in the deep sea is not understood. A general opinion is that they are generated by wind stress or by variations in wind stress. There is, for example, the suggestion by *Day and Webster* [1965] that their generation is related to the passage of storms. *Saelen* [1963] observed that inertial motions were found following the passage of a storm. *Hunkins* [1967] found that the inertial motions he observed had amplitudes varying with local wind stress.

The amplitude of inertial motions generated by surface winds might be expected to diminish with depth [e.g., *Veronis*, 1956]. Observations of currents from moored current meters do not support such expectations. For example, the measurements summarized in Figures 2 and 3 show well-developed inertial motions at 617 meters but not at 55 meters.

At great depths, inertial-period motions have been observed with amplitudes comparable to those found near the surface. An example of this is a series of measurements collected at 38°28.8′N, 70°00.5′W where inertial motions were observed 30 meters above the bottom in water 3300 meters deep. Figure 4 shows the spectrum of these motions. The dominant peak in the spectrum is at the inertial frequency.

Transient inertial motions. Often where ocean current measurements have been collected over a period long enough to resolve inertial motions, such motions have not been observed. Their absence is not because inertial motions do not generally occur at the point of measurement, but because they are really transient processes. Such a conclusion is consistent with recent observations. Long-period records of currents made with moored current meters characteristically show that the amplitude of inertial-period processes varies with time.

Results giving examples of the variation of inertial-period amplitude with time have been presented by *Day and Webster* [1965], *Webster* [1968], and *Hunkins* [1967]. Another example will be given here.

Figure 5 shows time plots of north and east components of velocity in which inertial-period variations can be observed. They can be seen, for example, from October 8 to 16 and from October 28 to November 9. To make the identification of these motions more quantitative, the data were analyzed by the method of complex demodulation [*Tukey*, 1961; *Webster*, 1968] to give the amplitude of the inertial motion as a function of time. The results, given in Figure 6, show that the amplitude of inertial motions computed with the complex demodulation method is large for the periods in which inertial motions can be clearly seen in Figure 5. In addition, it can be seen that the inertial motions can vary by an order of magnitude within a few days. From such evidence, it seems that

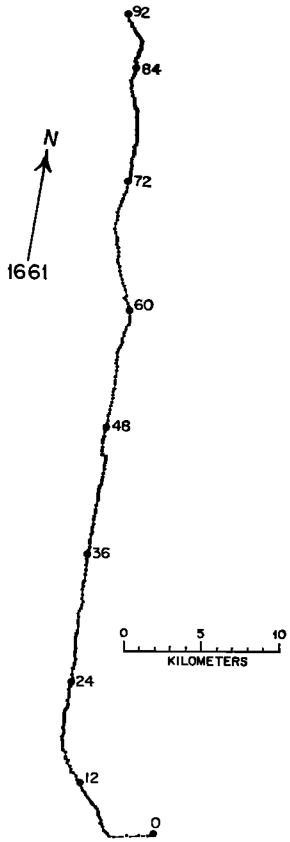

Fig. 3. Progressive vector diagram for data from film 1661. Depth of observations: 55 meters. Time origin: 0111 UT July 29, 1964. Although these measurements were collected at the same time and from the same mooring as those shown in Figure 2, there is here no indication of inertial-period motion.

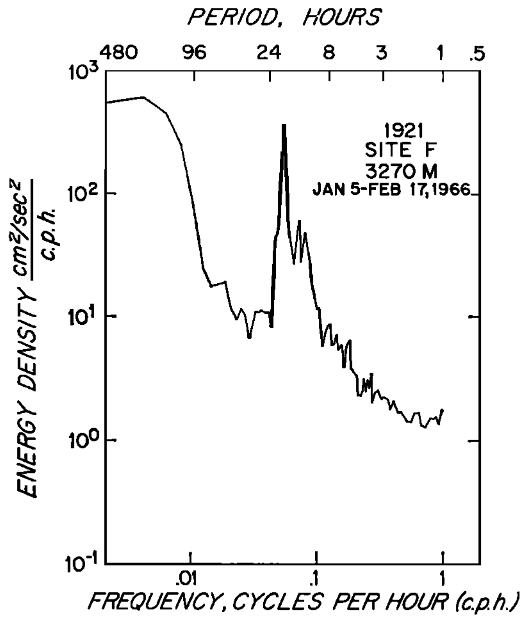

Fig. 4. Kinetic energy-density spectrum for measurements collected at a near-bottom site. The spectrum is plotted on a log-log scale. Site F is located at 38°30′N, 70°W. The local inertial period is 19.3 hours. The dominant peak in the spectrum has a period near 18.5 hours. The reason for the difference between these periods is not understood.

inertial motions may be basically a transient phenomenon. Typically, long-duration direct current measurements show that inertial motions can be generated and reach their maximum amplitude in a few days and in turn can die out within a few days.

It is extremely difficult to measure precisely the generation and decay times of inertial motions. Measurements have shown that inertial motions are quasi-periodic processes imbedded within a continuum of processes of all scales. These other scales might be considered as 'noise' from the point of view of identifying

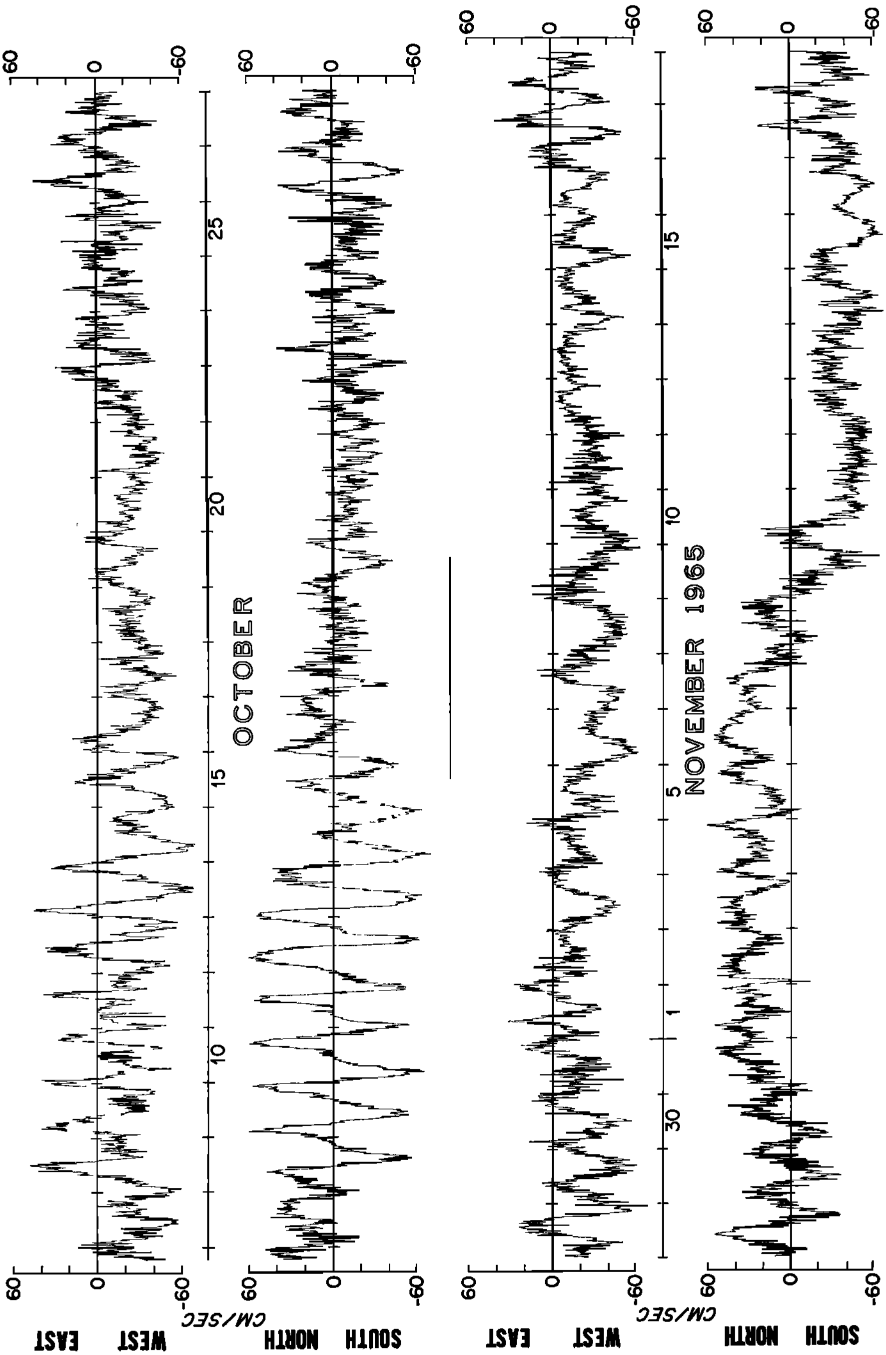

Fig. 5. North and east components of velocity as a function of time for data from 7-meter depth at site D (39°20'N, 70°W). The large-amplitude oscillations have inertial period.

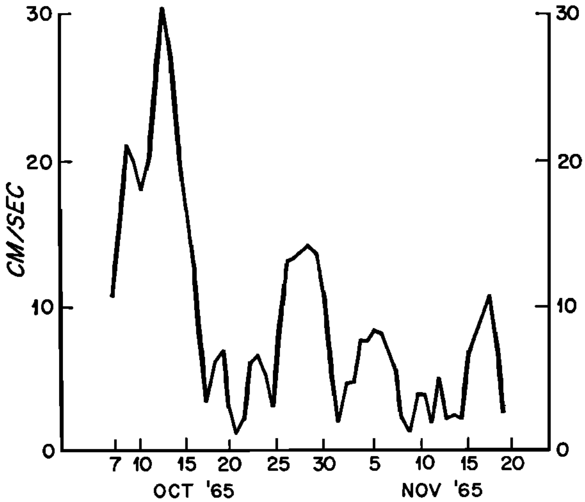

Fig. 6. Variation of inertial-period amplitude of 1882 with time for the data shown in Figure 5.

the inertial motions. To separate the inertial signal from the 'noise' of other processes, a record having a length of several inertial periods is necessary. The more sharply the inertial-motion detector is to be 'tuned'; the longer is the total length of sample required. However, it is often not possible to obtain enough data to provide sharp 'tuning' because the generation and decay times of inertial motions may be only one or two inertial periods in length. Consequently, a relentless uncertainty principle controls experimental studies of inertial motions. The length of data necessary to resolve inertial motions (from, say, the tides) may be longer than the lifetime of the motions. If an effort is made to resolve the generation and decay times, the frequency resolution is poor. As a consequence, the results of complex demodulation given here and in the other papers cited are subject to strong contamination from tidal motions. There is fundamentally no way to prevent this contamination in studies of periodic processes whose amplitudes change significantly over short periods.

Scales of spatial coherence. A simple array of moored current meters at site D (39°20′N, 70°W) has been used to investigate the spatial coherence of inertial-period motions. Two moorings 3 km apart were used. Their arrangement is shown in Figure 7. A surface buoy with wind recorder was used on one mooring; at the other mooring a subsurface float at a depth of about 90 meters was used to reduce the effects of surface wave motion. The array of instruments was maintained on the site for 54 days, from October 6 to November 29, 1965. Not all instruments collected records for the entire experiment, but the shortest record extended from the beginning of the period until November 11, 1965.

Coherences at inertial frequency were computed between pairs of instruments in the array. Before considering the results of these computations, it will be useful to describe the framework used to describe the results. Methods of spectral and cross-spectral analysis and their interpretation are described by *Granger* [1964]. Although the methods of numerical estimation have recently been revolutionized by the fast Fourier transform algorithm (see, for example, *IEEE Transactions on*

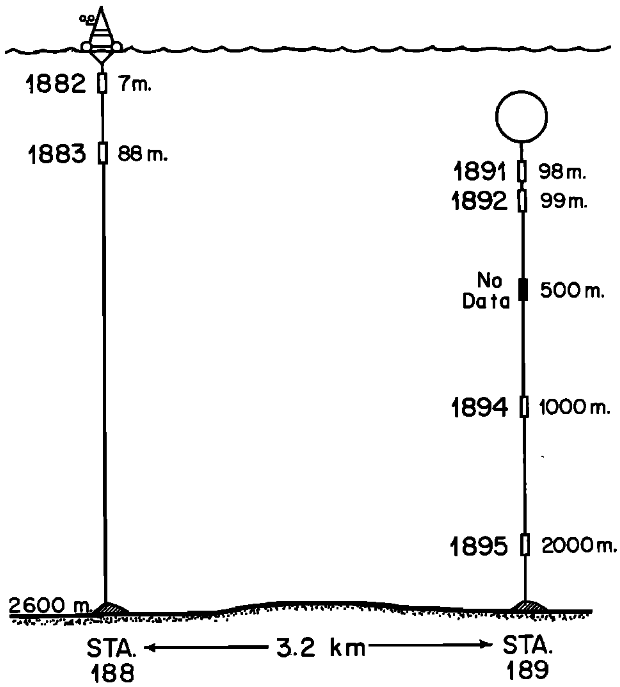

Fig. 7. Instrument configuration for the simple array used to study scales of coherence of inertial motions. The positions were: station 188, 39°20.5′N, 69° 59.0′W; station 189, 39°20.0′N, 60°57.0′W.

Audio and Electroacoustics, vol. *AU-15* (2), June 1967), Granger's book remains a useful explanation of cross-spectral techniques.

The spectral density function of a scalar time series $\alpha(t)$ can be expressed as a real function of frequency $P_{\alpha\alpha}(f)$. That is, at frequency f, a single number can describe the spectral energy density. With a vector time series, such as that obtained with a moored current meter, a full description of the spectral density requires three functions of frequency: two real and one complex. Suppose the original vector time series is $\alpha(t) + i\beta(t)$. The corresponding spectral density functions are $P_{\alpha\alpha}(f)$, $P_{\beta\beta}(f)$, and $P_{\alpha\beta}(f) + i\, Q_{\alpha\beta}(f)$. In the usual nomenclature, these are called the autospectrum of $\alpha(t)$, the autospectrum of $\beta(t)$, and the (complex) cross spectrum between $\alpha(t)$ and $\beta(t)$. The real part of the latter function $P_{\alpha\beta}(f)$ is called the co-spectrum between α and β. The complex part, $Q_{\alpha\beta}(f)$, is called the quadrature spectrum between α and β.

It is often convenient to express the complex cross spectrum in polar form as

$$R_{\alpha\beta}(f) \exp i\phi_{\alpha\beta}(f)$$

where $R_{\alpha\beta}\ (f)$ is the cross-spectral amplitude and $\phi_{\alpha\beta}\ (f)$ is the phase of the cross spectrum between $\alpha(t)$ and $\beta(t)$ at frequency f. Whether or not the Cartesian or polar form of the cross spectrum is used, four numbers are needed at frequency f to describe the complete spectrum of the vector process $\alpha(t) + i\beta(t)$. These numbers can conveniently be written in the form of a matrix. In Cartesian form, the diagonal terms are the autospectra of the components and the off-diagonal terms are coefficients of the real and imaginary parts of the cross spectrum between the components:

$$\begin{bmatrix} P_{\alpha\alpha} & P_{\alpha\beta} \\ Q_{\alpha\beta} & P_{\beta\beta} \end{bmatrix}$$

In polar form, the off-diagonal terms are the amplitude and phase of the cross spectrum between components of the vector time series:

$$\begin{bmatrix} R_{\alpha\alpha} & R_{\alpha\beta} \\ \phi_{\alpha\beta} & R_{\beta\beta} \end{bmatrix}$$

To express the cross-spectral relationships between pairs of vector time series, more terms are required, but the principle is only an extension of the method used for describing the spectrum of a single vector time series.

Suppose one vector series is $\alpha(t) + i\beta(t)$ and a second is $\gamma(t) + i\delta(t)$. Then four real autospectral density functions and six complex cross-spectral density functions are needed for a complete description. Using the notation as before, these become in a 4×4 matrix

$$\begin{bmatrix} P_{\alpha\alpha} & P_{\alpha\beta} & P_{\alpha\gamma} & P_{\alpha\delta} \\ Q_{\alpha\beta} & P_{\beta\beta} & P_{\beta\gamma} & P_{\beta\delta} \\ Q_{\alpha\gamma} & Q_{\beta\gamma} & P_{\gamma\gamma} & P_{\gamma\delta} \\ Q_{\alpha\delta} & Q_{\beta\delta} & Q_{\gamma\delta} & P_{\delta\delta} \end{bmatrix} \quad \text{(in Cartesian form)}$$

$$\begin{bmatrix} R_{\alpha\alpha} & R_{\alpha\beta} & R_{\alpha\gamma} & R_{\alpha\delta} \\ \phi_{\alpha\beta} & R_{\beta\beta} & R_{\beta\gamma} & R_{\beta\delta} \\ \phi_{\alpha\gamma} & \phi_{\beta\gamma} & R_{\gamma\gamma} & R_{\gamma\delta} \\ \phi_{\alpha\delta} & \phi_{\beta\delta} & \phi_{\gamma\delta} & R_{\delta\delta} \end{bmatrix} \quad \text{(in polar form)}$$

A common practice is to normalize the complex amplitudes to form

$$C_{ij} = R_{ij}/[P_{ii}\,P_{jj}]^{1/2}$$

C_{ij} is known as the coherence. Unfortunately, an ambiguous usage has grown up, and the quantity C_{ij}^2 is often called coherence or coherency. There seems little to recommend the quantity C_{ij}^2 over C_{ij} except a wide (but not universal) usage. Note that $C_{jj} = 1$ and that

$$C_{ij} = \left[\frac{P_{ij}^2 + Q_{ij}^2}{P_{ii} \cdot P_{jj}}\right]^{1/2}$$

Thus the normalized cross-spectral matrix can be expressed in polar form as

$$\begin{bmatrix} C_{\alpha\alpha} & C_{\alpha\beta} & C_{\alpha\gamma} & C_{\alpha\delta} \\ \phi_{\alpha\beta} & C_{\beta\beta} & C_{\beta\gamma} & C_{\beta\delta} \\ \phi_{\alpha\gamma} & \phi_{\beta\gamma} & C_{\gamma\gamma} & C_{\gamma\delta} \\ \phi_{\alpha\delta} & \phi_{\beta\delta} & \phi_{\gamma\delta} & C_{\delta\delta} \end{bmatrix}$$

The role of the elements in this matrix can be better understood if the 4×4 matrix is divided into four 2×2 matrices, one for each quadrant:

$$\left[\begin{array}{c|c} 1 & 3 \\ \hline 4 & 2 \end{array}\right]$$

Submatrix 1 is the cross-spectral matrix of vector time series one; submatrix 2 is the cross-spectral matrix of vector time series 2. Submatrix 3 contains only elements describing the coherence between series 1 and series 2. Submatrix 4 contains only elements describing the phase relationships between the components of series 1 with the components of series 2.

The present investigation answers the question: How is the separation between pairs of instruments related to the magnitude of the cross-spectral coherences at inertial frequency? Specifically, the amplitude of terms in submatrix 3 of the cross-spectral matrix will be examined in relation to the horizontal and vertical separation of current meters in the array.

Over the array described here, the cross-spectral matrices were computed at inertial frequency between four pairs of instruments. At each instrument, a vector time series having components of horizontal velocity in the east and north directions was available. For the first instrument of each pair, the components are called u_1 and v_1. For the second instrument, they are called u_2 and v_2. Thus, corresponding to the notation used above, the components u_1, v_1, u_2, and v_2 correspond to the components α, β, γ, and δ. The cross-spectral energy-density coefficients are thus arranged in the polar-form matrix as

$$\begin{bmatrix} C_{u_1u_1} & C_{u_1v_1} & C_{u_1u_2} & C_{u_1v_2} \\ \phi_{u_1v_1} & C_{v_1v_1} & C_{v_1u_2} & C_{v_1v_2} \\ \phi_{u_1u_2} & \phi_{v_1u_2} & C_{u_2u_2} & C_{u_2v_2} \\ \phi_{u_1v_2} & \phi_{v_1v_2} & \phi_{u_2v_2} & C_{v_2v_2} \end{bmatrix}$$

To interpret the cross-spectral matrices, it is helpful to consider the matrix of a simple idealized model. If the currents observed at a pair of current meters are purely inertial and if the motion is uniform between the instruments, the cross-spectral matrix at inertial frequency in the northern hemisphere should have the value

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 270^\circ & 1 & 1 & 1 \\ 0^\circ & 90^\circ & 1 & 1 \\ 270^\circ & 0^\circ & 270^\circ & 1 \end{bmatrix}$$

A cross-spectral matrix of this simple form would probably never be observed. It does provide a model for comparison with observations. The nearest approximation to the idealized matrix is found for the cross spectrum between 1891 and 1892, separated by 1 meter in depth:

$$\begin{bmatrix} 1 & 0.75 & 0.95 & 0.83 \\ 270^\circ & 1 & 0.82 & 0.96 \\ -12^\circ & 81^\circ & 1 & 0.84 \\ 257^\circ & -10^\circ & 270^\circ & 1 \end{bmatrix}$$

As can be seen from the cross-spectral amplitudes in quadrants 1 and 2, the extent of the coherence between components at a single point of measurement is less than that expected in a simple inertial-motion model. Defects in the current meters might introduce a noise signal that degrades the quality of the signal. In addition, higher-frequency turbulent processes may decrease the inertial-frequency coherence between components. The submatrix in quadrant 3 shows a characteristic value of about 0.9.

The phase angles of quadrant 4 will fit the model more closely if 11° is added to all the angles. The consistent 11° phase angle is probably associated with a slight offset between time origins of the two sets of measurements. For an inertial period of 18.88 hours (site D), an 11° phase lag corresponds to a time difference of 35 minutes. The sign of the phase angle indicates that the clock time for the first series (1891) is running 35 minutes slow with respect to the second series (1892)

As the separation between instruments is increased, the cross-spectral amplitudes decrease. The decrease in amplitude is greater, however, for a given amount of vertical separation than for a similar horizontal separation. On the surface-float mooring, 1882 and 1883 are separated vertically by 81 meters. The cross-spectral matrix between them is

$$\begin{bmatrix} 1 & 0.84 & 0.28 & 0.30 \\ 270^\circ & 1 & 0.35 & 0.40 \\ -126^\circ & 138^\circ & 1 & 0.89 \\ 200^\circ & -160^\circ & 270^\circ & 1 \end{bmatrix}$$

In quadrant 3, the cross-spectral amplitudes have a charteristic value of 0.33.

On the surface-float mooring, 1883 is at a depth of about 87 meters. On the subsurface-float mooring, 1891 is suspended 8 meters below the subsurface float. No precise measurement of the true float depth is available. A 90-meter float depth was intended and is assumed here. The actual depth could, however, have varied as much as 10 meters either way. The depth of current meter 1891 is thus 98 ± 10 meters. It was located approximately 3 km east of current meter 1883. The cross-spectral matrix at inertial frequency between 1883 and 1891 is

$$\begin{bmatrix} 1 & 0.82 & 0.65 & 0.67 \\ 270^\circ & 1 & 0.67 & 0.73 \\ 34^\circ & 123^\circ & 1 & 0.76 \\ 314^\circ & 34^\circ & 270^\circ & 1 \end{bmatrix}$$

In quadrant 3, the cross-spectral amplitudes have a characteristic value of 0.7.

The cross-spectral amplitudes over greater vertical extent are small at inertial frequency. On the subsurface-float mooring, 1894 and 1895 are separated vertically by 1000 meters. The cross spectrum between them is

$$\begin{bmatrix} 1 & 0.98 & 0.22 & 0.30 \\ 268° & 1 & 0.24 & 0.32 \\ -21° & 77° & 1 & 0.91 \\ 271° & 6° & 267° & 1 \end{bmatrix}$$

In quadrant 3, the cross-spectral amplitudes have a characteristic value of 0.27. For the data used in this experiment there are, roughly, 16 degrees of freedom for each estimate. The corresponding 10% probability level for coherence is 0.6 [*Granger*, 1964, p. 79]. The cross-spectral estimates between 1894 and 1895 do not exceed the 10% probability level, and the coherence is probably not significant.

The results of this array experiment can easily be summarized. At inertial frequency, greater coherence was found horizontally over 3-km separation than vertically over an 80-meter separation. Characteristic coherences are 0.7 over the 3-km horizontal separation and 0.3 over the 80-meter vertical separation. Over 1000-meter separation at greater depth, no significant inertial-period coherence was found.

CONCLUSIONS

Inertial-period motions are essentially transient phenomena of thin vertical extent. They may also have narrow latitudinal extent.

In consequence, because inertial motions are confined in both space and time, they may or may not be found in a short set of measurements at a given position. This occurrence does not seem to be a function of depth, bottom topography, or latitude. The generation of inertial motions may be related to the passage of storms and associated changes in sea-surface pressure or surface wind stress.

Future observational and theoretical studies of inertial-period processes are needed. In particular, observations with closely spaced vertical arrays should be made to define more accurately the vertical extent of inertial motions and to explore the possible vertical modal structure.

The mechanism of generation of inertial motions might be understood better by simultaneous measurements of surface wind and atmospheric pressure. Such observations might serve to explain only the local generation of inertial motions. The question of local versus global generation is more difficult to attack by observation, but perhaps critical experiments could be designed. In this regard, further theoretical studies are needed to define such critical experiments.

Acknowledgments. This work was supported by the Office of Naval Research under contract N000 14-66-CO241, NR 083-004.

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(Manuscript received March 15, 1968.)