Ekman Currents and Mixing due to Surface Gravity Waves

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ABSTRACT

The steady Ekman boundary-layer current is studied theoretically for the case when the eddy viscosity is proportional to the shear of the wave orbital velocity in a turbulent wave, times the square of a mixing length (Kitaigorodsky, 1961). Assuming a fully developed sea, the wave characteristics, and hence the eddy viscosity distribution with depth, are determined by the wind. The momentum equation is solved numerically to yield the Ekman current as a function of the wind speed. The results show that the magnitude of the Ekman surface current lies between 2.1 and 3% of the wind speed at 10 m height. The deflection angle away from the wind direction is a monotonic decreasing function of wind speed. It varies from 36 to 25° for winds between 5 and 30 m s^{-1}.

1. Introduction

It appears from the literature to be some disagreement concerning various aspects of Ekman currents in the open ocean. One particular matter of discussion is the value of the deflection angle of the surface current away from the wind direction. Basically, this problem arises because we know little about the turbulent downward flux of horizontal momentum in the ocean. By relegating the problem into an eddy viscosity coefficient as is often done, one is faced with the difficulty of predicting the size and variation with depth of the latter.

According to Ekman (1905), the surface current and the volume flux in an infinitely deep ocean should be deflected 45 and 90°, respectively, to the right of the wind (on the Northern Hemisphere). The latter result is independent of the vertical variation of the eddy viscosity, while the result for the surface current is based on a constant eddy viscosity with depth. This assumption, however, is not very realistic and several attempts have been made to improve the theory by assuming various forms for the eddy viscosity. Dobroklonskiy (1969) considers the turbulent mixing due to surface waves, and assumes an eddy coefficient which decreases exponentially from the surface. The fact that the eddy viscosity is largest near the surface and decreases downward, also seems to be supported by measurements of the distribution of radon in the sea (Klug, 1974). Lai and Rao (1976) consider essentially the same model as Dobroklonskiy (1969), except that they first let the eddy viscosity increase as the square of depth before it starts to fall off exponentially. Both Dobroklonskiy (1969) and Lai and Rao (1976) obtain large deflection angles of the surface current away from the wind; ~70° in both cases. On the other hand, Madsen (1977) takes, by analogy with shear-generated turbulence in pipes, etc., the eddy coefficient to increase linearly with depth in the whole water column. This model results in a deflection angle of the steady surface current of ~10°.

In the present paper we shall investigate the Ekman current using Kitaigorodsky's (1961) model for the eddy viscosity. This model takes into account the mixing effects due to surface gravity waves, which is the same starting point as that of Dobroklonskiy (1969) and Lai and Rao (1976). According to Kitaigorodskiy (1961) the distribution of eddy viscosity with depth may be obtained by assuming that the eddy coefficients are proportional to the shear of the wave orbital velocity in a turbulent wave, times the square of a mixing length. The latter is assumed to vary linearly with depth. This model implies that the production of turbulence by mean shear and wave breaking is negligible, and that the small-scale turbulence has a characteristic time scale much smaller than the period of the dominant waves. Jacobs (1978) used Kitaigorodsky's model, among several other models, in a numerical simulation of air-sea interaction, and this particular model succeeded quite well in reproducing the observed vertical distributions of temperature during Period III of BOMEX. It seems therefore that turbulent mixing due to surface gravity waves is of importance in the upper ocean. The purpose of the present note is to investigate how this process, modeled through a variable eddy viscosity, influences the stress-driven current.

2. Model and method of solution

Consider two-dimensional horizontal steady motion in a homogeneous ocean of constant depth H and of infinite lateral extent. Let the x, y, z axes form a right-handed system with the z-axis positive downward and the origin at the surface. There is no surface elevation or horizontal pressure gradients. Introducing a complex velocity $W = u + iv$, where u and v are the mean turbulent velocity components

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in the x and y direction, respectively, the horizontal momentum equation may be written

$$(AW')' + ifW = 0, \tag{1}$$

where the prime denote differentiation with respect to z. Furthermore, A is the eddy viscosity in the vertical direction and f the constant Coriolis parameter.

At the surface the boundary condition is

$$W' = -T, \quad z = 0, \tag{2}$$

where $T = [\tau_0^{(x)} + i\tau_0^{(y)}]/A_0$. Here $\tau_0^{(x)}$ and $\tau_0^{(y)}$ are the wind stresses per unit density of sea water in the x and y directions and the subscript zero denotes surface value ($z = 0$). The wind stress $\boldsymbol{\tau}_0$ is related to the wind velocity $\mathbf{V}_{10}$ at 10 m above the sea level by $\boldsymbol{\tau}_0 = (\rho_a/\rho)c_{10}\mathbf{V}_{10}|\mathbf{V}_{10}|$, where ρ and ρ_a are the densities of sea water and air, respectively. The drag coefficient c_{10} depends on the wind velocity as well as on the vertical stability of the air.

The purely wind-driven current effectively vanishes below the Ekman depth $D_E \sim (\bar{A}/f)^{1/2}$, where $\bar{A}$ is some suitably vertically averaged eddy viscosity. In shallow water where $D_E > H$, the frictional influence of the bottom, in general, can not be neglected. Since the motion usually is turbulent, a no-slip condition ($u = v = 0$) would constrain the motion too much and therefore be unrealistic. The presence of turbulence then leads to a bottom stress proportional to the square of the fluid velocity. However, if the depth H is much larger than D_E, the bottom is really never "felt," and $u = v = 0$ are appropriate boundary conditions at the bottom. It is this latter case which will be considered here, so we take

$$W = 0 \quad \text{at} \quad z = H. \tag{3}$$

For given vertical variation of the eddy viscosity, Eq. (1), subject to the boundary conditions (2) and (3), was solved numerically by a standard "shooting" method for solving boundary-value problems; see, e.g., Conte (1966). To obtain solutions for an "infinitely" deep ocean, the value of H, for given wind stress, was increased systematically, until no noticeable change of the results was achieved.

Numerically problems arise when the Ekman depth becomes much smaller than the fluid depth. Then initially independent solutions in the integration procedure become dependent due to insufficient accuracy of the computer. To handle this problem, the original integration interval had to be divided into subintervals, in which the solutions were orthonormalized before continued integration.

3. Ekman flow results

As stated in the introduction, we shall apply a model for the eddy viscosity which essentially is that developed by Kitaigorodsky (1961). Here it is assumed that the eddy coefficients are proportional to the shear of the wave orbital velocity in a turbulent wave, times the square of a mixing length, which varies linearly with depth. Any effects on wave frequency due to mean drift currents are disregarded. This is a reasonable assumption since mean drift velocities only amount to a few percent of the wind speed, which is much less than the wave orbital velocities. Also, since the frequencies of the dominant waves are typically of order 1 s^{-1} which is much larger than the inertial frequency, the effect of rotation on wave frequency can safely be neglected. We then may write for the eddy viscosity distribution (see also Jacobs, 1978; Delnore, 1980)

$$A = a(z + b)^2 e^{-cz}, \tag{4}$$

where

$$a = K\delta(2\pi g/\lambda)^{1/2}, \quad b = \delta\lambda/2, \quad c = 2\pi/\lambda. \tag{5}$$

The empirical constant K in (5) was taken by Kitaigorodsky to be 0.02 in the case of heat diffusion. This value also was used for the eddy momentum coefficient in Jacobs' (1978) numerical simulation, and will be adopted here. Furthermore, δ in (5) is the steepness of fully developed waves (=0.055) and g is the acceleration due to gravity. λ is the dominant wavelength for surface waves in a well-developed sea. It is connected with the wind at 19.5 m above sea level by

$$\lambda = (2.803 \times 10^{-3} U_{19.5}^2 \text{ s}^2 \text{ cm}^{-2}) \text{ cm} \tag{6}$$

(Jacobs, 1978; Delnore, 1980), where we for simplicity have directed the x axis along the wind. The theory rests on the assumption of deep water. Hence, we must require $H > \lambda$.

By specifying the drag coefficient c_{10} and assuming that the velocity profile in the air above the sea surface is logarithmic, i.e.,

$$U_{19.5} = U_{10}[1 + (c_{10}^{1/2}/\kappa)\ln(19.5/10)], \tag{7}$$

where $\kappa = 0.4$ (Pierson, 1964), all the parameters of the Ekman problem are determined by the wind speed at 10 m height. There have been many attempts to determine c_{10}, and the results have considerable scatter; see, for example, the discussions and comparisons by Pierson (1964) and Timmerman (1977). For the present purpose, we take $c_{10} = 1.8 \times 10^{-3}$ when $U_{10} < 15$ m s^{-1} and $c_{10} = 2.7 \times 10^{-3}$ when $U_{10} > 20$ m s^{-1}, with a linear interpolation in between. This choice has been shown to yield excellent agreement between computed wind surge and observed surface elevation in the North Sea (Timmerman, 1977).

A typical eddy viscosity distribution with depth is shown in Fig. 1a for $U_{10} = 10$ m s^{-1}. The depth d at which the eddy viscosity maximum occurs is given by $d = 2/c - b$ from (4), which is 9.3 m in this

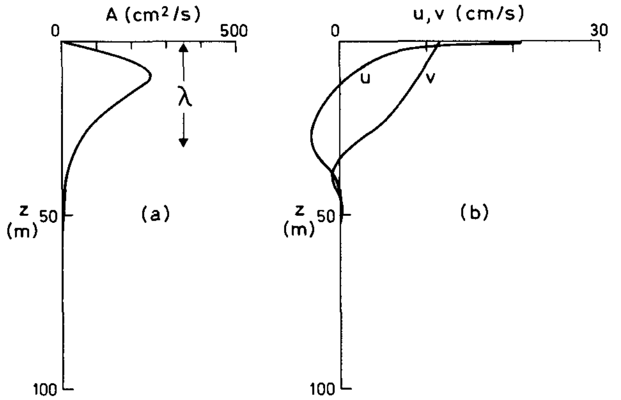

FIG. 1a. Kinematic eddy viscosity A versus depth for $U_{10} = 10$ m s^{-1}. The wavelength λ of dominant surface waves is given by (6).

FIG. 1b. Variations with depth of the steady Ekman current when $U_{10} = 10$ m s^{-1}. The viscosity distribution is that of Fig. 1a.

example. The dominant wavelength λ of surface waves in a well developed sea is here 32 m. In Fig. 1b we have displayed the steady vertical variation of the horizontal velocity with depth for this case. In the computations the depth H was taken to be 100 m, and we note that the velocity variations essentially take place in the upper 50 m. We also find that the velocity shear in the y direction tends to zero only in the upper meters. This is not very clear from Fig. 1b, however, due to the poor vertical resolution.

A suitably average eddy viscosity $\bar{A}$ for this problem may be defined by

$$\bar{A} = \frac{1}{\lambda}\int_0^\infty A dz. \tag{8}$$

The Ekman depth is then given by

$$D_E \propto (\bar{A}/f)^{1/2}, \tag{9}$$

where the coefficient of proportionality is of order unity.

In Fig. 2 we have plotted the steady Ekman surface current as function of wind speed at 10 m height. The displayed curve consists of three line segments, each of which is close to a straight line. The steepening of the curve for U_{10} between 15 and 20 m s^{-1} is due to the linearly increasing drag coefficient in this region. In the displayed domain the steady surface current has a value which lies between 2.1 and 3% of the wind speed.

The deflection of the steady surface current to the right of the wind direction (on the Northern Hemisphere) is displayed in Fig. 3, where the deflection angle is plotted as a function of the wind speed. We note the interesting result that the deflection angle decreases monotonically with increasing wind speed. The decrease is most pronounced for light winds, while for U_{10} between 10 and 30 m s^{-1} α_0 decreases from about 29 to 25°.

Recalling that the wind speed determines the distribution of eddy viscosity with depth, it is easy to understand why the deflection angle should depend on the wind speed. When the wind is weak, the viscosity maximum is close to the surface. The ratio of the depth d of the viscosity maximum to the Ekman depth D_E is small ($d/D_E \sim U_{10}^{1/2}$) and the viscosity distribution is basically a decreasing one. This is analogous to the situation considered by Dobroklonskiy (1969) and Lai and Rao (1976), and leads to deflection angles which are larger than 45°. When the wind is stronger, the ratio d/D_E is larger and the viscosity is essentially increasing with depth. This is similar to the situation studied by Madsen (1977) (where the eddy viscosity was increasing for different reasons). The deflection angles now became <45°.

The fact that increasing eddy viscosity with depth leads to decreasing deflection angles and vice versa can be demonstrated analytically, also. Take, for simplicity, a linear eddy distribution, i.e.,

$$A = A_0 + \epsilon z, \tag{10}$$

where A_0 is constant and ϵ is a small constant parameter which can be positive or negative. Since the real length scale of the problem is proportional

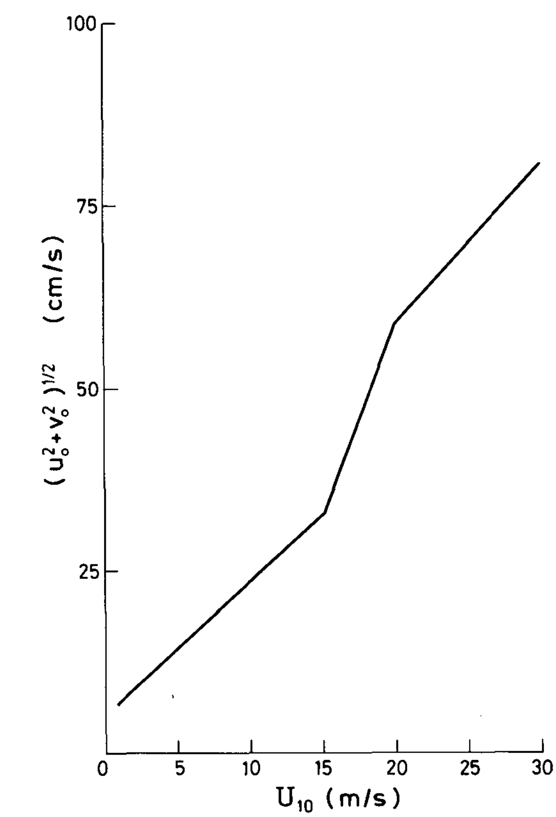

FIG. 2. Surface speed of the Ekman current versus wind speed at 10 m height.

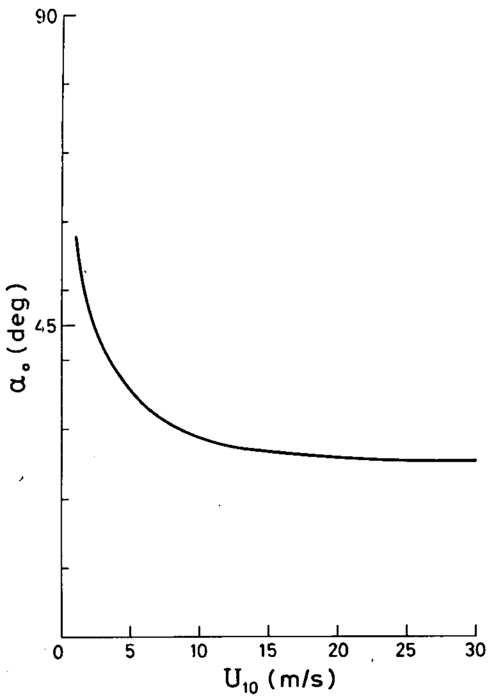

FIG. 3. Deflection angle of the Ekman surface current versus wind speed at 10 m height.

to $(A_0/f)^{1/2}$, the requirement that ϵ must be small can be stated more precisely as $|\epsilon| \ll (A_0 f)^{1/2}$. The ocean is taken to be deep, i.e., $H \gg (A_0/f)^{1/2}$. Hence we assume that $W \to 0$, $z \to \infty$. The wind stress is constant and directed along the x axis. Solutions of (1) will be obtained by expanding W in a series after ϵ as a small parameter, i.e.,

$$W = W^{(0)} + \epsilon W^{(1)} + \epsilon^2 W^{(2)} + \cdots. \quad (11a)$$

The boundary conditions are

$$W'^{(n)} = \begin{cases} -\tau_0/A_0, & n = 0 \\ 0, & n = 1, 2, 3 \cdots \end{cases} \quad (11b)$$

at $z = 0$, while we assume

$$W^{(n)} \to 0, \quad n = 0, 1, 2, 3 \cdots \quad (11c)$$

when $z \to \infty$.

The zeroth-order solution is just the ordinary Ekman solution for constant eddy viscosity. To O(ϵ) we obtain

$$W^{(1)} = -\frac{i\tau_0}{4A_0 f}[1 + \gamma z - \gamma^2 z^2]e^{-\gamma z}, \quad (12)$$

where

$$\gamma = \frac{1 - i}{(2A_0/f)^{1/2}}.$$

For the deflection angle at the surface we then find to O(ϵ)

$$\alpha_0 = \tan^{-1}(v_0/u_0) = \tan^{-1}[1 - \epsilon/(8A_0 f)^{1/2}]. \quad (13)$$

Hence, we see for $\epsilon > 0$ (increasing eddy viscosity) that $\alpha_0 < 45°$, while for $\epsilon < 0$ (decreasing eddy viscosity) we obtain that $\alpha_0 > 45°$.

4. Summary and concluding remarks

The main difficulty in determining the vertical structure of the oceanic Ekman boundary-layer current comes from the problem of relating the value of the eddy viscosity coefficient to the other parameters of the system. This difficulty is overcome in the present paper by assuming that the turbulent mixing is due to surface gravity waves. Hence the eddy viscosity distribution with depth for a fully developed sea can be assessed as function of the wind speed. In turn, this means that the Ekman current at any level is completely determined by the wind at 10 m height.

In particular we find that the value of the surface current lies between 2 and 3% of the wind speed, while the deflection angle away from the wind is a monotonic decreasing function of wind speed. It varies from 29 to 25° for winds between 10 and 30 m s^{-1}. Although these values are not inconsistent with drift data from the open sea (Haug, 1970), direct comparison may not be quite relevant. This is because the total drift current also must include a wave-induced part. However, the role of the wave-drift in a rotating ocean is by no means clear. On one hand, the non-viscous calculations of Ursell (1950) and Pollard (1970) lead to zero net mass transport. On the other hand, Madsen (1978) finds that the wave-drift at the surface for a viscous fluid is of the same order of magnitude as the shear-induced current, and directed approximately 45° to the right of the wave propagation direction (on the Northern Hemisphere). Furthermore, the effect of a variable eddy viscosity complicates the wave-induced flow problem even more. This topic therefore will be left for future research.

Finally, it should be emphasized that there are several uncertain points concerning the computation of the stress-driven current in this paper. In particular, we have chosen a model for the eddy viscosity, where not only the values of the different coefficients in the formula (5) are somewhat uncertain, but which also, and more seriously, excludes turbulence generated by mean shear and wave-breaking. On the other hand, Jacobs' (1978) results (see also, Delnore, 1980) indicate that turbulent mixing due to surface gravity waves is important in the sea. Therefore, we believe that the present approach is a step in the right direction toward a realistic description of the Ekman boundary-layer current in the open ocean.

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Comments on "Signatures of Mixing from the Bermuda Slope, the Sargasso Sea and the Gulf Stream"

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For some years now, practitioners of the art of "microstructure" have been gathering wiggly lines, records of temperature gradients encountered by a freely falling instrument as it descends through the oceanic water column. Most have suspected that some of the wiggles are caused by turbulent overturns acting on a mean gradient. We have wondered: 1) which of the wiggles are caused by "turbulence," and 2) to what extent does this turbulence obey laws similar to the "universal" forms obeyed by the homogeneous, isotropic turbulence of laboratory fluid mechanics. Early observations with towed sensors in a tidal channel showed considerable agreement with the universal forms (Grant *et al.*, 1968), but some later authors have perceived discrepancies (Nasmyth, 1970; Elliott and Oakey, 1976). Our own results have been more encouraging (Caldwell *et al.*, 1980; Dillon and Caldwell, 1980; Caldwell *et al.*, 1981; Newberger and Caldwell, 1981), so we were surprised to find a negative result in what appeared to be a favorable situation for the presence of classical turbulence (Gregg and Sanford, 1980). Examination of Sections 4a and 6b of the paper by Gregg and Sanford reveals some problems with their arguments. In the following, these difficulties are discussed and alternative interpretations are offered.

Gregg and Sanford (1980, hereafter referred to as GS) discuss the applicability of the theory of universal turbulence spectra to one vertical temperature-gradient "microstructure" spectrum. In an actively mixing surface layer in the Sargasso Sea, a cast of the microstructure recorder was made close in space and time to a profile of kinetic-energy dissipation rate ϵ obtained by Gargett *et al.* (1979). Gargett *et al.* found the dissipation to be relatively vertically homogeneous in a layer extending from the surface to a depth of approximately 135 m, the average value of ϵ being $1.4-1.5 \times 10^{-7}$ W kg^{-1}. Using this value for ϵ together with an estimate of the dissipation rate of temperature variance, χ (5.42×10^{-9} °C^2 s^{-1}), determined by GS, quantitative comparison between the vertical temperature-gradient spectrum and the universal turbulence form is possible, subject to uncertainty as to the value of the universal constants (and to uncertainty as to changes in the water column in the time-space interval between the two casts). GS conclude that in the convective subrange (which corresponds to the inertial subrange in velocity spectra) the agreement is excellent. At higher wavenumbers, in the presumed "viscous-convective" and "diffusive" subranges where the universal spectrum takes on the

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