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"The Earth's Hum" is Driven by Ocean Waves over the Continental Shelves

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The first section of this document describes the calculation of the near surface pressure field beneath low frequency ocean waves over the shelf and the excitation of normal modes by this pressure field. The second section discusses problems with a previous theory of the excitation of Earth normal modes by linear ocean waves. The last section provides an estimate of the high phase speed components of the pressure field under atmospheric turbulence and compares this estimate with previous estimates to show that atmospheric turbulence is of negligible importance as the source of excitation of Earth seismic modes.

1. The Hasselmann Surface Pressure Spectrum in Shallow Water

Hasselmann[1] derives a formula for the wavenumber and frequency spectrum of the near surface pressure induced by the nonlinear coupling of waves in a random ocean wave field to estimate the forcing of microseisms by ocean waves. The formula is valid for small wavenumber (high phase velocity) and for deep water (relative to the wavelength of the ocean waves). Hasselmann's derivation is extended to apply to shallow water ocean waves and then adapted to the specific problem of the forcing of planetary modes. There is a large enhancement of the mechanism in shallow water so that waves over the continental shelf dominate the excitation of Earth normal modes. In the calculation, the shelves of the Earth are divided up into a large number of small source regions of width and length equal to the local shelf width. The forcing is summed over all regions assuming the contributions from each region are independent. This theory, given a reasonable estimate for the wave height spectrum of infragravity waves on the shelf predicts a spectrum of Earth normal mode vertical acceleration that closely matches observations at low frequency. To match the higher frequencies, the high order modes are described by traveling wave equivalents and a factor describing attenuation is added to the equations to account for attenuation of short period modes during propagation into the centers of continents. With this added term, the theory accurately models the entire background spectrum for "quite sites" at frequencies from 2mHz and to the single frequency microseism peak above 40 mHz.

Longuet-Higgins[2] provides an easily understood description of how interacting ocean waves can excite much higher phase velocity seismic waves. Consider a pair of oppositely traveling waves interacting so as to generate a standing wave over the entire shelf. During one part of the cycle of the standing wave, the surface of the ocean will be flat. A quarter cycle later, the two waves constructively interfere and the ocean surface is a sinusoid. A quarter cycle later, the ocean is again flat, and a quarter cycle later the surface is again a sinusoid. During the transition from a flat surface to a sinusoid, the water move from the troughs into the peaks so during each half cycle, the center of mass of the water lying over the entire shelf goes up and down once. This movement of the center of mass requires a force that is exerted simultaneously over the entire shelf. During a full cycle of the standing wave, the center of mass goes up and down twice producing a pressure signal below the sea surface at a frequency equal to twice the frequency of the ocean waves making up the standing wave. Interacting ocean waves also exert a force on the atmosphere above the ocean. We can determine the atmospheric forcing by noting that the each time the center of mass of the water over the shelf moves up and down, the air in a layer just above the waves must be moved out of the way. The volume of air that moves is equal to the moving volume of water. The force per unit area required to move the center of mass of this air layer will be equal to the ratio of the density of air to the density of water times the force per unit area (pressure) needed to move the water. The forcing of coupled atmosphere-solid Earth modes by nonlinear ocean waves is then best described by a source term that is a "pressure glut[4]" or jump in pressure between a level just above the sea surface and a level just below the sea surface.

Nishida et al.[3] discovered that the OS29 and OS37 normal modes were excited in the Earth's hum above the adjacent modes by about 20 and 10% respectively during the northern summer. Only for the 0S29 mode at 3.7mHz and 0S37 mode at 4.4 mHz are the frequencies of the atmospheric modes nearly coincident with the corresponding solid Earth normal mode. They attributed the larger amplitudes of these two modes to coupling between the atmospheric and solid Earth modes. The relative enhancement disappeared during the winter months due to decoupling from changes in the frequencies of the atmospheric modes with changes in the temperature structure of the atmosphere.

Rather than calculating the mode forcing from nonlinear wave-wave interaction in a full coupled atmospheric-solid Earth model, I instead approximate the forcing as a vertical point force acting on the surface of the Earth and estimate the effect of atmosphere coupling on these two modes separately.

These atmospheric modes can be well described as the sum of a (nearly) vertically upward propagating and nearly downward propagating acoustic wave. The ocean wave interaction produces a pressure field that excites upward propagating acoustic waves in the atmosphere that at altitude reflect into a downward propagating wave. For components of the wave forcing that are resonant with atmospheric modes, the atmospheric mode amplitudes will increase until the rate of forcing by the ocean waves is balanced by the dissipation in the atmospheric mode, but the dissipation of the two relevant atmospheric modes is mostly due to losses into the Earth[4].

The acoustic impedance at the surface of the Earth is much higher than in the atmosphere so the downward propagating acoustic wave is nearly perfectly reflected at the Earth's surface, except the upward propagating wave amplitude will be smaller by the fraction π / Q because of losses into the Earth. In regions where wave forcing is occurring, the attenuation in the Earth is exactly balanced by the wave forcing, so that the difference in amplitude of the upward and downward going wave will be equal to the excitation pressure signal in the atmosphere p_a. The total pressure signal exerted by the atmospheric modes on the Earth will be $p_s = p_u + p_d \approx 2p_d \approx 2p_a Q_a / \pi$. As described above, the atmospheric forcing pressure is related to the forcing under the ocean waves by the ratio of the density of air at the Earth's surface to the density of seawater. The effect of the excitation of the atmospheric mode on the strongly coupled solid Earth modes can be described as an enhancement to the pressure signal forcing the modes by the ratio: $p_e / p = \left[1 + 2Q_a \rho_a / (\pi \rho_w)\right]$. Lognonne, et al.[4] calculated the Q's of the 0S29 and 0S37 modes to be about 115 and 21. The predicted enhancements of the amplitude of these two modes are the factors 1.09 and 1.02, smaller than enhancement in amplitudes measured (1.2 and 1.1[3]). However, to date no other studies have reproduced the Nishida et al.[3] result. Tanimoto[5] notes Nishida et al.[3] show a annual cycle to mode amplitudes whereas all other published measurements show a biannual signal and suggests the discrepancy is related to the method of data analysis. Even the small predicted enhancements appear surprising large given the much smaller pressure signal in the atmosphere compared to in the ocean, but the rate of work per unit area done on the modes by the wave interaction pressure signal is proportional to $p_a^2 / (\rho_a c_a)$ and relatively more energy is transferred because the speed of sound (c_a) and density are small in air.

To compute the nonlinear coupling of ocean waves into other components, Hasselmann[1,6] expands the potential describing the ocean wave field in a perturbation series: $\phi(\vec{x}, z, t) = \phi_1 + \phi_2 + ...$ The first order (linear) ocean wave field is described by the potential $\phi_1(\vec{x}, z, t)$, valid for small amplitude waves (relative to water depth). The second order term that can force planetary modes is described as equivalent to a pressure signal appearing just below the sea surface equal to $p_2 = \rho(\nabla\phi_1)^2\big|_{z=0}$. In both the microseism and the Earth normal mode problems, it is necessary to include the compressibility of the water (allowing the propagation of sound) and the elastic structure of the seafloor in the solution of the second order potential. Rather than formally continuing the solution using the perturbation expansion, the calculation is simplified by dividing it into two steps. The formula relating the ocean wave height frequency-directional spectrum $f_\varsigma(\sigma,\theta)$ to the frequency-wavenumber spectrum $F_p(\vec{K},\omega)$ of the sea surface pressure signal arising from ocean wave coupling is first developed. Then this calculation of the surface pressure spectrum is used to calculate the forcing of Earth normal modes using the standard computational machinery for calculating the excitation of Earth normal modes by earthquakes.

Adapting Hasselmann's[1,6] derivation, the first order potential valid for both shallow and deep water beneath a random wave field becomes:

1) $$\varphi(\vec{x},z,t) = \iint\left[d\Phi_+(\vec{k})e^{-i\sigma t} + d\Phi_-(\vec{k})e^{+i\sigma t}\right]\exp(i\vec{k}\cdot\vec{x})\cosh\left[k(z-h)\right]/\cosh\left[kh\right],$$

The expression for the first order term in the surface elevation is:

2) $$\zeta(\vec{x},t) = \iint\left[dZ_+(\vec{k})e^{-i\sigma t} + dZ_-(\vec{k})e^{+i\sigma t}\right]\exp\left[i(\vec{k}\cdot\vec{x})\right] \text{ and } Z_\pm = \pm\frac{i\sigma}{g}\Phi_\pm .$$

The exponential form of the z dependence appropriate for infinite water depth is replaced by a cosh term to satisfy the bottom boundary condition of zero flow through the seafloor at $z = h$.

The Fourier transform in time of the wave-wave interaction pressure signal acting near the sea surface is:

3) $$p_2(\omega,\vec{x}) = \rho(u^2 + w^2)\big|_{z=0} = \rho\sum_{s'}\sum_{s''}\iint d\Phi_{s'}(\vec{k}')d\Phi_{s''}(\vec{k}'')\cdot$$
$$[k'k''\tanh(k'h)\tanh(k''h) - \vec{k}'\cdot\vec{k}'']\exp(i(\vec{k}'+\vec{k}'')\cdot x)\delta(s'\sigma'+s''\sigma''-\omega)$$

again following Hasselmann[1,6]. Here s' and s'' =-1 or +1. The general expression (to second order) for the nonlinear component of the wavenumber- frequency spectrum of the near surface pressure field $F_p(\vec{K},\omega)$ caused by coupling of ocean waves with the wave height spectrum $F_\varsigma(\vec{k})$ (described here as a function of the ocean wave horizontal wavenumber $\vec{K}$) becomes:

4) $$F_p(\vec{K},\omega) = \rho^2 g^4 \iint\iint \frac{F_\varsigma(\vec{k}')F_\varsigma(\vec{k}'')}{(\sigma'\sigma'')^2}\left[k'k''\tanh(k'h)\tanh(k''h) - \vec{k}'\cdot\vec{k}''\right]^2$$
$$\delta(\vec{k}'+\vec{k}''-\vec{k})\delta(\sigma'+\sigma''-\omega)d\vec{k}'d\vec{k}''$$

from (Hasselmann[1], equation 2.13, except a second term corresponding to forcing at small phase velocity has been dropped). The symbols: k' and k'' (without vector notation) represent the vertical wavenumbers of pairs of ocean waves associated with horizontal wavenumbers $\vec{k}'$ and $\vec{k}''$. The wave frequencies σ' and σ'' are related to the wavenumbers by the dispersion relation $\sigma^2 = gk\tanh(kh)$. Here water density is ρ and g is the gravitational acceleration at the Earth's surface. Hasselmann recasts his result in terms of the frequency-directional spectrum of the original ocean wave field $f_\varsigma(\sigma,\theta)$. The new result for the forcing pressure spectrum (valid only at small horizontal wavenumber $\vec{K}$) is:

5) $$F_p(\vec{K},\omega) \approx \frac{\rho^2 g^2\omega}{2}G(\omega/2,h)\int_{-\pi}^{\pi} f_\varsigma(\omega/2,\theta)f_\varsigma(\omega/2,\theta+\pi)d\theta$$

Here the ocean wave height frequency-directional spectrum $f_\zeta(\omega/2,\theta)$ is evaluated at half the frequency of the forced wave (or planetary mode) reflecting the frequency doubling in the coupling from ocean waves to seismic waves. This expression describes the pressure field forcing either of microseisms or planetary modes in water of any depth. A new factor $G(\omega,h)$ has been added to the original expression of Hasselmann and accounts for the depth dependence of the forcing. In the limit of large water depth (defined as the $kh >> 1$), $G(\omega,h)=1$ and the result reduces to the original Hasselman result.

For the microseism problem, ocean waves are always sufficiently short in wavelength to be considered "deep water waves" except in very shallow water (<40m). (The excitation of microseisms can then be evaluated from equation 5 using Green's functions or other methods[1,7]). Infragravity waves at frequencies relevant to the forcing of Earth normal modes (1 to 30 mHz) are inherently shallow water waves and $G(\omega,h)$ is needed to account for the noncircular particle orbits and depth dependence of the dispersion relation in shallow water. The magnitude of the horizontal and vertical particle velocities in terms of the wave height ζ in finite water depth can be written as: $[|\bar{u}|,|w|]=[\coth(kh),1]\sigma|\zeta|$. For large kh (deep water), the vertical and horizontal particle velocities are equal in magnitude. At increasingly shallower water depths (relative to the wavelength), the particle motions are more and more elliptical, with the water moving mostly horizontally for $kh<<1$. The near surface pressure signal caused by the quadratic nonlinearity of the ocean wave equations is proportional to mean squared particle motion at the free surface. Thus the larger particle motion (for a given wave height) in shallow water leads to an enhancement in the strength of the wave-wave interaction.

An expression for $G(\omega,h)$ is derived in terms of the water wave phase C and group velocities U as a function of frequency and depth:

$$6)\quad G(\omega/2,h)=\left[\frac{C(\omega/2,\infty)}{C(\omega/2,h)}\right]^3\frac{U(\omega/2,h)}{U(\omega/2,\infty)}\left[\frac{1+(C(\omega/2,h)/C(\omega/2,\infty))^2}{2}\right]^2$$

The three terms in G in equation 6 can be related to 1) the effect of larger wavenumber at smaller water depth (for fixed frequency) on the terms proportional to k^4 in equation 4, 2) a term related to changing variables from a wave height wavenumber spectrum to a frequency spectrum, and 3) a term related to the ellipticity of the particle motion in shallow water.

The continental shelves of the ocean basins range in width from a few km to more than a thousand kilometers with an average width of about 100km. Depths on the shelf range from the coastline (0m) to more than 200m. It is difficult to define a "mean depth" for the shelves, but I use 30m in the calculations below representing a typical depth near the shoreline. Roughly 1/3 of the world's shelf area is shallower than this depth[8]. Infragravity wave height spectra are considerably more energetic on the shelf than in deep water. The source of infragravity waves is at the coastline, with very little energy leaking off the shelf. Infragravity wave amplitudes decrease with increasing water depth because the conservation of energy flux requires amplitudes to decrease as waves travel in deeper water and because strong bathymetric trapping (by the depth dependent phase velocity) acts to refract most of the wave energy back toward the coastline[9-11]. Typical spectral levels near 10 mHz measured by ocean bottom pressure gauges in deep water in the Pacific basin are about $3\text{x}10^4 Pa^2/Hz$ corresponding to surface wave height spectral values of $3\text{x}10^{-4} m^2/Hz$. Deep water infragravity wave heights in the Pacific are quite stable, whereas levels in the western north Atlantic are lower and more variable with little known about other oceans[12].

Shelf spectra are quite variable spatially and temporally. Infragravity wave spectral levels increase toward shallower water and can be very large near the shoreline during storm events. Observations of infragravity waves on the shelf reveal both a "bound" or forced wave component[9] and a free wave component[10]. The bound wave also originates from the nonlinearity of the surface gravity equations. The bound wave component is typically smaller than the free wave component except very near the coastline. Bound waves may be a modest source of Earth normal mode excitation but are ignored in the calculation below.

Measurements show the free and forced infragravity wave energies are related to the square of the incoming swell energy. Observations of infragravity waves on the shelf off California suggest spectral values of $10^{-2}-10^{-1} m^2/Hz$, but wave conditions off California are relatively benign. Observations of infragravity waves made with tide gauges in Queen Charlotte Sound, B.C. show levels exceeding $50 m^2/Hz$ at 1 mHz during a winter storm[13]. The quadratic dependence of the forcing on spectral levels suggest mode forcing will be dominated by the effects of very large waves over the shelf in relatively limited regions. I use a flat spectrum ($2\, m^2/Hz$) as an estimate of the most energetic wave height spectrum over the shelves, recognizing this estimate as quite uncertain. The root mean square wave amplitude in the band from 1 mHz to 30 mHz is then 0.25m.

Earth normal modes are well described as spherical harmonics in a spherical geometry (r,θ,ϕ). The spectral amplitude (a_{nl}^m) of a normal mode at frequency ω excited by a time varying pressure field can be calculated from a spatial integral of the temporal Fourier transform of the pressure field $p(\omega,\theta,\phi)$ over the source region D_j. Ignoring the slight splitting of the eigenfrequencies for different values of *m,* the amplitude of a mode excited by this pressure field will be

$$7)\quad a_{nl}^m(\omega)=\iint_{D_j} d\Omega\,\frac{\omega_{nl}\,p(\omega,\theta,\phi)}{\omega^2\Gamma_{nl}(\omega)}Y_l^m(\theta,\phi);\quad \Gamma_{nl}(\omega)=\left(\frac{\omega_{nl}}{\omega}\right)^2-\left(1+i\frac{\omega_{nl}}{2Q_{nl}\omega}\right)^2$$

from Tanimoto[14], (note $d\Omega = R^2\sin\theta d\varphi d\theta$, R is an Earth radius).

The resulting vertical displacement at any point described by the spherical coordinates (R,θ,ϕ) on the Earth's surface is:

$$8)\quad u_r(\omega,\theta,\phi)=\sum_n\sum_l\frac{U_{nl}^2(R)}{\omega_{nl}}\sum_m a_{nl}^m(\omega)Y_l^m(\theta,\phi)$$

The power spectrum of acceleration at location (θ,ϕ) due to excitation by the small region D_j is then

$$9)\quad A_j(\omega,\theta,\phi)=\omega^4\left\langle u_r^*(\omega,\theta,\phi)u_r(\omega,\theta,\phi)\right\rangle$$

$$=\omega^4\sum_n\sum_{n'}\sum_{l'}\sum_l\frac{U_{nl}^2(R)U_{n'l'}^2(R)}{\omega_{nl}\omega_{n'l'}}\sum_m\sum_{m'}\left\langle a_{nl}^{m*}a_{n'l'}^{m'}\right\rangle Y_l^{m*}(\theta,\phi)Y_{l'}^{m'}(\theta,\phi)$$

and

$$10)\quad \left\langle a_{nl}^{m*}a_{n'l'}^{m'}\right\rangle=\iint_{D_j}\iint_{D_j}\frac{\omega_{nl}\omega_{n'l'}Y_l^{m*}(\theta,\phi)Y_{l'}^{m'}(\theta',\phi')}{\omega^4\Gamma_{nml}(\omega)\Gamma_{n'm'l'}^*(\omega)}\left\langle p^*(\omega,\theta,\phi)p(\omega,\theta',\phi')\right\rangle d\Omega d\Omega'$$

The extent of any shelf region with coherent pressure fluctuations under an interacting ocean wave field is assumed to be small compared to the wavelengths of the relevant Earth normal modes so that the spherical harmonic describing each Earth normal mode can be approximated as constant across the source region reducing the equation above to:

$$11)\quad \left\langle a_{nl}^{m*}a_{n'l'}^{m'}\right\rangle=\frac{\omega_{nl}\omega_{n'l'}Y_l^{m*}(\theta_j,\phi_j)Y_{l'}^{m'}(\theta_j,\phi_j)}{\omega^4\Gamma_{nml}(\omega)\Gamma_{n'm'l'}^*(\omega)}\iint_{D_j}\iint_{D_j}\left\langle p^*(\omega,\theta,\phi)p(\omega,\theta',\phi')\right\rangle d\Omega d\Omega'$$

where (θ_j,ϕ_j) is the center point of a source region. This approximation becomes poorer toward higher mode order and frequency because the spatial scale of the modes becomes more comparable to the size of the source regions. However the error is probably small compared to the uncertainty in our knowledge of the infragravity wave climatology.

The total forcing of normal modes over the ocean shelves will be calculated by dividing the shelves into small regions which are shelf width L in width and of the same scale in the along shore direction. The across shelf width is set by the rapid decrease in wave height as waves propagate into deep water. The ocean wave field so far has been described in a Cartesian coordinate system appropriate for a flat Earth. The widths of the continental shelves are small compared to the radius of the Earth so the "flat Earth" approximation is valid when considering forcing on the shelves. It is possible to recast the wave-wave interaction problem into spherical coordinates but this leads to excessively complicated mathematics. A paper describing this calculation in spherical geometry is in preparation. The main result is to demonstrate that forcing by infragravity waves within the ocean basins is small compared to the forcing occurring on the shelves.

The integrals over the sphere for the forcing from each region D_j are replaced by integrals in a Cartesian coordinate system (i.e. "flat Earth") $\vec{x} \approx \left[R \sin\theta_j (\phi - \phi_j), R(\theta - \theta_j) \right]$ centered at (R, θ_j, ϕ_j).

The term $\left\langle p^*(\omega,\theta,\phi) p(\omega,\theta',\phi') \right\rangle$ is the Fourier transform in time of the spatial cross correlation function of the near surface pressure field. In Cartesian coordinates it is related to the pressure horizontal wavenumber, frequency power spectrum as:

12) $$\left\langle p^*(\omega,\vec{x}) p(\omega,\vec{x}') \right\rangle = \frac{1}{(2\pi)^2} \iint \exp[i(\vec{K}\cdot(\vec{x}-\vec{x}'))] F(\vec{K},\omega) d\vec{K}$$

Instead of summing over a large number of regions of different area, I assume the specific spatial limits describing each D_j can be replaced by a Gaussian term in the two horizontal directions to represent the range of areas associated with varying shelf widths around the world. The integrals in equation 11 become:

13) $$\iint_{D_j} \iint_{D_j} \left\langle p^*(\omega,\theta,\phi) p(\omega,\theta',\phi') \right\rangle d\Omega d\Omega' \approx$$

$$\iint_{D_j} \mathrm{e}^{-|\vec{x}|^2/2L^2} dx \iint_{D_j} \mathrm{e}^{-|\vec{x}'|^2/2L^2} dx' \iint \exp[i(\vec{K}\cdot(\vec{x}-\vec{x}'))] F(\vec{K},\omega) d\vec{K}$$

Performing the integrals over space first this reduces to:

14) $$\iint_{D_j} \iint_{D_j} \left\langle p^*(\omega,\theta,\phi) p(\omega,\theta',\phi') \right\rangle d\Omega d\Omega' \approx (2\pi L^2)^2 \iint \mathrm{e}^{-|\vec{K}|^2 L^2} F(\vec{K},\omega) d\vec{K}$$

$F(\vec{K},\omega)$ is constant for small $\vec{K}$ so the equation reduces to

15) $$\iint_{D_j} \iint_{D_j} \left\langle p^*(\omega,\theta,\phi) p(\omega,\theta',\phi') \right\rangle d\Omega d\Omega' \approx 4\pi^3 L^2 F(\vec{0},\omega)$$

Equation 9, representing the forcing from a single region becomes:

16) $$A_j(\omega,\theta,\phi) = 4\pi^3 L^2 F(\vec{0},\omega) \sum_n \sum_{n'} \sum_{l'} \sum_l \frac{U_{nl}^2(R) U_{n'l'}^2(R)}{\Gamma_{nl}(\omega) \Gamma_{n'l'}^*(\omega)} \cdot$$

$$\sum_m \sum_{m'} Y_l^{m*}(\theta,\phi) Y_{l'}^{m'}(\theta,\phi) Y_l^{m*}(\theta_j,\phi_j) Y_{l'}^{m'}(\theta_j,\phi_j)$$

The slight dependence of Γ on *m* and *m'* has been ignored in the equation above. The acceleration spectrum is found by summing the contributions from many small regions covering the continental shelves. If we had detailed knowledge of the infragravity wave spectrum over the worlds shelves, equation 10 could be calculated explicitly for each region and the result summed following Tanimoto[14], but we currently know little about the infragravity wave spectrum over the shelf along most of the Earth's coastlines. In particular, measurements of infragravity wave spectra from under very large storm waves from the northernmost coastlines (in winter) and southernmost coastlines (in southern winter) are needed. Given only limited knowledge of the distribution of source regions, I make the simplifying assumption that regions are randomly distributed around the globe. This is clearly incorrect, as evidenced by the biannual cycle seen in mode energy[14,15].

I divide the coastal regions into N source regions of area πL^2, so that the number of regions is:

$$17)\quad N = \frac{\Omega_S}{\pi L^2}$$

The term Ω_S describes the total Earth area covered by continental shelves divided by the effective area of each source region defined above. The fraction f of the total Earth's surface covered by shelves is about 5%, but only a subset of this shelf area is likely to have energetic infragravity waves at any given time and therefore be important for exciting normal modes. The acceleration spectrum excited by the entire continental shelf will be the sum of the contributions from each small region because the waves are assumed to be incoherent between regions. The energies from each source region D_j sum:

$$18)\quad A(\omega,\theta,\phi) = \sum_j^N A_j(\omega,\theta,\phi)$$

The only dependence of the sum on the source region center points is

$$19)\quad \sum_{j=1}^N Y_l^{m*}(\theta_j,\phi_j) Y_{l'}^{m'}(\theta_j,\phi_j) \approx N\left\langle Y_l^{m*}(\theta_j,\phi_j) Y_{l'}^{m'}(\theta_j,\phi_j)\right\rangle \approx \frac{N\delta_{ll'}\delta_{mm'}}{4\pi}$$

This last result is valid if the regions are randomly distributed over the globe so that the expected value of the products of the spherical harmonics is the integral of the spherical harmonics over the unit sphere divided by the area of the sphere. The spherical harmonics are defined to be orthonormal over the unit sphere.

Equation 17 then can be expanded as

$$20)\quad A_j(\omega,\theta,\phi) = 4\pi^3 L^2 F(\vec{0},\omega) \sum_n \sum_{n'} \sum_{l'} \sum_l \frac{U_{nl}^2(R) U_{n'l'}^2(R)}{\Gamma_{nl}(\omega)\Gamma_{n'l'}^*(\omega)} \cdot \sum_m \sum_{m'} Y_l^{m*}(\theta,\phi) Y_{l'}^{m'}(\theta,\phi) \frac{N\delta_{ll'}\delta_{mm'}}{4\pi}$$

using equation 19, equation 20 simplifies to:

$$21)\quad A_j(\omega,\theta,\phi) = \pi^2 L^2 N F(\vec{0},\omega) \sum_n \sum_{n'} \sum_l \frac{U_{nl}^2(R) U_{n'l}^2(R)}{\Gamma_{nl}(\omega)\Gamma_{n'l}^*(\omega)} \sum_m \left|Y_l^{m*}(\theta,\phi)\right|^2$$

Evaluate the sum over m:

$$22)\quad \sum_{m=-l}^{m=l} \left|Y_l^m(\theta,\phi)\right|^2 = \frac{2l+1}{4\pi} P_l(1) = \frac{(2l+1)}{4\pi}$$

An additional simplification is to note that the minimums of the functions $\Gamma_{nl}(\omega)$ occur at the eigenfrequencies of the Earth normal modes, and because the eigenfrequencies are different for different orders $n \neq n'$ the cross terms contribute relatively little to the spectrum reducing equation 16 further to:

$$23)\quad A(\omega,\theta,\phi) \approx \pi^2 L^2 N F(\vec{0},\omega) \sum_n \sum_l \frac{(2l+1)U_{nl}^4(R)}{4\pi|\Gamma_{nl}(\omega)|^2}$$

The vertical acceleration spectrum (equation 22) is independent of the location on Earth as expected for a source that is randomly distributed over the Earth's surface:

$$24)\quad A(\omega) = \pi^2 L^2 N F(\vec{0},\omega) E(\omega) = \pi \Omega_S F_p(\vec{0},\omega) E(\omega)$$

$$E(\omega) = \sum_n \sum_l \frac{(2l+1)U_{nl}^4(R)}{4\pi|\Gamma_{nl}(\omega)|^2}$$

$$F_P(\vec{K},\omega) = \frac{\rho^2 g^2 \omega}{2} G(\omega/2,h) \int_{-\pi}^{\pi} f_\varsigma(\omega/2,\theta) f_\varsigma(\omega/2,\theta+\pi) d\theta$$

Ω_S is the continental shelf area of the Earth.

The excitation is not really randomly distributed over the Earth, because the shelves are not randomly distributed and the ocean wave spectrum is not uniform over the shelves. Some variation in the normal mode spectrum is expected between locations, but the formula provides a useful first approximation.

Figure 3a shows $A(\omega)$ using an estimate for the typical infragravity wave spectrum on the shelf (Fig.1b). The values of the other variables used in the calculation are shown in Table 1. The result depends on an integral of the directional spectrum times the directional spectrum evaluated 180^o around in azimuth (at $\theta+\pi$). Directional spectra from the shelf typically show most of the wave energy at sites more than a few kilometers offshore is directed either toward or away from the coast. Evaluating the azimuthal integral in equation 24 using a measurement of the shelf directional spectrum for infragravity waves (From ref. 10, Fig. 6) yields a factor of 0.17 times the wave height spectrum squared. Our knowledge of the "mean" infragravity wave spectrum on the shelf (both its directional dependence and frequency dependence) is poor. In particular, we have no knowledge of the infragravity wave spectrum in southern regions where the biggest waves occur and it is such areas likely to dominate the excitation of Earth normal modes.

Table 1.

Infragravity wave spectrum	$f_\varsigma(\mathtt{f}) \approx 2m^2/Hz$
Scale of coherent regions on shelf	L=100km
Water depth on shelf	h=30m
Fraction of Earth's area in shelves	$f = 0.05$

The good fit between the shape of the spectrum and the absolute magnitude realized from equation 24 using a reasonable guess for the infragravity spectrum strongly suggests that infragravity waves on the continental shelves are indeed what drives the normal mode signal observed on quiet days seismically (Fig. 3).

The width of the envelope of $E(\omega)$ is controlled by the Q's of the modes. Modal Q's decrease with increasing mode number and frequency. The width of the envelope goes from wide to narrow with increasing frequency accurately reproducing the observed spectrum. I have used the standard PREM model[17] of Earth elastic structure but augmented the model by using the attenuation (Q) structure[18] PA5 which provides a more accurate model of the attenuation structure under the oceans above the transition zone (above 660km depth). PREM does not include a low Q low velocity zone in the aesthenosphere and so overestimates mode Q for modes above 10 mHz.

The dependence of the width of the Earth hum spectral envelope on attenuation has been previously explored by other authors[14,15].

A model of the effect of the gravitational attraction of the changing atmospheric mass[19,20] above a sensor is estimated from a figure in Widmer-Schnidrig[16]. This noise term, which affects only the frequencies below 2mHz was added to the model to better match the spectrum at very low frequency (Fig. 3).

The slope of the function $E(\omega)$ decreases above 3 mHz, so that the predicted spectrum rolls over gently, but a similar gentle hump in the spectrum is found in quiet spectra from most sites. The background noise above the clearly identifiable normal mode spectral peaks has been shown to be describable as propagating Rayleigh waves[22].

The model (equation 24) overestimates the spectra for quiet sites at frequencies above 10 mHz. Quiet sites are always deep within continents. Sites near the shoreline are invariably more energetic reflecting the noise from nearby ocean waves. Missing from the development above is the attenuation of the higher frequency modes with distance into the continents. This problem can be corrected by dropping the assumption of a uniform distribution of source sites, and numerically calculating the forcing over source regions (following Tanimoto[14]), but because the true distribution of sources is not known, I instead model the effect of attenuation into the continents. Each spherical harmonic has an asymptotic approximation as a pair of propagating (Rayleigh) waves valid at large range (Δ) :

$$25)\quad Y_l^0(\theta,\phi)e^{-i\omega t} = P_l(\cos\Delta)e^{ii\omega t}$$

$$\approx \frac{e^{-i\omega t}}{\sqrt{2\pi l\sin\Delta}}\left(\exp[i(l+1/2)\Delta-\frac{i\pi}{4}]+\exp[-i(l+1/2)\Delta+\frac{i\pi}{4}]\right)$$

The attenuation of Rayleigh wave amplitude with distance can be modeled as[23]:

$$26)\quad \exp\left[-\frac{\omega\Delta}{2QU}\right]$$

Here U is the group velocity. The attenuation of the Earth hum signal with distance Δ into a continent can then be modeled by modifying the function $E(\omega)$:

$$E(\omega)=\sum_n\sum_l\frac{(2l+1)U_{nl}^4(R)}{4\pi\left|\Gamma_{nl}(\omega)\right|^2}\exp\left[-\frac{\omega\Delta}{Q_{nl}U_{nl}}\right]$$

The curves in Figure 3c represent attenuation over 0, 2000, and 4000km. This attenuation leads to the formation of a small peak in the predicted spectrum between 8 and 20 mHz. This feature of spectra from quiet sites has been known for some time[24]. The peak is controlled by the attenuation structure of the Earth, and without the higher attenuation of short period Rayleigh waves caused by the LVZ zone near100km depth, the spectrum would be expected to be more energetic near 20 mHz at these quiet sites. The importance of attenuation in forming this peak in the noise spectrum from quiet sites has been previously described by Tanimoto[14].

The good agreement between the estimated Earth Normal mode spectrum and measurements confirms infragravity waves on the continental shelf are the likely source of the Earth's hum. Deep water infragravity waves are a negligible source because of the strong dependence of the $G(\omega/2,h)$ term on inverse water depth and because of the much lower spectral levels in the deep ocean basins, despite the much larger fraction (93%) of the ocean surface underlain by water deeper than 200m (beyond the shelves) and despite the small size of the shelves which limits the horizontal extent of the source region of coherent wave-wave interaction to regions of scales small compared to the wavelengths of normal modes. The contributions from the shelves overwhelm the excitation of normal modes by ocean waves elsewhere over the oceans.

2. A Previous Theory on Excitation by Infragravity Waves and Alternative Mechanisms

A previous report by Tanimoto[14] examined infragravity waves as the energy source for Earth normal modes in the absence of large Earthquakes, but did not identify a mechanism for the forcing. The paper assumed without justification a form for the spatial cross correlation of seafloor pressure variations beneath infragravity waves. The model is not a valid physical model to describe either this process or atmospheric turbulence (see section below). Linear infragravity waves cannot couple significantly into Earth normal modes because the phase speeds of ocean waves and Earth normal modes are very different. Linear infragravity waves drive a simple (inhomogeneous) response in the Earth. This response has been used to study magma beneath ocean ridges[25], but the deformation signal extends only to depths comparable to the ocean wave wavelength.

The Tanimoto paper does provide a convincing case that it is a combination of energetic infragravity waves in the southern ocean in the southern winter (July-Sept) and energetic infragravity waves in the northern ocean in the northern winter (Dec-Feb.) that produces the twice yearly cycle of the Earth's hum.

The transfer of energy between infragravity waves and Earth normal modes requires a nonlinear process to couple energy from the relatively shorter wavelength, slow phase speed infragravity waves into the much faster, longer wavelength Earth normal modes. There are two obvious ways this can happen. The first way is through the inherent nonlinearity of the surface wave equations as described in this letter. The second way is through the interaction of ocean waves with bathymetry. It is known that ocean wave energy couples into "single frequency" microseisms (the peak at 0.07 Hz in worldwide seismic spectra) and it is believed that this coupling happens through the interaction of ocean waves with bathymetry close to shore. Hasselmann[1] addresses the interaction of waves with bathymetry. The problem is quite difficult and obtaining a result involves many simplifications. To couple energy into high phase velocities, waves must interact with bathymetry of comparable wavenumber. The coupling of infragravity waves with bathymetry is necessarily weak in deep water, so the supposition in the Tanimoto paper that there is a significant contribution to the forcing from deep water infragravity waves is unlikely. It is possible that coupling between infragravity waves and shallow water bathymetry does contribute to the excitation of Earth normal modes but the Hasselmann wave-wave interaction mechanism described in this letter appears to be fully adequate to explain the amplitude observed without invoking coupling through bathymetry. The two mechanisms differ in their dependence of the mechanism on frequency. Hasselmann suggests an ω^{-6} dependence for the forcing of microseisms by waves interacting with bathymetry. I found a similar dependence in the infragravity wave band using numerical calculations that followed Hasselmann's formulation but which used a realistic model for the continental shelf, slope and ocean floor. My calculations suggest both that the mechanism is too weak and also that the frequency dependence is wrong in order to fit what is observed for the spectrum of the Earth's hum, but a definitive answer must await further modeling. In particular, the modeling to date assumes normal incidence of the ocean waves to the bathymetry, but real variations in the bathymetry in the along coast direction seem likely to minimize the forcing of planetary modes by shifting the coupling to shorter wavelength (scales comparable to the scales of the along coastline bathymetric variations).

It may be possible to discriminate with observations between the two nonlinear mechanisms by determining the dependence of the forcing on the ocean wave spectrum. The forcing due to the interaction of waves with the bathymetry interaction is expected to depend on the spectrum to the first power, and the nonlinear wave-wave interaction mechanism depends on the ocean wave spectrum squared.

For simplicity, the description of infragravity waves in this manuscript has been for a "flat" (Cartesian) Earth. It is straightforward to extend the description of infragravity waves to a spherical Earth of constant water depth for which the waves can be described as "tsunami' modes of the Earth[23]. These modes are dominated by the effects of gravity and lie on dispersion curves separate from the seismic modes associated with the Earth's hum.

3. Atmospheric Turbulence as a Source of Normal Mode Excitation

A series of papers[5,15,26-27] have invoked atmospheric turbulence as the source of the Earth's background free oscillations. These papers follow earlier work on the excitation of solar normal mode vibrations by turbulence within stars in their derivations. The stellar papers derive the excitation of normal modes within stars using a model for the spatial and temporal cross correlation of pressure fluctuations driven by high Mach number turbulence[28-31]. As with the high phase velocity terms associated with ocean waves, the high phase velocity terms associated with turbulence are controlled by the nonlinear terms in the momentum equation.

The study of sound driven by turbulence goes back to the fundamental paper of Lighthill[32] who showed that sound generation in free space was related to the fourth (and higher) order correlations between velocity fluctuations, thus making turbulence an inefficient generator of sound at low Mach number. Turbulence acting on a boundary or within stratification may be more efficient (see below). Many authors have extended the Lighthill theory to explain the excitation of stellar oscillations by turbulence. The stellar oscillation mechanism predicts a high dependence on Mach number[31]. The strong Mach number dependence suggests low Mach number atmospheric turbulence on the Earth is not likely to be associated with an energetic normal mode spectrum. This Mach number dependence has been ignored in the previous papers on the Earth's hum. The strong Mach number dependence arises because only higher order correlations of the particle velocities in turbulence contribute significantly to forcing of planetary modes. The direct action of turbulent pressure fluctuations acting on the Earth's surface approximately averages to zero over large areas (corresponding to the forcing at the large wavelengths of planetary modes), because producing a nonzero force acting on a patch of the Earth surface requires the transfer of vertical momentum from the turbulence to the Earth. Averaged over areas comparable to planetary wavelengths the vertical momentum in any isolated volume of turbulence averages toward zero because there is no net movement of the center of mass of volumes that are large compared to the largest scales of the turbulence[33].

The strong stratification of the lower atmosphere establishes a thin (1km) turbulent shear layer, "the atmospheric boundary layer" (ABL). Flow fluctuations at the Earth's surface in the normal mode band (periods less than one hour) are dominated by atmospheric boundary layer turbulence[34]. While all levels in the atmosphere experience turbulence, the stratification limits the vertical scale of the turbulence (and hence the horizontal scale, at least at short periods). Turbulence in the boundary layer is driven by wind shear and by heating at the surface.

The pressure signals seen at a sensor on the Earth's surface can be divided into three components: 1) pressure fluctuations from flow very near the sensor, 2) pressure fluctuations from boundary layer turbulence and 3) infrasound[35]. There is a component of the pressure fluctuations under atmospheric turbulence at very small wavenumber (and hence high phase velocity) that can resonantly excite the Earth normal mode spectrum, but it is a tiny component of the pressure spectrum measured at any site.

Measurements of pressure fluctuations beneath boundary layers in the lab might provide an appropriate model for the pressure fluctuations beneath the atmospheric boundary layer, but it has proven remarkably difficult to either measure or predict the high phase velocity components beneath a boundary layer. Recent theory begins to explain much of the wavenumber-frequency spectrum of pressure fluctuations under turbulent boundary layers, but large discrepancies remain between measurements and between theories at small wavenumber[36-39]. The phase velocities associated with Earth normal modes are much faster than the speed of sound in air, so it is the supersonic component of the pressure field beneath atmospheric turbulence that is relevant to this problem.

Recent work converges to the view that the supersonic component of the pressure field at a shear flow boundary is related to the viscous stress so that the pressure frequency-wavenumber spectrum is proportional to the square of the shear stress ($\tau \approx \rho C_D U^2$) at the boundary multiplied by some power of the Mach number ($M = U / c$). One model[39] valid in the supersonic range is:

27) $P(\omega,k) = X(k)\tau^2 M^2 \delta^3 / u_*$

The drag coefficient $C_D \approx 0.0015$ relates the stress to the free stream velocity U. In equation 27, δ is the boundary layer thickness, and c is the speed of sound in air. The scaled spectrum is defined as $X(k)$. The efficiency of production of the supersonic components ($\omega / k > c$) is observed to be higher above a rough surface because of turbulent flow around roughness elements. A model[39] of the wavenumber-frequency spectrum for flow over a rough wall suggests a flat wavenumber spectrum for small wavenumber with $X_0 \approx 10^{-7}$. An estimate of the power in small wavenumber components in the spectrum of the pressure field is obtained by assuming the spectrum is white at small wavenumbers and integrating the wavenumber spectrum over $k < \omega / C_m$ where C_m is the phase speed of a typical Earth normal mode. The relevant component of the pressure spectrum under atmospheric boundary layer turbulence is then about:

28) $P_{|k|<(\omega/U_m)}(\omega) \approx C_D^2 \rho^2 U^4 X_0 \delta^3 M^2 \pi / u_* (\omega / C_m)^2$.

The theory[39] predicts the pressure spectrum (for low Mach number flow) will be at a minimum at small wavenumber and much more energetic at wavenumbers corresponding to the advection velocity: $k_a \approx \omega / U$ forming an advective peak in the spectrum, before falling again toward higher wavenumber. If the turbulence is instead at high Mach number (as is the case for stellar oscillations) then U becomes similar to C_m (flow velocities comparable to the speed of planetary modes), and the planetary mode wavenumbers are instead within the advective peak. For this case, the "stellar model" is an appropriate model for the forcing.

The proponents of mode forcing by atmospheric turbulence assume the pressure fluctuations under turbulence that can drive planetary free oscillations are of order $p_0 = \rho U^2$. This term correctly describes pressure fluctuations that are felt by a structure impeding the flow, but grossly overestimates the source of forcing planetary modes because this pressure term changes sign and will average roughly to zero over a large area[33] (certainly on a scale comparable to a planetary mode wavelength) Kobayashi and Nishida[26] make the assumption that the pressure spatial correlation function has a scale length H equal to the scale depth of the atmosphere:

29) $\langle P(\vec{x},t)P(\vec{x}',t)\rangle \propto \exp(-|\vec{x},-\vec{x}'| / H)$

This implies the horizontal wavenumber spectrum is:

$$30) \quad P(\omega,\vec{k}) = \frac{2\pi P(\omega)}{H\left(4\pi^2(k^2 + 1/H^2)\right)^{3/2}}$$

which approaches a constant value at small wavenumber ($|k| << 1/H$). This form for the spatial correlation function is valid for wavenumbers that are comparable to the reciprocal of the eddy scale in the turbulence (that is of order $k \approx \omega / U$) but wrong at the very small wavenumbers corresponding to planetary modes. Integrating equation 30 over wavenumbers $|k| < \omega / U_m$ suggests the frequency spectrum of pressure fluctuations with wavenumbers small enough to force normal modes should be:

31) $P_{k<U/c}(\omega) \approx 4\pi^2 P(\omega) H^2 \omega^2 / U_m^2$

The authors[26] state "the pressure fluctuations at a frequency f are $\delta p = p_0 f_0 / f$", ($f_0 = U / H$), implying a pressure power spectrum proportional to $(f_0 / f)^2$. Setting the variance in the spectrum equal to $p_0^{\,2}$ for $f \geq f_0$ gives the equivalent pressure spectrum driving the Earth normal modes as

32) $P_{kn}(\omega) \approx 4\pi^2 p_0^2 f_0 / f^2 H^2 \omega^2 / U_m^2 \approx p_0^2 HU / U_m^2$.

For any reasonable choice of parameters (see Table 2), the spectral estimates for the high phase velocity component of the pressure frequency spectrum under turbulence from the expression derived from boundary layer theory (equation 28) will be roughly 150 dB lower than the result from equation 32). The >150 dB difference between the model in this paper and previous models comes from 1) $X_0 \approx -70dB$ (spectral levels are down 70 dB relative to the advective peak), 2) $u_*^4 / U^4 = C_D^2 \approx -55dB$ (the ratio of the friction velocity to the flow velocity to the fourth power) and 3) $M^2 \approx -39dB$ (Mach number squared) with all other terms of order 1 at $f = f_0$. Atmospheric turbulence from the boundary layer is a negligible forcing term for Earth seismic normal modes as there just is very little energy in pressure fluctuations at high phase velocity for low Mach turbulence charactering the Earth's atmosphere.

Atmospheric turbulence above the boundary can generate infrasound[40] and the high phase velocity component of this could couple in planetary modes. However, the excitation of sound by atmospheric turbulence is expected to depend on the Mach number to the fifth power suggesting the boundary layer component (which depends on Mach number squared) will dominate the excitation of planetary modes for low Mach number turbulence.

An array of microbarographs was recently used to investigate the wavenumber and frequency spectrum of pressure fluctuations at the surface of the Earth[41]. The array detected infrasound propagating across the array at velocities between the approximate speed of sound in air (350 m/s) and 1 km/s (associated with waves arriving from above the horizon). It also detected more slowly propagating (<100 m/s) internal gravity waves in the atmospheric boundary layer. However, the aperture of the array (20km) was far to small to resolve wavelengths comparable to low order Earth normal modes (400km for ***L***=100, ***N***=0, *f*=7 mHz) so the estimate provided in the report[40] of the high phase velocity components of the pressure field (that might be associated with Earth normal mode excitation) is dominated by energy at wavenumbers too large (and phase velocities too slow) to excite Earth normal modes. It is expected that the infrasound component corresponding to planetary mode wavelengths and phase speeds will be negligible except for the infrasound driven by solid earth modes after excitation by earthquakes.

Table 2.

Density of air	$\rho_{air} = 1.3kg / m^3$
Drag coefficient	$C_D = 0.0015$
Free stream velocity	U=3.8 m/s
Mach Number	M=0.011
Speed of Sound in Air	C=350 m/s
Scale height of atmosphere	H=8.7 km
Boundary layer thickness	$\delta = 1km$
Shear stress	$\tau = 0.03kg / (ms^2)$
Typical mode phase velocity	$C_m \approx 5km / s$

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