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The Earth's hum: the excitation of Earth normal modes by ocean waves

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SUMMARY

The recent discovery that the seismic normal modes of the Earth are excited to a nearly constant level during seismically quiet days ('the hum of the Earth') has lead to much speculation as to what drives the observed background in the absence of large earthquakes. Other authors have shown that the hum cannot be explained by the many small earthquakes occurring each day and have also precluded excitation by 'slow' earthquakes as the source. The non-linear interaction of low frequency ocean waves (infragravity waves) generates high phase velocity components that excite the seismic normal modes of the Earth. I show the forcing of the normal modes of the Earth by infragravity waves interacting over the continental shelves and over the deep ocean basins is sufficient to explain the background seismic acceleration spectrum observed during seismically quiet days. Atmospheric turbulence is shown to be a negligible source of the Earth's hum. Observations suggesting enhanced spectral levels for two seismic modes that couple strongly with atmospheric modes (${}_0S_{27}$ and ${}_0S_{39}$) have been cited as evidence that the source of the hum lies in the atmosphere. However, the wave interaction mechanism also couples energy into infrasound and thus may explain these observations. A calculation of the coupling between ocean waves and normal modes expected from ocean waves interacting over the shelves reproduces the vertical acceleration spectrum observed at quiet seismic sites from 2 to 40 mHz including the small 'hump' between 5 and 15 mHz and the observed rise at higher frequencies. The model diverges above 40 mHz because the 'single frequency' microseism peak is generated by a different mechanism. The shape of the hum spectrum is controlled primarily by the elastic properties of the Earth with the attenuation structure controlling mode amplitudes. A second calculation shows the excitation of Earth's normal modes by infragravity waves interacting over the deep ocean basins can also explain the observed spectrum of the Earth's hum. Infragravity waves are less energetic and the forcing mechanism weaker over deep water, but the integrative effect of forcing over a large area offsets the weaker forcing per unit area. It is likely that both pelagic and shelf ocean wave sources contribute significantly to the forcing of the Earth's hum and both coastal and pelagic source regions have been identified using seismic techniques. Infragravity wave spectra in deep water over the Pacific basin vary little, thus explaining the small variation in the hum that is observed. However, time varying source regions are also observed in the hum, and may be best explained by intense near shore sources. The great similarity between the spectra predicted from continental shelf forcing and from forcing over the deep ocean basins precludes discriminating between coastal and pelagic sources based on the shape of the spectrum. A more complete knowledge of the climatology of infragravity waves coupled with better seismic observations are needed to resolve this question.

Key words: Surface waves and free oscillations; Theoretical seismology.

1 INTRODUCTION

Much of our understanding of the interior of the Earth has been inferred from measurements of the frequencies of the normal modes of the Earth following excitation by large earthquakes. Advances in seismometry and analysis lead to the discovery that some Earth modes were excited to detectable levels in the absence of large Earthquakes (Fig. 1). It was soon shown that the excitation expected from the many small Earthquakes that occur each day was insufficient to explain the observed levels (Tanimoto 2001). A search for 'slow' earthquakes that could provide the necessary energy in the 200–400 s band confirmed these events also did not explain the background normal mode spectrum

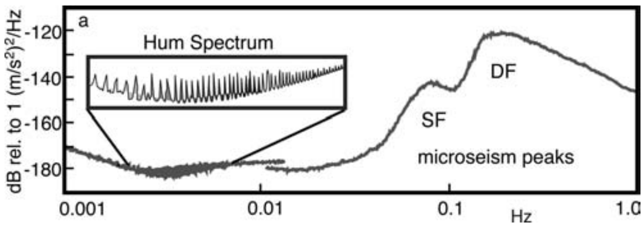

Figure 1. Vertical Acceleration spectrum from a quiet site (BFO) redrawn from figure by R. Widmer-Schnidvig. Insert: expanded view showing resonance peaks associated with earth normal modes.

observed (Ekström 2001). The background excitation spectrum is nearly constant during the year except for a small biannual cycle with maxima in mid-January and mid-July (e.g. Ekström 2001). Some sites may exhibit an annual signal (Nishida *et al.* 2000; Roult & Crawford 2000). The existence of a seasonal cycle also eliminates earthquakes as the source of the background signal. The persistent background spectrum has been called the 'hum of the Earth' (Nishida *et al.* 2000).

It has been known for more than 60 yr that ocean waves drive a persistent, energetic seismic noise peak in a narrow frequency band between 5 and 7 s period that is known as the 'microseism peak' (Fig. 1). Longuet-Higgins (1950) explained how pairs of ocean waves of similar frequency but travelling in nearly opposing directions will interact through the quadratic non-linearity of the ocean surface gravity wave surface boundary conditions to force high phase velocity components at close to twice the frequency. These components couple into acoustic waves in the ocean and seismic waves below the seafloor.

Longuet-Higgins (1950) provides a easily understood description of how interacting ocean waves can excite much higher phase velocity seismic waves. Consider a pair of oppositely travelling waves interacting so as to generate a standing wave over the entire shelf. During one part of the cycle of the standing wave, the surface of the ocean will be flat. A quarter cycle later, the two waves constructively interfere and the ocean surface is a sinusoid. A quarter cycle later, the ocean is again flat, and a quarter cycle later the surface is again a sinusoid. During the transition from a flat surface to a sinusoid, the water moves from the troughs into the peaks so during each half cycle, the centre of mass of the water lying over the entire shelf goes up and down once. This movement of the centre of mass requires a force that is exerted simultaneously over the entire shelf. During a full cycle of the standing wave, the centre of mass goes up and down twice, producing a pressure signal below the sea surface at a frequency equal to twice the frequency of the ocean waves making up the standing wave.

It is therefore, reasonable to hypothesize that a similar mechanism might be coupling energy from low frequency ocean waves into Earth normal modes. Recent observations (Rhie & Romanowicz 2004, 2006) using beamforming and noise cross-correlation (Shapiro *et al.* 2006; Nishida & Fukao 2007) suggest source regions are found primarily, and probably exclusively over the oceans. In Section 2 of this paper, I estimate the forcing of Earth normal modes by low frequency ocean waves over the continental shelves. Ocean waves at periods longer than can be generated directly by the wind (longer than about 25 s) are called 'infragravity waves'. The spectrum of waves driven directly by the wind is limited by the finite size of ocean basins (fetch) and the maximum sustained winds occurring over the oceans. Infragravity waves are generated from wind and swell waves by non-linear mechanisms in shallow water.

I extend to water of finite depth (shallow water waves) Hasselmann's (1963) calculation of the forcing of seismic waves by the Longuet-Higgins mechanism. Tanimoto (2007) provides a related derivation in a normal mode context. I find substantial enhancement of the mechanism at low frequencies (see also Webb 2007). This derivation is applicable to the excitation of normal modes by ocean waves on the continental shelves. The efficiency per unit area (for a given wave height) of the coupling from ocean waves to long wavelength pressure fluctuations (and hence to Earth normal modes) is proportional to the inverse water depth and, therefore, the mechanism is stronger in shallow water. Infragravity waves are also usually much more energetic over the shelf than over the ocean basins. Thus although shelves represent a small fraction of the Earth's surface, the forcing over the shelves can explain the forcing of the Earth's hum.

The 'flat Earth' derivations of Section 2 are not immediately applicable to estimating the excitation over the large ocean basins. Section 3 of this paper adapts the Hasselmann (1963) formulation to a spherical geometry. In a spherical geometry, infragravity waves are best described as 'tsunami modes' of the whole Earth (Ward 1980; Dahlen & Tromp 1998). Ocean waves modes act coherently over the ocean basins to drive Earth normal modes so that the total excitation is proportional to the area of the ocean basins squared. So although infragravity waves are much smaller in deep water, the large area of the ocean basins counteracts the effects of less energetic wave height spectrum, thus the excitation of the Earth normal mode background spectrum can also be explained by ocean waves acting over the deep ocean basins. It is likely that the observed Earth's hum spectrum is driven by a combination of coastal and pelagic sources.

Infrasound and the acoustic modes of the atmosphere are also excited by interacting ocean waves. The generation of 'microbaroms' at 5–7 s period by waves at sea has been described previously by Donn & Rind (1972) and more recently by Garces *et al.* (2003). Nishida *et al.* (2000) proposed that two seismic modes: the ${}_0S_{29}$ and ${}_0S_{37}$ normal modes of the Earth were excited above the adjacent seismic modes in the Earth's hum because these modes were strongly coupled to the corresponding acoustic modes of the atmosphere. This result has been cited as evidence that atmospheric turbulence is the primary source of the hum rather than ocean waves (Nishida & Fukao 2007). Kurrle & Widmer-Schnidrig (2008) show slightly raised vertical component amplitudes at both 3.7 and 4.4 mHz corresponding to ${}_0S_{29}$ and ${}_0S_{37}$. They also detected a large peak at 4.4 mHz on the horizontal components and suggest this may be associated with a coupling between the ${}_0S_{37}$ and

$_0T_{35}$ modes. I estimate the coupling of ocean waves into infrasound in Section 4 of this paper and show the wave interaction mechanism also predicts some slight enhancement of the background spectrum at the frequencies corresponding to these coupled atmospheric-seismic modes.

A previous paper invoked linear long period (infragravity) ocean waves as the source of continuous normal mode excitation (Tanimoto 2005), but the model for the spatial cross-correlation of pressure fluctuations under linear infragravity waves was incorrect. Section 5 briefly describes previous work on mode forcing by ocean waves.

Many papers have suggested that pressure fluctuations under atmospheric turbulence are the primary driving force for background normal mode oscillations, (Kobayashi & Nishida 1998; Tanimoto & Um 1999; Tanimoto 2001; Fukao *et al.* 2002). In Section 6, I show these papers greatly overestimate the strength of this source because they ignore the strong Mach number dependence of coupling between turbulence and sound (e.g. Lighthill 1952, Stein 1967). Recent work on the production of infrasound by convection in thunderstorms includes thermo-acoustic sources, and confirms the low efficiency of sound production by atmospheric turbulence at low Mach number (Akhalkatsi *et al.* 2004). Sound generated by atmospheric turbulence above the surface must first travel through the atmosphere as infrasound before coupling into the ground.

In Section 6, I estimate the forcing of seismic modes by turbulence within the atmospheric boundary layer where pressure fluctuations can act directly on the surface. Pressure fluctuations proportional to the wind velocity squared are expected at the Earth's surface within the atmospheric boundary layer, but only a small component will be associated with the high phase velocities required to force Earth seismic normal modes. High phase velocity components originate out of higher order correlations of velocity within the boundary layer, but at low efficiency for small Mach number (e.g. Bull 1996). Atmospheric turbulence is not likely to drive an important component of the observed Earth normal mode background spectrum.

Thomson *et al.* (2007) detected a possible correlation between magnetospheric fluctuations and band pass filtered (31 min $\pm$ 15 min pass band, 0.36–1 mHz) seismic observations at one site (PFO) and have suggested magnetic field variations driven by the solar wind as a source of the hum. They cite evidence for a daily cycle in this seismic record as evidence that the source cannot be due to ocean waves. However, Forbriger (2007) demonstrates that many broad band seismic instruments, including the STS-2 (such as at PFO) have significant sensitivity to magnetic field fluctuations, responding erroneously to magnetic storms. Thus the correlation observed by Thomson *et al.* (2007) could simply be an instrumental artifact as the intensity of magnetic fluctuations exhibits a strong daily cycle due to solar heating of the ionosphere. Thomson *et al.* (2007) citing Suda *et al.* (1998) states 'there are lines in the spectrum that do not correspond to seismic normal modes', but Tanimoto (2005) noted 'a close match between modal peaks and the eigenfrequencies of the PREM model unambiguously shows that all peaks are fundamental spheroidal modes' of the Earth.

Thomson & Vernon (2007) and Vernon & Thomson (2007) provide additional evidence of very low frequency ($<$1 mHz) seismic motions driven by the magnetospheric fluctuations. In particular they find evidence in horizontal component data of forcing of torodial oscillations by magnetospheric fluctuations. Solar modes are very high Q relative to Earth modes, so the solar forcing is evident as very narrow lines in the seismic spectra. Kurrle & Widmer-Schnidrig (2008) provide more evidence of continuous excitation of both torodial and spheriodal modes at long period ($<$5 mHz). The excitation of torodial modes does require a mechanism other than wave forcing. The coupling between spheroidal and torodial modes is too weak to explain torodial mode amplitudes comparable to spheroidal mode amplitudes.

Fluctuations in the Earth's external magnetic field can exert a spatially and temporally varying force on the Earth's crust through the interaction of induced currents in the mantle. The converse is also known to be true. Webb & Cox (1986) show evidence of magnetic fields (and associated electrical currents) induced in the ocean and uppermost oceanic crust by seismic motions. Magnetic field fluctuations provide an explanation on how it is possible to excite torodial modes because the forces generated between mostly horizontal induced electric currents in the conductive upper mantle should be perpendicular to the Earth's surface. (One can think of the process as similar to the action of an induction motor with a 'squirrel cage' rotor). Only the upper mantle should be involved because the time varying fields at Earth mode frequencies will decay rapidly with depth into the Earth. The time varying field decays exponentially into the Earth with the skin depth which is proportional to the square root of the period divided by the conductivity. The skin depth at 1 mHz for an upper mantle conductivity of 0.03 S m^{-1} will be on the order of 100 km, so primarily only the fundamental mode would be excited.

The skin depth effect and the red spectrum that characterizes the magnetic field at the Earth's surface suggests that the solar wind forcing should be important primarily a low frequency. Thus the hum is likely to be controlled by ocean waves at higher frequencies in agreement with the many observations showing a correlation between waves and the hum, but the solar wind may be important at low frequencies. Currently there is no evidence for significant forcing by solar oscillations at frequencies above 5 mHz, and all recent observations from seismic arrays appear to demonstrate oceanic sources as the primary cause of the Earth's 'hum'.

2 EXCITATION OF THE NORMAL MODES BY INFRAGRAVITY WAVES ON THE CONTINENTAL SHELF

The equations governing ocean surface gravity waves are non-linear at second order. To compute the non-linear coupling of ocean waves into other components, Hasselmann (1962, 1963) expands the potential describing the ocean wavefield in a perturbation series: $\phi(\vec{x}, z, t) = \phi_1 + \phi_2 + \cdots$. The first order (linear) ocean wavefield is described by the potential $\phi_1(\vec{x}, z, t)$, valid for small amplitude waves (relative to water depth or wavelength). The coupling between ocean waves and seismic waves occurs primarily through the non-linear surface boundary condition:

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial x} = -\frac{\partial}{\partial t}(\nabla\phi)^2 + \cdots \tag{1}$$

Hasselmann (1963) shows the right hand side of these equation is equivalent to a pressure signal appearing just below the sea surface equal to:

$$p_2 = \rho(\nabla\phi_1)^2\,|_{z=0} \approx \rho(u^2 + w^2)\,|_{z=0}. \tag{2}$$

Here ϕ_1 is a potential describing the linear ocean waves (to first order in wave steepness) and (u^2+w^2) the sum of the squares of the vertical and horizontal wave particle velocities evaluated near the sea surface (ρ is the water density). Tanimoto (2007) shows eq. (1) may underestimate the forcing above 0.1 Hz, but is a good approximation at normal mode frequencies.

The quadratic non-linearity in the equations causes one sinusoidal component of the ocean wavefield with horizontal wavenumber $\vec{k}'$ and frequency σ' to interact with a second wave component of horizontal wavenumber $\vec{k}''$ and frequency σ'' to force waves of frequency and wavenumber equal to the sums and differences of the original wave components:

$$\phi_2 \propto a_1\cos[(\vec{k}' + \vec{k}'')\cdot\vec{x} - (\sigma' + \sigma'')t] + a_2\cos[(\vec{k}' - \vec{k}'')\cdot\vec{x} - (\sigma' - \sigma'')t]. \tag{3}$$

If two waves are of very similar frequency, $(\sigma' \approx \sigma'')$ but travelling in nearly opposing directions so that $\vec{k}_1 \approx -\vec{k}_2$, the resultant frequency of the first forced wave component in the above equation will correspond to a wave of roughly double the initial frequency of the ocean waves: $\omega = \sigma' + \sigma'' \approx 2\sigma'$, but with a wavenumber $\vec{k} = \vec{k}' + \vec{k}''$ that is much smaller than the initial wavenumbers in magnitude $(|\vec{k}| \ll |\vec{k}'|)$ corresponding to a much higher phase velocity $c_p = \omega/|\vec{k}|$ than the phase velocity of the ocean waves; $c_w = \sigma'/|\vec{k}'|$. The non-linearity of the surface wave equations thus creates a sea surface pressure field containing very long wavelength components through the interaction of pairs of relatively short wavelength infragravity waves.

Webb (2007), following Hasselmann's (1963) derivation derives an equation for the wavenumber–frequency spectrum $F_p(\vec{K}, \omega)$ of the near surface pressure field that results from the non-linear interaction of ocean waves within a ocean wavefield described by the wave height frequency direction spectrum $f_\varsigma(\omega, \theta)$:

$$F_p(\vec{K}, \omega) \approx \frac{\rho^2 g^2 \omega}{2} G(\omega/2, h) \int_{-\pi}^{\pi} f_\varsigma(\omega/2, \theta) f_\varsigma(\omega/2, \theta + \pi)\,\mathrm{d}\theta$$

$$G(\omega/2, h) = \left[\frac{C(\omega/2, \infty)}{C(\omega/2, h)}\right]^3 \frac{U(\omega/2, h)}{U(\omega/2, \infty)} \left\{\frac{1 + [C(\omega/2, h)/C(\omega/2, \infty)]^2}{2}\right\}^2. \tag{4}$$

The function $G(\omega, h)$ depends on the water wave phase C and group velocities U (as functions frequency and depth). This result is valid only at small horizontal wavenumber ($\vec{k}$ much smaller than an ocean wave wavenumber). See the supplemental information accompanying Webb (2007) for the derivation of this equation. Here the ocean wave height frequency-directional spectrum $f_\varsigma(\omega/2, \theta)$ is evaluated at half the frequency of the forced wave (or planetary mode) reflecting the frequency doubling in the coupling from ocean waves to seismic waves. This expression describes the pressure field forcing either of microseisms or planetary modes in water of any depth. The factor $G(\omega, h)$ accounts for dependence of the ocean wave properties on water depth (Fig. 2). In the limit of large water depth and high frequencies (defined as the $kh \gg 1$), $G(\omega, h) = 1$.

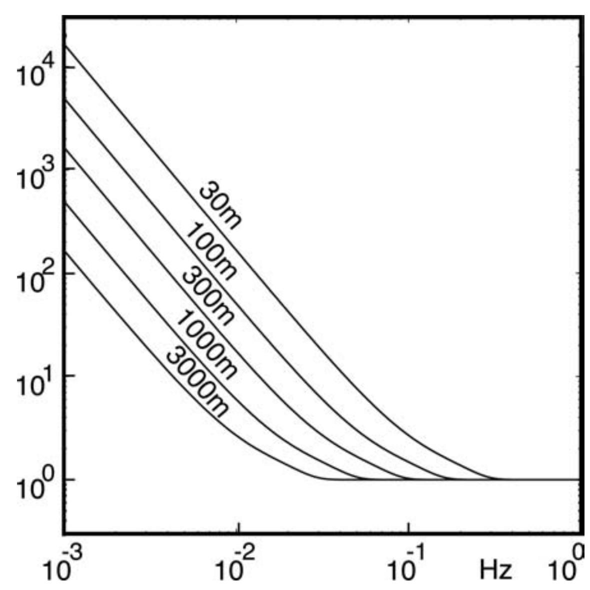

Figure 2. The function $G(f, H)$ describing the relative strength of non-linear wave interaction is shown for five depths H versus frequency f.

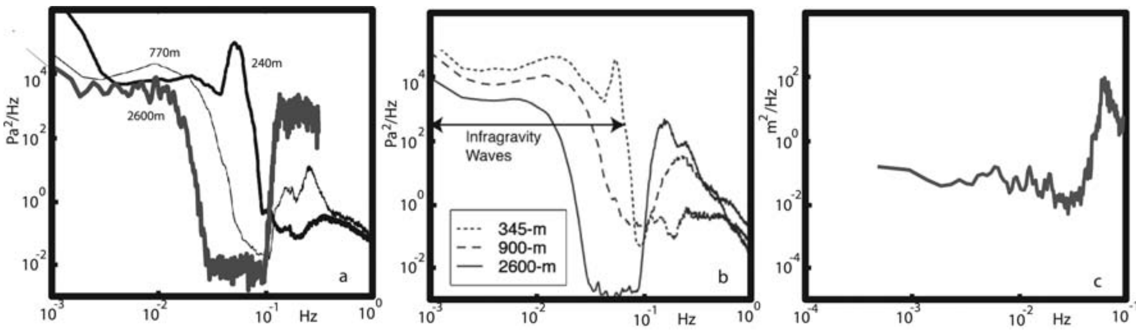

Figure 3. (a) Pressure spectra from three water depths in the Northern Pacific. The two shallow sites are from near San Diego, the deep site is from the East Pacific Rise near 10°N. (b) Pressure spectra from three water depths in the Northern Atlantic near the Faroes Islands (Crawford *et al.* 2005). (c) Energetic wave height spectra from offshore of Florida (redrawn from Herbers 2006).

For the microseism problem, ocean waves are always sufficiently short in wavelength to be considered 'deep water waves' except in very shallow water (<40 m). The excitation of microseisms can then be evaluated from eq. (4) using Green's functions or other methods: Hasselmann 1963; Webb 1992). Infragravity waves at frequencies relevant to the forcing of Earth normal modes (1–30 mHz) are inherently shallow water waves and $G(\omega, h)$ is needed to account for the noncircular particle orbits and depth dependence of the dispersion relation in shallow water. The magnitude of the horizontal and vertical particle velocities in terms of the wave height ζ in finite water depth can be written as: $[|\vec{u}|, |w|] = [\coth(kh), 1]\sigma\zeta$. For large kh (deep water), the vertical and horizontal particle velocities are equal in magnitude. At increasingly shallower water depths (relative to the wavelength), the particle motions are more and more elliptical, with the water moving mostly horizontally for $kh \ll 1$. The near surface pressure signal caused by the quadratic non-linearity of the ocean wave equations is proportional to mean squared particle motion at the free surface. The larger particle motion (for a given wave height) in shallow water leads to an enhancement in the strength of the wave–wave interaction forcing, suggesting the interaction in shallow water may dominate the forcing of seismic waves by ocean waves near the continents.

The continental shelves of the ocean basins range in width from a few km to several hundred kilometres with an average width of perhaps 100 km. Depths on the shelf range from the coastline (0 m) to 200 m. It is difficult to define a 'mean depth' for the shelves, but I use 30 m in the calculations here, representing a typical depth near the shoreline. Roughly 1/3 of the world's shelf area is shallower than this depth (Wunsch 2005).

Infragravity wave height spectra are considerably more energetic on the shelf than in deep water. The source of infragravity waves is at the coastline and only a little energy leaks off the shelf. Infragravity wave amplitudes decrease with increasing water depth because the conservation of energy flux requires amplitudes to decrease as waves travel into deeper water and because strong bathymetric trapping (by the depth dependent phase velocity) acts to refract most of the wave energy back toward the coastline (Herbers *et al.* 1994, 1995a, b). The phase speed of an infragravity wave changes with depth (h) following the dispersion relation. In the shallow water limit ($kh \ll 1$), the phase and group velocities for ocean waves are equal: $C = U = \sqrt{gh}$. Increasing water depth (and thus phase velocity) with distance from shore creates a waveguide with most of the Infragravity wave energy trapped as 'edge waves' near the coast. Slightly further from the coastline, on the flatter parts of shelf, observations show most of the wave energy is associated with waves travelling roughly perpendicular to the coast (Herbers *et al.* 1995a).

Fig. 3(a) shows a typical infragravity wave bottom pressure gauge spectrum from deep water in the Pacific. Spectral levels at 10 mHz in deep water are about 10^4 Pa^2 Hz^{-1} corresponding to surface wave height spectral values near 10^{-4} m^2 Hz^{-1}. Deep water infragravity wave heights in the Pacific are quite stable, whereas levels in the western north Atlantic are lower and more variable with little known about other oceans (Webb *et al.* 1991). Crawford *et al.* (2005) find values above 10^3 Pa^2 Hz^{-1} near the Faroes Islands in the North Atlantic in winter (Fig. 3b).

Shelf spectra are quite variable spatially and temporally. Infragravity wave spectral levels increase toward shallower water and can be very large near the shoreline during storm events. Fig. 3(c), shows a surface wave height spectrum inferred from a bottom pressure gauge offshore of Florida following passage of a hurricane. Observations of infragravity waves on the shelf reveal both a 'bound' or forced wave component and a free wave component (Herbers *et al.* 1994, 1995a). The bound wave also originates from the non-linearity of the surface gravity equations. The bound wave component is typically smaller than the free wave component except near the coastline. Bound waves may be a modest source of Earth normal mode excitation but are ignored in the calculation below.

Measurements show free and forced infragravity wave energies are related to the square of the incoming swell energy. Observations of infragravity waves on the shelf off California suggest spectral values of $10^{-2} - 10^{-1}$ m^2 Hz^{-1}, but wave conditions off California are relatively benign. Observations of infragravity waves made with tide gauges in Queen Charlotte Sound, B.C. show levels exceeding 50 m^2 Hz^{-1} at 1 mHz during a winter storm (Rabinovich & Stephenson 2004). The quadratic dependence of the forcing on spectral levels suggest mode forcing will be dominated by the effects of very large waves over the shelf in a few relatively limited regions. I use a flat spectrum (2 m^2 Hz^{-1}) as an estimate of the most energetic wave height spectrum over the shelves, recognizing this estimate as quite uncertain. The corresponding root mean square wave amplitude in the band from 1 to 30 mHz is 0.25 m.

Earth normal modes are well described as spherical harmonics in a spherical geometry (r, θ, ϕ). The spectral amplitude (a_{nl}^m) of a normal mode at frequency ω excited by a time varying pressure field can be calculated from a spatial integral of the temporal Fourier transform of the pressure field $p(\omega, \theta, \phi)$ over the source region D_j. Ignoring the slight splitting of the eigenfrequencies for different values of *m,* the complex amplitude of a mode excited by a near surface pressure field will be

$$a_{nl}^m(\omega) = \iint_{D_j} \frac{\omega_{nl}\, p(\omega, \theta, \phi)}{\omega^2 \Gamma_{nl}(\omega)} Y_l^m(\theta, \phi); \quad \Gamma_{nl}(\omega) = \left(\frac{\omega_{nl}}{\omega}\right)^2 - \left(1 + i\frac{\omega_{nl}}{2Q_{nl}\omega}\right)^2 \tag{5}$$

from Tanimoto (2005), here $\mathrm{d}\Omega = R^2 \sin\theta \mathrm{d}\varphi \mathrm{d}\theta$, and R is an Earth radius. The resulting vertical displacement at any point described by the spherical coordinates (R, θ, ϕ) on the Earth's surface is:

$$u_r(\omega, \theta, \phi) = \sum_n \sum_l \frac{U_{nl}^2(R)}{\omega_{nl}} \sum_m a_{nl}^m(\omega) Y_l^m(\theta, \phi). \tag{6}$$

Here $U_{nl}(R)$ is the normalized vertical displacement of a mode at the free surface. The power spectrum of acceleration at location (θ, ϕ) due to excitation by the small region D_j is then

$$\begin{aligned} A_j(\omega, \theta, \phi) &= \omega^4 \left\langle u_r^*(\omega, \theta, \phi) u_r(\omega, \theta, \phi)\right\rangle \\ &= \omega^4 \sum_n \sum_{n'} \sum_{l'} \sum_l \frac{U_{nl}^2(R) U_{n'l'}^2(R)}{\omega_{nl}\omega_{n'l'}} \sum_m \sum_{m'} \left\langle a_{nl}^{m^*} a_{n'l'}^{m'} \right\rangle Y_l^{m^*}(\theta, \phi) Y_{l'}^{m'}(\theta, \phi) \end{aligned} \tag{7}$$

and

$$\left\langle a_{nl}^{m^*} a_{n'l'}^{m'} \right\rangle = \iint_{D_j} \iint_{D_j} \frac{\omega_{nl}\omega_{n'l'} Y_l^{m*}(\theta, \phi) Y_{l'}^{m'}(\theta', \phi')}{\omega^4 \Gamma_{nml}(\omega) \Gamma_{n'm'l'}^*(\omega)} \left\langle p^*(\omega, \theta, \phi) p(\omega, \theta', \phi')\right\rangle \mathrm{d}\Omega\ \mathrm{d}\Omega'. \tag{8}$$

The forcing thus depends on the frequency spectrum of the spatial cross-correlation function of the near surface pressure from wave forcing $\langle p^*(\omega, \theta, \phi) p(\omega, \theta', \phi')\rangle$ and is related to the wavenumber spectrum derived above as:

$$\left\langle p^*(\omega, \vec{x}) p(\omega, \vec{x}')\right\rangle = \frac{1}{(2\pi)^2} \int\int \exp[i(\vec{K} \cdot (\vec{x} - \vec{x}'))] F(\vec{K}, \omega) \mathrm{d}\vec{K}. \tag{9}$$

This equation is derived for an infinite ocean. The second order pressure signal under waves near the coast will decorrelate because the amplitude of infragravity waves will decrease rapidly as the waves propagate into deep water, and decorrelate in the along-shelf direction on a similar scale because adjacent sections of the coastline will act as independent sources of infragravity waves. The pressure cross-correlation function is, therefore, approximated as a Gaussian function in both directions with a decorrelation scale equal to *L*, the typical shelf width. To estimate the infragravity forcing, the shelves are then divided in independent source regions of area L^2 (see Webb 2007, supplementary information for details of this calculation). The contributions from the regions are then summed. The forcing from each region can be approximated as a point vertical source, because *L* is small compared to wavelength of the relevant Earth normal modes and small compared to the Earth's radius.

With these approximations, the forcing from the *j*th coastal region centred on angular coordinates: (θ_j, ϕ_j) becomes:

$$\begin{aligned} A_j(\omega, \theta, \phi) = 4\pi^3 L^2 F(\vec{0}, \omega) \sum_n \sum_{n'} \sum_{l'} \sum_l \frac{U_{nl}^2(R) U_{n'l'}^2(R)}{\Gamma_{nl}(\omega) \Gamma_{n'l'}^*(\omega)}. \\ \sum_m \sum_{m'} Y_l^{m^*}(\theta, \phi) Y_{l'}^{m'}(\theta, \phi) Y_l^{m^*}(\theta_j, \phi_j) Y_{l'}^{m'}(\theta_j, \phi_j) \end{aligned} \tag{10}$$

The slight dependence of Γ on *m* and *m′* has been ignored in the equation above. The acceleration spectrum is found by summing the contributions from many small regions covering the continental shelves. If we had detailed knowledge of the infragravity wave spectrum over the world's shelves, eq. (8) could be calculated explicitly for each region and the result summed following Tanimoto (2005), but we currently know little about the infragravity wave spectrum over the shelf along most of the Earth's coastlines. In particular, measurements of infragravity wave spectra from under very large storm waves from the northernmost coastlines (in winter) and southernmost coastlines (in southern winter) are needed. Given only limited knowledge of the distribution of source regions, I make the simplifying assumption that regions are randomly distributed around the globe. This is clearly incorrect, as evidenced by the biannual cycle seen in mode energy (Fukao *et al.* 2002; Tanimoto 2005).

I divide the coastal regions into N source regions of area πL^2, so that the number of regions is: $N = \Omega_S/\pi L^2$. The term Ω_S describes the total Earth area covered by continental shelves and is divided by the effective area of each source region defined above. The fraction *f* of the total Earth's surface covered by shelves is about 5 per cent, but only a subset of this shelf area is likely to have energetic infragravity waves at any given time and, therefore, be important for exciting normal modes. The acceleration spectrum excited by the entire continental shelf will be the sum of the contributions from each small region because the waves are assumed to be incoherent between regions. The energies from each source region D_j sum:

$$A(\omega, \theta, \phi) = \sum_j^N A_j(\omega, \theta, \phi). \tag{11}$$

The only dependence of the sum on the source region centre points is

$$\sum_{j=1}^{N} Y_l^{m^*}(\theta_j,\phi_j)Y_{l'}^{m'}(\theta_j,\phi_j) \approx N\left\langle Y_l^{m^*}(\theta_j,\phi_j)Y_{l'}^{m'}(\theta_j,\phi_j)\right\rangle \approx \frac{N\delta_{ll'}\delta_{mm'}}{4\pi}. \tag{12}$$

This last result is valid if the regions are randomly distributed over the globe so that the expected value of the product of spherical harmonics is the integral of the product over the unit sphere divided by the area of the sphere. The spherical harmonics are defined to be orthonormal over the unit sphere.

Eq. (11) then can be written as

$$A_j(\omega,\theta,\phi) = \pi^2 L^2 N F(\vec{0},\omega)\sum_n\sum_{n'}\sum_l \frac{U_{nl}^2(R)U_{n'l}^2(R)}{\Gamma_{nl}(\omega)\Gamma_{n'l}^*(\omega)}\sum_m \left|Y_l^{m^*}(\theta,\phi)\right|^2. \tag{13}$$

Evaluating the sum over m:

$$\sum_{m=-l}^{m=l}\left|Y_l^m(\theta,\phi)\right|^2 = \frac{2l+1}{4\pi}P_l(1) = \frac{(2l+1)}{4\pi}. \tag{14}$$

An additional simplification is to note that the minimums of the functions $\Gamma_{nl}(\omega)$ occur at the eigenfrequencies of the Earth normal modes, and because the eigenfrequencies are different for different orders $n \neq n'$ the cross-terms contribute relatively little to the spectrum reducing eq. (13) further to:

$$A(\omega,\theta,\phi) \approx \pi^2 L^2 N F(\vec{0},\omega)\sum_n\sum_l \frac{(2l+1)U_{nl}^4(R)}{4\pi\,|\Gamma_{nl}(\omega)|^2}. \tag{15}$$

The vertical acceleration spectrum (eq. 15) is independent of the location on Earth as expected for a source that is randomly distributed over the Earth's surface:

$$A(\omega) = \pi\Omega_S F_p(\vec{0},\omega)E(\omega). \tag{16}$$

The forcing is proportional to Ω_S, the continental shelf area of the Earth. The term

$$E(\omega) = \sum_n\sum_l \frac{(2l+1)U_{nl}^4(R)}{4\pi\,|\Gamma_{nl}(\omega)|^2} \tag{17}$$

describes the excitation of Earth under a time varying point vertical force. The strength of the wave–wave interaction forcing is related to the wavenumber frequency spectrum evaluated at zero wavenumber (eq. 4). This can be rewritten as a product of the square of the ocean wave height frequency spectrum $f_\varsigma(\sigma)$ and an integral I_g of the product of the normalized directional spectrum $g_\varsigma(\sigma,\theta)$ with itself evaluated 180° apart:

$$f_\varsigma(\sigma,\theta) = f_\varsigma(\sigma)g_\varsigma(\sigma,\theta) \qquad \int_{-\pi}^{\pi} g_\varsigma(\sigma,\theta)\mathrm{d}\theta = 1$$

$$F_P(0,\omega) = \frac{\rho^2 g^2\omega}{2}\,G(\omega/2,h)[f_\varsigma(\omega/2)]^2 I_g(\omega/2)$$

$$I_g(\omega/2) = \int_{-\pi}^{\pi} g_\varsigma(\omega/2,\theta)g_\varsigma(\omega/2,\theta+\pi)\mathrm{d}\theta. \tag{18}$$

Eq. (16) can be rewritten in terms of the area of the source Ω_S, $E(\omega)$, the square of the ocean wave spectrum and another term $\hat{G}(\omega/2,h)$ that includes everything else:

$$A(\omega) = \Omega_S E(\omega)[f_\varsigma(\omega/2)]^2\hat{G}(\omega/2,h)$$

$$\hat{G}(\omega/2,h) = \frac{\pi\rho^2 g^2\omega}{2}\,G(\omega/2,h)I_g(\omega/2). \tag{19}$$

The assumption of an isotropic source allows us to also estimate the spectrum of horizontal acceleration at any point on the Earth by replacing $E(\omega)$ with $E_h(\omega)$ in eq. (19):

$$E_h(\omega) = \sum_n\sum_l \frac{(2l+1)U_{nl}^2(R)V_{nl}^2(R)}{8\pi\,|\Gamma_{nl}(\omega)|^2}. \tag{20}$$

Here V_{nl} is the normalized surface displacement in the horizontal (azimuthal) direction for the spheroidal mode. The horizontal acceleration at any site is azimuthally isotropic, because the wave forcing is described as a uniform distribution of vertical point force sources everywhere on the Earth. This leads to an added factor of ½ in $E_h(\omega)$ compared to $E(\omega)$. For low frequency spheroidal modes $[V_{nl}(R)/U_{nl}(R)] \approx 0.7$ so that $A_h(\omega)/A_v(\omega)$ will be roughly equal to one quarter. Kurrle & Widmer-Schnidvig (2008) detected horizontal motions that were larger than vertical motions and concluded that much of the horizontal energy was in torodial modes and thus wave forcing could not explain the horizontal spectra observed at quiet sites below 5 mHz. The toroidal modes do not cause vertical motions and so are invisible on the vertical component.

The excitation from the shelves is not really randomly distributed over the Earth, because the shelves are not randomly distributed nor is the ocean wave spectrum uniform over the shelves. Some variation in the normal mode spectrum is expected between locations, but the formula (eq. 19) provides a useful first approximation for predicting the vertical component hum.

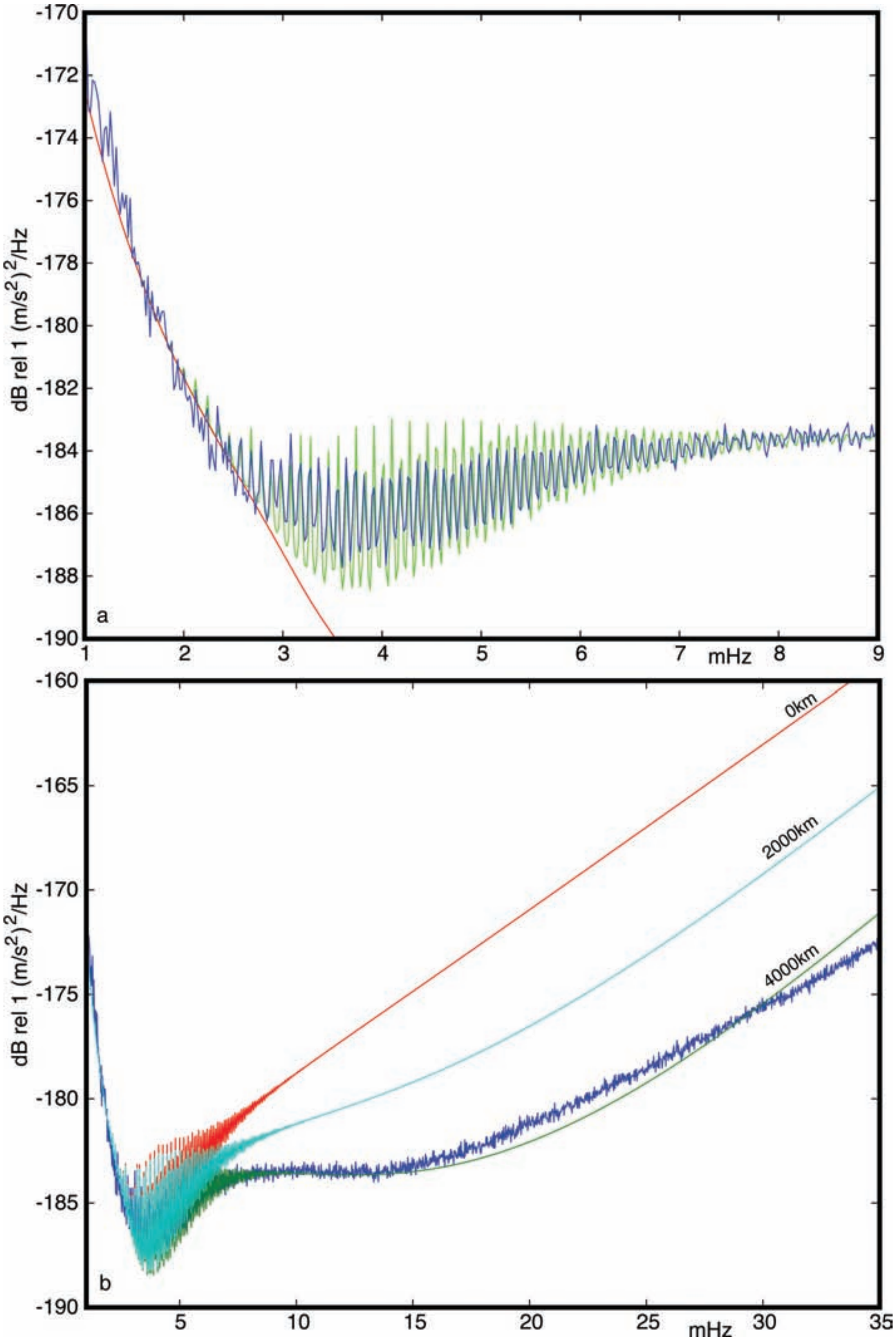

Figure 4. (a) Spectrum predicted from the interaction of energetic infragravity waves over the continental shelf (green) compared with observations of the Earth's hum derived from selected spectra from quiet sites (blue line, Berger *et al.* 2004). A model for the spectra of the time varying gravitational attraction of the atmosphere has been added to the model (red line). Fit assumes a distance of 4000 km from nearest sources to the receiver. (b) Same as (a), except shows predicted spectra for different distances from the receiver to the nearest energetic source.

Fig. 4 shows $A(\omega)$ from eq. (19) using 2 m^2 Hz^{-1} for the infragravity wave spectrum on the shelf. The values of the other variables used in the calculation are shown in Table 1. The result depends on an integral of the directional spectrum times the directional spectrum evaluated 180° around in azimuth (at $\theta + \pi$). Directional spectra from the shelf typically show most of the wave energy at sites more than a few kilometres offshore is directed mostly toward or away from the coast. Evaluating the azimuthal integral I_g in eq. (18) using a measurement of the shelf directional spectrum for infragravity waves (from Herbers *et al.* 1995a, Fig. 4) yields a value of 0.17. In comparison, the value of this integral for an isotropic spectrum would be 0.15. Although the wavefield is nearly bidirectional, the component of the offshore proparating energy is small and the wave interaction is fairly weak. The integral can be arbitrarily large in the limit of a directional spectrum that is two delta functions, although the assumptions of the Hasselmann derivation for the wave interaction spectrum (eq. 4) would no longer be

Table 1. Continental shelf model parameters.

Infragravity wave spectrum	$f_\varsigma(f) \approx 2\mathrm{m}^2\ \mathrm{Hz}^{-1}$
Scale of coherent regions on shelf	$L = 100$ km
Water depth on shelf	$h = 30$ m
Fraction of Earth's area in shelves	$f = 0.05$

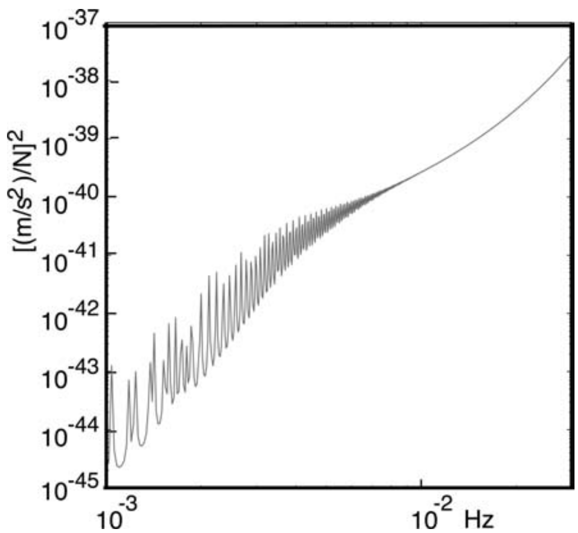

Figure 5. The function $E(f)$ describing the Earth normal excitation by a time varying vertical point force at the Earth's surface versus frequency f.

valid. The minimum value for the integral is zero, corresponding to a wave directional wave spectrum with no waves travelling in opposing directions.

Our knowledge of the 'mean' infragravity wave spectrum on the shelf (both its directional dependence and frequency dependence) is poor. In particular, we have no knowledge of the infragravity wave spectrum in the far southern or northern regions where the biggest waves occur and it is such areas that are likely to dominate the excitation of Earth normal modes.

The good fit between the shape and magnitude of the Earth's hum spectrum at low frequencies and the spectrum predicted from eq. (19) using a reasonable guess for the infragravity spectrum demonstrates that infragravity waves on the continental shelves can explain the hum (Fig. 4). A model for the effect of the gravitational attraction of the changing atmospheric mass (Warburton & Goodkind 1977; Zürn & Widmer 1995) above a sensor was estimated from a figure in Widmer-Schnidrig (2003). This noise term, which affects only the frequencies below 2 mHz was added to the model to better match the spectrum below 2 mHz (Fig. 4).

The shape of the predicted spectrum is primarily determined by the elastic wave velocity and attenuation structure of the Earth as described by the function $E(\omega)$. The width of the envelope of $E(\omega)$ is controlled by the Q's of the modes (Fig. 5). Modal Q's decrease with increasing mode number and frequency. The width of the envelope goes from wide to narrow with increasing frequency accurately reproducing the observed spectrum. I have used the standard PREM model (Dziewonski & Anderson 1981) of Earth elastic structure but augmented the model by using a model of the attenuation (Q) structure: PA5 (Gaherty & Jordan 1996) which is known to provide a more accurate model of the attenuation structure under the oceans above the transition zone (above 660 km depth). PREM does not use a sufficiently low Q in the low velocity zone and so overestimates the mode Q for modes above 10 mHz. The dependence of the width of the Earth hum spectral envelope on attenuation has previously been explored by other authors (Fukao *et al.* 2002; Tanimoto 2005).

The slope of the function $E(\omega)$ decreases above 3 mHz, so that the predicted spectrum rolls over gently, but a similar gentle hump in the spectrum is found in quiet spectra from most sites. The model overestimates the spectra for quiet sites at frequencies above 10 mHz. Quiet sites are always deep within continents. Sites near the shoreline are invariably more energetic because of the noise source from nearby ocean waves. Missing from the development above is the attenuation of the higher frequency modes with distance into the continents. The source regions were assumed to be isotropic. If one knew the true distribution of major source regions, one could drop the assumption of a uniform distribution of source sites, and numerically calculate the forcing over the primary source regions following Tanimoto (2005). Because the true distribution of sources is not known, I instead model the effect of attenuation into the continents to correct the predicted spectrum for interior sites. Implicitly modelling the attenuation into the interior also demonstrates that the shape and amplitude of the hum spectrum at higher frequencies is controlled by attenuation.

The background noise at frequencies above the clearly identifiable normal mode spectral peaks is better described as propagating Rayleigh waves (e.g. Ekström 2001). Each spherical harmonic has an asymptotic approximation as a pair of propagating (Rayleigh) waves valid at large

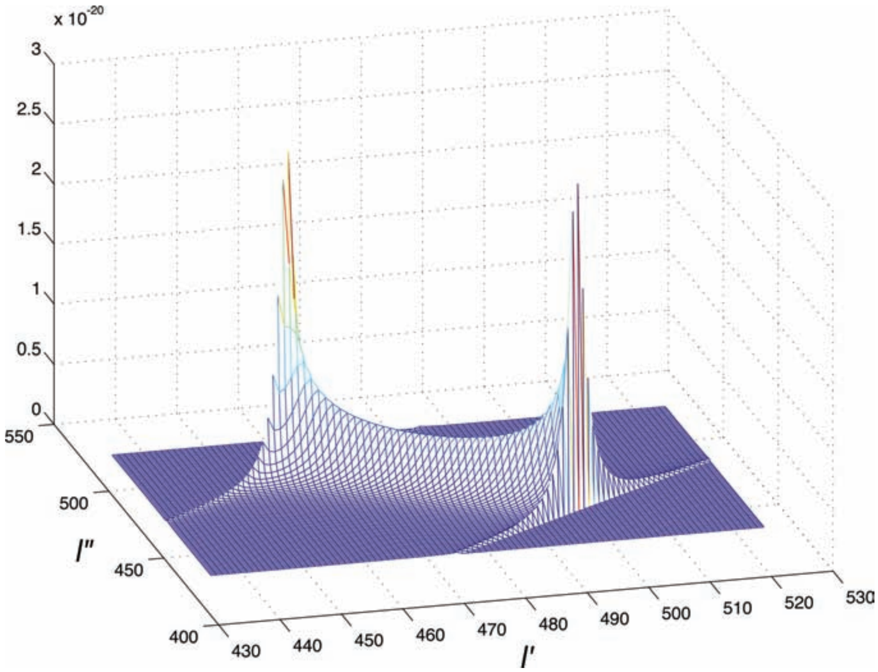

Figure 6. The resulting forcing of Earth normal modes of order $L = 40$, by the interaction of two infragravity wave modes of order $l'$and l''. For any pair l', l'', the forcing occurs at a frequency: $\omega = (l' + l'')\Delta s$.

range (Δ):

$$Y_l^0(\theta,\phi)e^{-i\omega t} = P_l(\cos\Delta)e^{i\omega t} \approx \frac{e^{-i\omega t}}{\sqrt{2\pi l \sin\Delta}}\left\{\exp\left[i(l+1/2)\Delta - \frac{i\pi}{4}\right] + \exp\left[-i(l+1/2)\Delta + \frac{i\pi}{4}\right]\right\}. \quad (21)$$

The attenuation of Rayleigh wave amplitude with distance can be modelled as:

$$\exp\left[-\frac{\omega\Delta}{2Qu}\right] \quad (22)$$

(Dahlen & Tromp 1998). Here u is the group velocity. The attenuation of the Earth hum signal with distanceΔ into a continent can then be modelled by modifying the function $E(\omega)$:

$$E(\omega) = \sum_n \sum_l \frac{(2l+1)U_{nl}^4(R)}{4\pi\,|\Gamma_{nl}(\omega)|^2}\exp\left[-\frac{\omega\Delta}{Q_{nl}u_{nl}}\right]. \quad (23)$$

The three curves in Fig. 4 represent attenuation over distances of 0, 2000 and 4000 km. This attenuation leads to the formation of a small peak in the predicted spectrum between 5 and 15 mHz. This feature of spectra from quiet sites has been known for some time (Peterson 1993). The peak is controlled by the attenuation structure of the Earth, and without the higher attenuation of short period Raleigh waves caused by the LVZ zone near100 km depth, the spectrum would be expected to be more energetic near 20 mHz at these quiet sites. The importance of attenuation in forming this broad peak in the noise spectrum has been previously described by Tanimoto (2005). The good agreement between the estimated Earth Normal mode spectrum and measurements suggests infragravity waves on the continental shelf can be the source of the Earth's hum.

3 FORCING BY INFRAGRAVITY WAVES OVER THE OCEAN BASINS

I consider next the effect of the interaction of infragravity waves over the large ocean basins, on a scale for which the sphericity of the Earth becomes important for the wave–wave interaction mechanism. The first order equation for a potential function describing the ocean waves is the Laplacian $\nabla^2\Phi = 0$. In a constant depth basin, the solution can be described as a sum of spherical harmonics (Ward 1980):

$$\Phi = \sum_l \sum_{m=-1}^{l}\left[A_{lm}\left(\frac{R}{r}\right)^{l+1} + B_{lm}\left(\frac{r}{R}\right)^{l}\right]Y_{lm}(\theta,\phi)e^{is_{lm}t}. \quad (24)$$

On a 'water earth' with a spherical shell of constant water depth these are the tsunami modes of the Earth (Dahlen & Tromp 1998). The solution is useful to describe waves within any region of approximately constant depth on the real Earth.

An approximate solution near the seafloor $\boldsymbol{r} = \boldsymbol{R}$, valid to second order in $(r - R)/R$ is:

$$\Phi \approx \sum_l \sum_{m=-1}^{l} A_{lm} \left[1 - (l+1)\left(\frac{r-R}{R}\right) + \frac{(l+2)(l+1)}{2}\left(\frac{r-R}{R}\right)^2 \right] + B_{lm}\left[1 + l\left(\frac{r-R}{R}\right) + \frac{l(l-1)}{2}\left(\frac{r-R}{R}\right)^2 \right] Y_{lm}(\theta,\phi)e^{is_{lm}t}. \quad (25)$$

Imposing the bottom boundary condition of zero vertical velocity at $\boldsymbol{r} = \boldsymbol{R}$ (accurate to first order in $(r - R)/R$:

$$\left.\frac{\partial\Phi}{\partial r}\right|_{r=R} \approx \sum_l \sum_{m=-1}^{l} [-A_{lm}(l+1) + lB_{lm}] = 0; \quad (l+1)A_{lm} = lB_{lm}. \quad (26)$$

At the sea surface $\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{h}$, the vertical and horizontal velocities (to first order in $\boldsymbol{h}/\boldsymbol{R}$) can be written in terms of the tsunami mode wave heights $Z_{lm} = A_{lm}(2l+1)(l+1)H/(is_l R^2)$ as

$$\begin{aligned} w|_{r=R+h} &= \sum_l \sum_{m=-1}^{l} is_{lm} Z_{lm} Y_{lm}(\theta,\phi)\, e^{is_{lm}t} \\ \vec{u}|_{r=R+h} &= \sum_l \sum_{m=-1}^{l} \frac{is_{lm} Z_{lm} R^2}{hl(l+1)} \frac{1}{R} \nabla_1 Y_{lm}(\theta,\phi) e^{is_{lm}t}. \end{aligned} \quad (27)$$

Here $\nabla_1 = \hat{\theta}\,\partial_\theta + \hat{\phi}(\sin\theta)^{-1}\partial_\phi$ is the horizontal gradient operator (Dahlen & Tromp 1998).

The dispersion relation for the tsunami modes results from applying the kinematic boundary condition at the sea surface:

$$\begin{aligned} &\left.\left(\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial r}\right)\right|_{r=R+h} = 0 \\ &s_l^2 = s_{lm}^2 = \frac{gh\, l(l+1)}{R^2} = ghK^2; \quad K^2 = \frac{l(l+1)}{R^2}. \end{aligned} \quad (28)$$

The dispersion relation and phase velocity for a shallow water wave on a flat Earth are $\sigma^2 = ghk^2; c_p \approx \sqrt{gh}$. For the spherical Earth the dispersion curve for large l is: $s_l \approx (l + 1/2)\sqrt{gh}/R$. For large l in a constant water depth basin, the spacing between the modes is uniform in frequency: $\Delta s = s_{l+1m} - s_{lm} \approx c_p/R$. (The frequencies of the infragravity wave modes are independent of $\boldsymbol{m}$**)**. For a water depth of 4000 m, $c_p \approx 200$ m s^{-1} and $\Delta s/2\pi \approx 0.005$ mHz. The particle velocities at the sea surface can be rewritten using the dispersion relation as:

$$\begin{aligned} w|_{z=R} &= \sum_l \sum_{m=-1}^{l} is_l Z_{lm} Y_{lm}(\theta,\phi) e^{is_l t} \\ \vec{u}|_{z=R} &= \sum_l \sum_{m=-1}^{l} \frac{igZ_{lm}}{Rs_l} \nabla_1 Y_{lm}(\theta,\phi) e^{is_l t}. \end{aligned} \quad (29)$$

The ratio in the second order pressure term between the contributions from the horizontal and vertical velocities is of order $gl/s^2R \approx \sqrt{g/(s^2h)}$. The contribution from the horizontal term dominates the vertical term in shallow water, or at very low frequency when the modes are well described as shallow water waves. The two components are comparable for a water depth of $\boldsymbol{h} = 4000$ m, except for $s/2\pi \ll 0.008$ Hz showing both components are important to deep water forcing of Earth normal modes.

The predicted attenuation of infragravity waves in the deep, open ocean is very small (Lighthill 1979) which would imply very narrow spectral lines in the wave height spectrum in an ocean basin of constant water depth. In a real ocean, water depths vary significantly within a basin causing the wavelength at a given frequency to vary with position. This causes coupling between the modes and mode mixing. The effective Q of any mode should be proportional to the horizontal scale of depth variability within a basin. At 10 mHz, an infragravity wave has a wavelength of 20 km. If the scale length of depth variation across an ocean basin is 1000 km, the effective mode Q's are likely to be less than 50. Later, I will assume the energy is distributed over a band equal to the spacing between modes Δs, because of mode mixing. Adjacent modes are coupled together by bathymetry.

The horizontal gradient of spherical harmonics can be more easily calculated in terms of generalized spherical harmonics Y_{LM}^N:

$$\nabla_1 Y_{lm} = \sqrt{l(l+1)/2}\,\left(Y_{lm}^{-1}\hat{e}_- + Y_{lm}^{+1}\hat{e}_+\right) \quad (30)$$

(Dahlen & Tromp 1998), also $(Y_{lm}^N)^* = (-1)^{m+N} Y_{l-m}^{-N}$, so that

$$\begin{aligned} (\nabla_1 Y_{l'm'})^* \cdot \nabla_1 Y_{l''m''} &= \sqrt{l'(l'+1)/2}\,\sqrt{l''(l''+1)/2}\left(Y_{l'm'}^{-1}\hat{e}_- + Y_{l'm'}^{+1}\hat{e}_+\right)^* \cdot \left(Y_{l''m''}^{-1}\hat{e}_- + Y_{l''m''}^{+1}\hat{e}_+\right) \\ &= -\frac{R^2 s_{l'} s_{l''}}{2gh}(-1)^{m'}\left[Y_{l'-m'}^{+1}Y_{l''m''}^{-1} + Y_{l'-m'}^{-1}Y_{l''m''}^{+1}\right]. \end{aligned} \quad (31)$$

Following Hasselmann (1963), the forcing appears as an apparent pressure just below the sea surface equal to:

$$p(\theta,\phi,t) = \rho(\nabla\Phi\cdot\nabla\Phi)|_{r=R+h} = \rho(\vec{u}\cdot\vec{u}+|w|^2)|_{r=R+h}$$

$$p(\theta,\phi,\omega) = \rho\sum_{l'}\sum_{l''}\sum_{m'=-l'}^{l'}\sum_{m''=l''}^{l''} Z^*_{l'm'} Z_{l''m''}\delta(\omega-(s_{l'}+s_{l''}))$$
$$\times\left[(-1)^{m'}\frac{g}{2h}\left[Y^{+1}_{l'-m'}Y^{-1}_{l''m''}+Y^{-1}_{l'-m'}Y^{+1}_{l''m''}\right]+s_{l'}s_{l''}Y_{l'm'}Y_{l''m''}\right]. \quad (32)$$

There are also terms that depend on the differences between mode frequencies. These terms are dropped from eq. (32) because these are associated with phase velocities too slow to excite Earth normal modes. The terms associated with the difference in frequency between the two infragravity modes dropped from eq. (32) would imply a second order pressure field component at frequency $\omega = \Delta s\,|l'-l''|$. An exclusion principle (shown later) implies non-zero terms only for $\omega \le \Delta s L$ and, therefore, this component of the forcing is associated with phase velocities less than the phase velocity of infragravity waves: $\omega R/L \le c_p$. This term is related to the forcing of bound wave infragravity waves by shorter period ocean waves. Bound infragravity waves travel with the wave groups at a phase speed equal to the group velocity of the short period ocean waves (Herbers *et al.* 1994).

To reduce confusion, from here on, I use capital letters (*L, M, N*) for the indices of Earth normal modes, and small letters (*l, m*) for the indices associated with the infragravity waves. The frequency of an Earth mode is ω_{NL} and the frequency of an infragravity wave mode is s_l.

The mode amplitudes and expected vertical acceleration spectrum at a location due to excitation by distributed pressure field are described by eq. (5)–(8). The spectral amplitude depends on an integral of the pressure field over the source regions as before, but the integrals are now over entire ocean basins, or parts of the ocean basins. The spectral amplitude depends on the spatial integral of the pressure cross-correlation function over the Earth's surface (eq. 8).

If I make the useful, although not necessary, approximation that the phase velocity of the infragravity waves is independent of frequency, then the forcing frequency excited by pairs of infragravity waves of order *l* and *l'* will be $\omega = s_{l'}+s_l = \Delta s(l'+l'')$. The high Q's characterizing Earth normal modes suggests that only those values near resonance such that $\omega_{NL} \approx \Delta s(l'+l'')$ will contribute significantly to the excitation of that mode or $(l'+l'') = l_{\mathrm{sum}} \approx \omega_{NL}/\Delta s$. For water depths of 4000 m, an Earth fundamental normal mode with $L = 26$ has an eigenfrequency $\omega_{NL}/2\pi = 5$ mHz, and is forced by pairs of ocean modes with $l_{\mathrm{sum}} \approx 1000$. I show later that nonzero contributions come only for $|l'-l''| \le L$ so given $l+l' = 1000$ and $L = 26$, then only for $487 \le l' \le 513$ and $487 \le l \le 513$. The mode order of the infragravity waves interacting to excite an Earth normal mode will always be much larger than the mode order of the Earth normal mode. This is as expected as the mechanism invokes two waves of relatively short wavelength interacting to excite a wave of much longer wavelength and at roughly twice the frequency of the infragravity waves.

The power spectrum of acceleration at a location due to excitation over a region is related to the expected value $\langle a^*_{NLM}a_{N'L'M'}\rangle$, and therefore, the spectrum (eq. 8) depends on the fourth order correlation of the mode amplitudes Z_{lm} through the dependence of the second order pressure field (eq. 32) on the product of wave heights Z_{lm}. The equations can be simplified by assuming the Z_{lm} amplitudes are independent, a reasonable expectation for infragravity waves driven from multiple coastal source regions. Then the fourth order correlation simplifies to products of the mode variances:

$$\left\langle Z^*_{l'm'}Z_{l''m''}Z_{l'''m'''}Z^*_{l''''m''''}\right\rangle = \left\langle|Z_{l'm'}|^2\right\rangle\left\langle|Z_{l''m''}|^2\right\rangle\delta_{l'l'''}\delta_{m'm'''}\delta_{l''l''''}\delta_{m''m''''}. \quad (33)$$

The frequency-directional spectrum of infragravity waves in the open ocean has been measured only once (Webb *et al.* 1991). These observations suggest a broadly distributed directional spectrum for infragravity waves in that basin. The seafloor frequency spectrum of pressure fluctuations from infragravity waves has been measured at many deep water locations in the Pacific. The spectrum appears to be quite stable at low frequencies in the Pacific, and is roughly flat at about 3×10^4 Pa^2 Hz^{-1} in the band from 0.002 to 0.03 Hz. This is equivalent to a wave height spectrum equal to about 3×10^{-4} m^2 Hz^{-1}. (The root mean square wave height in the band from 0.002 to 0.03 Hz is only about 3 mm). The spectrum rises as f^{-2} at frequencies below 0.002 Hz (Filloux 1983). The spacing in frequency between modes at low frequency in deep water was shown earlier to be about $\Delta s \approx c_p/R$. If we assume the energy in the modes of a given mode order *l*, is distributed between the modes Z_{lm} evenly over the *m* (in agreement with observations suggesting the directional spectrum is roughly isotropic). Then the spectrum can be written as $|Z_{lm}|^2 = F_l$ and the variance in each mode is:

$$f_\zeta(s_l)\,\Delta s = \sum_{m=-l}^{l}|Z_{lm}|^2\left\langle\left|Y_l^m\right|^2\right\rangle = \sum_{m=-l}^{l}F_l\frac{1}{4\pi};\quad F_l = \frac{4\pi}{2l+1}f_\zeta(s_l)\,\Delta s. \quad (34)$$

The cross-spectrum of the amplitudes of the Earth normal modes $\langle a^*_{NLM}a_{N'L'M'}\rangle$ then can be written as

$$\langle a^*_{NLM}a_{N'L'M'}\rangle = \sum_{l'}\sum_{l''}\sum_{m'=-l'}^{l'}\sum_{m''=-l''}^{l''}\rho^2\frac{\omega_{NL}\omega_{N'L'}F_{l'}F_{l''}}{\omega^4\Gamma_{NL}(s_{l'}+s_{l''})\Gamma^*_{N'L'}(s_{l'}+s_{l''})}\delta(\omega-(l'+l'')\Delta s)$$
$$\times\left[\frac{g}{h}\left(I^H_{l'm'l''m''LM}\right)+s_{l'}s_{l'''}\left(I^V_{l'm'l''m''LM}\right)\right]^*\left[\frac{g}{h}\left(I^H_{l'm'l''m''L'M'}\right)+s_{l'}s_{l'''}\left(I^V_{l'm'l''m''L'M'}\right)\right]. \quad (35)$$

Here the I^V, I^H are integrals over the source region describing the vertical and horizontal velocity contributions to the forcing of the modes:

$$I^H_{l'm'l''m''LM} = \iint_{D_0} \mathrm{d}\Omega Y_{LM} \{1/2(-1)^{m'} \left[\left(Y^{+1}_{l'-m'} \right) Y^{-1}_{l''m''} + \left(Y^{-1}_{l'-m'} \right) Y^{+1}_{l''m''} \right]\}$$

$$I^V_{l'm'l''m''LM} = \iint_{D_0} \mathrm{d}\Omega Y_{LM} Y_{l'm'} Y_{l''m''}. \tag{36}$$

To estimate these integrals, the source regions are assumed evenly distributed over the Earth, so that the expected value of the integrals is just the value of the integral over the whole Earth times the fraction of the area of the Earth covered by the source region. This estimate is approximate for many reasons: the depths of ocean basins varies, so that the infragravity wave modes are not fully described by the spherical harmonics, and the infragravity wave spectrum is expected to vary between basins, and to a lesser extent, within a basin. Spherical harmonic spectral expansions on localized regions of the sphere are analysed by Wieczorek & Simons (2005). The effect of a finite source region is similar to the effect of a finite length window in Fourier analysis: energy leaks into adjacent wavenumber bins. I subsume all of this uncertainty into a fudge factor f_R, which describes the effective fraction of the Earth covered by that ocean basin.

$$\begin{aligned}
I^H_{l'm'l''m''LM} &\approx f_R R^2 1/2 \iint \sin\theta \mathrm{d}\theta \mathrm{d}\phi Y_{LM} \left[(-1)^{m'} \left(Y^{+1}_{l'-m'} Y^{-1}_{l''m''} + Y^{-1}_{l'-m'} Y^{+1}_{l''m''} \right) \right] \\
&\approx \frac{f_R R^2 \, (-1)^{m'}}{2} \left[\frac{(2L+1)(2l'+1)(2l''+1)}{4\pi} \right]^{1/2} \times \begin{pmatrix} L & l' & l'' \\ 0 & +1 & -1 \end{pmatrix} \\
&\quad \times \left[\begin{pmatrix} L & l' & l'' \\ M & -m' & m'' \end{pmatrix} + (-1)^{L+l'+l''} \begin{pmatrix} L & l' & l'' \\ M & -m' & m'' \end{pmatrix} \right] \\
I^V_{l'm'l''m''LM} &\approx f_R R^2 \int\int \sin\theta \mathrm{d}\theta \mathrm{d}\phi Y_{LM} Y_{l'm'} Y_{l''m''} \\
&\approx f_R R^2 \left[\frac{(2L+1)(2l'+1)(2l''+1)}{4\pi} \right]^{1/2} \times \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & l' & l'' \\ M & m' & m'' \end{pmatrix}.
\end{aligned} \tag{37}$$

Here the last two sets of brackets in the equations following the '×" sign are Wigner 3-*j* symbols (Dahlen & Tromp 1998).

These symbols have a series of exclusion rules, in particular, the Wigner 3-*j* symbols are equal to zero, unless $|l' - l''| \leq L$ and $M = m' + m''$. These rules will be shown to require that two waves of similar wavenumber and frequency interact to force a much smaller wavenumber Earth normal mode at a frequency equal to the sum of the other ocean wave frequencies, (approximately twice the frequency of either ocean mode).

This summation property of Wigner 3-*j* symbols:

$$\sum_{m'=-l'}^{l'} \sum_{m''=-l''}^{l''} \begin{pmatrix} L & l' & l'' \\ M & m' & m'' \end{pmatrix} \begin{pmatrix} L' & l' & l'' \\ M' & m' & m'' \end{pmatrix} = \frac{\delta_{LL'} \delta_{MM'}}{2L+1} \tag{38}$$

can be used to simplify eq. (35) because the variance in the modes F_{lm} is assumed to be independent of $\boldsymbol{m}$.

The cross-terms between I^V, I^H in eq. (35) are zero except for $m' = 0$ and $M' = M = -m''$ because otherwise $M + m' + m''$ and $M' - m' + m''$ cannot be zero simultaneously. The cross-terms involving $\langle a_{NLM}{}^* a_{N'L'M'} \rangle$ for $N' \neq N$ are small because $\Gamma_{NL}(\omega)$ and $\Gamma^*_{N'L}(\omega)$ can't both be simultaneously small unless $\omega_{N'L} \approx \omega_{NL}$ but the modes are separated in eigenfrequency for different N or L. Dropping the cross-terms and using the above summation formula to reduce the summations over the $(I^H_{l'm'l''m''LM})^* (I^H_{l'm'l''m''L'M'})$ and $(I^V_{l'm'l''m''LM})^* (I^V_{l'm'l''m''L'M'})$ terms, eq. (35) reduces to:

$$\begin{aligned}
\langle a^*_{NLM} a_{N'L'M'} \rangle &\approx \delta_{NN'} \delta_{LL'} \delta_{MM'} \sum_{l'} \sum_{l''} \rho^2 f_R^2 R^4 4\pi \frac{(\Delta s)^2 \omega^2_{NL} f_\zeta(s_{l'}) \, f_\zeta(s_{l''})}{\omega^4 \, |\Gamma_{NL}(\omega)|^2} \delta(\omega - (l' + l'')\Delta s) \\
&\quad \times \left[\frac{g}{h} \begin{pmatrix} L & l' & l'' \\ 0 & +1 & -1 \end{pmatrix} \right]^2 + \left[s_{l'} s_{l''} \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix} \right]^2 \quad \text{for} \quad L + l' + l'' \text{even}
\end{aligned} \tag{39}$$

Here it is assumed that $s_{l'} + s_{l''} = \Delta s(l' + l'')$. Also

$$\begin{aligned}
\begin{pmatrix} L & l' & l'' \\ 0 & +1 & -1 \end{pmatrix}^2 &= \frac{[l'(l'+1) + l''(l''+1) - L(L+1)]^2}{4l'(l'+1)l''(l''+1)} \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}^2 \\
&\approx \frac{\left(s^2_{l'} + s^2_{l''}\right)^2}{4 s^2_{l'} s^2_{l''}} \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}^2 \approx \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}^2
\end{aligned} \tag{40}$$

for $L + l' + l''$ even. Note $s_{l'}^2 \approx s_{l''}^2$ because $|l' - l''| \leq L$ for $(l', l'' \gg L)$. Thus

$$\langle a^*_{NLM} a_{N'L'M'} \rangle \approx \delta_{NN'}\delta_{LL'}\delta_{MM'} \sum_{l'}\sum_{l''} \rho^2 f_R^2 R^4 4\pi \frac{(\Delta s)^2 \omega_{NL}^2 f_\zeta(s_{l'})\ f_\zeta(s_{l''})}{\omega^4\, |\Gamma_{NL}(\omega)|^2}\, \delta(\omega - (l' + l'')\Delta s)$$
$$\times\ s_{l'}^2 s_{l''}^2 \left(1 + \frac{g^2}{s_{l'}^2 s_{l''}^2 h^2}\right) \left| \begin{pmatrix} L & l' & l'' \\ 0 & 0 & 0 \end{pmatrix} \right|^2 \qquad (41)$$

which shows the horizontal velocities dominate the interaction integral at low frequencies or shallow depth as expected.

The squared amplitude of infragravity wave modes was calculated from the continuous ocean wave spectrum from the spectral power in a band of width Δs (eq. 34). Assuming discrete modes results in a delta function in frequency in eq. (41) and implies the energy is concentrated in spectral lines. To convert back to a continuous power spectrum, I assume that the mode energy is distributed over a band of width Δs. The spectrum is then $S_{NML}(\omega_{l_{\rm sum}}) = \langle a^*_{NLM} a_{NLM} \rangle / \Delta s$. All infragravity waves in band of width Δs corresponding to modes with frequencies such that $\omega_{l_{\rm sum}} = (l' + l'')\Delta s = l_{\rm sum}\Delta s$ combine to excite Earth seismic modes in a band of width Δs.

A difficulty arises because the integrals in eq. (37) are zero for odd values of $L + l' + l''$. Because the frequency of the forcing is related as: $\omega_{l_{\rm sum}} = (l' + l'')\Delta s$, there will be no predicted contribution to the excitation of even order Earth normal modes in the frequency bins corresponding to the odd values of $(l' + l'')$ or similarly no contribution to frequency bins corresponding to even values of $(l' + l'')$ for odd order Earth normal modes. Unless some smoothing is applied, this results in a significant sawtooth pattern to the predicted spectrum on an interval equal to Δs. On a real Earth, the local apparent frequency corresponding to each infragravity wave mode will be doppler shifted by low frequency ocean currents and tides and the spectrum broadened by attenuation. Attenuation of infragravity wave modes is dominated by losses over the continental shelves. The e-folding time for mode as inferred from observations of tsunamis corresponds to roughly the traveltime across the Pacific basin (about 12 hr), implying a mode Q for a 1 mHz frequency mode of roughly 50. Thus the spectral peaks of adjacent modes are expected to overlap over a interval of width of at least several times Δs in frequency. I added the result of evaluating the integrals in eq. (37) at L, l', l'' to the result of evaluating the integrals at $L, l' + 1/2, l'' + 1/2$ to generate a smooth estimate of the predicted forcing across frequency (see Fig. 6).

A seond argument for using this procedure is to note the zero values of the integrals in eq. (37) for odd values of $L + l' + l''$ are the result of the precise spherical symmetry assumed for the forcing. On a real Earth, the forcing will be dominated by the hemisphere containing Pacific with little forcing occurring in the opposite, mostly continental hemisphere. Thus the integrals in eq. (37) might be better replaced by integrals over a single hemisphere. The first two leading terms of a spherical harmonic expansion of a function that is equal to a spherical harmonic Y_{lm} in one hemisphere and zero in the other will be Y_{lm} and Y_{l+1m} with succeeding terms of much lower amplitude (Mochizuki 1992). In the asymptotic limit of large order the summation described above appears to be equivalent to summing the values of the same integrals evaluated at L, l', l'' and $L, l' + 1, l''$ producing a nearly identical final result.

The acceleration power spectrum is then:

$$A(\omega_{l_{\rm sum}}, \theta, \phi) = \omega^4 \langle u_r^*(\omega, \theta, \phi) u_r(\omega, \theta, \phi) \rangle$$
$$= \omega_{l_{\rm sum}}^4 \sum_N \sum_L \frac{U_{NL}^4(R)}{\omega_{NL}^2} \sum_{M=-L}^{L} S_{NLM}(\omega_j) \left[Y_L^M(\theta, \phi) \right]^2$$
$$= \omega_{l_{\rm sum}}^4 \sum_N \sum_L \frac{U_{NL}^4(R)}{\omega_{NL}^2} \frac{2L+1}{4\pi} S_{NLM}(\omega_{l_{\rm sum}}). \qquad (42)$$

Using eq. (14), the summation over M simplifies because S_{NLM} is independent of M.

Fig. 6 shows the contributions to the acceleration spectrum from the interaction of infragravity modes of order l' and l'' and a single Earth mode of order $L = 40$. The figure shows the cut-off of the interaction between modes for $|l' - l''| > L$. The forcing peaks along the lines $|l' - l''| = L$, resembling the result for the flat Earth case where $\vec{k}' + \vec{k} = \vec{K}$, given $\vec{k}' \approx -\vec{k}$. The frequency of the Earth normal mode forcing is equal to the sum of the ocean wave mode frequencies: $\omega = (l' + l'')\Delta s$. The forcing peaks along the line $l' + l'' = 957$, corresponding to $\omega = 0.0297$ rad s^{-1} which is the eigenfrequency of the $L = 40$ Earth normal mode. Increasing distance from the line $l' + l'' = 957$ corresponds to an increasing deviation in frequency from the resonance frequency of the mode and the amplitude of the forcing decreases following $\Gamma_{NL}(\omega)$; $\quad \omega = (l' + l'')\Delta s$.

The double summation over l' and l'' and the delta function in frequency (eq. 41) can be replaced by a single summation over either variable. Eqs (41) and (42) are combined to yield:

$$A_B(\omega_{l_{\rm sum}}, \theta, \phi) = \frac{\rho^2 g^2 f_R^2 R^4 (\Delta s)}{h^2} \sum_N \sum_L (2L+1) U_{NL}^4(R)$$
$$\times \sum_{l_{\rm sum} - 2l' \leq -L}^{2l' - l_{\rm sum} \leq L} \frac{f_\zeta(s_{l'})\ f_\zeta(s_{l_{\rm sum} - l'})}{\left|\Gamma_{NL}(\omega_{l_{\rm sum}})\right|^2} \left| \begin{pmatrix} L & l' & l_{\rm sum} - l' \\ 0 & 0 & 0 \end{pmatrix} \right|^2. \qquad (43)$$

Fig. 7 shows the result of this calculation for the values of the parameters shown in Table 2 for water depths of 400 and 4000 m. The strength of the normal mode forcing varies inversely with the water depth (assuming a constant ocean wave spectrum). The explicit $1/h^2$ dependence in eq. (43), is reduced to $1/h$ because of both the explicit dependence on Δs, and because the number of terms in the summation

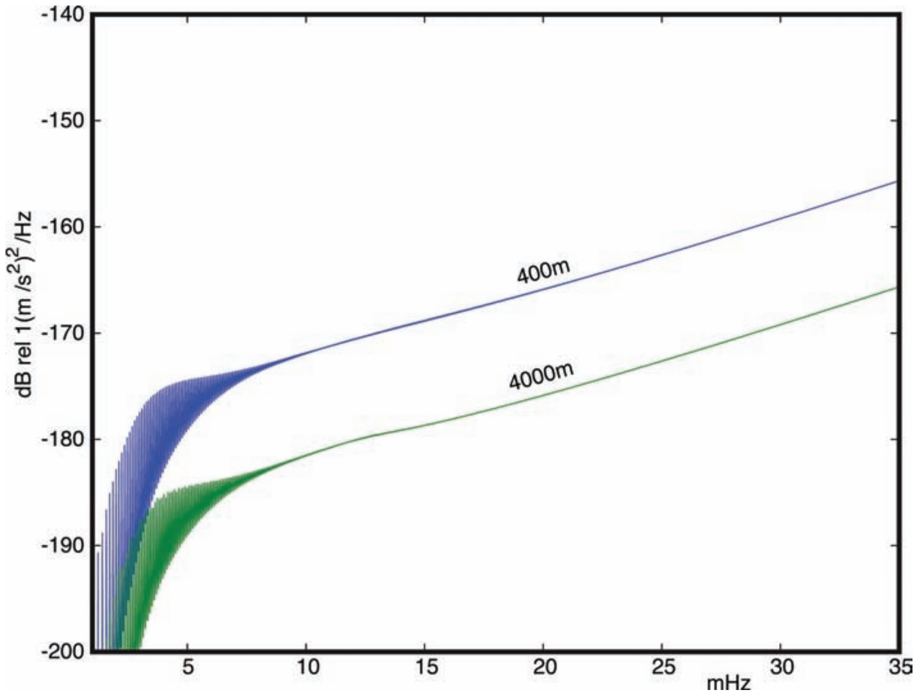

Figure 7. Estimates of the vertical acceleration spectrum that results from Earth seismic normal modes driven by the interaction of infragravity waves over the ocean basins predicted for two different water depths. The reciprocal dependence of the spectrum on water depth is the same as found for the continental shelf result (Fig. 2).

Table 2. Ocean Basin Model Parameters.

Deep Water Infragravity Wave spectrum	$f_\zeta(f) = 3.3 \times 10^{-4}$ m^2 Hz^{-1}
Fraction of Earth with Energetic Ocean Waves	$f_R = 0.5$
Mean water depth	$H = 4000$ m

is proportional to Δs (and $\Delta s \propto \sqrt{h}$). $1/h$ is the same dependence on water depth as predicted by the flat Earth calculation of Section 2 (Fig. 2).

Given the values shown in Table 2, the excitation of Earth normal modes predicted by forcing of ocean waves over ocean basins corresponding to 50 per cent of the Earth's area, adequately explains the observed spectrum of the Earths hum at low frequency (Fig. 8). The same model for the time varying gravitation attraction of the atmosphere as used in Fig. 4 has been added to the model to better explain the very low frequency spectrum. Much as with excitation from coastal sources (Fig. 4), the model overpredicts the spectrum above 10 mHz compared to this realization of the hum spectrum based on spectra from very quiet sites and quiet intervals (Berger *et al.* 2004). Again it is reasonable to assume the discrepancy occurs because the model assumes a uniform distribution of sources. The data the model is being compared to come from sites in the deep interior of continents. Following the same arguments as used to justify eq. (23), I add a term to describe attenuation from the source regions into the deep interior of continents to eq. (43):

$$A_B(\omega_{l_{\rm sum}}, \theta, \phi) = \frac{\rho^2 g^2 f_R^2 R^4 (\Delta s)}{h^2} \sum_N \sum_L (2L+1) U_{NL}^4(R) \times \sum_{l' \geq (l_{\rm sum}-L)/2}^{l' \leq (l_{\rm sum}+L)/2} \frac{f_\zeta(s_{l'})\, f_\zeta(s_{l_{\rm sum}-l'})}{\left|\Gamma_{NL}(\omega_{l_{\rm sum}})\right|^2} \left|\begin{pmatrix} L & l' & l_{\rm sum}-l' \\ 0 & 0 & 0 \end{pmatrix}\right|^2 \exp\left[-\frac{\omega \Delta_x}{Q_{NL} U_{NL}}\right]. \tag{44}$$

Here Δ_x is the distance into the interior, Q_{NL} is the quality factor for the mode and U_{NL} is the group velocity of the mode. The deep water forcing has a slightly different spectrum than the continental shelf spectrum so that the best fit for the deep water spectrum to the data is with $\Delta_x = 2000$, (Fig. 8) whereas the best fit for the continental shelf model assumed a distance of 4000 km into the interior (Fig. 7).

The above result minimizes the potential acceleration spectrum by partitioning the infragravity wave energy evenly between all the possible values of *m* for a given *l*. If instead, the energy is concentrated into a few values of *m*, the interaction will be stronger, and the predicted acceleration spectrum will be larger. This is similar to the observation that the microseism spectral level is higher when the directional spectrum of the ocean waves has energy focused into two opposing directions (e.g. Webb 1992). The maximum spectral levels are found when the energy is restricted to a single value of *m* so that the amplitude spectrum of the modes becomes: $F_{lm} = f_\zeta(s_l)\Delta s\, \delta_{m0}$. This would roughly correspond to waves travelling in a single pair of directions in the high order limit. In this example, only the $M = 0$ Earth normal modes will

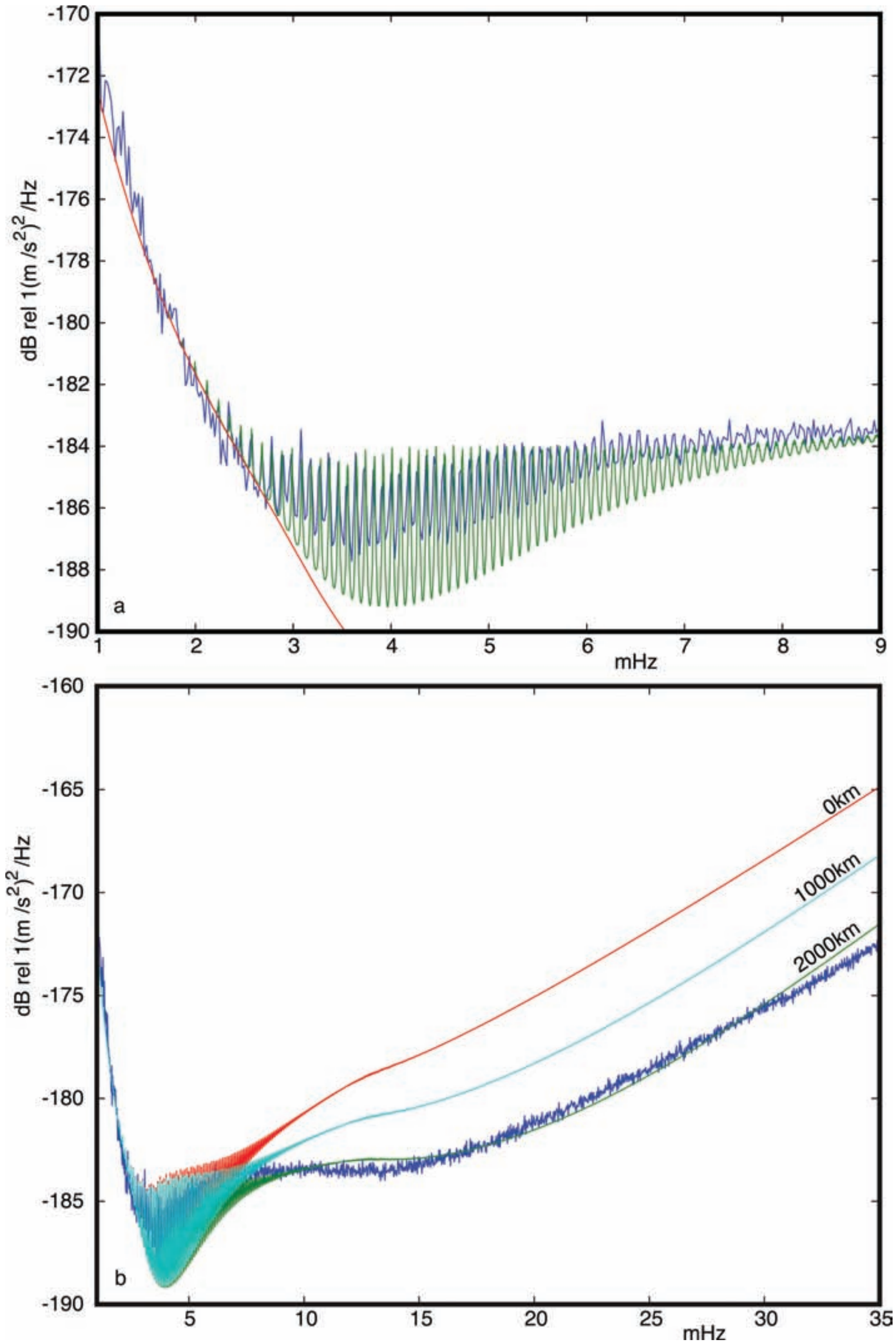

Figure 8. Same as Fig. 4 except here comparing the vertical acceleration spectrum resulting from infragravity waves interacting over the deep ocean basins. (a) assumes the distance from the nearest source region to the site is 2000 km. (b) same as (a), except showing the predicted spectrum for different distances from the receiver to the nearest energetic source.

be excited. The mean spectrum averaged over the Earth will be:

$$\left\langle A(\omega_j)\right\rangle = \frac{\rho^2 g^2 f_R^2 R^4(\Delta s)}{h^2} \sum_N \sum_L U_{NL}^4(R) \times \sum_{l' \geq (l_{\mathrm{sum}}-L)/2}^{l' \leq (l_{\mathrm{sum}}+L)/2} \frac{f_\zeta(s_{l'})\; f_\zeta(s_{l_{\mathrm{sum}}-l'})(2l'+1)(2(l_{\mathrm{sum}}-l')+1)}{\left|\Gamma_{NL}(\omega_{l_{\mathrm{sum}}})\right|^2} \left|\begin{pmatrix} L & l' & l_{\mathrm{sum}}-l' \\ 0 & 0 & 0 \end{pmatrix}\right|^4. \tag{45}$$

The predicted mean spectrum over the Earth is compared to the isotropic case in Fig. 9. As expected the predicted values for the very directional ($m = 0$) case are much higher than the result for the equipartitioned infragravity wave spectrum. The two sets of results map out an approximate range of the acceleration spectrum with variations in the distribution of energy between infragravity wave modes. Source regions

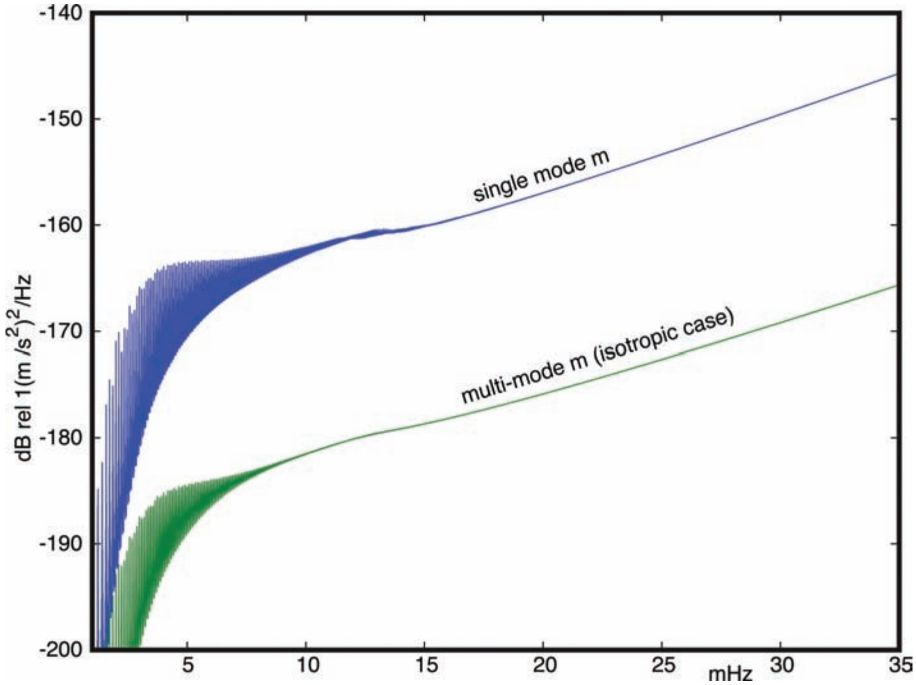

Figure 9. A comparison of the predicted vertical acceleration spectrum resulting from infragavity waves interacting over the ocean basins, but making two different assumptions about the distribution of energy between harmonics. The single mode curve assumes all of the energy is in a single (zonal) mode $m = 0$. The multimode curve assumes energy is evenly partitioned between modes *m*.

for infragravity waves are likely to be distributed along the coast. Topographic effects and the uneven boundaries of coastal lines are likely to strongly scatter energy between modes for a given *l*. These factors probably ensure infragravity wavefield always approaches the isotropic model. Thus the forcing of the Earth normal modes by ocean wave interaction is probably best described by the isotropic case in Fig. 9.

4 THE FORCING OF COUPLED ATMOSPHERE- SOLID EARTH MODES BY INTERACTING INFRAGRAVITY WAVES

The measurements of Nishida *et al.* (2002) suggest the ${}_0S_{29}$ and ${}_0S_{37}$ normal modes are excited in the Earth's hum above the adjacent modes by about 20 and 10 per cent, respectively, during the northern summer. They attributed the larger amplitudes of these two modes to coupling between atmospheric acoustic modes and the Earth seismic modes. Only for the ${}_0S_{29}$ mode at 3.7 mHz and ${}_0S_{37}$ mode at 4.4 mHz are the frequencies of the atmospheric modes nearly coincident with the corresponding solid Earth normal mode. That no enhancement of these modes was observed during the winter months they attributed to decoupling from changes in the frequencies of the atmospheric modes with changes in the temperature structure of the atmosphere. However most measurements of hum spectra in the literature do not show higher amplitudes for these two modes. In many measurements, the spectral peaks are smaller than adjacent modal peaks, presumably the result of the normal variability expected for a random process.

Kurrle & Widmer-Schnidrig (2008) show slightly higher peaks ($<$1 dB or $<$12 per cent in amplitude) for the the ${}_0S_{29}$ and ${}_0S_{37}$ normal modes compared to the adjacent modes in Black Forest Observatory (BFO) vertical component spectra. They also detected a significant spectral peak at 4.4 mHz on the horizontal channels at four STS-1 equipped stations in Europe and Japan that they associated with the ${}_0T_{35}$ torodial mode which is nearly coincident with the ${}_0S_{37}$ spheroidal mode.

Interacting ocean waves also couple into the atmosphere above the ocean. There are two components of this forcing (Waxler & Gilbert 2006). The first is due to propagation of the sound induced by wave interaction from the water into the atmosphere. The second component is due to changes in the sea surface height displacing the atmosphere above the ocean. Longuet-Higgins' (1950) description of the microseisms mechanism (see the introduction of this paper) shows the centre of mass of the water beneath a standing wave must move up and down twice per period of the standing wave. It follows that an equal volume of air (per unit area) in a layer just above the waves must be moved out of the way twice per cycle as well. The force required to displace this volume of air (per unit area) should be equal to the force induced by the moving volume of water below the waves, but reduced by the ratio of the density of air to the density of water. I will assume (at least at long wavelength) that the wavenumber frequency spectrum of the pressure field applied to the bottom of the atmosphere is equal to wavenumber spectrum of the wave interaction appearing below the sea surface (eq. 4) times the ratio of the density of air (at the sea surface) to the density of seawater (roughly 1/300). Waxler & Gilbert (2006) suggest the forcing of atmospheric microbaroms (the equivalent of microseisms in the atmosphere) should have a strong frequency dependence with the efficiency of forcing rising quickly toward low frequency. The analysis is invalid at Earth hum frequencies because it assumes the infragravity waves follow the deep water dispersion curve. At very low frequency and for small displacement, the rate of change in potential energy associated with moving a thin layer of air vertically against the gravitational field appears to be large compared to the momentum transport per unit area in a sound wave of the same frequency and displacement. Thus it seems likely that the force per unit area required to move the air is primarily associated with the rate of change of potential energy. The forcing of coupled atmosphere-solid Earth modes by non-linear ocean waves is best described by a source term that is a 'pressure glut' or jump in pressure between a level just above the sea surface and a level just below the sea surface. The term pressure glut is introduced by Lognonné *et al.* (1998).

Rather than calculating the mode forcing from non-linear wave–wave interaction in a full coupled atmospheric-solid earth model, I instead estimate the rate energy is coupled into atmosphere from interacting ocean waves, and then use estimates of the rate of attenuation of the relevant acoustic modes to estimate the rate energy is coupled into the Earth. Energy losses in coupled acoustic-Earth modes are dominated by attenuation within the much larger volume of the Earth. Atmospheric modes can be well described as the sum of an approximately vertically upward propagating and a nearly downward propagating acoustic wave. The interaction of infragravity waves induces high phase velocity, long wavelength components of the near surface pressure field that will excite upward propagating acoustic waves in the atmosphere. At altitude these waves will reflect into downward propagating waves. For components of the wave forcing that are resonant with atmospheric modes, the atmospheric mode amplitudes will increase until the rate of forcing by the ocean waves is balanced by the dissipation in the atmospheric mode. It is observed that the dissipation of the two relevant atmospheric modes is mostly due to losses into the Earth (Lognonné *et al.* 1998).

The acoustic impedance at the surface of the Earth is much higher than in the atmosphere so the downward propagating acoustic wave is nearly perfectly reflected at the Earth's surface, except the upward propagating wave amplitude will be smaller by the fraction π/Q because of losses into the Earth. In regions where wave forcing is occurring, the attenuation in the Earth is exactly balanced by the wave forcing, so that the difference in amplitude of the upward and downward going wave will be equal to the excitation pressure signal in the atmosphere p_a. The total pressure signal exerted by the atmospheric modes on the Earth will be $p_s = p_u + p_d \approx 2p_d \approx 2p_a Q_a/\pi$. As described above, the atmospheric forcing pressure is related to the forcing under the ocean waves by the ratio of the density of air at the Earth's surface to the density of seawater. The effect of the excitation of the atmospheric mode on the strongly coupled solid Earth modes can be described as an enhancement to the pressure signal forcing the modes in the Earth by the ratio: $p_e/p = [1 + 2Q_a\rho_a/(\pi\rho_w)]$. Lognonné *et al.* (1998) estimated the Q's of the ${}_0S_{29}$ and ${}_0S_{37}$ modes to be about 115 and 21. The predicted enhancements of the amplitude of these two modes from the equation above will be a factors 1.09 and 1.02 for the two modes, smaller than the enhancement in amplitudes estimated (1.2 and 1.1) by Nishida *et al.* (2000). Tanimoto (2001) notes Nishida *et al.* (2000) show an annual cycle to mode amplitudes whereas all other published measurements show a biannual signal and suggests the discrepancy is related to the method of data analysis. Even the small predicted enhancements appear surprising large given the much smaller pressure signal in the atmosphere compared to in the ocean, but the rate of work per unit area done on the modes by the wave interaction pressure signal is proportional to $p_a^2/(\rho_a c_a)$ and relatively more energy is transferred because the speed of sound (c_a) and density are small in air.

5 PREVIOUS WORK ON THE EXCITATION OF NORMAL MODES BY INFRAGRAVITY WAVES

Tanimoto (2007) provides an independent derivation of the wave–wave interaction mechanism within a more formal normal mode framework and also calculates the amplitude ratio of the forcing due to the horizontal and vertical components of the ocean wavefield. This latter calculation is closely related to the G term (eq. 4) and the two derivations appear to be equivalent. Tanimoto (2007) however, suggests there should be a significant anisotropic radiation pattern to the forcing derived from the horizontal velocity component of the ocean wavefield. This result is incorrect in the limit of small differences in wavenumber between the interacting components.

The relevant component of the forcing is derived from terms in the governing equations of the form: $(\vec{u}\cdot\nabla)\vec{u}$ where $\vec{u}$ is the particle velocity under the wavefield. I consider two interacting waves so the first order potential is the sum of the two potentials describing the waves:

$$\vec{u} = \vec{u}_1 + \vec{u}_2 = \nabla\phi_1 + \nabla\phi_2; \quad \phi_j(\vec{x}, z) = \omega a(\vec{k}_j)\cosh[|\vec{k}_j|(z+d)]\exp[i(\vec{k}_j\cdot\vec{x} - \omega t)]. \tag{46}$$

The interaction term $(\vec{u}\cdot\nabla)\vec{u}$ can be written as:

$$[(\vec{u}_1+\vec{u}_2)\cdot\nabla](\vec{u}_1+\vec{u}_2) = (\vec{u}_1\cdot\nabla)\vec{u}_1 + (\vec{u}_2\cdot\nabla)\vec{u}_1 + (\vec{u}_1\cdot\nabla)\vec{u}_2 + (\vec{u}_2\cdot\nabla)\vec{u}_2. \tag{47}$$

Only the cross-terms are relevant to the excitation of normal modes with wavelengths that are long compared to the ocean waves. One of the horizontal components of the forcing terms can be written as:

$$(\vec{u}_i\cdot\nabla)\vec{u}_j = \frac{\vec{k}_i\cdot\vec{k}_j}{k_i k_j} k_i\, \vec{k}_j \phi_i(\vec{x}, z)\phi_j(\vec{x}, z); \qquad \cos\Psi = \frac{\vec{k}_i\cdot\vec{k}_j}{k_i k_j}; \qquad k_i = |\vec{k}_i|. \tag{48}$$

This term has the same form as the anisotropic horizontal forcing suggested by eq. (39) in Tanimoto (2007) and describes the physics behind the apparent isotropic component. Ψ is as defined in that equation. However the forcing terms are necessarily in pairs. If we take $\vec{k}_1 \approx -\vec{k}_2$, $k \approx |\vec{k}_1| \approx |\vec{k}_2|$ then $\cos\Psi = 1$ and

$$(\vec{u}_1\cdot\nabla)\vec{u}_2 + (\vec{u}_2\cdot\nabla)\vec{u}_1 \approx [\vec{k}_1 + \vec{k}_2]k\omega^2 a(\vec{k}_1)a(\vec{k}_2)\cosh[k(z+d)]\exp[i([\vec{k}_1+\vec{k}_2]_j\cdot\vec{x} - \omega t)]. \tag{49}$$

The direction of the forcing is given by $[\vec{k}_1 + \vec{k}_2]$. The direction of this vector for two arbitrary waves with $|\vec{k}_1|, |\vec{k}_2| \gg |\vec{k}_1 + \vec{k}_2|$ if picked from a wavefield with a smoothly varying spectrum will be uniformly distributed in direction. Thus while a single term $(\vec{u}_i\cdot\nabla)\vec{u}_j$

generates an apparent directionality to the forcing, the sum of the two terms $(\vec{u}_i \cdot \nabla)\vec{u}_j + (\vec{u}_j \cdot \nabla)\vec{u}_i$ will result in an average forcing term that is isotropic. If the difference wavenumber is relatively large, $|\vec{k}_1 - \vec{k}_2| \approx |\vec{k}_1|, |\vec{k}_2|$, or if the spectrum is not smoothly varying, then the forcing can be significantly anisotropic if the ocean wavefield is anisotropic. Jacobs *et al.* (1988) have calculated the near surface, second order wavenumber spectrum generated by the interaction of ocean waves and showed it differs significantly from an isotropic spectrum at wavenumbers approaching the ocean wave wavenumbers. The second order pressure spectrum differs from the isotropic approximation of eq. (4) at wavenumbers $\vec{K}$ large enough that that the infragravity wave wavenumber spectrum at $\vec{k}_1 + \vec{K}_2$ or at $\vec{k}_2 + \vec{K}$ varies significantly from the values at wavenumbers $\vec{k}_1,\ \vec{k}_2$, but for the components relevant to the Earth's hum $|\vec{k}_1|,\ |\vec{k}_2| \gg |\vec{K}|$ so that the infragravity wavenumber spectrum can be assumed to be slowly varying in wavenumber.

An earlier paper by Tanimoto (2005) also proposed infragravity waves as the energy source for Earth normal modes in the absence of large Earthquakes. The Tanimoto (2005) paper examined the apparent seasonal covariability of the normal mode energy and ocean wave climate to provide support for this hypothesis. The physical mechanism proposed in the Tanimoto (2005) paper for the excitation of normal modes by infragravity waves depends however, on an invalid model for the spatial and temporal cross-correlation of seafloor pressure variations beneath linear infragravity waves. The theory assumes the spatial cross-correlation decays exponentially with horizontal distance between sites on the seafloor. However linear infragravity waves follow a dispersion relation and the seafloor cross-correlation of pressure between two sites separated by a horizontal distance r in direction ϕ can be derived exactly from the frequency-directional spectrum of the infragravity waves as:

$$S_p(\vec{x}, \omega) = \int_{-\pi}^{\pi} f_\varsigma(\omega, \theta) \exp[ik(\omega)r\cos(\theta - \phi)]\mathrm{d}\theta. \tag{50}$$

We can explicitly calculate the frequency Fourier transform of the sea surface pressure spatial cross-correlation under the infragravity wavefield for some simple models of the directional spectrum. For example, Webb (1986) derives the spatial cross-correlation for a wavefield that is isotropic in direction:

$$S_p(\vec{x}, \omega) = f_\varsigma(\omega) \int_{-\pi}^{\pi} \exp[ik(\omega)r\cos(\theta - \phi)]\mathrm{d}\theta = f_\varsigma(\omega) J_0(kr). \tag{51}$$

In the Tanimoto (2005) formulation the normal mode excitation spectrum is related to the integral of the pressure spatial cross-correlation function against the product of two spherical harmonic wavefunctions associated with the Earth normal modes (Tanimoto 2005; eq. (3)). For most models of the wavefield, the cross-correlation becomes very small for distances that are small compared to an Earth normal mode wavelength and the integral can be approximated by the value of the spherical harmonic at a location times a integral of the pressure spatial cross-correlation function over a local area. For the isotropic wavefield example given above this integral is:

$$\int_{-\pi}^{\pi} \int_0^{\infty} J_0(kr) r\,\mathrm{d}r\,\mathrm{d}\theta = 0. \tag{52}$$

The equivalent integral of the pressure spatial cross-correlation function for any model of the directional spectrum of infragravity waves will be zero reflecting that linear infragravity waves drive pressure fluctuations only at wavenumbers corresponding to the infragravity wave dispersion relation and not at the small wavenumbers associated with Earth normal modes. This correct formulation for the pressure spatial cross-correlation function contrasts with the function used in the Tanimoto (2005) formulation which has a finite integral over the horizontal plane representing finite energy at zero wavenumber.

Linear infragravity waves drive a simple (inhomogeneous) response in the Earth. This response has been used to study magma beneath ocean ridges (Crawford & Webb 2002), but the deformation signal extends only to depths comparable to the ocean wave wavelength.

6 ATMOSPHERIC TURBULENCE AS A SOURCE OF NORMAL MODE EXCITATION

Papers by Tanimoto & Um (1999), Tanimoto (2001), Fukao *et al.* (2002) and Kobayashi & Nishida (1998) invoke atmospheric turbulence as the source of the Earth's background free oscillations. These papers follow earlier work on the excitation of solar normal mode vibrations by turbulence within stars in their derivations. The stellar papers derive the excitation of normal modes within stars from models of the spatial and temporal cross-correlation of pressure fluctuations driven by turbulence.

The study of sound driven by turbulence goes back to the fundamental paper of Lighthill (1952) who showed that sound generation in free space was related to the fourth (and higher) order correlations between velocity fluctuations, thus making turbulence an inefficient generator of sound at low Mach number. Turbulence acting on a boundary or within stratification may be more efficient (see below). Many authors have extended the Lighthill theory to explain the excitation of stellar oscillations by turbulence (e.g. Goldreich & Keeley 1977; Balmforth 1992; Houdek *et al.* 1999). The stellar oscillation mechanism predicts a high dependence on Mach number (Stein 1967). The strong Mach number dependence suggests low Mach number atmospheric turbulence on the Earth is not likely to be associated with an energetic normal mode spectrum. This Mach number dependence has been ignored in the previous papers on the Earth's hum. The strong Mach number dependence arises because only higher order correlations of the particle velocities in turbulence contribute significantly to forcing of planetary modes. The direct action of turbulent pressure fluctuations acting on the Earth's surface approximately averages to zero over large areas (corresponding to the forcing at the large wavelengths of planetary modes), because producing a nonzero force acting on a patch of the Earth surface requires the transfer of vertical momentum from the turbulence to the Earth. Averaged over areas comparable to planetary mode wavelengths the vertical momentum in any isolated volume of turbulence averages toward zero because there is no net movement of the centre of mass of volumes that are large compared to the largest scales of the turbulence (Lighthill 1979).

One can divide the forcing of Earth seismic modes by turbulence into the effects of pressure fluctuations acting directly on the Earth's surface (beneath the atmospheric boundary layer), and the effects of turbulence above the boundary layer associated with infrasound production by convection in the atmosphere. Recent work on the production of infrasound by convection in thunderstorms confirms the low efficiency of sound production by atmospheric turbulence, predicting a Mach number to the fifth power dependence for sound production. (Akhalkatsi *et al.* 2004). Infrasound produced within thunderstorms will couple into the ground if the apparent phase velocity of the infrasound signal across the ground is comparable to the phase velocities of the seismic modes, but because the phase velocities of Earth normal modes are mostly much higher than the speed of sound in the atmosphere (350 m s^{-1}) it will be only those infrasound components travelling nearly vertically (and therefore, with high apparent phase velocity across the Earth's surface) that will excite these modes (e.g. Artru *et al.* 2004). The low efficiency of infrasound production by vigorous storms suggests that infrasound is not likely to be important to the excitation of Earth normal modes.

The strong dependence of sound production on Mach number suggests one should look to the most energetic parts of the atmosphere as the source regions for sound. Bedard (2005) describes observations of infrasound produced by vortices in thunderstorms. Tatom *et al.* (1995) and Tatom & Vitton (2001) describe detection of tornados with seismometers. The infrasound and seismic vibrations produced by tornados have peak energies near 1 Hz with the peak frequency associated with the width of the tornado. Convective storms do produce large pressure fluctuations at low frequency (Georges 1973), but these signals are mostly associated with other types of atmospheric waves (gravity waves) rather than infrasound. These waves propagate slowly (Bedard 2005) compared to Earth normal mode phase velocities. Tatom & Vitton (2001) estimate the rate of energy transfer to the ground is about 10^4 MW for the largest tornados (150 m s^{-1} F5 tornado) falling to less than 10 MW for the more common F2 tornado. Given the rarity of scale F5 tornados (fortunately) and the predicted sharp fall off in energy transfer predicted below 0.1 Hz (Tatom *et al.* 1995), tornados are unlikely to be a significant source of normal mode energy. Tanimoto (2001) estimates the power (dissipation) associated with the background normal mode oscillations (the hum) is between 500 and 10 000 MW.

Hurricanes and large weather systems over the ocean do produce significant infrasound in the band from 0.1 to 0.5 Hz ('microbaroms'), but these signals are driven by ocean waves through the same mechanism that drives microseisms (e.g. Donn & Rind 1972; LePichon *et al.* 2004). The production of microbaroms by wave–wave interaction mechanism was described Section 5. Breaking surf can also generate infrasound (Garces *et al.* 2003).

Meecham (1971) estimates the infrasound produced by turbulent fluctuations in the jet stream using the Lighthill theory applied to a jet. He also shows balloon observations of pressure fluctuations in the upper atmosphere that may be related to this source. He estimates the power radiated by fluctuating winds high in the atmosphere is comparable to the predicted noise measurements at the ground of infrasound to obtain a sound spectral level at the ground of about 10^{-2} Pa^2 Hz^{-1} at 1 Hz. If we assume the production of sound from this source is essentially isotropic in direction, then the fraction of the spectrum associated with horizontal wavenumbers for down-going waves with phase speeds greater than 5 km s^{-1} (or comparable to Earth normal mode phase velocities) will be approximately $[c^2/U_m^2/2 \approx [350/5000]^2/2 \approx 0.002]$ times smaller or about 10^{-6} Pa^2 Hz^{-1}. This level appears insufficient to drive the level of Earth normal modes observed. Meecham predicts infrasound from this source will peak near 0.3 Hz falling rapidly toward lower frequencies. I have found no more recent discussions of the jet stream as a source of infrasound in the literature, although there is an extensive literature describing the production of much slower moving atmospheric gravity waves by jet stream fluctuations (e.g. Herron & Tolstoy 1969; Einaudi & Finnigan 1993; Hauf *et al.* 1996). It seems likely the jet stream is a negligible source of infrasound.

The stratification of the lower atmosphere establishes a thin (1 km) turbulent shear layer, 'the atmospheric boundary layer'. Flow fluctuations at the Earth's surface in the normal mode band (periods less than one hour) are dominated by atmospheric boundary layer turbulence (Wyngaard 1992). While all levels in the atmosphere experience turbulence, the stratification limits the vertical scale of the turbulence (and hence the horizontal scale, at least at short periods). Turbulence in the boundary layer is driven by wind shear and by heating at the surface.

Nishida *et al.* (2005) used an array of microbarographs to investigate the wavenumber and frequency spectrum of pressure fluctuations at the surface of the Earth. They were able to detect infrasound propagating across the array at phase velocities between the approximate speed of sound in air (350 m s^{-1}) and 1 km s^{-1} (associated with waves arriving from above the horizon). They also detected more slowly too propagating ($<$100 m s^{-1}) internal gravity waves in the atmospheric boundary layer. However, the aperture of the array (20 km) was far to small to resolve wavelengths comparable to low order Earth normal modes (400 km for $\boldsymbol{L} = 100$, $\boldsymbol{N} = 0$, $f = 7$ m Hz). The authors estimated the pressure spectrum associated with Earth normal mode excitation by including all energy corresponding to horizontal wavenumbers smaller than 10^{-3} m^{-1}, but this estimate is dominated by waves associated with wavenumbers corresponding to phase velocities far too slow to excite Earth normal modes and thus they have greatly overestimated this component.

The pressure signals seen at a sensor on the Earth's surface can be divided into three components: (1) pressure fluctuations from flow very near the sensor, (2) pressure fluctuations from boundary layer turbulence and (3) infrasound (Meecham 1971). In a general sense, the pressure fluctuations from boundary layer turbulence can be described as being caused either by advection by the mean flow of compressed regions (with higher mass densities) and less compressed regions (with lower densities) past a site or else are associated with gravity wave propagation. Within a finite region of turbulence, regions with more mass are balanced by regions of less mass (since mass is conserved) so that the resulting integral of the surface pressure over any region that is large compared to the scale of the largest boundary layer eddies (a few kilometres) will be essentially constant over timescales comparable to Earth normal mode periods (less than one hour). The net effect is that the 'linear' pressure field under atmospheric turbulence advects along at the typical wind speed and does not contain significant energy in the high phase velocity components required to drive Earth normal modes.

There is a component of the pressure fluctuations under atmospheric turbulence at very small wavenumber (and hence high phase velocity) that can resonantly excite the Earth normal mode spectrum, but it is a tiny component of the pressure spectrum measured at a site. Measurements of pressure fluctuations beneath boundary layers might provide a appropriate model for the pressure fluctuations beneath the atmospheric boundary layer, but it has proven remarkably difficult to either measure or predict the high phase velocity components beneath a shear layer (beneath a boundary layer). Many quite clever experiments have been devised to measure the supersonic component of the pressure field, but the measurements remain problematic because the pressure fluctuations at any given sensor are dominated by the very much larger, short wavelength components. Recent theory begins to explain much of the wavenumber–frequency spectrum of pressure fluctuation under a turbulent boundary layer, but large discrepancies remain between measurements and between theories at small wavenumber (Chase 1993; Bull 1996; Herbert *et al.* 1999).

The phase velocities associated with Earth normal modes are much faster than the speed of sound in air, so it is the supersonic component of the pressure field beneath atmospheric turbulence that is relevant to this problem. Recent work converges to the view that the supersonic component of the pressure field at the boundary is related to the viscous stress so that the pressure frequency–wavenumber spectrum is proportional to the square of the shear stress ($\tau \approx \rho C_D U^2$) at the boundary multiplied by some power of the Mach number ($M = U/c$):

$$P(k, \omega) = X(k)\tau^2 M^2 \delta^3 / u_* \tag{53}$$

(Howe 1991), where δ is the boundary layer thickness, c is the speed of sound in air. The drag coefficient $C_D = u_*/U \approx 0.0015$ relates the friction velocity (u_*), to the free stream velocity U. The scaled spectrum is defined as $X(k)$. The efficiency of production of the supersonic components ($\omega/k > c$) is observed to be higher above a rough surface because of turbulent flow around roughness elements. A model (Howe 1991) of the wavenumber–frequency spectrum for flow over a rough wall suggests a flat wavenumber spectrum for small wavenumber with $X_0 \approx 10^{-7}$. An estimate of the power in small wavenumber components in the spectrum of the pressure field is obtained by assuming the spectrum is white at small wavenumbers and integrating the wavenumber spectrum over $k < \omega/C_m$ where C_m is the phase speed of a typical Earth normal mode. The relevant component of the pressure spectrum under atmospheric boundary layer turbulence is then about:

$$P_{|k|<(\omega/U_m)}(\omega) \approx C_D^2 \rho^2 U^4 X_0 \delta^3 M^2 \pi / u_* \, (\omega/U_m)^2. \tag{54}$$

The theory (Howe 1991) predicts the pressure spectrum (for low Mach number flow) will be at a minimum at small wavenumber and much more energetic at wavenumbers corresponding to the advection velocity: $k_a \approx \omega/U$ forming an advective peak in the spectrum, before falling again toward higher wavenumber. If the turbulence is instead at high Mach number (as is the case for stellar oscillations) then U becomes similar to C_m (flow velocities comparable to the speed of planetary modes), and the planetary mode wavenumbers are instead within the advective peak. For this case, the 'stellar model' is an appropriate model for the forcing.

With the values shown in Table 3 for the various components, the eq. (54) estimate of the high phase velocity component of the pressure spectrum under boundary layer turbulence is shown in Fig. 10.

Kobayashi & Nishida (1998) assumed the pressure fluctuations under turbulence potentially driving planetary free oscillations would be of order $p_0 = \rho U^2$. This assumption is in conflict with other work on sound generation by turbulence (e.g. Lighthill 1952; Meecham 1971) and with work on atmospheric boundary layers. The authors assume a spatial correlation scale H equal to the scale depth of the atmosphere thereby implying the spatial cross-correlation function of the pressure fluctuations goes as

$$\left\langle P(\vec{x}, t) P(\vec{x}', t) \right\rangle \propto \exp(-|\vec{x} - \vec{x}'|/H). \tag{55}$$

This in turn implies the horizontal wavenumber spectrum must be

$$P(\omega, \vec{k}) = \frac{2\pi \, P(\omega)}{H[4\pi^2(k^2 + 1/H^2)]^{3/2}}. \tag{56}$$

This approaches a constant value at small wavenumber ($|k| \ll 1/H$). Integrating over wavenumbers $|k| < \omega/U_m$ gives the frequency spectrum of pressure fluctuations with wavenumbers small enough to force normal modes as:

$$P_{k<U/c}(\omega) \approx 4\pi^2 P(\omega) H^2 \omega^2 / U_m^2. \tag{57}$$

Table 3. Atmospheric Turbulance Model Parameters.

Density of air	$\rho_{\text{air}} = 1.3$ kgm^{-3}
Drag coefficient	$C_D = 0.0015$
Free stream velocity	$U = 3.8$ m s^{-1}
Mach number	$M = 0.011$
Speed of sound in air	$C = 350$ m s^{-1}
Scale height of atmosphere	$H = 8.7$ km
Boundary layer thickness	$\delta = 1$ km
Shear stress	$\tau = 0.03$ kg(ms^2)$^{-1}$
Typical mode phase velocity	$U_m \approx 5$ kms^{-1}

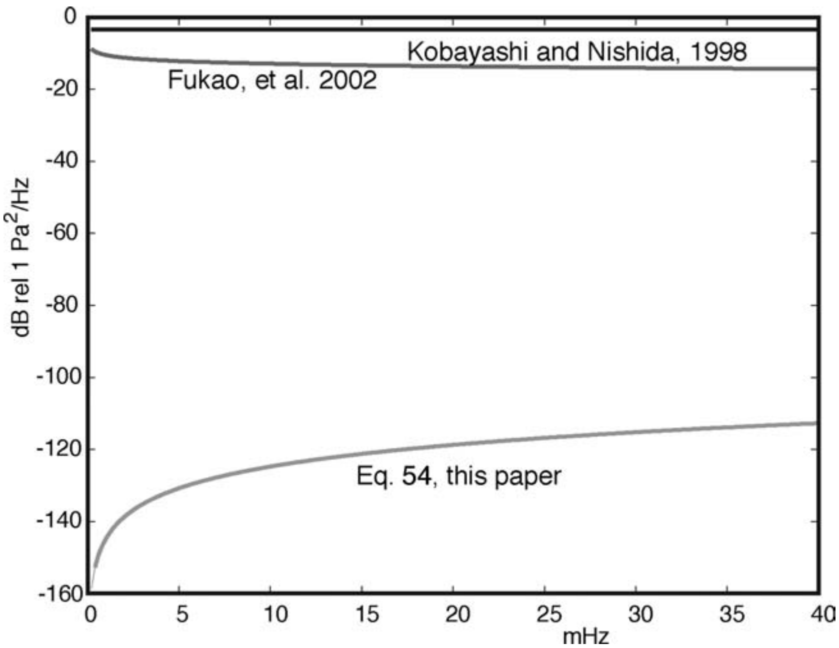

Figure 10. Estimates of the component of the surface pressure spectra under atmospheric turbulence at phase velocity high enough to be relevant to driving the Earth's hum as inferred from three models of turbulence. A label below each curve notes the relevant reference. The two previous models greatly overestimated the long wavelength component of atmospheric turbulence.

The authors state 'the pressure fluctuations at a frequency f are $\delta p = p_0 f_0/f$', ($f_0 = U/H$), implying a pressure power spectrum proportional to $(f_0/f)^2$. Setting the variance in the spectrum equal to p_0^2 for $f \geq f_0$ gives the equivalent pressure spectrum driving the Earth normal modes as

$$P_{kn}(\omega) \approx 4\pi^2 p_0^2 f_0/f^2 H^2 \omega^2/U_m^2 \approx p_0^2 HU/U_m^2. \quad (58)$$

Here f_0 is defined in terms of the scale height of the atmosphere divided by the typical wind velocity. Kobayashi & Nishida (1998) take $H = 8.7$ km, $v = 3.8$ m s^{-1}, so $f_0 = 0.0004$ s^{-1}. Fig. 10 compares the estimate from eq. (58) with the inferred pressure spectrum from eq. (54). The Kobayashi & Nishida (1998) equivalent spectrum is more than 120 db larger than the estimate from the boundary layer calculation, suggesting the Kobayashi and Nishida estimate vastly overestimates the importance of atmospheric turbulence in driving planetary oscillations.

Fukao *et al.* (2002) use a frequency dependent spatial cross-correlation function assuming a constant value for $|\vec{x} - \vec{x}'| < L(\omega)$, and zero elsewhere [$L(f) = L_0(f_0/f)^\beta$ ($L_0 = 600$ m, $\beta = -0.12$ and $f_0 = 0.001$ Hz], but an equivalent calculation as eq. (58) yields very similar results (Fig. 10) implying these authors also greatly overestimate the efficiency of atmospheric turbulence in exciting planetary normal modes. Tanimoto & Um (1999) use a similar model as Fukao *et al.* (2002) for the pressure spatial cross-correlation function to model the pressure fluctuations under atmospheric turbulence with similar results (model not shown here).

Fig. 10 shows these papers on the excitation of Earth free oscillation by atmospheric turbulence enormously overestimate (>120 dB) the strength of atmospheric turbulence as a source to drive normal modes when compared to the estimate obtained from a theory of boundary layer turbulence. The previous papers on excitation by turbulence approximately follow work on the excitation of solar oscillations (e.g. Goldreich & Keeley 1977; Balmforth 1992). The problem comes not from the models of the pressure spatial cross-correlation function in the papers (for there is indeed a long wavelength or high phase velocity component of the pressure field that can excite Earth normal modes), but rather from the assumption that the pressure fluctuations associated with these high phase velocity components of the pressure field are of the same magnitude as the pressure fluctuations associated with the 'linear' pressure field: (of order ρU^2). The 'linear' component of the field produces pressure fluctuations only at phase velocities comparable to the flow velocity and does not couple energy into Earth normal modes. It is only the higher order correlations (non-linear components) that produce long wavelength, high phase velocity components that can drive free oscillations of the Earth, but these components are associated with much smaller pressure fluctuations. The >120 dB difference between the model in this paper and previous models comes from (1) $X_0 \approx -70$ dB (spectral levels are down 70 dB relative to the advective peak), (2) the ratio of the friction velocity to the flow velocity to the fourth power $u_*^4/U^4 = C_D^2 \approx -55$ dB and (3) Mach number squared $M^2 \approx -39$ dB with other terms order 1 at $f = f_0$. Atmospheric turbulence from the boundary layer is a negligible forcing term for Earth normal modes.

One motivation for making broadband seismometer measurements on Mars has been the detection of the Martian planetary seismic modes (Lognonné *et al.* 2000). Measurements of the frequencies of these modes would provide strong constraints on the radial structure of the planet. The frequency of earthquakes on Mars is expected to be less than on Earth because of the thicker lithosphere. Nawa *et al.* (1998) and Suda *et al.* (1998) have been cited as demonstrating that the planetary modes of Mars would be continuously excited at detectable levels. However, as shown above, these authors overestimate the strength of this source by many orders of magnitude. When planetary modes are detected on Mars, it will be because these modes are excited by some other source (Marsquakes).

7 CONCLUSIONS

1. An estimate of the wavenumber–frequency spectrum of the pressure fluctuations beneath turbulence in the atmospheric boundary layer suggest the high phase velocity component needed to drive Earth normal modes is utterly insufficient to explain the persistent excitation of Earth normal modes observed.

2. An estimate of the infrasound generated by turbulence in the atmosphere suggests atmospheric turbulence cannot explain the persistent excitation of Earth normal modes at the amplitudes observed. Previous estimates ignoring the high Mach number dependence of sound production from turbulence greatly overestimated the strength of turbulence as a source for Normal mode excitation.

3. Previous papers predicting significant energy in free oscillations on other planets excited by atmospheric turbulence are almost certainly in error for the same reasons. If energetic normal modes are observed on other planets it will be through another mechanism.

4. Longuet-Higgins (1950) and Hasselmann's (1963) theories, when extended to the non-linear interaction of low frequency (infragravity) waves over the oceans predict pressure fluctuations over the shallow (continental shelf) seabed have sufficient energy at low frequency and high phase velocity to explain the levels observed. This is the same mechanism by which the familiar microseism peak near 7 s period in the seismic spectrum is generated.

5. The forcing of Earth normal modes by infragravity waves interacting in the large, deep ocean basins can be estimated by recasting the problem in a spherical geometry and describing the infragravity wavefield as a sum of tsunami modes of the basins. Despite the much less energetic wave height spectrum of infragravity waves in deep water and an inverse depth dependence of the efficiency of wave interaction mechanism, an estimate of the forcing of Earth normal modes by infragravity acting over the world's ocean basins accurately predicts the complete Earth vertical acceleration background spectrum (the Earth's hum) from 1 to 40 mHz. The predicted spectrum diverges from the observed spectrum above 40 mHz because the single frequency peak (49–90 mHz) is generated by a different mechanism.

6. The shape of the hum spectrum is primarily determined by the elastic structure of the Earth. The infragravity spectrum is nearly frequency independent above 0.5 mHz at most sites and so imposes no structure on the hum spectrum. Mode amplitudes are determined by a balance between the forcing and the rate of dissipation and so depend directly on mode Q. The modelling of mode forcing in the paper assumes the source regions are isotropically distributed around the Earth. It is necessary to add a factor correcting for the attenuation of the modes during propagation into the interior of the continents to correctly estimate the spectrum at quiet interior sites. Alternatively, one could calculate the forcing assuming particular source regions but that would require a more complete knowledge of infragravity wave climate. Source regions are necessarily distant from the quiet sites where the Earth's hum can be observed.

This paper shows that calculations of both infragravity waves interacting over the shelf and infragravity waves interacting over the deep ocean basins can explain the observed background acceleration spectrum of the Earth (the Earth's hum). A previous paper by this author suggested forcing from infragravity waves over the ocean basins was not likely to be sufficiently energetic to explain the observed hum spectrum (Webb 2007). This paper shows that despite the much smaller wave heights seen over the deep ocean basins, the integration of the forcing over a much larger area than the shelves compensates for the weaker forcing per unit area. Over large areas, the interaction of infragravity waves is best described as the interaction between tsunami modes, and the energy coupled from ocean waves into Earth modes increases as the source area squared.

It is likely that infragavity waves interacting over the large Pacific basin dominate the forcing of the Earth's hum. It has been known for many decades that the infragravity wave spectrum over the Pacific basin remains remarkably constant, varying by only about 5 dB near 4 mHz (Filloux 1983) on timescales from weeks to months, although the variation may be larger at lower frequency. This low variability in wave spectral amplitudes may be maintained by a combination of relatively steady sources for infragravity waves derived from trade wind driven ocean waves combined with storm sources in both hemispheres and a long decay time for infragravity wave energy. The decay time for infragravity wave energy in the Pacific is about 22 hr as inferred from observations of the decay of waves following tsunamis (e.g. Webb *et al.* 1991). It is possible that infragravity wave energy may be modulated by an interaction of infragravity wave energy and the tides acting on the shallow shelf preventing infragravity wave energies much above the average value to be maintained for very long. Thomson *et al.* (2006) describe tidal modulation of infragavity waves on the shelf in a local context.

Observations of the variation in the hum averaged seasonally show about 5 dB variation in the energy in the hum (Nishida et al. 2000) although the variability is higher (>10 dB) averaged over shorter timescales (e.g. Ekström 2001; Rhie & Romanowicz 2004, 2006). It seems reasonable to assume that the relatively constancy of the Earth's hum is a consequence of the relatively constancy of the infragravity wave spectrum in the Pacific.

Efforts to localize the Earth's hum using seismic arrays primarily show source regions around the rim of the Pacific or south Atlantic (Rhie & Romanowicz 2004, 2006; Nishida & Fukao 2007) with the dominant regions found in the North Pacific in the northern winter and the south Pacific or Atlantic in the southern winter. It seems likely that large infragravity waves acting over the shelf, generate locally 'bright' source regions forcing seismic modes that array processing techniques preferentially detect. Short timescale variations in hum energy and, therefore, peaks in hum energy will be associated with localized coastal source regions, whereas the interaction over the much larger ocean basins will vary on much longer timescales, a simple consequence of the relatively long propagation times of infragravity waves across the Pacific basin (order 22 hr).

The relative importance of coastal and pelagic sources for the Earth's hum remains uncertain. It should be possible to better constrain this problem through a combination of more complete observations of infragravity wave climate, and better constraints on hum source regions from analysis of seismic array data.

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