THE EFFECT OF THE FINITE DEPTH OF THE OCEAN ON THE MICROBAROM SIGNAL

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The recently proposed theory of the generation of microbaroms by ocean surface waves is generalized to include the effects of the finite depth of the ocean. Only the case of waves over the deep ocean are considered. It is predicted that the effect of resonances in the water column due to reflections from the sea floor and surface are present in the microbarom signal.

The broad peaks at about 0.2 Hz observed in infrasound and seismic spectra are known as the microbarom and microseism peaks respectively.[1, 2, 3, 4, 5, 6] The source of the microbarom and microseism signals is believed to be colliding ocean surface waves. Due to the non-linear nature of fluid mechanics traveling surface waves interact as they collide. It has been shown that the harmonics produced during this interaction have significant radiating components when the colliding waves have nearly equal and opposite wave vectors.[7, 8, 9, 10] Indeed, under conditions of high sustained winds and long fetch ocean wave spectra tend to peak at about 0.1 Hz,[11, 12, 13, 14] supporting the hypothesis that the microbarom and microseism peaks are a harmonic of the ocean surface wave field.

Previous treatments of the microbarom radiation have assumed the ocean to be infinitely deep.[9, 10] At microbarom frequencies however, the acoustic wavelength in the ocean is about 7.5 km, close to typical deep-water ocean depths. One or more vertical water column resonances should lie in the microbarom and microseism frequency band. These resonances have a significant effect on both microseism and microbarom spectra.

In this presentation the theoretical treatment of Ref. [10] is generalized to take account of the finite depth of the ocean. It will be assumed that the water is deep enough that the surface waves themselves don't interact with the ocean floor so that the linear solutions of the equations of fluid mechanics are as in the infinitely deep case.[15] As in Refs. [8, 10] the motion of the interface is described statistically by the frequency angle spectrum $F(f, \theta)$. The frequency angle spectrum can be thought of as the density of ocean surface wave power of frequency f propagating in the direction θ.[13, 14] Under this assumption the calculation of the microbarom and microseism source spectra reduces to finding the vertically propagating, frequency doubled component of the second order velocity potential.[7, 8, 10] This component is produced during the collision of two counter propagating surface waves of equal frequency and opposite direction. Since a vertically propagating component is required only the excitation of compressional waves in the sea floor need be considered. The sea floor can then be modeled as a third fluid with the density and compressional wave speed of bedrock.

In the deep water case considered here §III B of Ref. [10] requires generalization. As in that reference let ρ_a, ρ_w and c_a, c_w be the densities and sound speeds of air and water respectively. Let D be the depth of the water and consider the sea floor to be an elastic solid with density ρ_b and compressional wave speed c_b. Pressure and velocity are assumed to be continuous across the ocean / sea-floor interface. The acceleration due to gravity is denoted by g.

Consider the collision of two counter propagating surface waves of wave vector $\pm\mathbf{k}$ and angular frequency $\omega(\mathbf{k}) = 2\pi f(\mathbf{k})$. Assume that for the surface waves the deep water dispersion relation $\omega = \sqrt{g|\mathbf{k}|}$ is valid so that $\mathbf{k} = (4\pi^2 f^2/g)(\cos\theta, \sin\theta)$. As in Ref. [10] introduce $\phi(z,t) = \hat{\phi}(z)\, e^{-i2\omega t}$ and $\xi(t) = \hat{\xi}\, e^{-i2\omega t}$. ϕ is proportional to the relevant component of the second order velocity potential and ξ is proportional to the second order correction to the surface displacement. They satisfy

$$\hat{\phi}(z) = \begin{cases} C_\mathrm{a}\, e^{i\frac{2\omega}{c_\mathrm{a}} z} & \text{if } 0 < z \\ C_\mathrm{w}\Big(\cos(\frac{2\omega}{c_\mathrm{w}}(z+D)) + i\frac{\rho_\mathrm{w} c_\mathrm{w}}{\rho_\mathrm{b} c_\mathrm{b}} \sin(\frac{2\omega}{c_\mathrm{w}}(z+D))\Big) & \text{if } -D < z < 0 \\ C_\mathrm{w}\, e^{-i\frac{2\omega}{c_\mathrm{b}}(z+D)} & \text{if } z < -D \end{cases}$$

for some constants C_σ with the air/sea interface conditions

$$\big(-2i\omega\hat{\phi} + \rho_\sigma g\hat{\xi}\big)\Big|_{z=-0^+}^{0^+} = 2\rho_\mathrm{w}\omega^2$$

$$\frac{\partial\hat{\phi}}{\partial z}\Big|_{z=-0^+}^{0^+} = 3i\frac{\omega^3}{|\mathbf{k}|c_\mathrm{a}^2}$$

and

$$-2i\omega\hat{\xi} = \big(\frac{\partial\hat{\phi}}{\partial z} - 3i\frac{\omega^3}{|\mathbf{k}|c_\mathrm{a}}\big)\Big|_{z=\pm 0^+}.$$

The source strength power spectrum is given by[8, 10]

$$\mathcal{D}(2f) = \frac{g^2}{f} H(f) \begin{cases} \rho_\mathrm{a}^2 \left|\frac{d\hat{\phi}}{dz}\Big|_{z=0^+}\right|^2 & \text{into the air} \\ \rho_\mathrm{w}^2 \left|\frac{d\hat{\phi}}{dz}\Big|_{z=-0^+}\right|^2 & \text{into the sea} \end{cases}$$

where

$$H(f) = \int F(f,\theta)F(f,\theta+\pi)\, d\theta$$

measures the density of counter propagating waves of frequency f on the sea surface.[8]

Solving one finds

$$C_\mathrm{w} \approx i\omega \frac{1}{\cos(\frac{2\omega D}{c_\mathrm{w}}) + i\frac{\rho_\mathrm{w} c_\mathrm{w}}{\rho_\mathrm{b} c_\mathrm{b}}\sin(\frac{2\omega D}{c_\mathrm{w}})}$$

and

$$C_\mathrm{a} \approx -i\omega\Big(i\frac{3}{2}\frac{\omega}{|\mathbf{k}|c_\mathrm{a}} + \frac{c_\mathrm{a}}{c_\mathrm{w}} \cdot \frac{\frac{\rho_\mathrm{w} c_\mathrm{w}}{\rho_\mathrm{b} c_\mathrm{b}}\cos(\frac{2\omega D}{c_\mathrm{w}}) + i\sin(\frac{2\omega D}{c_\mathrm{w}})}{\cos(\frac{2\omega D}{c_\mathrm{w}}) + i\frac{\rho_\mathrm{w} c_\mathrm{w}}{\rho_\mathrm{b} c_\mathrm{b}}\sin(\frac{2\omega D}{c_\mathrm{w}})}\Big).$$

It follows that the source strength power spectrum is given by

$$\mathcal{D}(f) = 4g^2\pi^4 f^3 H(\frac{f}{2}) \begin{cases} \frac{\rho_a^2}{c_a^2}\Big(\frac{9g^2}{4\pi^2 c_a^2 f^2} \\ \quad + \frac{3g}{2\pi c_{\rm w} f} \frac{\big(1-(\frac{\rho_{\rm w}c_{\rm w}}{\rho_{\rm b}c_{\rm b}})^2\big)\sin(\frac{2\pi f D}{c_{\rm w}})\cos(\frac{2\pi f D}{c_{\rm w}})}{\cos^2(\frac{2\pi f D}{c_{\rm w}}) + (\frac{\rho_{\rm w}c_{\rm w}}{\rho_{\rm b}c_{\rm b}})^2\sin^2(\frac{2\pi f D}{c_{\rm w}})} & \text{into the air} \\ \quad\quad + \frac{c_a^2}{c_w^2}\frac{(\frac{\rho_{\rm w}c_{\rm w}}{\rho_b c_b})^2\cos^2(\frac{2\pi f D}{c_w}) + \sin^2(\frac{2\pi f D}{c_w})}{\cos^2(\frac{2\pi f D}{c_w}) + (\frac{\rho_{\rm w}c_{\rm w}}{\rho_b c_b})^2\sin^2(\frac{2\pi f D}{c_w})}\Big) \\ \\ \frac{\rho_w^2}{c_w^2}\frac{(\frac{\rho_{\rm w}c_{\rm w}}{\rho_b c_b})^2\cos^2(\frac{2\pi f D}{c_w}) + \sin^2(\frac{2\pi f D}{c_w})}{\cos^2(\frac{2\pi f D}{c_w}) + (\frac{\rho_{\rm w}c_{\rm w}}{\rho_b c_b})^2\sin^2(\frac{2\pi f D}{c_w})} & \text{into the sea.} \end{cases}$$

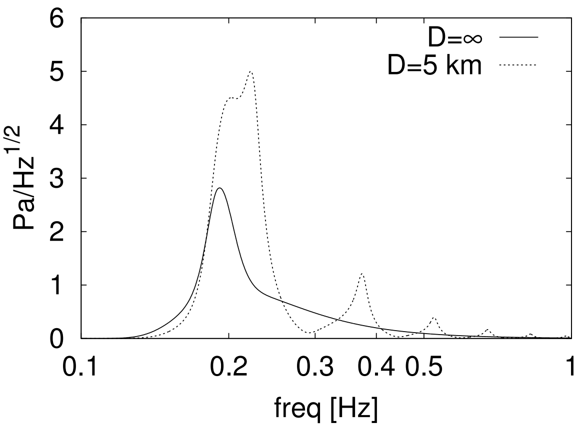

Fig.1 *The microbarom source spectrum* $\sqrt{D_{\rm inf}/b}$ *for the infinitely deep ocean and source spectrum* $\sqrt{D/b}$ *for an ocean of depth* $D = 5$ *km.*

In the figure the microbarom source strength for an ocean of depth 5 km is compared to that for an infinitely deep ocean. The simple model considered in Ref. [10] for the frequency angle spectrum of the surface wave field is used: $F(f,\theta) = \bar{F}(f)a(\theta)$ where $\bar{F}(f)$ is taken to have the JONSWAP form[12, 14] (significant wave height of 12 m and central frequency of 0.095 Hz) and the integral $b = \int a(\theta)^2\, d\theta$ is taken to be an unknown parameter. In the figure the source spectra divided by $\sqrt{b}$, $\sqrt{D(f)/b}$ and $\sqrt{D_\infty(f)/b}$ for radiation into the air are plotted. The impedance contrast between the ocean and the bedrock, $\frac{\rho_{\rm w}c_{\rm w}}{\rho_b c_b}$, is assumed to be $\frac{1}{5}$. The effect of the resonances in the water column is clearly visible.

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