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An application of Miles' theory to Bragg scattering of water waves by doubly composite artificial bars

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Abstract

In the present paper, Miles' (1981) theory is implemented to derive formulae for describing the Bragg scattering of water waves for doubly composite artificial bars with different shapes, spacings, relative bar heights, relative bar footprint and the number of bars. The theory has clear advantage in estimating Bragg reflection coefficient for practical applications concerning coastal problems. Experiments of Bragg reflections over doubly composite rectangular artificial bars have also been performed in a wave flume. Key parameters that may lead to the optimal selection of a doubly composite artificial bar are studied. Theoretical solutions are seen to compare fairly well with the numerical computations and the laboratory experiments. Our simulated results reveal that the Bragg resonance for doubly composite artificial bars effectively increases the bandwidth of the reflection coefficient.

Keywords: Miles' theory; Bragg resonance; Artificial bars; Boundary integral equation

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1. Introduction

During the past decade, the surface wave scattering by rippled seabed has been studied extensively through experiments, numerical simulations and theoretically. Concerning the problems of waves passing through a region of the sinusoidal undulation, the interesting phenomenon of Bragg resonance occurs when the surface wavenumber k is equal to one-half of bed wavenumber K, i.e. $2k/K=1$. It means that the reflected waves will return in equal phases and reinforce each other when they match the situation as stated above. The behavior of the Bragg reflection leads to the possibility that the offshore bars could protect the beach face from the full impact of the incident waves. Some explanations concerning the Bragg scattering can be found in the earlier paper of Mei (1985). The case of linear surface waves incident upon a horizontally one-dimensional sinusoidal bottom was examined by Davies and Heathershaw (1984). Investigations as conducted by Davies et al. (1989), and O'Hare and Davies (1993) reveal that there takes place not only the primary resonance at $2k/K=1$ but also the second-harmonic at $2k/K=2$ can be found for a single sinusoidal bed. To this point, Miles' (1981) theory is frequently used to compare the results of Bragg scattering, as induced by sinusoidal bars, with numerical results or experimental data. The theory is derived on the basis of linear wave conditions to predict wave scattering over horizontal bottom superposed by small undulations.

In the case of a bed consisting the superimposition of two sinusoidal bottoms having different wavenumbers K_1 and K_2 ($K_2>K_1$) and the relative amplitude of the bottom undulation D/h (D is the amplitude of the bottom undulation, h is the water depth in the mild-slope sense), the resonances occur at $k=(K_2-K_1)/2$ and $k=(K_2+K_1)/2$, which were referred to as sub-harmonic and higher-harmonic resonances, respectively, as addressed by Belzons et al. (1991) and Guazzelli et al. (1992). A step-approximation model (Guazzelli et al., 1992) and a successive application matrix method (O'Hare and Davies, 1993) were developed to reproduce the resonant reflection, in which the bed was divided into a number of very small horizontal shelves. A numerical model of extension of mild-slope equation (EMSE), has been developed by Kriby (1986) to represent the Bragg resonances by adding amplitude deviation terms in the slowly varying water depth. Zhang et al. (1999) developed a hybrid model (HM) by extending the EMSE to the case of monochromatic waves over a steep undulating bottom. Hsu and Wen (2001) developed a parabolic mild-slope equation (PMSE) to accommodate the rapidly varying topography in order to study the Bragg reflection for sinusoidal bottoms. An evolution equation of mild-slope equation (EEMSE) was also developed by Hsu et al. (2003), in which the higher-order terms (which is neglected in PMSE) relevant for steep undulating bottoms are retained. However, it is a very complex and laborious work to derive the reflection coefficients from numerical methods.

The studies mentioned above are mostly concerned with Bragg scattering of surface waves over sinusoidal bars which are commonly formed offshore by partial or full standing waves. However, a practical application of the patches of sinusoidal sand bars is not feasible in coastal engineering techniques due to many planning difficulties. Mei et al. (1988) proposed the concept of the Bragg breakwater to protect the oil drilling platform from wave attack. The potential effectiveness against waves appears to be reasonable on their studies. Kirby and Anton (1990) presented the theory on the basis of Miles' (1981) theory and Kirby's (1986) EMSE model to study the Bragg reflection of surface waves induced by artificial bars placed discretely on the seabed. They discussed the limitations of both Miles' (1981) theory and EMSE and compared to experimental results. Bailard et al. (1990) explored the feasibility of the Bragg reflection of artificial bars placed offshore on a natural beach. Their results concluded that the Bragg reflection of artificial bars may have merits as an appropriate shore protection method, but its Bragg resonance has only primary harmonic resonances of waves reflected from a single sinusoidal undulation for a monochromatic wave with a given wave period. Zhang et al. (1999) proved that the limitation could be improved to produce both primary and higher-harmonic resonances, i.e. to increase the bandwidth of the Bragg reflection, by using doubly superposed sinusoidal bottom undulations.

From engineering point of view, convenience and advantage are both key elements in practice. Many researches (Hsu et al., 2002, 2003) have shown that different shapes of artificial bars such as triangular, rectangular and rectified cosine geometries could produce Bragg scatting as natural ripple seabed and could be constructed easier in the field application. For this case, Miles' (1981) theory provides a simple method to explain the Bragg resonance than any other complex numerical methods. The Bragg reflection coefficient could be obtained in an easy way after integrating a formula for any undulation bottom. For the application of Miles' (1981) theory, the major studies in the earlier stage focused on the sinusoidal or artificial bars with the same wavenumbers. Up to now there has been no research for the study of Bragg reflection of combined artificial bars. In this paper, we extended Miles' (1981) theory to explore Bragg scattering of monochromatic water waves over doubly composite artificial bars with varying affecting parameters. Experiments, Hsu and Wen's (2001) PMSE (numerical) model, and the EEMSE (numerical) model of Hsu et al. (2003) were also carried out in recent years to compare the Bragg reflection over doubly composite artificial bars with the existing theoretical predictions. By varying the key parameters of doubly composite artificial bars, such as the number of bars, relative bar height, relative bar spacing, formulae derived from Miles' (1981) theory is examined in the present study.

2. Theoretical formulation

Notably, Miles (1981) presented an integral equation formulation of wave reflection due to a cylindrical obstacle using the Laplace equation subject to bottom and free surface conditions. Later researchers (e.g. Mei, 1985; Kirby, 1986), by using different methods, have shown that the Bragg resonance could occur for a small amplitude undulation over a horizontal bottom. The reflection coefficient, R, for an arbitrary topography is given by the boundary integral equation of Miles (1981)

$$R = \left| -2ik\alpha \int_{\infty}^{\infty} \delta(x) e^{2ikx} \, dx \right| \tag{1}$$

where $\delta(x)$ represents the bottom undulation varying in the x direction, $i=\sqrt{-1}$ is the complex unit, k is the wave number and α is a parameter defined as

$$\alpha = \frac{k}{2kh + \sinh 2kh} \tag{2}$$

In the present study, in order to improve the narrow bandwidth of Bragg resonance in water waves, two groups of doubly composite artificial bars are used to produce both primary and higher-harmonic resonances. Three shapes of artificial bars, consisting of rectangular, triangular and rectified cosine shapes, with different spacing are considered (Fig. 1) in the theoretical analysis using Miles' (1981) theory for Bragg resonance of water waves. Each group of combination is periodic over an equal spacing S_1 and S_2, respectively. In such combinations, the spacing is varied to investigate the performance of Bragg resonance, where S is the interval between two combinations of bars. In Fig. 1, N is the total number of artificial bars and $N/2$ artificial bars in each group, B is the footprint of the bar on the seabed, D is the bar height and h is the mean water depth.

The undulation term $\delta(x)$ is arbitrary aside from the small amplitude restriction, i.e. $D/h \ll 1$. For convenience, we set the bar spacing in the second combination as $S = S_2$ in the theoretical formulation. The expressions of undulation term $\delta(x)$ for these three different shapes of doubly composite artificial bars are written, respectively, as follows.

Rectangular bar:

$$\delta(x) = \begin{cases} D, & nS_1 \le x \le nS_1 + B, \qquad n = 0, 1, \ldots, \frac{N}{2} - 1 \\ D, & \left(\frac{N}{2} - 1\right) S_1 + nS_2 \le x \le \left(\frac{N}{2} - 1\right) S_1 + nS_2 + B, \qquad n = 1, 2, \ldots, \frac{N}{2} - 1 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

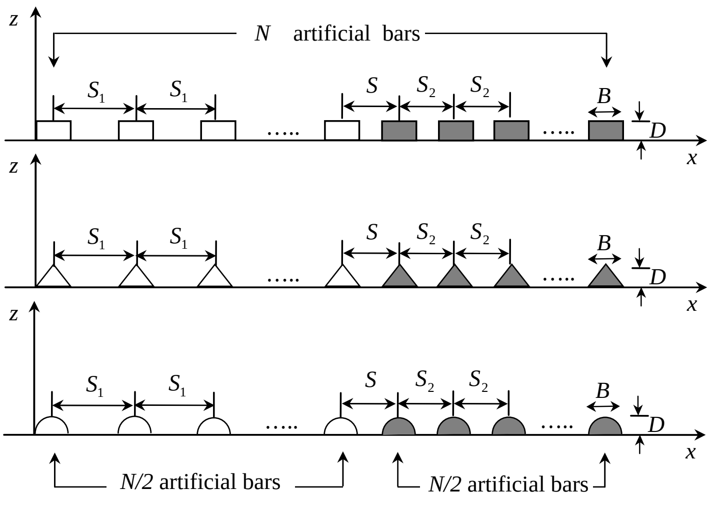

Fig. 1. Shapes of doubly composite artificial bars with different spacings: (a) rectangular bar; (b) triangular bar; and (c) rectified cosine bar.

Triangular bar:

$$\delta(x)=\begin{cases}\frac{2D}{B}(x-nS_1), & nS_1\le x\le nS_1+\frac{B}{2}, \quad n=0,1,\ldots,\frac{N}{2}-1\\ -\frac{2D}{B}[x-(B-nS_1)], & nS_1+\frac{B}{2}\le x\le nS_1+B, \quad n=0,1,\ldots,\frac{N}{2}-1\\ \frac{2D}{B}\left\{x-\left[\left(\frac{N}{2}-1\right)S_1+nS_2\right]\right\}, & \\ \left(\frac{N}{2}-1\right)S_1+nS_2\le x\le\left(\frac{N}{2}-1\right)S_1+nS_2+\frac{B}{2}, & n=1,2,\ldots,\frac{N}{2}\\ -\frac{2D}{B}\left\{x-\left[B+\left(\frac{N}{2}-1\right)S_1+nS_2\right]\right\}, & \\ \left(\frac{N}{2}-1\right)S_1+nS_2+\frac{B}{2}\le x\le\left(\frac{N}{2}-1\right)S_1+nS_2+B, & n=1,2,\ldots,\frac{N}{2}\\ 0, & \text{otherwise}\end{cases} \tag{4}$$

Rectified cosine bar:

$$\delta(x)=\begin{cases}D\cos\left\{-\frac{\pi}{B}\left[x-\left(\frac{B}{2}+nS_1\right)\right]\right\}, & nS_1\le x\le nS_1+B, \quad n=0,1,\ldots,\frac{N}{2}-1\\ D\cos\left\{-\frac{\pi}{B}\left[x-\left(\frac{B}{2}+nS_2+\left(\frac{N}{2}-1\right)S_1\right)\right]\right\}, & \\ \left(\frac{N}{2}-1\right)S_1+nS_2\le x\le\left(\frac{N}{2}-1\right)S_1+nS_2+B, & n=1,2,\ldots,\frac{N}{2}\\ 0, & \text{otherwise}\end{cases} \tag{5}$$

Substitution of these expressions into the integral equation (e.g. Eq. (1)) yields the reflection coefficients for these three doubly composite artificial bars. After some algebraic manipulation, the Bragg reflection coefficients of these three doubly composite artificial bars are given, respectively, by:

Rectangular bar

$$R=2\alpha D\sin kB\sqrt{A_1^2+2\cos\left[\frac{kS_1(N-2)}{2}+\frac{kS_2(N+2)}{2}\right]A_1A_2+A_2^2} \tag{6}$$

where the coefficients A_1 and A_2 are given by

$$A_1=\frac{\sin\left(\frac{kS_1N}{2}\right)}{\sin kS_1} \tag{7}$$

$$A_2 = \frac{\sin\left(\frac{kS_2N}{2}\right)}{\sin kS_2} \tag{8}$$

Triangular bar:

$$R = 2\alpha D(1-\cos kB)\sqrt{A_1^2 + 2\cos\left[\frac{kS_1(N-2)}{2} + \frac{kS_2(N+2)}{2}\right]A_1A_2 + A_2^2} \tag{9}$$

Rectified cosine bar:

$$R = \begin{cases} \left|\dfrac{4\pi k\alpha D\cos kB}{B\left[\left(\dfrac{\pi}{B}\right)^2 - 4k^2\right]}\right|\sqrt{A_1^2 + 2\cos\left[\dfrac{kS_1(N-2)}{2} + \dfrac{kS_2(N+2)}{2}\right]A_1A_2 + A_2^2}, & \dfrac{\pi}{B} \neq 2k \\ k\alpha DB\sqrt{A_1^2 + 2\cos\left[\dfrac{kS_1(N-2)}{2} + \dfrac{kS_2(N+2)}{2}\right]A_1A_2 + A_2^2}, & \dfrac{\pi}{B} = 2k \end{cases} \tag{10}$$

From Eqs. (6), (9), and (10), it is interesting to note that the Bragg reflection coefficients depend on key parameters, such as, the footprint of the bar B, the bar spacing S_1 and S_2, the total numbers of bar N, and the bar height D.

In the case of equal bar spacing, i.e. $S_1 = S_2 = S$, the doubly composite artificial bars will be reduced to a series of bars, where the relations of $A_1 = A_2$ holds good. This result implies that the reflection coefficients are consistent with Hsu et al.'s (2002) formulae of a series of artificial bars, indicating that the formulae in the present paper could easily be applied to a single series of artificial bars.

3. The evolution equation of the mild-slope equation (EEMSE)

The EEMSE model is developed by Hsu et al. (2003) by extending PMSE (Hsu and Wen, 2001) and HM (Zhang et al., 1999). The higher-order terms of steep bottom undulation, neglected in PMSE, are retained in the model. The model has the merits to achieve the faster convergence and to save computer time for a large coastal area. Here, the EEMSE model is adopted to study the interaction between surface waves and doubly composite artificial bars. The numerical results are compared with theoretical computations obtained from Miles' (1981) theory.

Following the procedure outlined in Hsu and Wen (2001), Hsu et al. (2003), the evolution equation of the mild-slope equation (EEMSE) is written as

$$\left[\frac{-2\omega \mathrm{i}}{CC_\mathrm{g}-g(1-\lambda^2)\delta}\right]\left(\frac{\partial\phi}{\partial t}\right)$$

$$= \nabla_\mathrm{h}^2\phi + k_\mathrm{c}^2\phi + \frac{g}{\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}}\left[2\boldsymbol{G}_1\cdot\delta\nabla_\mathrm{h}\frac{\phi}{\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}}\right]$$

$$+\frac{\delta}{\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}}\nabla_\mathrm{h}[(1-\lambda^2)]\cdot\nabla_\mathrm{h}\left[\frac{\phi}{\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}}\right] \quad (11)$$

where

$$k_\mathrm{c}^2 = \left[\frac{g\boldsymbol{G}_1\cdot\nabla_\mathrm{h}\delta + g\boldsymbol{G}_2 + k^2CC_\mathrm{g}}{CC_\mathrm{g}-g(1-\lambda)\delta}\right] - \frac{\nabla_\mathrm{h}^2\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}}{\sqrt{CC_\mathrm{g}-g(1-\lambda)\delta}} \quad (12)$$

is a pseudo wave number, ω is the angular frequency, C and C_g are the wave celerity and the group velocity, respectively, g is the gravitational acceleration, $\lambda = \tanh kh$, δ represents a rapidly varying component over a slowly varying depth h, and $\nabla_\mathrm{h} = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator. The functions $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are expressed as

$$\boldsymbol{G}_1 = \lambda(1-\lambda^2)(k\nabla_\mathrm{h}h + h\nabla_\mathrm{h}k) \quad (13)$$

$$\boldsymbol{G}_2 = \alpha_1(\nabla_\mathrm{h}h)^2k + \alpha_2\nabla_\mathrm{h}^2h + \alpha_3\nabla_\mathrm{h}k\cdot\nabla_\mathrm{h}h/k + \alpha_4\nabla_\mathrm{h}^2k/k^2 + \alpha_5(\nabla_\mathrm{h}k)^2/k^3 \quad (14)$$

The parameters α_i in Eq. (14) appear as follows:

$$\alpha_1 = -\lambda(1-\lambda^2)(1-\lambda kh) - 2(1-\lambda^2)\lambda^2k\delta \quad (15)$$

$$\alpha_2 = -\lambda kh(1-\lambda^2)/2 + (1-\lambda^2)\lambda k\delta \quad (16)$$

$$\alpha_3 = kh(1-\lambda^2)(2kh\lambda^2 - 5\lambda/2 - kh/2) - 2(1-\lambda^2)(2\lambda^2kh - \lambda - kh)k\delta \quad (17)$$

$$\alpha_4 = kh(1-\lambda^2)(1-2\lambda kh)/4 - \lambda/4 + (1-\lambda^2)\lambda k^2h\delta \quad (18)$$

$$\alpha_5 = kh(1-\lambda^2)(4\lambda^2k^2h^2 - 4k^2h^2/3 - 2\lambda kh - 1)/4 + (1-\lambda^2)k^2h^2(1-2\lambda^2)k\delta \quad (19)$$

The paper of Hsu et al. (2003) could be referred for detailed derivations. The rapidly varying terms, $\nabla_\mathrm{h}h$ and ∇_h^2h, neglected in the EMSE model (Kirby, 1986), are retained in EEMSE to take into account the steep bottom undulations. Notably, the EEMSE model has the advantage of saving the storage and computing time when compared with the hyperbolic equation. The equations of PMSE (Hsu and Wen, 2001) can be recovered if $\delta = 0$.

The radiation boundary condition for the problem is specified as follows

$$\frac{\partial\phi}{\partial x} = \pm(-1)^m\mathrm{i}\beta k\phi + 2\mathrm{i}k\phi_\mathrm{i}, \quad \text{on} \pm x \text{ direction} \quad (20)$$

where $\beta = (1-R)/(1+R)$ is an absorption coefficient, R is the reflection coefficient, ϕ_i denotes the velocity potential of the incident waves. For the partial reflection boundary,

$\phi_{\rm i}=0$, $m=0$ and $0\le\beta\le1$. For the given boundary condition, $m=1$, $\beta=1$ and $\phi_i=(igH_0\sqrt{CC_{\rm g}}/2\omega){\rm e}^{{\rm i}s}$, where H_0 is the incident wave height, $s=kx-\omega t$ is the phase function.

4. Experiments

In order to verify the validity of the present theory, experiments are conducted in a wave flume with a dimension of length = 100 m, width = 1.5 m and height = 2.0 m which is located in the Center of Harbor and Marine Technology, Institute of Transportation, Ministry of Transportation and Communications, Taichung, Taiwan. A piston-type wave generator is equipped in one side (end) of the flume to generate the sinusoidal waves and the other side (end) is placed with the absorbing material to dissipate the wave energy from reflection. Doubly composite artificial bars are placed discretely on a flat bottom, which are located in the middle region of the wave flume. The first artificial bar is placed 55 m far from the wave generator to avoid wave reflection back to it. The spacing of artificial beds is varied only in the x direction along the wave flume, and thus horizontally one-dimensional wave motion is generated, propagating normally over the artificial bars.

In total, 11 wave gauges with capacity type are displayed at different locations to measure the surface elevation in the experiments. The schematic diagram of doubly composite artificial bars on the wave flume and the setup are shown in Fig. 2. One wave gauge placed in region **A** is used to measure and to calibrate the incident wave conditions, one wave gauge is installed in region **E** to monitor the transmitted waves, and the other six wave gauges are setup in region **B** to estimate the reflected wave. Four wave gauges are displayed in regions **C** and **D** for measuring wave profiles over artificial bars. The placement of wave gauges in region **B** matches the requirement of the analysis of wave reflection coefficient for the least square method developed by Mansard and Funke (1980). The surface wave elevations were recorded at a 30 Hz frequency to achieve an accurate resolution in the range of the designed wave period.

Only rectangular artificial bars are chosen as typical examples in the laboratory experiments for the study of Bragg resonance by doubly composite artificial bars. The bar footprint B is adopted as 60 cm and the water depth is taken to be 60 cm for all

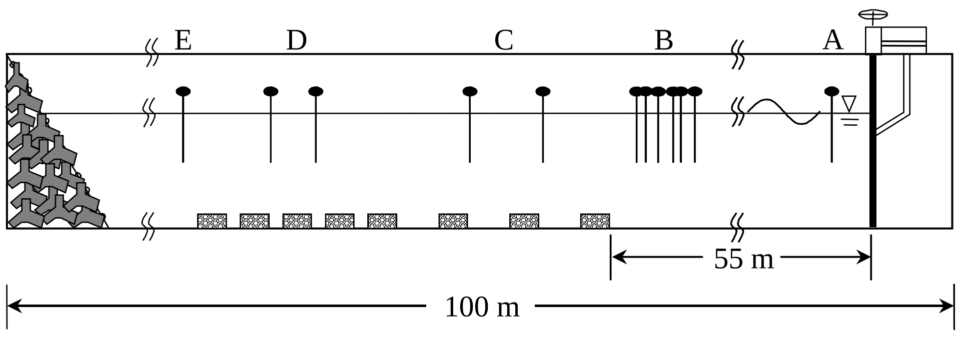

Fig. 2. Wave flume system and bar field placement.

experiments. The bar spacing of first series S_1 is fixed to 240 cm and the second spacing S_2 is varied as 180, 240, and 300 cm, respectively. Total numbers of bars used in the experiments are $N=4$ and 8. The bar height D is taken as 12 and 24 cm so that the values of the relative bar heights D/h became 0.2 and 0.4 in the experiments. Sinusoidal waves are thereby generated from the wave board. The wave height H_0 is taken as 4 cm in all the experiments and the wave periods T is varied from 1.03 to 4.03 s. The detailed experimental conditions are summarized in Table 1.

Wave steepness ka,with range $0.0013<ka<0.077$, and Stokes parameter $\hat{s}=a/k^2h^3$ ($0.006<\hat{s}<0.212$) used in the experiments are both within the range of linear theory, and $a=H_0/2$ is the wave amplitude. Fig. 3 presents the measured water surface elevation and it ensures that the waves are linear gravity waves. These results justify that the theoretical and the numerical model are based on the linear potential theory form a reasonable basis for comparison with experiments. The analysis of region **E** reveals that a reflection from the absorbing beach is within the order of 6% for all wave periods. This result further indicates that the influences due to the reflections at the end/side of the wave flume are negligible, and the experimental results for the Bragg reflection due to the doubly composite artificial bars are acceptable.

5. Theoretical verification

Using Miles' (1981) theory, the estimation of reflection coefficients for Bragg resonance under any shapes of bottom undulation could be achieved easily, even for the case of Bragg reflection induced by doubly composite artificial bars. With the formulae obtained from the integral equations in the former section, the phenomena of Bragg resonance could be easily captured. To have a better understanding of the Bragg resonance, three cases were considered:(a) a single series bars for $N=4$, $S=2.4$ m; (b)a single series bars for $N=4$, $S=1.8$ m; and (c) doubly composite bars for $N_1=4$, $N_2=4$, $S_1=2.4$ m, $S_2=1.8$ m. All cases are investigated at the same situations of $B=0.6$ m, $D=0.24$ m, and $h=0.6$ m. The reflection coefficients of the three cases are obtained from Eq. (6) and the results are shown in Fig. 4. The bar spacing has abscissa $2S_1/L$ in Fig. 4, with $S_1=2.4$ m.

Previous studies have proved that the Bragg resonance of sinusoidal bed include primary resonance, second-harmonic resonance, sub-harmonic resonance and higher-harmonic resonance(e.g. Davies et al., 1989; O'Hare and Davies, 1993; Belzons et al.,

Table 1
Experimental conditions

Case	Conditions of artificial bars							Wave conditions		
	N_1	N_2	N	S_1 (cm)	S_2 (cm)	D (cm)	B (cm)	h (cm)	T (s)	H_0 (cm)
1	4	4	8	240	240	24	60	60	1.03–4.	4.0
2	4	4	8	240	180	24	60	60	03	
3	4	4	8	240	180	12	60	60		
4	2	2	4	240	180	24	60	60		
5	4	4	8	240	300	24	60	60		

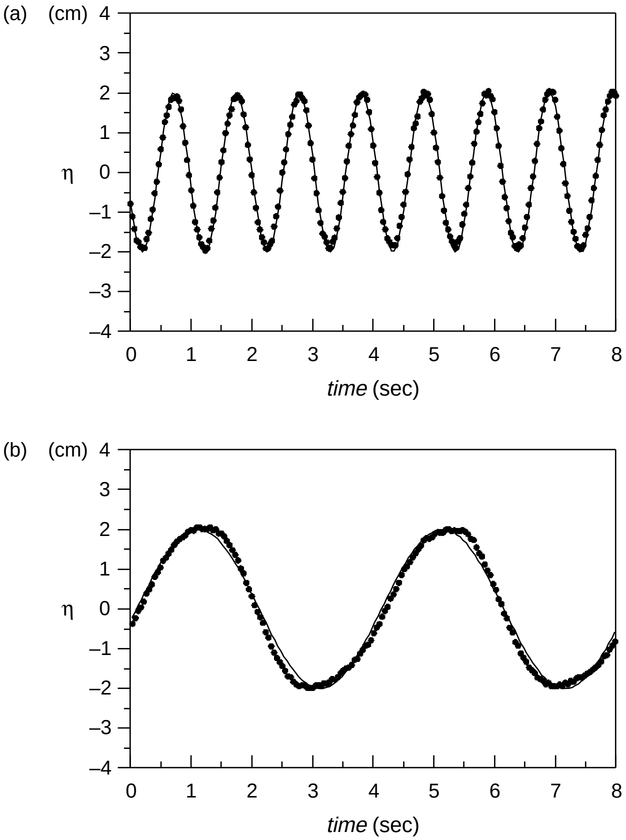

Fig. 3. Comparison of measured surface elevations with the linear wave theory: (a) $T=1.03$ s; (b) $T=4.03$ s; –, linear theory; ●, measurements.

1991; Guazzelli et al., 1992). Similar to the sinusoidal bed, the rectangular shape of doubly composite artificial bars also exhibit such characteristics and could be found in Fig. 4. For cases (a) and (b), the primary resonance occurs at $2k/K_1=1$ and $2k/K_2=1$, i.e. $L=2S_1$ and $L=2S_2$, respectively. Apparently, it appears in Fig. 4 at $2S_1/L=1$ and $2S_1/L=1.33$. The second-harmonic resonance is seen to occur at $2k/K_1=2$ and $2k/K_2=2$ for cases (a) and (b). As evident from Fig. 4, the second peaks occur at $2S_1/L=2$ and $2S_1/L=2.67$, respectively. The undulation bottom of case (c) is the superposition of two series rectangular artificial bars, i.e. cases (a) and (b). The higher-harmonic resonance in this case occurs at $k=(K_1+K_2)/2$, i.e. $L=2S_1S_2/(S_1+S_2)$. Although it is not conspicuous in Fig. 4, we also have higher-harmonic resonance at the frequency of $2S_1/L=2.33$. The sub-harmonic, lower frequency of resonance, appears at $k=(K_1-K_2)/2$, i.e. $L=2S_1S_2/(S_1-S_2)$. Using $S_1=2.4$ m and $S_2=1.8$ m, we get the sub-harmonic resonance frequency at $2S_1/L=0.33$.

Zhang et al. (1999) has observed the enhancement of bandwidth of the Bragg resonance with superposed sinusoidal undulation. The doubly composite rectangular artificial bars

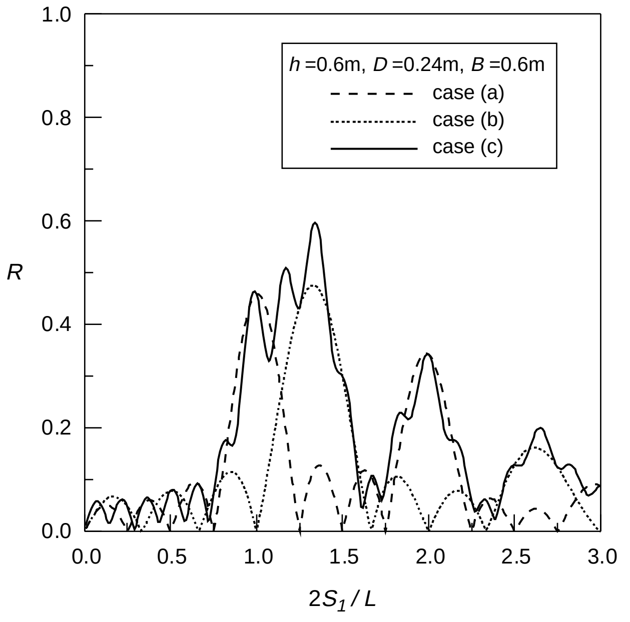

Fig. 4. Reflection coefficients with doubly composite rectangular artificial bars: (a) $N=4$, $S=2.4$ m; (b) $N=4$, $S=1.8$ m; (c) $N_1=4$, $N_2=4$, $S_1=2.4$ m, $S_2=1.8$ m.

seem to have the good efficiency, too. Because of the superposition, the reflection coefficients in the case (c) are reduplicated with cases (a) and (b). The resonances caused by the combinations of two different spacing S_1 and S_2 produce a larger bandwidth and enhance the wave-blocking efficiency.

The reflection coefficients of experimental results for cases 1–5 are shown in Figs. 5–9. Results are compared using the deductions in Eq. (6), the PMSE model (Hsu and Wen, 2001), and the EEMSE numerical method (Hsu et al., 2003). In spite of frequency shift, the magnitude of reflection coefficients as obtained from Eq. (6) appear in good agreement with the experiments, especially in cases 2 and 3. In cases 4 and 5, the Miles' (1981) theory makes a little underestimate for the primary resonance but the PMSE and EEMSE method, similar to many other numerical methods, appears to have larger errors with overestimating in all the cases. For safety reason in engineering, underestimation of the efficiency allow us to strengthen the countermeasures and will improve the safety. It implies that Miles' (1981) theory seems to be a very simple and quick method to grasp the Bragg resonance and could be applied in coastal engineering effectively.

6. Results and discussion

Based on the Miles' (1981) method, in the present investigation we intend to study the effect of the key parameters and observe their influence on the Bragg resonance.

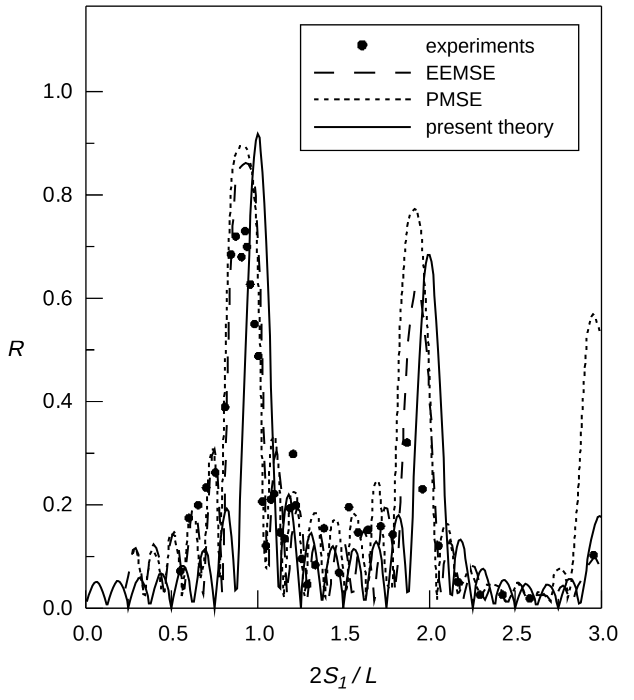

Fig. 5. Results of reflection coefficient for case 1 of Table 1.

The artificial bars are constructed in the field. Here, we keep the water depth to be fixed ($h=0.6$ m) to realize the influence of the other key parameters. Three different combinations of rectangular artificial bars $N=4$ ($N_1=N_2=2$), $N=6$ ($N_1=N_2=3$), and $N=8$ ($N_1=N_2=4$) are considered and the cases are investigated under the conditions $S_2/S_1=0.75$, $D/h=0.25$ and $B/S_1=0.25$. The reflection coefficients as obtained from Eq. (6) for the three different cases are shown in Fig. 10. Results indicate that the peak amplitude of primary and higher-harmonic resonance is increased as the numbers of bars increased. Such results are in agreement with the previous studies on the doubly sinusoidal bed of Guazzelli et al. (1992) and Zhang et al. (1999). Fig. 10 also shows that the bandwidth of the resonances increases at primary and higher-harmonic resonance while increasing the number of bars. It is because, the positions of resonances as caused by two groups with interval S_1 and S_2 are very close. Such a situation produces a larger bandwidth of harmonic resonance especially at the high frequency region.

Notably, Guazzelli et al. (1992) and Zhang et al. (1999) observed that a larger amplitude of sinusoidal bottom could increase the peak amplitude and bandwidth of Bragg resonance. In order to investigate the influence of the relative bar heights of doubly composite rectangular artificial bars, three varieties of relative bar heights $D/h=0.3$, $D/h=0.4$, and $D/h=0.5$ are considered here with $N=8$ ($N_1=N_2=4$), $S_2/S_1=0.75$ and $B/S_1=0.25$. As may be observed from Fig. 11, the present results match very well with the previous investigations. It is also important to note that the effect on the peak amplitude is

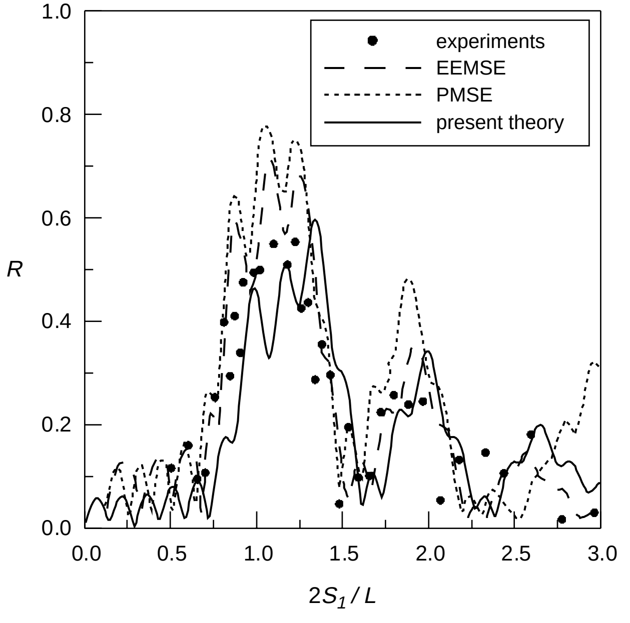

Fig. 6. Results of reflection coefficient for case 2 of Table 1.

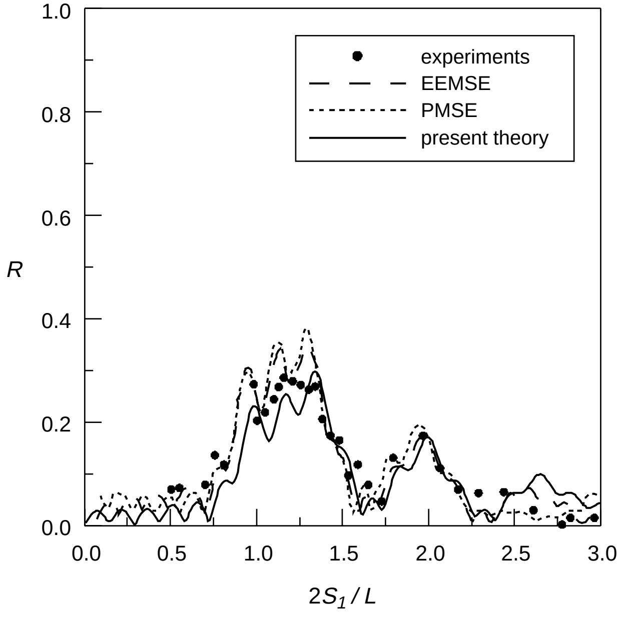

Fig. 7. Results of reflection coefficient for case 3 of Table 1.

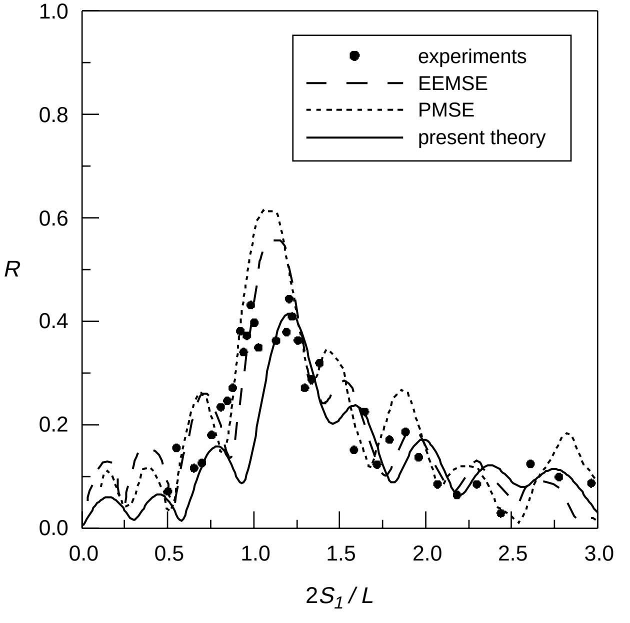

Fig. 8. Results of reflection coefficient for case 4 of Table 1.

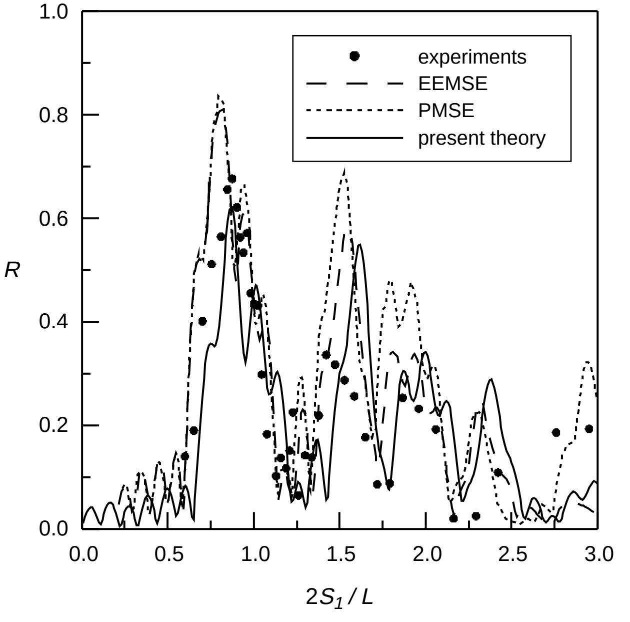

Fig. 9. Results of reflection coefficient for case 5 of Table 1.

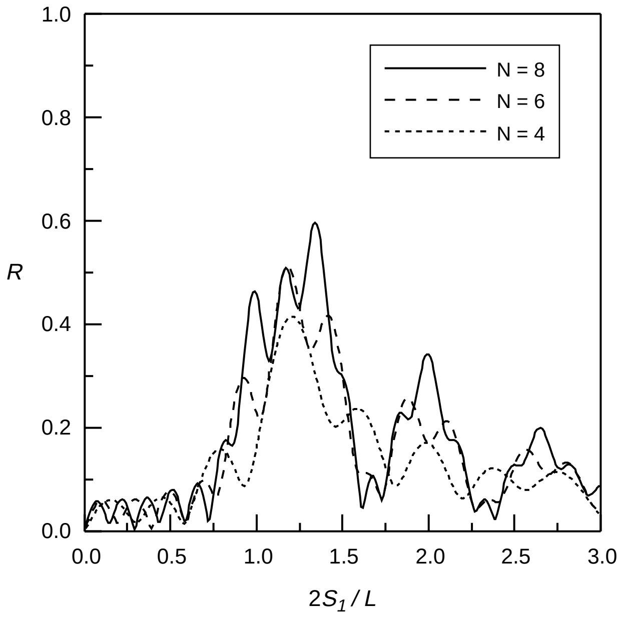

Fig. 10. Reflection coefficients with doubly composite rectangular artificial bars with different number of bars ($S_2/S_1=0.75$, $D/h=0.25$, and $B/S_1=0.25$).

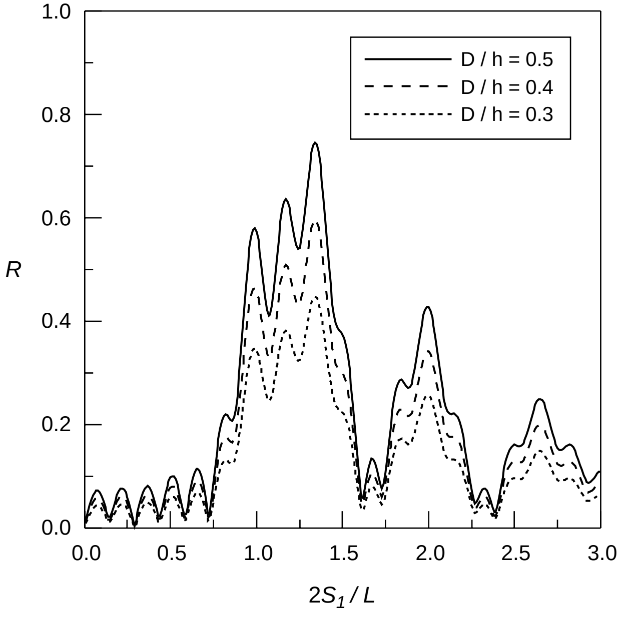

Fig. 11. Reflection coefficients with doubly composite rectangular artificial bars with different relative bar heights ($N=8$, $S_2/S_1=0.75$, and $B/S_1=0.25$).

more pronounced than on the bandwidth, as we increase the relative bar heights. In addition, a comparison between Figs. 10 and 11, show that the larger reflection is caused by increasing the relative bar heights than the number of bars. It implies that the relative bar heights could enhance the wave-blocking efficiency.

Furthermore, the investigation by Kriby and Anton (1990) reveal that the peak values of primary and higher-harmonic resonance can be adjusted by changing the bar spacing. In order to investigate such phenomenon with doubly composite rectangular artificial bars, three cases of relative bar with footprints $B/S_1 = 0.2$, $B/S_1 = 0.3$ and $B/S_1 = 0.4$ are studied here with $N = 8$, $S_2/S_1 = 0.75$ and $D/h = 0.4$. Fig. 12 shows that while the primary resonance increases with larger relative bar footprint, the higher-harmonic resonance decreases in such a situation. It is therefore concluded that pushing bars closer together could reduce the importance of higher-harmonic resonances.

Fig. 13 presents the effects of the ratio of bar spacing on Bragg reflection. Doubly composite artificial rectangular bars with different ratio of bar spacing, $S_2/S_1 = 0.8$, $S_2/S_1 = 1.0$ and $S_2/S_1 = 1.2$ are investigated under the condition $N = 8$, $D/h = 0.4$ and $B/S_1 = 0.25$. Notably, compared to the equal bar spacing ($S_2/S_1 = 1.0$), the bandwidth of both primary and higher-harmonic resonances become larger under the doubly composite artificial bars conditions ($S_2/S_1 = 0.8$ and $S_2/S_1 = 1.2$). The reason, as discussed above, is that different bar spacing could produce distributive positions of the Bragg reflection. It is interesting to note here that the primary and the second resonance do not occur at $2S_1/L = 1$ and $2S_1/L = 2$ but at $2S_1/L = 0.83$ and $2S_1/L = 1.67$ under the condition of $S_2/S_1 = 1.2$. It seems that

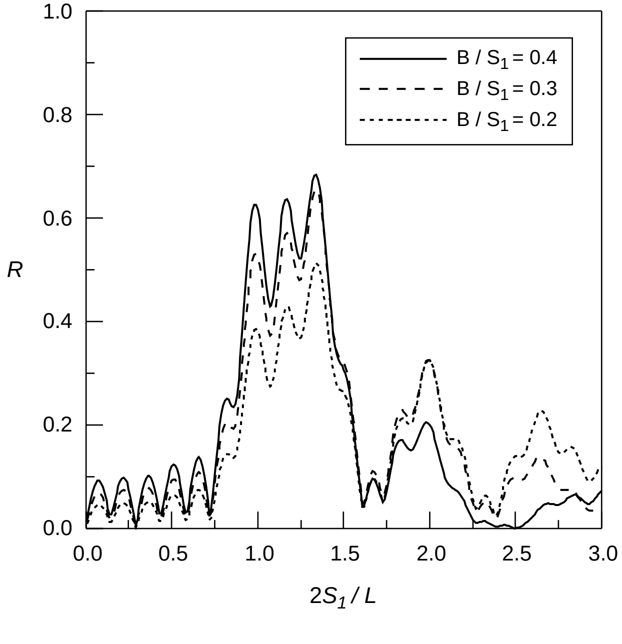

Fig. 12. Reflection coefficients with doubly composite rectangular artificial bars with different relative bar footprints ($N = 8$, $S_2/S_1 = 0.75$, and $D/h = 0.4$).

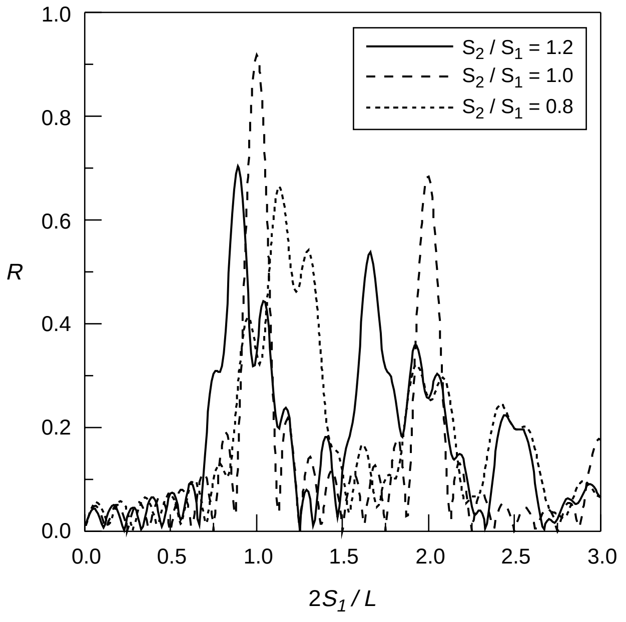

Fig. 13. Reflection coefficients with doubly composite rectangular artificial bars with different ratios of bar spacing ($N=8$, $D/h=0.4$ and $B/S_1=0.25$).

the Bragg resonance is particularly influenced by the bigger spacing when doubly composite artificial bars are used.

Finally, we examine the efficiencies of different shapes of artificial bars. The reflection coefficients for three differently shaped artificial bars, i.e. rectangle, triangle and rectified cosine, with $N=8$, $S_2/S_1=0.75$, $D/h=0.4$, and $B/S_1=0.25$ are presented in Fig. 14. The rectangular artificial bars are seems to have more influence than the other two shapes. The reason is that, the rectangular bars have a large volume and a vertical contour in the front face. Consequently, it produces higher reflection coefficients and therefore have important role to play.

7. Conclusions

Based on the Miles' (1981) theory, in the present study, the reflection coefficients (as induced by the Bragg scattering for wave propagation) over doubly composite artificial submerged breakwaters with different spacing were derived. The distributive positions of the Bragg reflection by doubly composite artificial bars could be illustrated reasonably. Comparison of the experimental measurements, Hsu and Wen's (2001) PMSE model and Hsu et al's (2003) EEMSE method, reveals that the present method is capable of producing good results. It seems that Miles' (1981) theory could be a much simple and quick method to interpret the Bragg resonances than many other complex numerical methods. It can be applied effectively and conveniently in practical engineering.

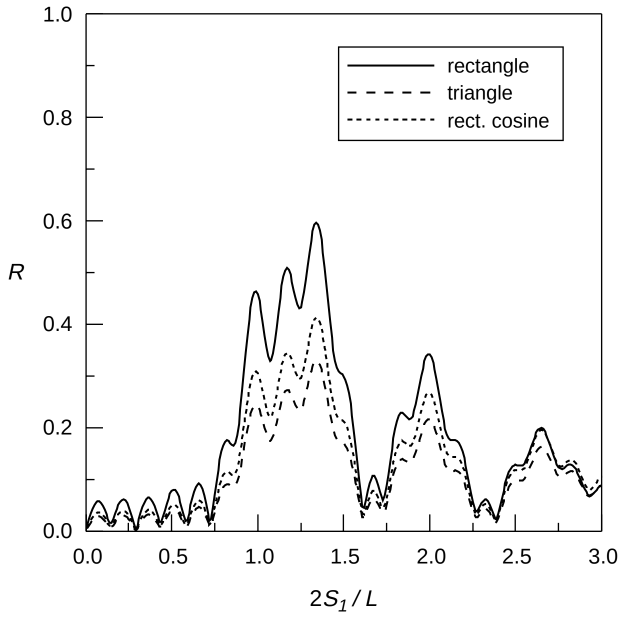

Fig. 14. Reflection coefficients of different shapes of doubly composite artificial bars ($N=8$, $S_2/S_1=0.75$, $D/h=0.4$, and $B/S_1=0.25$).

Three shapes of doubly composite artificial bars are considered to examine the efficiency of resonance. Rectangular bars produce higher reflection coefficients in both primary and higher-harmonic resonance than triangular and rectified cosine bars. It also shows that the Bragg resonances are governed by some key parameters, the number of bars, relative bar height, relative bar footprint and ratio of bar spacing. In addition to resonance peak, doubly composite artificial bars are observed to significantly influence the higher-harmonic resonances and the bandwidth at the high performance region. It also found that when two different spacing of doubly composite bars were used the Bragg resonances are dominated by the bigger spacing. An increase in the numbers of bars and the relative bar heights, both the amplitude and bandwidth increase at primary and higher-harmonic resonances. But the magnitude and bandwidth increases at primary resonance and decreases at the higher-harmonic resonance when the relative bar footprint increased. It is concluded that key parameters could be suitably chosen to control the Bragg resonance. Doubly composite artificial bars with appropriate key parameters may used to protect the beach face from erosion.

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