Theory of HF Ground Wave Backscatter from Sea Waves

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A simplified but quantitative theory is presented for backscatter of high-frequency radio waves from a gently rippled surface. A principal resonance occurs when the electromagnetic wavelength is just double the predominant wavelength of the sea waves. The results have some relevance to reported experimental observations.

A number of interesting observations, at HF, concerning radar backscatter from sea waves have been reported in recent years. Unlike the more conventional 'clutter' observed with microwave radar, HF observations indicate that the quasi-periodic character of ocean waves will, in some cases, lead to resonance-type reflections. In a significant paper, *Crombie* [1955] indicated that a principal resonance occurs when the electromagnetic wavelength is just double the predominant wavelength of the sea waves. Further observations and discussions of this phenomenon were given by *Dowden* [1957] and *Haubert* [1958].

In this paper a quantitative theory for this type of backscatter is outlined. A rather idealized model is chosen and some gross simplifications are made in order to achieve tractability.

The model is shown in Figure 1, which is a plan view of the situation with respect to a Cartesian (x, y, z) coordinate system. The transmitter, which is idealized as a vertical electric dipole, is located at point A, which has coordinates $(-d, 0, 0)$. The mean surface of the sea and the transmitter are assumed to be the plane $z = 0$. We now wish to calculate the change δZ of the self-impedance of the dipole due to a gently rippled surface extending from $x = 0$ to $x = d_0$ as indicated in Figure 1. As we shall see, this can be related to the strength of the backscatter echo when A is transmitting pulses.

The ripples (e.g., the sea waves) are taken to be uniform in the y direction. Thus the surface is adequately characterized by its slope $\gamma(x)$ being only a function of x. Using a previous formulation [*Wait and Jackson*, 1963], for the 'sloping-beach problem,' we can immediately write

$$\delta Z = -\frac{\eta_0}{I^2}\int_{y=-\infty}^{\infty}\int_{x=0}^{d_0}\gamma(x)H_{ay}H_{ay}'\,dx\,dy \quad (1)$$

where $\eta_0 = 120\pi$, I is the current at the terminals of the antenna, H_{ay} is the magnetic field over the mean surface $z = 0$ in the absence of the waves (i.e., $\gamma = 0$), and H_{ay}' is the corresponding value of the magnetic field when the surface is perturbed. Now, presumably, H_{ay} is known, but H_{ay}' is not. However, for a perturbation theory, we can replace H_{ay}' by H_{ay} which, in effect, neglects multiple scattering.

From the theory of ground wave propagation, we write

$$H_{ay} = \frac{ikIh_e}{2\pi r}e^{-ikr}(\sin\theta)W(r) \quad (2)$$

where $k = 2\pi/(\text{radio wavelength})$, h_e is the effective height of the source dipole, $r = [(x + d)^2 + y^2]^{1/2}$, $\sin\theta = (x + d)/r$, and $W(r)$ is the ground wave attenuation function [e.g., *Wait*, 1964]. This expression for H_{ay} is valid for $kr \gg 1$, and, of course, $W(r)$ depends on the electrical characteristic of the surface. In what follows, we shall set $W(r) = 1$, which corresponds to neglecting the attenuation of the ground wave. The consequence of this assumption is mentioned briefly below. Using (2), we now express (1) in the following form:

$$\delta Z = \frac{k^2\eta_0 h_e^2}{4\pi^2}\int_{x=0}^{d_0}(x + d)^2\gamma(x) \cdot\left[\int_{-\infty}^{+\infty}\frac{\exp[-2ik[(x + d)^2 + y^2]^{1/2}]\,dy}{[(x + d)^2 + y^2]^2}\right]dx \quad (3)$$

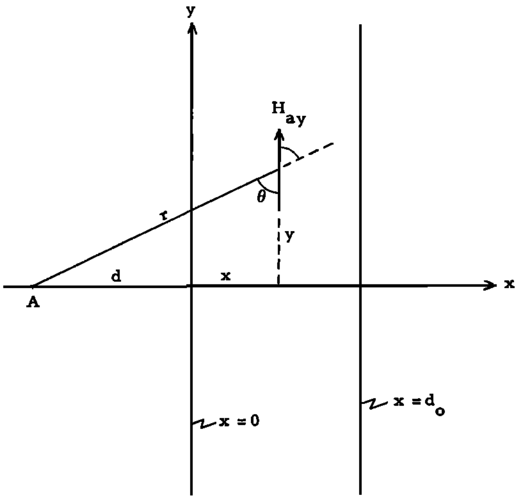

Fig. 1. Plan view of rippled surface with source location at A.

Although the inner integral may be expressed in terms of Hankel functions, we content ourselves here with an approximated evaluation which, in effect, replaces $[(x + d)^2 + y^2]^{1/2}$ in the exponent by $(x + d) + (y^2/2)/(x + d)$; at the same time the denominator of the integrand is merely replaced by $(x + d)^4$. Then, using the result that

$$\int_0^\infty \exp(-z^2)\, dz = \pi^{1/2}/2$$

we find it a simple matter to show that

$$\delta Z = \frac{k\eta_0 h_e^{\,2}}{4\pi}\left(\frac{k}{i\pi}\right)^{1/2} e^{-2ikd} \int_0^{d_0} \frac{\gamma(x)e^{-2ikx}}{(x + d)^{3/2}}\, dx \tag{4}$$

The physical significance of (4) is clear. The power reflected from an elemental strip of width dx is proportional to $(x + d)^{-3}$. This inverse cube law has been observed by *Dowden* [1957]. It is interesting to note from (4) that the backscatter power for an elemental strip is proportional to $\gamma^2(x)$, which is the square of the slope. It should also be mentioned that if the attenuation of the ground wave is to be considered, the function $[W(x + d)]^2$ should be included in the integrand of (4).

To cast (4) into a slightly more meaningful form, we normalize it by dividing by the mutual impedance Z_0 for two dipoles at a distance $2d$. The latter is given by

$$Z_0 = \frac{ik\eta_0}{4\pi d} h_e^{\,2} e^{-2ikd} \tag{5}$$

where, again, we have neglected the attenuation of the ground wave. The dimensionless quantity $\delta Z/Z_0 = R$ may be regarded as a reflection coefficient referred to the vertical plane at $x = 0$. Explicitly,

$$R = (k/\pi)^{1/2}\, d e^{-i3\pi/4} \int_0^{d_0} \frac{\gamma(x)e^{-2ikx}}{(x + d)^{3/2}}\, dx \tag{6}$$

It should be mentioned that because of the limitations of the theory (e.g., smallness of $\gamma(x)$ and neglect of multiple scatter), this result for R is valid only for situations which yield $|R| \ll 1$.

The profile $h(x)$ of the sea surface may be represented in the form

$$h(x) = \sum_{n=1}^{\infty} h_n \sin n\beta x \tag{7}$$

where $\beta = 2\pi/l$, l is the basic sea wavelength, n is an integer, and h_n is a coefficient. Then, of course, the slope is

$$\gamma(x) = \frac{dh(x)}{dx} = \beta \sum_{n=1}^{\infty} nh_n \cos n\beta x \tag{8}$$

Using this result, we can write (6) as

$$R = \sum_{n=1}^{\infty} R_n \tag{9}$$

where

$$R_n = e^{-i3\pi/4}\left(\frac{k}{\pi}\right)^{1/2} dh_n(n\beta) \cdot \int_0^{d_0} \frac{\cos(n\beta x)}{(x + d)^{3/2}} e^{-2ikx}\, dx \tag{10}$$

By integrating by parts, this can be expressed in the form

$$R_n = e^{-i3\pi/4}(n\beta)h_n d\left(\frac{k}{\pi}\right)^{1/2} \cdot \Bigg[2\left(\frac{1}{d^{1/2}} - \frac{\cos(n\beta d_0)e^{-i2kd_0}}{(d + d_0)^{1/2}}\right) - i(2k - n\beta)\int_0^{d_0} \frac{e^{-i(2k-n\beta)x}}{(x + d)^{1/2}}\, dx - i(2k + n\beta)\int_0^{d_0} \frac{e^{-i(2k+n\beta)x}}{(x + d)^{1/2}}\, dx\Bigg] \tag{11}$$

By a change of variable, via $(2k \mp n\beta)(x + d) = \pi z^2/2$, the latter two integrals may be expressed in terms of the extensively tabulated Fresnel integral

$$F(z_0) = \int_0^{z_0^{1/2}} \exp\left(-i\frac{\pi}{2}z^2\right) dz = C(z_0^{1/2}) - iS(z_0^{1/2}) \tag{12}$$

Thus

$$\begin{aligned} R_n = e^{-i3\pi/4}(n\beta)h_n \cdot \Big\{ & 2d\left(\frac{k}{\pi}\right)^{1/2}\left[\frac{1}{d^{1/2}} - \frac{\cos(n\beta d_0)e^{-i2kd_0}}{(d+d_0)^{1/2}}\right] \\ & - i[2k(2k - n\beta)]^{1/2} d e^{i(2k-n\beta)d} \\ & \cdot [F[(2/\pi)(2k - n\beta)(d + d_0)] \\ & - F[(2/\pi)(2k - n\beta)d]] \\ & - i[2k(2k + n\beta)]^{1/2} d e^{i(2k+n\beta)d} \\ & \cdot [F[(2/\pi)(2k + n\beta)(d + d_0)] \\ & - F[(2/\pi)(2k + n\beta)d]]\Big\} \end{aligned} \tag{13}$$

This expression may be used to compute R_n, which is the reflection coefficient for a spatial harmonic of order n.

Considerable simplification is achieved if we are permitted to assume that $d_0 \ll d$. Then (10) may be approximated by

$$R_n \approx e^{-i3\pi/4}\left(\frac{k}{\pi d}\right)^{1/2}(n\beta)h_n \cdot \int_0^{d_0} \cos(\beta n x)e^{-2ikx}\, dx \tag{14}$$

which is readily evaluated to give

$$\begin{aligned} R_n \approx e^{-i3\pi/4}\left(\frac{k}{\pi d}\right)^{1/2}(n\beta)\frac{d_0}{2}h_n \cdot \Big\{ & e^{-i(2k-n\beta)d_0/2}\frac{\sin[(2k - n\beta)(d_0/2)]}{(2k - n\beta)(d_0/2)} \\ & + e^{-i(2k+n\beta)d_0/2}\frac{\sin[(2k + n\beta)(d_0/2)]}{(2k + n\beta)(d_0/2)}\Big\} \end{aligned} \tag{15}$$

From this result it is evident that there is only an appreciable response when $2k \approx n\beta$, in full accord with the work of *Crombie* [1955]. In fact, if $k\,d_0 \gg 1$, we see that

$$|R_n| \approx \frac{1}{2}\left(\frac{k}{\pi d}\right)^{1/2}(n\beta)d_0 h_n \frac{\sin[(2k - n\beta)(d_0/2)]}{(2k - n\beta)(d_0/2)} \tag{16}$$

which has the familiar $(\sin X)/X$ response associated with uniform antenna arrays. The maximum, of course, occurs if $2k = n\beta$, whence

$$|R_n| \approx (kd_0/\pi)^{1/2}(kh_n)(d_0/d)^{1/2} \tag{17}$$

which is a remarkably simple result.

To illustrate the magnitude of the echo strength, we consider the contribution from ten successive sea waves with a wavelength of 20 m. The operating frequency for resonance scatter is thus 7.5 Mc/s (i.e., $\lambda = 40$ m). Then, if we choose a wave height $h_n = 1$ m and a range $d = 20$ km, (17) yields

$$|R_n| \approx 0.05$$

which is the order of magnitude of echoes reported by *Dowden* [1957]. It should be mentioned, however, that, according to Dowden's explanation, coherent reflections from many more sea waves are required to obtain reflection coefficients of this order. We believe that under actual conditions resonance scatter is not effective over the whole length of the transmitter pulse because the dominant period is quite variable. As suggested by *Crombie* [1955], it is more likely that sea waves occur in relatively short trains. At least this view is compatible with the spectrum of his Doppler-shifted echoes of 13.56 Mc/s. Certainly, Dowden's model of 10^3 equi-spaced crests is difficult to accept. I believe his method of calculating backscatter from a single wave leads to a gross underestimate. For this reason, he apparently needs to sum over an excessively large number of waves to obtain an effective reflection coefficient of the order of 0.05. Despite this apparent discrepancy, I believe the present analysis confirms the conclusions put forth by Dowden concerning the nature of ground wave backscatter at HF.

Finally, I should mention that the inclusion of the ground wave attenuation function $[W(x + d)]^2$, in the integrand of (4) and again as a factor $W(2\,d)$ on the right-hand side of (5), will modify the results. However, for HF propagation over seawater, W does not depart from unity by more than 10% if the ranges are less than 20 km. Because of my method of normalizing, the reflection coefficient R has only a weak dependence on the ground wave attenuation, but it is worth while, in further work on this subject, to include this modification. Also, of course, other factors such as multiple scattering and the effect of steeply inclined wave crests should be considered.

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