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The effect of the modulation of Bragg scattering in small-slope approximation

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Abstract
Small-slope approximation (SSA) belongs to a class of 'unifying' scattering theories which reproduce small perturbations and semiclassic (Kirchhoff) results within appropriate limits. However, the most stringent test for such theories involves a two-scale situation when a small-scale roughness is located on a tilted plane. A 'unifying' theory should properly account for the effects of modulation of the scattering cross section associated with a large-scale tilt. This paper shows that SSA does properly take into account these modulation effects.

1. Introduction

The calculation of wave scattering from rough surfaces based on a two-scale model (TSM) was first introduced for acoustic waves in [1], and then for electromagnetic waves in [2] and [3]. Now it is most widely applied to theoretical interpretation of radar scattering data. The reasons for this are its relative simplicity and, more importantly, its transparent physical background.

A TSM separates the whole spectrum of roughness into large- and small-scale components. The small-scale component scatters incident waves according to the Bragg (resonant scattering) mechanism. The large-scale component tilts the facets carrying the small-scale scatterers and modulates the Bragg scattering appropriately. One can single out three different mechanisms of such modulation. *First*, it changes the resonant (Bragg) wavevector that is responsible for resonant scattering of the given incident wave into a given direction in the absolute coordinate system (this is a purely kinematic effect). *Second*, the value of the scattering cross section, which depends on local incidence and scattering angles measured from normal to a given facet, is modulated. *Third*, for vector waves, local tilts modify polarization states, and this effect also contributes to overall scattering cross section modulation.

Providing a physically correct description of the process of scattering TSM suffers from two inherent drawbacks. First, the power-type spectra usually met in applications do not suggest a natural division of scales, and the scale-dividing parameter can be arbitrarily chosen within certain limits. Different recipes could be invoked to fix the value of this parameter; however, those recipes are also arbitrary. The recipe chosen can affect the results when one is comparing experimental data and theoretical modelling, especially when it comes down to relatively small differences of the order of 1 dB or less. Second, the TSM practically does not allow one to calculate corrections to it. For this reason the overall accuracy of its predictions is difficult to control. As a result, one cannot tell whether observed differences between experimental data and theoretical measurements should be attributed to inaccurate scattering calculations or to an inadequate model of roughness. This hinders further development of appropriate remote sensing techniques.

For these reasons, numerous scattering theories that go beyond the TSM have been suggested. One possible approach is called the small-slope approximation (SSA) [4, 5]. The SSA represents a systematic expansion of a scattering amplitude (SA) (or scattering cross section, in a statistical case) with respect to slopes of roughness. The SSA does not require the introduction of any scale-dividing parameter, and it allows calculation of both a basic approximation of the theory (SSA-1) and second-order corrections to it (SSA-2), thus allowing one to estimate the accuracy of theoretical calculations. The SSA have been successfully applied to numerical calculations of the scattering of electromagnetic waves from the sea surface [6].

On the other hand, the modulation effects described above and accounted for by a TSM in a 'clean' two-scale situation (i.e. when only two clearly distinct scales of roughness are present) correspond to a physical reality, and the results provided by the TSM in this case are asymptotically correct. The question then arises whether these modulation effects are accounted for by the SSA. The way the SSA was derived [4, 5] does not allow one to answer this question. This paper addresses exactly this issue, and provides a positive answer to it.

Generally speaking, the two-scale situation provides the most stringent analytical test for 'unifying' scattering theories. The theory, which in appropriate limiting cases reproduces small perturbations and Kirchhoff results will not necessarily do so in this more general case. To pass the test, a theory has to account for the all the above-described 'modulation' effects properly.

2. Modulation effects in the SSA

This section provides proof that all three modulation effects mentioned in the previous section are accounted for by the SSA. To this effect, we will consider the situation when small-amplitude roughness is located on a slightly tilted plane. Scattering from the small-scale roughness is asymptotically exactly described by first-order perturbation theory (Bragg scattering). The presence of the tilted plane in this case can be accounted for by the simple rotation of a coordinate system. To the lowest nontrivial order, the result is proportional to the product of the amplitude of small-scale roughness and the tilt of the plane.

On the other hand, this two-scale situation can be directly considered with the help of an explicit SSA expression. Since the terms proportional to the product of two small slopes are in fact involved, it is clear that to obtain a correct answer we will need to consider the second-order SSA (SSA-2). We will show that both results do coincide to the accuracy of terms proportional to the product of the slope of the tilted plane and the amplitude of the small-scale roughness.

Let us assume that the incident EM wave has the form

$$\Psi_{\text{in}} = \exp(\mathrm{i}\vec{k}_0\vec{r} + \mathrm{i}q_0 z)\Psi_0 \tag{1}$$

(the z-axis is directed downward). Here, $\vec{k}_0, q_0 = \sqrt{\omega^2/c^2 - k_0^2}$ are projections of the wavevector of the incident wave on the horizontal plane $\vec{r} = (x, y)$ and the z-axis, accordingly. The scattered field Ψ_{sc} can be represented in the form

$$\Psi_{sc} = q_0^{1/2} \int q_k^{-1/2} \exp(i\vec{k}\vec{r} - iq_k z) S(\vec{k}, \vec{k}_0)\, d\vec{k} \cdot \Psi_0, \tag{2}$$

where $\vec{k}, q_k = \sqrt{\omega^2/c^2 - k^2}$ are horizontal and vertical projections of the wavevector of the scattered wave. Here, Ψ_0, Ψ are two-component column polarization-state vectors

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

whose coordinates represent complex amplitudes of vertically (a_1) and horizontally (a_2) polarized waves. A pair of these complex amplitudes provide a complete description of the field. For instance, an electric vector $\vec{E}$ in the incident wave is

$$\vec{E} = (a_1 \vec{E}_1 + a_2 \vec{E}_2) \exp(i\vec{k}_0\vec{r} + iq_0 z). \tag{3}$$

We will choose basis vectors $\vec{E}_1$, $\vec{E}_2$ according to a standard definition of the waves of vertical and horizontal polarization. For vertically polarized waves, a unit vector of electric field $\vec{E}_1$ lies in the plane formed by a unit normal to the horizontal plane $\vec{N} = (0, 0, 1)$ and a three-dimensional wavevector of the incident wave $\vec{K}_0$:

$$\vec{K}_0 = \vec{k}_0 + q_0 \vec{N}. \tag{4}$$

For horizontally polarized waves, a unit electric vector $\vec{E}_2$ is parallel to the horizontal plane. Thus,

$$\vec{E}_2 = \frac{[\vec{N}, \vec{K}_0]}{|[\vec{N}, \vec{K}_0]|}, \qquad \vec{E}_1 = \frac{c}{\omega}[\vec{K}_0, \vec{E}_2]. \tag{5}$$

After substituting equation (4) into (5) we obtain [4]

$$\vec{E}_1 = \frac{c}{\omega} \frac{k_0^2 \vec{N} - q_0 \vec{k}_0}{k_0}, \qquad \vec{E}_2 = \frac{[\vec{N}, \vec{k}_0]}{k_0}. \tag{6}$$

(Throughout the paper $[\vec{a}, \vec{b}]$ denotes the vector product of appropriate vectors and $k_0 = |\vec{k}_0|$, $k = |\vec{k}|$.) An expression for basis electric field vectors for the scattered field coincides with equations (5), (6) with the sign of the vertical component of the wavevector changed: $\vec{K}_0 \to \vec{K} = \vec{k} - q_k \vec{N}$, $q_0 \to -q_k$.

SA $S(\vec{k}, \vec{k}_0)$ represents a 2×2 matrix, correspondingly. In the SSA the expression for $S(\vec{k}, \vec{k}_0)$ is as follows [4]:

$$S(\vec{k}, \vec{k}_0) = \frac{2(q_k q_0)^{1/2}}{q_k + q_0} \int \frac{d\vec{r}}{(2\pi)^2} \exp[-i(\vec{k} - \vec{k}_0)\vec{r} + i(q_k + q_0) h(\vec{r})] \times \left(B(\vec{k}, \vec{k}_0) - \frac{i}{4} \int M(\vec{k}, \vec{k}_0; \vec{\xi}) \hat{h}(\vec{\xi}) \exp(i\vec{\xi}\vec{r})\, d\vec{\xi} \right) \tag{7}$$

where $h = h(\vec{r})$ is the shape of undulations and

$$\hat{h}(\vec{\xi}) = \int h(\vec{r}) \exp(-i\vec{\xi}\vec{r}) \frac{d\vec{r}}{(2\pi)^2} \tag{8}$$

is its Fourier transform. The first term $B(\vec{k}, \vec{k}_0)$ in the bracket in equation (7) corresponds to the lowest (first) order approximation, and the second (integral) term provides a correction. The function M is as follows:

$$M(\vec{k}, \vec{k}_0; \vec{\xi}) = B_2(\vec{k}, \vec{k}_0; \vec{k} - \vec{\xi}) + B_2(\vec{k}, \vec{k}_0; \vec{k}_0 + \vec{\xi}) + 2(q_k + q_0) B(\vec{k}, \vec{k}_0); \tag{9}$$

the explicit expressions for kernels B and B_2 are given in [6], and for easy reference are reproduced here (see the appendix).

It can be proven [7] that in the general case

$$M(\vec{k}, \vec{k}_0; 0) = 0. \tag{10}$$

Considering the case of a small roughness located on a horizontal plane: $z = 0$ and expanding the exponential in equation (7) with account of equation (10), we find to the lowest order in elevations

$$S(\vec{k}, \vec{k}_0) = B(\vec{k}, \vec{k})\delta(\vec{k} - \vec{k}_0) + 2\mathrm{i}(q_k q_0)^{1/2} B(\vec{k}, \vec{k}_0)\hat{h}(\vec{k} - \vec{k}_0). \tag{11}$$

The scattered field associated with Bragg scattering (the second term in equation (11)) becomes

$$\Delta\Psi_B = 2\mathrm{i}q_0 \int B(\vec{k}, \vec{k}_0)\hat{h}(\vec{k} - \vec{k}_0) \exp(\mathrm{i}\vec{k}\vec{r} - \mathrm{i}q_k z)\, \mathrm{d}\vec{k} \cdot \Psi_0. \tag{12}$$

Let us consider the case of an incident wave of vertical and horizontal polarization

$$\Psi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{or} \qquad \Psi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

We can combine both cases into one, assuming that Ψ_0 in equation (12) is a unit 2×2 matrix; $\Delta\Psi_B$ becomes a second-order matrix as well.

Let us now rotate both the roughness and the coordinate system by a small angle, keeping the incident three-dimensional wavevector $\vec{K}_0$ the same. The horizontal (x, y) plane now transforms into the plane $z = \vec{a}\vec{r}$. Let us suppose that all calculations below are performed to the first order with respect to a small, dimensionless vector $\vec{a}$. The coordinates and incident wavenumbers with respect to the rotated system are obviously as follows:

$$\begin{aligned} \vec{r}' &= \vec{r} + \vec{a}z \qquad & \vec{k}_0' &= \vec{k}_0 + \vec{a}q_0 \\ z' &= z - \vec{a}\vec{r}\,, \qquad & q_0' &= q_0 - \vec{a}\vec{k}_0. \end{aligned} \tag{13}$$

According to equation (12), the expression for a scattered field calculated to the first order with respect to roughness amplitude h from the viewpoint of the rotated coordinate system is

$$\Delta\Psi_B' = 2\mathrm{i}q_0' \int B(\vec{k}', \vec{k}_0')\hat{h}(\vec{k}' - \vec{k}_0') \exp(\mathrm{i}\vec{k}'\vec{r}' - \mathrm{i}q_{k'}z')\, \mathrm{d}\vec{k}' \cdot \Psi_0'. \tag{14}$$

Substituting expressions for $\vec{r}'$, z' from equation (13) we find

$$\vec{k}'\vec{r}' - q_k' z' = (\vec{k}' + \vec{a}q_k')\vec{r} - (q_k' - \vec{a}\vec{k}')z. \tag{15}$$

Introducing a new variable of integration $\vec{k}$:

$$\vec{k} = \vec{k}' + \vec{a}q_k', \qquad \mathrm{d}\vec{k}' = \mathrm{d}\vec{k}\,\frac{q_k'}{q_k}$$

where

$$q_k' = q_k + \vec{a}\vec{k},$$

we cast equation (14) into the form

$$\begin{aligned} \Delta\Psi_B' = 2\mathrm{i}(q_0 - \vec{a}\vec{k}_0) \int &\frac{(q_k + \vec{a}\vec{k})}{q_k} B(\vec{k} - \vec{a}q_k, \vec{k}_0 + \vec{a}q_0)\Psi_0' \\ &\times \hat{h}(\vec{k} - \vec{k}_0 - \vec{a}(q_k + q_0)) \exp(\mathrm{i}\vec{k}\vec{r} - \mathrm{i}q_k z)\, \mathrm{d}\vec{k}. \end{aligned} \tag{16}$$

Note, that polarization state matrix Ψ_0 in the new coordinate system has also changed to Ψ_0'. To calculate Ψ_0' one simply needs to project the same incident field given by equation (3) onto new basis vectors $\vec{E}_1'$ and $\vec{E}_2'$. Thus, polarization states transform as follows:

$$\Psi_0' = \begin{pmatrix} \vec{E}_1' \cdot \vec{E}_1 & \vec{E}_1' \cdot \vec{E}_2 \\ \vec{E}_2' \cdot \vec{E}_1 & \vec{E}_2' \cdot \vec{E}_2 \end{pmatrix} \Psi_0 = T(\vec{k}_0, \vec{a})\Psi_0. \tag{17}$$

Vectors $\vec{E}_1'$ and $\vec{E}_2'$ are given by the same equation (5) with $\vec{N}$ replaced by $\vec{N}'$. Taking into account that

$$\vec{N}' = (\vec{N} - \vec{a})(1 + a^2)^{-1/2} = \vec{N} - \vec{a} + \mathrm{O}(a^2) \tag{18}$$

after simple calculations we obtain

$$\vec{E}_1' = \frac{c}{\omega}\frac{k_0^2\vec{N} - q_0\vec{k}_0}{k_0} + \frac{\omega}{c}\frac{(\vec{a}\vec{k}_0)\vec{k}_0 - k_0^2\vec{a}}{k_0^3}, \tag{19}$$

$$\vec{E}_2' = \frac{k_0^2 - q_0(\vec{a}\vec{k}_0)}{k_0^3}[\vec{N}, \vec{k}_0] + \frac{[\vec{k}_0 + q_0\vec{N}, \vec{a}]}{k_0}. \tag{20}$$

Substituting equations (19), (20), and (6) into (17) we find the following explicit expression for matrix T:

$$T(\vec{k}_0, \vec{a}) = \begin{pmatrix} 1 & -\frac{\omega}{ck_0^2}[\vec{N}, \vec{k}_0]\cdot\vec{a} \\ \frac{\omega}{ck_0^2}[\vec{N}, \vec{k}_0]\cdot\vec{a} & 1 \end{pmatrix}. \tag{21}$$

One can easily see that to $\mathrm{O}(a)$ accuracy this is an orthogonal matrix (as it should be, since it has to preserve $|\vec{E}|^2$). Multiplying equation (16) by $T^{-1}(\vec{k}, \vec{a}) = T(\vec{k}, -\vec{a})$, and thus expressing the field in the initial reference system, we finally obtain

$$\Delta\Psi_B = 2\mathrm{i}\frac{(q_0 - \vec{a}\vec{k}_0)(q_k + \vec{a}\vec{k})}{q_k}\int T(\vec{k}, -\vec{a})B(\vec{k} - \vec{a}q_k, \vec{k}_0 + \vec{a}q_0)T(\vec{k}_0, \vec{a})$$
$$\times\,\hat{h}(\vec{k} - \vec{k}_0 - \vec{a}(q_k + q_0))\exp(\mathrm{i}\vec{k}\vec{r} - \mathrm{i}q_k z)\,\mathrm{d}\vec{k}.$$

This expression for the scattered field in the original reference system is correct to the order of the terms proportional to the product of the amplitude of small-scale roughness h and the tilt of the plane a.

Now let us perform a calculation of the same scattered field $\Delta\Psi_B$ in the original coordinate system, using SSA of the second order, equation (7). To first-order accuracy with respect to a, we can account for the presence of a sloped plane using the following substitution:

$$h(\vec{r}) \to \vec{a}\vec{r} + h(\vec{r}).$$

Now, according to equation (8)

$$\hat{h}(\vec{\xi}) \to \mathrm{i}\vec{a}\nabla\delta(\vec{\xi}) + \hat{h}(\vec{\xi}).$$

After simple calculation we find from equations (2), (7) to first-order accuracy with respect to h:

$$\Delta\Psi_B = \frac{2\mathrm{i}q_0}{q_k + q_0}\int\Bigg[(q_k + q_0)\Bigg(B(\vec{k}, \vec{k}_0) - \frac{1}{4}\frac{\mathrm{d}M(\vec{k}, \vec{k}_0; \vec{\xi})}{\mathrm{d}\vec{\xi}}\bigg|_{\vec{\xi}=0}\vec{a}\Bigg) - \frac{1}{4}M(\vec{k}, \vec{k}_0; \vec{k} - \vec{k}_0$$
$$- \vec{a}(q_k + q_0))\Bigg]\hat{h}(\vec{k} - \vec{k}_0 - \vec{a}(q_k + q_0)) \times \exp(\mathrm{i}\vec{k}\vec{r} - \mathrm{i}q_k z)\,\mathrm{d}\vec{k}. \tag{22}$$

Comparing equations (16) and (22) we find that if the SSA of the second order really takes care of the modulation effects, the following relation must hold to the first-order accuracy with respect to $\vec{a}$:

$$\frac{(q_0 - \vec{a}\vec{k}_0)(q_k + \vec{a}\vec{k})}{q_k q_0}T(\vec{k}, -\vec{a})B(\vec{k} - \vec{a}q_k, \vec{k}_0 + \vec{a}q_0)T(\vec{k}_0, \vec{a}) = B(\vec{k}, \vec{k}_0)$$
$$- \frac{1}{4}\frac{\mathrm{d}M(\vec{k}, \vec{k}_0; \vec{\xi})}{\mathrm{d}\vec{\xi}}\bigg|_{\vec{\xi}=0}\vec{a} - \frac{1}{4(q_k + q_0)}M(\vec{k}, \vec{k}_0; \vec{k} - \vec{k}_0 - \vec{a}(q_k + q_0)) + \mathrm{O}(a^2). \tag{23}$$

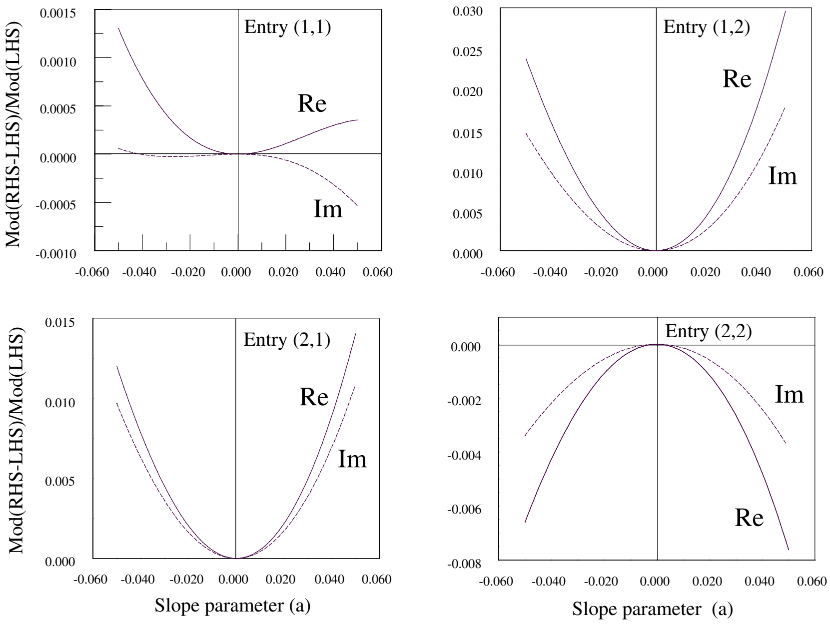

Figure 1. The dependencies of a relative difference between the right-hand side and left-hand side of equation (26) on slope parameter a for different entries. One can clearly see the absence of the O(a) order terms here (since all curves are tangent to the x-axis in the origin rather than crossing it at a non-zero angle).

Note, that according to equation (9),

$$M(\vec{k}, \vec{k}_0; \vec{k} - \vec{k}_0 - \vec{\xi}) = M(\vec{k}, \vec{k}_0; \vec{\xi}). \tag{24}$$

Taking into account equation (10) we also have

$$\left.\frac{\mathrm{d}M(\vec{k}, \vec{k}_0; \vec{\xi})}{\mathrm{d}\vec{\xi}}\right|_{\vec{\xi}=0} \vec{a} = \frac{1}{q_k + q_0} M(\vec{k}, \vec{k}_0; \vec{a}(q_k + q_0)) + \mathrm{O}(a^2). \tag{25}$$

Using equations (24), (25) we can represent equation (23) in the following equivalent form:

$$(q_0 - \vec{a}\vec{k}_0)(q_k + \vec{a}\vec{k})T(\vec{k}, -\vec{a})B(\vec{k} - \vec{a}q_k, \vec{k}_0 + \vec{a}q_0)T(\vec{k}, \vec{a})$$
$$= q_k q_0 B(\vec{k}, \vec{k}_0) - \frac{q_k q_0}{2(q_k + q_0)} M(\vec{k}, \vec{k}_0; \vec{a}(q_k + q_0)) + \mathrm{O}(a^2). \tag{26}$$

Direct analytical proof of equation (26) based on the explicit expressions for functions B, B_2 given in the appendix represents a straightforward but tedious procedure. Rather than performing this exercise, let us investigate this relation numerically. Figure 1 provides a numerical illustration of it, also giving an idea of the accuracy to which equation (26) holds as a function of a. The value of the difference between appropriate entries of the left- and right-hand sides of equation (26), divided by the absolute value of the corresponding left-hand side entry (i.e. the relative error in the calculation of the SA), is shown on the plots as a function of slope parameter $a : a_x = a\cos\varphi_a$, $a_y = a\sin\varphi_a$ for a fixed φ_a, and for some arbitrary generic set of parameters $\omega/c = 1.5$, $\varepsilon = 2 + 3\mathrm{i}$, $\theta = 20°$, $\varphi = 0°$, $\theta_0 = 50°$, $\varphi_0 = 30°$, $\varphi_a = 50°$ (here, θ, φ are the polar angles of the wavevectors of the scattered and incident waves: $k_x = \omega/c \sin\theta\cos\varphi$, $k_y = \omega/c \sin\theta\sin\varphi$, $q = \omega/c\cos\theta$ with similar expressions for $\vec{k}_0, q_0$). It is obvious from the behaviour of the curves at the origin that the left- and right-hand sides of equation (26) coincide with the O(a^2) accuracy for all entries.

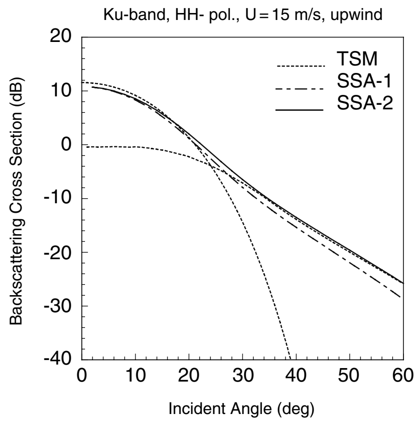

Figure 2. The dependency of the backscattering cross section on incidence angle for an upwind direction for HH-polarization. Short-dashed curves represent the Kirchhoff and the Bragg components of a TSM, the long-dashed curve is a SSA of first order (SSA-1), and the solid curve is a SSA of second order (SSA-2).

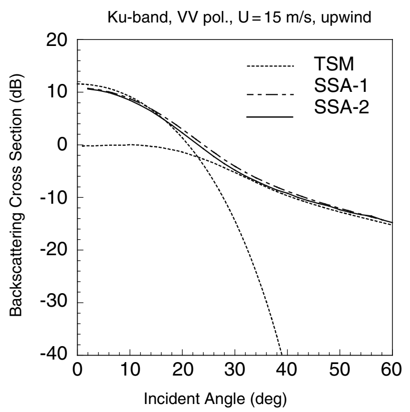

Figure 3. The same dependencies as presented in figure 2, however, for VV-polarization.

Figures 2 and 3 show the results of numerical calculation of the backscattering cross section of EM waves of the K_u-band as a function of the incidence angle for HH- and VV-polarizations, correspondingly for 15 m s^{-1} wind (details of calculations can be found in [6]). The calculations were performed with the help of the SSA of first and second orders and a TSM. The latter consists of two components: the Kirchhoff approximation for small incidence angles and Bragg scattering averaged over slopes induced by long waves. One can see that in the case of HH-polarization the difference between the two-scale and SSA-1 results, which is due to the modulation effects, practically disappears in the case of SSA-2. For VV-polarization, on the other hand, this difference was already negligible and SSA-2 provides very small correction, accordingly. The reason for the different behaviour of the two polarization is the stronger dependency of the backscattering cross section on the incidence angle for HH-polarization, which leads to more pronounced modulation effects as compared to the VV-polarization case. One can also see that for incidence angles of the order of 25°, the SSA insures a smooth transition between the Bragg and Kirchhoff regimes in both cases.

3. Conclusion

The situation when small-scale roughness is located on a large-scale undulating surface can be considered by direct application of the formula for SSA of the second-order equation (7). The effect of modification of resonant Bragg wavenumbers, which results in a replacement of $\vec{k}, \vec{k}_0$ by local tangent vectors $\vec{k}', \vec{k}_0'$ according to equation (13) has a purely kinematic origin and is in fact taken into account by the first-order SSA (see also [5, 8].) The dynamic effect of the modulation of kernel function B describing Bragg scattering (see equations (11), (12)) and the effect of transforming the polarization state are properly taken into account by SSA-2, due to equation (26).

Acknowledgments

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Appendix

The general expressions for kernel functions B and B_2 are given in [4] and are as follows. The first order:

$$B_{11}(\vec{k}, \vec{k}_0) = \frac{\varepsilon - 1}{(\varepsilon q_k^{(1)} + q_k^{(2)})(\varepsilon q_0^{(1)} + q_0^{(2)})} \left(q_k^{(2)} q_0^{(2)} \frac{\vec{k}\vec{k}_0}{kk_0} - \varepsilon k k_0 \right)$$

$$B_{12}(\vec{k}, \vec{k}_0) = \frac{\varepsilon - 1}{(\varepsilon q_k^{(1)} + q_k^{(2)})(q_0^{(1)} + q_0^{(2)})} \frac{\omega}{c} q_k^{(2)} \frac{\vec{N}[\vec{k}, \vec{k}_0]}{kk_0}$$

$$B_{21}(\vec{k}, \vec{k}_0) = \frac{\varepsilon - 1}{(q_k^{(1)} + q_k^{(2)})(\varepsilon q_0^{(1)} + q_0^{(2)})} \frac{\omega}{c} q_0^{(2)} \frac{\vec{N}[\vec{k}, \vec{k}_0]}{kk_0}$$

$$B_{22}(\vec{k}, \vec{k}_0) = -\frac{\varepsilon - 1}{(q_k^{(1)} + q_k^{(2)})(q_0^{(1)} + q_0^{(2)})} \frac{\omega^2}{c^2} \frac{\vec{k}\vec{k}_0}{kk_0}.$$

The second order:

$$\begin{aligned}
(B_2)_{11}(\vec{k}, \vec{k}_0; \vec{\xi}) = {} & \frac{\varepsilon - 1}{(\varepsilon q_k^{(1)} + q_k^{(2)})(\varepsilon q_0^{(1)} + q_0^{(2)})} \\
& \times \left[-2 \frac{\varepsilon - 1}{\varepsilon q_\xi^{(1)} + q_\xi^{(2)}} \left(q_k^{(2)} q_0^{(2)} \frac{\vec{k}\vec{\xi}}{k} \frac{\vec{\xi}\vec{k}_0}{k_0} + \varepsilon k k_0 \xi^2 \right) + 2\varepsilon \frac{q_\xi^{(1)} + q_\xi^{(2)}}{\varepsilon q_\xi^{(1)} + q_\xi^{(2)}} \right. \\
& \left. \times \left(k_0 q_k^{(2)} \frac{\vec{k}\vec{\xi}}{k} + k q_0^{(2)} \frac{\vec{\xi}\vec{k}_0}{k_0} \right) - \left(\varepsilon \frac{\omega^2}{c^2} (q_k^{(2)} + q_0^{(2)}) + 2 q_k^{(2)} q_0^{(2)} (q_\xi^{(1)} - q_\xi^{(2)}) \right) \frac{\vec{k}\vec{k}_0}{kk_0} \right]
\end{aligned}$$

$$\begin{aligned}
(B_2)_{12}(\vec{k}, \vec{k}_0; \vec{\xi}) = {} & \frac{(\varepsilon - 1)\omega/c}{(\varepsilon q_k^{(1)} + q_k^{(2)})(q_0^{(1)} + q_0^{(2)})} \left[-2 \frac{\varepsilon - 1}{\varepsilon q_\xi^{(1)} + q_\xi^{(2)}} q_k^{(2)} \frac{\vec{k}\vec{\xi}}{k} \frac{\vec{N}[\vec{\xi}, \vec{k}_0]}{k_0} \right. \\
& \left. + 2\varepsilon \frac{q_\xi^{(1)} + q_\xi^{(2)}}{\varepsilon q_\xi^{(1)} + q_\xi^{(2)}} k \frac{\vec{N}[\vec{\xi}, \vec{k}_0]}{k_0} - \left(\varepsilon \frac{\omega^2}{c^2} + q_k^{(2)} q_0^{(2)} + 2 q_k^{(2)} (q_\xi^{(1)} - q_\xi^{(2)}) \right) \frac{\vec{N}[\vec{k}, \vec{k}_0]}{kk_0} \right]
\end{aligned}$$

$$(B_2)_{21}(\vec{k},\vec{k}_0;\vec{\xi}) = \frac{(\varepsilon-1)\omega/c}{(q_k^{(1)}+q_k^{(2)})(\varepsilon q_0^{(1)}+q_0^{(2)})}\left[2\frac{\varepsilon-1}{\varepsilon q_\xi^{(1)}+q_\xi^{(2)}} q_0^{(2)}\frac{\vec{k}_0\vec{\xi}}{k_0}\frac{\vec{N}[\vec{\xi},\vec{k}]}{k}\right.$$
$$\left. -2\varepsilon\frac{q_\xi^{(1)}+q_\xi^{(2)}}{\varepsilon q_\xi^{(1)}+q_\xi^{(2)}}k_0\frac{\vec{N}[\vec{\xi},\vec{k}]}{k} - \left(\varepsilon\frac{\omega^2}{c^2}+q_k^{(2)}q_0^{(2)}+2q_0^{(2)}(q_\xi^{(1)}-q_\xi^{(2)})\right)\frac{\vec{N}[\vec{k},\vec{k}_0]}{kk_0}\right]$$

$$(B_2)_{22}(\vec{k},\vec{k}_0;\vec{\xi}) = \frac{\varepsilon-1}{(q_k^{(1)}+q_k^{(2)})(q_0^{(1)}+q_0^{(2)})}\frac{\omega^2}{c^2}$$
$$\times\left[-2\frac{\varepsilon-1}{\varepsilon q_\xi^{(1)}+q_\xi^{(2)}}\left(\frac{\vec{k}\vec{\xi}}{k}\frac{\vec{\xi}\vec{k}_0}{k_0}-\xi^2\frac{\vec{k}\vec{k}_0}{kk_0}\right)+(q_k^{(2)}+q_0^{(2)}+2(q_\xi^{(1)}-q_\xi^{(2)}))\frac{\vec{k}\vec{k}_0}{kk_0}\right].$$

Here, $q_k^{(1,2)}$ and $q_0^{(1,2)}$ are vertical components of the appropriate wavevectors in the first medium (air) and the second (dielectric) medium:

$$q_k^{(1)} = \sqrt{\frac{\omega^2}{c^2}-k^2}, \qquad q_k^{(2)} = \sqrt{\varepsilon\frac{\omega^2}{c^2}-k^2}, \qquad \mathrm{Im}\, q_k^{(1,2)} \geqslant 0$$
$$q_0^{(1)} = \sqrt{\frac{\omega^2}{c^2}-k_0^2}, \qquad q_0^{(2)} = \sqrt{\varepsilon\frac{\omega^2}{c^2}-k_0^2}, \qquad \mathrm{Im}\, q_0^{(1,2)} \geqslant 0.$$

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