FUNDAMENTAL AND CONCEPTUAL ASPECTS OF TURBULENT FLOWS

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We absolutely must leave room for doubt or there is no progress and no learning. There is no learning without posing a question. And a question requires doubt…Now the freedom of doubt, which is absolutely essential for the development of science, was born from a struggle with constituted authorities… FEYNMANN, 1964

LECTURES XI-XII

ANALOGIES, DIFFERENCES AND RELATIONS BETWEEN GENUINE TURBULENCE AND ITS 'ANALOGS'

Our understanding of the general character of the small-scale features of turbulent motion is very far from complete...Very few theoretical or experimental results have been established so that for the most part we must proceed analogy and plausible inference. BATCHELOR 1956, p 183

Analogies in turbulence research have a special status mainly due to unsatisfactory state of theory. Most analogies are aimed to look at similarity between genuine turbulence and some "analogous" system such as evolution of some passive object (e.g. scalar, vector, etc.) polymers, and some other (see below) in some prescribed random (usually Gaussian) velocity field. This led in many cases to exaggerated and consequently misleading claims on analogous behavior between the two and consequently to misconceptions. Hence the purpose of this lecture is twofold. Along with critical overview of similarities the main emphasis is given to differences rather than similarities. The primary reason for this is that (at least some) understanding of differences is expected to aid better understanding of both systems and avoid misconceptions associated with extending the analogies too far.

SOME EARLY ANALOGIES

Reynolds analogy on transport of momentum and heat, REYNOLDS, O. 1874 On the extent and action of the heating surface of steam boilers, *Proc. Lit. Phil. Soc. Manchester*, 14, 7-12.

Study of fluid motion by means of `colour bands', REYNOLDS. O. 1894 Study of fluid motion by means of coloured bands, *Nature*, 50, 161--164.

Frozennes of vorticity in the flow field in invisdid flows (and other solenoidal fields with vanishing diffusivity, e.g. magnetic field in perfectly conducting fluids), HELMHOLZ, H. 1858 On integrals of the hydrodynamical equations which express vortex motion. Translated from German by P.G.Tait, 1867 with a letter by Lord Kelvin (W.Thomson) in London Edinburgh Dublin *Phil. Mag. J. Sci., Fourth series*, 33, 485-512; KELVIN, LORD (THOMSON, W.) 1880 Vibration of columnar vortex, *London Edinburgh Dublin Phil. Mag. J. Sci.,Fifth series*, 33, 485-512.; (1910) *Mathematical and physical papers*, vol. 4, Cambr. Univ Press.

Finite diffusivity: analogy between amplification of vorticity and magnetic field by turbulent flow BATCHELOR, G.K. (1950) On the spontaneous magnetic field in a conducting liquid in turbulent motion, *Proc. Roy. Soc. London*, A201,

Vorticity Versus (Infinitesimal) Material Lines

Are they stretched in the same way and for the same reason? And what is the meaning of the "same way"

Vorticity amplification is a result of the kinematics of turbulence, TENNEKES, H. AND LUMLEY, J. L. (1972) First course in turbulence, MIT Press. In the context of concern here a similar view originates with TAYLOR 1937,1938 (see below)

The physics of vortex stretching is well understood…, see for instance, G.K. Batchelor, An introduction to Fluid Dynamics (Cambridge U.P., New York, 1967); and R. H. Kraichnan, J. Fluid Mech., 64, 737 (1974) A footnote in E. D. SIGGIA, (1977) `Origin of intermittency in fully developed turbulence', *Phys. Rev.*, 15(4), 1730.

...amplification of the vorticity by vortex stretching, a well-understood mechanism in 3D Euler flow." POMEAU, Y. AND SCIAMARELLA, D. (2005) An unfinished tale of nonlinear PDEs: Do solutions of 3D incompressible Euler equations blow-up in finite time? *Physica*, D205, 215

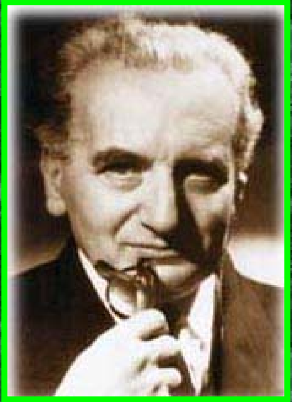
A BIT OF HISTORY - I

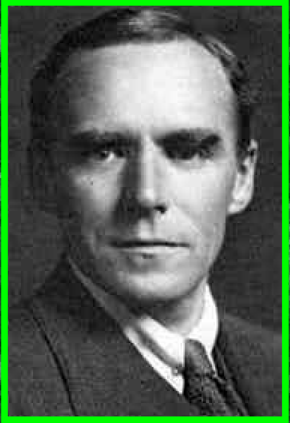
A SMALL DIGRESSION REMINDING A BIG MISTAKE

VON KARMAN (1937) assumed ***that the expression*** $\sum_j \sum_k \omega_i \omega_k \partial u_i/\partial u_k$ ***is zero in the mean*** and that he (vK) ***cannot see any physical reason for such a correlation.***

TAYLOR (1937) conjectured that there ***is a strong correlation between*** ω_3^2 ***and*** $\partial u_3/\partial x_3$ ***so that*** (the mean of) $\omega_3^2 \partial u_3/\partial x_3$ ***is not equal to zero*** (x_3 is directed along vorticity) and showed experimentally that this is really the case,

TAYLOR (1938)





It is a rather common view (and misconception) that the prevalence of vortex stretching is due to the predominance of stretching of material lines. This view originates with TAYLOR 1938:

Turbulent motion is found to be diffusive, so that particles which were originally neighbors move apart as motion proceeds. In a diffusive motion the average value of d^2/d_0^2 continually increases. It will be seen therefore ..., that the average value of ω^2/ω_o^2 continually increases.

#*... the interesting physical argument that $\langle\omega_i\omega_j s_{ij}\rangle$ is positive because two particles on average move apart from each other and therefore vortex lines are on average stretched rather than compressed*, HUNT 1973.

The relative diffusion of a pair of probe particles in grid turbulence at high Reynolds numbers is treated as the most clear-cut manifestaion of vortex stretching. MORI & TAKAYOSHI, 1983.

The latest example is found in DAVIDSON (2004, p.259.): *However, since material-line stretching seems to be a norm for the broader class of kinematically admissible fields, it should also be the norm for the narrower class of dynamically admissible velocity fields, and so one should not be surprised that vortex-line stretching, like material line stretching, is seen in practice.*

CHORIN (1994) points to the problematic aspect of such a view: *Vortex lines are special lines, and constitute a negligible fraction of all lines (there is one vortex direction at each point, but an infinite number of others). All arguments that involve averages with respect to a probability measure may fail to hold in a negligible fraction of cases, and thus one cannot conclude from* (5.1) (i.e $d/dt\langle|\delta x(t)|^2\rangle > 0$) *that vortex lines stretch, even in isotropic flow,* but ends with a the statement that *This conclusion is, however, eminently plausible.*

Indeed, it is plausible, since it is observed in the laboratory and in numerical simulations. But the underlying reasons/processes are still not understood unlike in case of passive material lines.

COCKE (1969) proved the following important results. The first result is that the length of an infinitesimal material line element, $l \equiv |\mathbf{l}|$, increases on average in any isotropic random velocity field*. Similarly, Cocke showed that an infinitesimal material surface element, N, identified by its vector normal, **N**, increases on average in an any isotropic random velocity field as well.** Two important points have to be stressed. **First**, the results of Cocke are based on statistics of 'all' material lines having the property that infinitely many lines pass through each point (whereas typically only one vorticity line is passing through a point – recall the statement by Chorin: ***All arguments that involve averages with respect to a probability measure*** (i.e. all material lines) ***may fail to hold in a negligible fraction of cases*** (i.e. vorticity lines only).

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* For references on more details and a review of other related issues see TSINOBER 2001, p. 48 and on.

**More precisely Cocke showed that $\ln[\langle l(t)\rangle/l(0)] \geq 0$, and $\ln[\langle N(t)\rangle/N(0)] \geq 0$ for all $t>0$ with equality holding only if there is no fluid motion at all. Arguments similar to those by Cocke (1969) show that $\langle l^p(t)\rangle \geq l^p(0)$ and $\langle N^p(t)\rangle \geq N^p(0)$ for any $p > 0$ (MONIN AND YAGLOM, 1975, pp. 579 - 580).

Second, the results by Cocke are purely kinematic: the flow does not 'know' about material lines and does not have to be a real one, i.e. to satisfy the Navier-Stokes equations and/or to be observable in laboratory or elsewhere - the only requirement is that the flow should be random and isotropic. For example, this result is true for a Gaussian velocity field as well, which is important for the purpose of comparison of material line elements, which are passive, and vorticity, which is not. Namely, the enstrophy production in a Gaussian isotropic field vanishes identically in the mean $\langle \omega_i \omega_k s_{ik} \rangle \equiv 0$, but $\langle l_i l_k s_{ik} \rangle > 0$!

On the qualitative level the results by Cocke were confirmed in a number of DNS experiments both for real and artificial flow fields (Drummond, 1993; Girimaji and Pope, 1990; Huang, 1996; Yeung, 1994) and laboratory experiments (Lüthi et al., 2005).

In an inviscid flow

$$D\omega/Dt = (\omega \cdot \nabla)u; \quad Dl/Dt = (l \cdot \nabla)u$$

$$D(\omega - l)/Dt = \{(\omega - l) \cdot \nabla\}u$$

So $\omega = l$ at all times if initially $\omega - l = 0$;

However, in a flow with $\nu \neq 0$ whatever small

$$\langle \omega_i \omega_k s_{ik} \rangle \approx \nu \langle \omega_i \nabla^2 \omega_i \rangle,$$

i.e. the vortex lines are not frozen into the fluid at whatever high Reynolds number – otherwise how the enstrophy production can be approximately balanced by viscous terms again at any – whatever large – Reynolds number. In othetr words in slightly viscous flows frozennes is meaningless. Just like the claim that turbulence is slightly viscous at whatever large Re. In this context the question: what happens with enstrophy/strain production as $\nu \rightarrow 0$ is of special interest.

The above is not entirely new (at least in part)

… *a material line which is initially coinsiding with a vortex line continues to do so. It is thus possible and convenient to regard a vortex-line as having a continuing identity and as moving with the fluid (In a viscous fluid it is, of course, possible to draw the pattern of vortex lines at any instant, but there is no way in which particular vortex-line can be identified at different instants).* BATCHELOR, 1967, p. 274

What are the main problems with the above misconception? -A summary

First, the vortex lines are not frozen into the fluid at whatever high Reynolds number - otherwise how the enstrophy production can be approximately balanced by viscous terms.

Second, even if frozen vorticity is not a marker, it reacts back strangly: everybody knows what is Bio-Savart law, or more generally $\nabla^2 \mathbf{u} = -\,\mathrm{curl}\,\boldsymbol{\omega}$.

Third, even if frozen those material lines coinciding with vorticity are special and not the other way around. Namely, the material line elements which initially and thereby consequently coincide with vorticity are special in the sense that they are not dynamically passive quantities anymore and react back on the flow precisely as does vorticity. In other words, the fact that vorticity is frozen in the inviscid flow field does not mean that vorticity behaves the same way as material lines, but the other way around: those material lines which coincide with vorticity behave like vorticity, because they are not passive anymore as are all the other material lines: the rules of democarcy do not apply to science.

MORE DIFFERENCES BETWEEN VORTICITY AND MATERIAL LINES

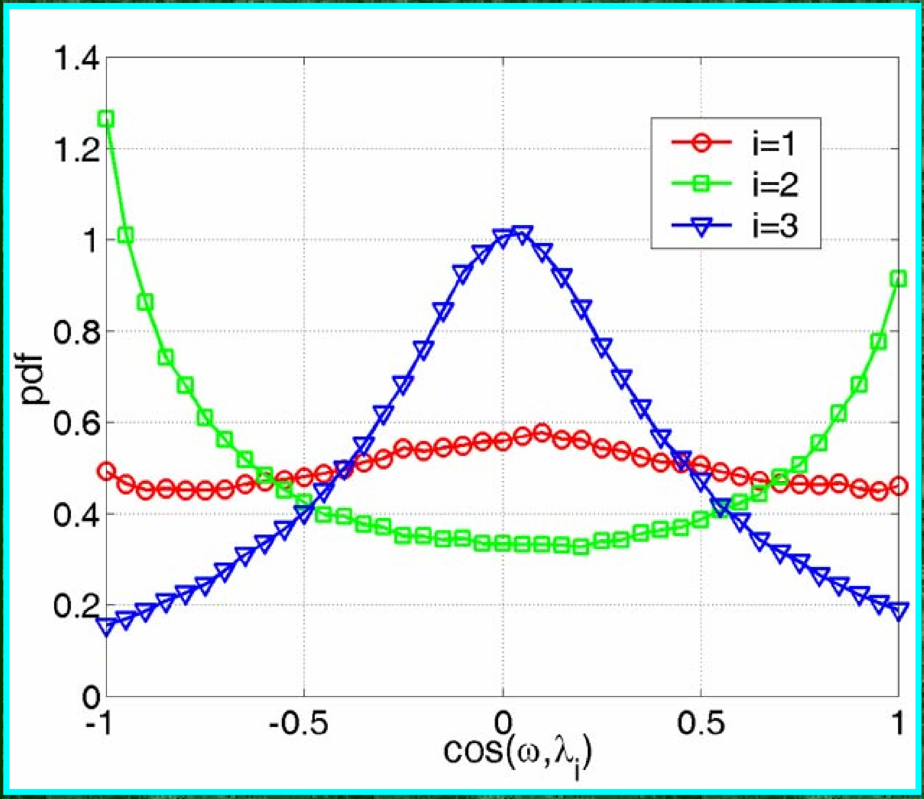
VORTICITY VERSUS MATERIAL LINES

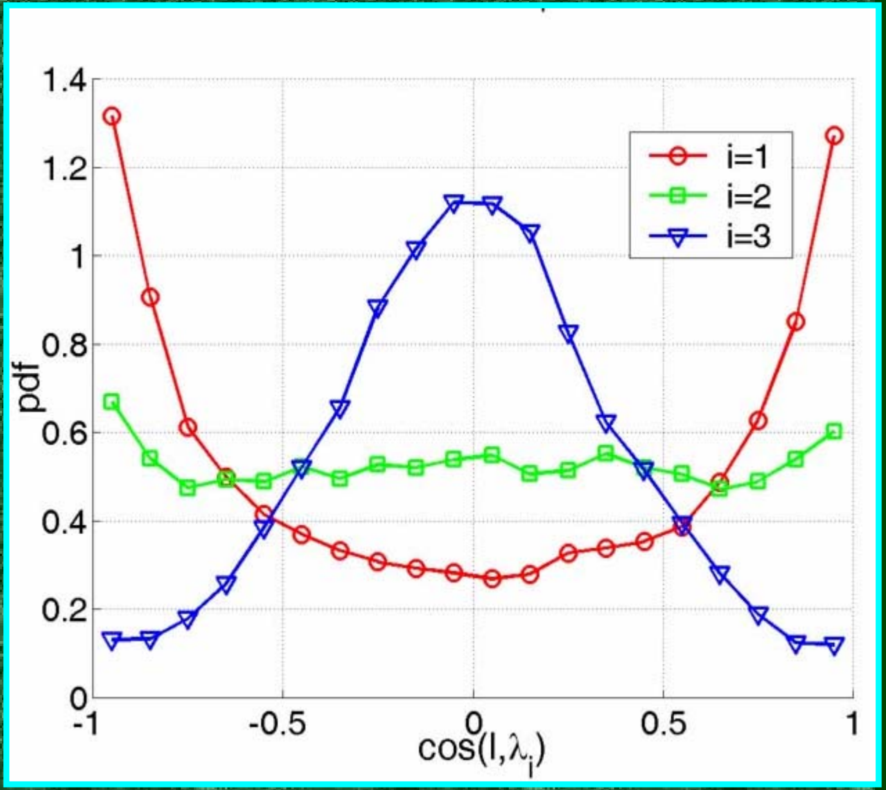
ALIGNMENT BETWEEN THE EIGENFRAME λ_i OF THE RATE OF STRAIN TENSOR S_{ij} AND

(from 3D-PTV; Reλ=60, LUTHI ET AL 2005)

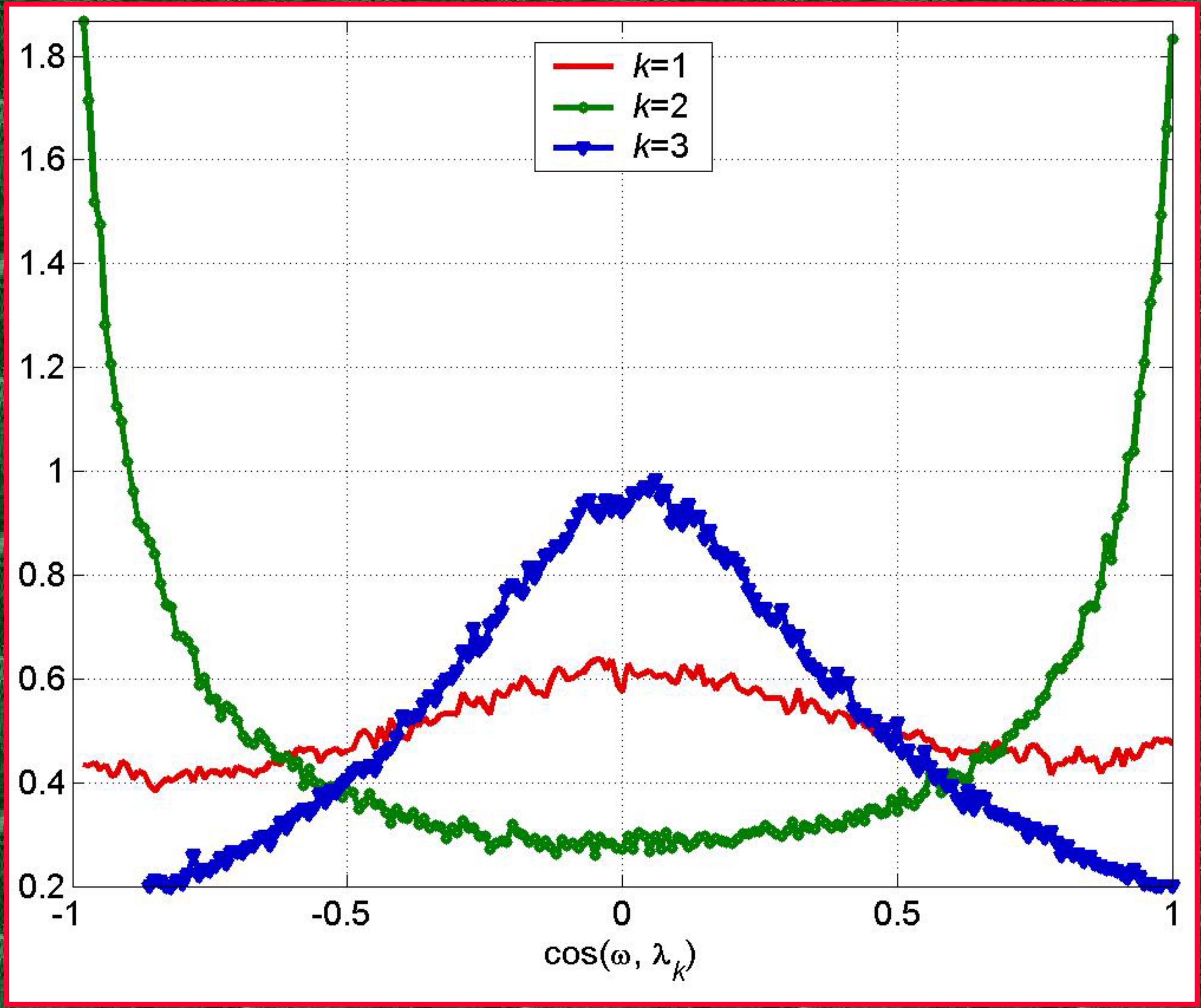
VORTICITY ω

MATERIAL LINES l





Geometrical Statistics Alignments



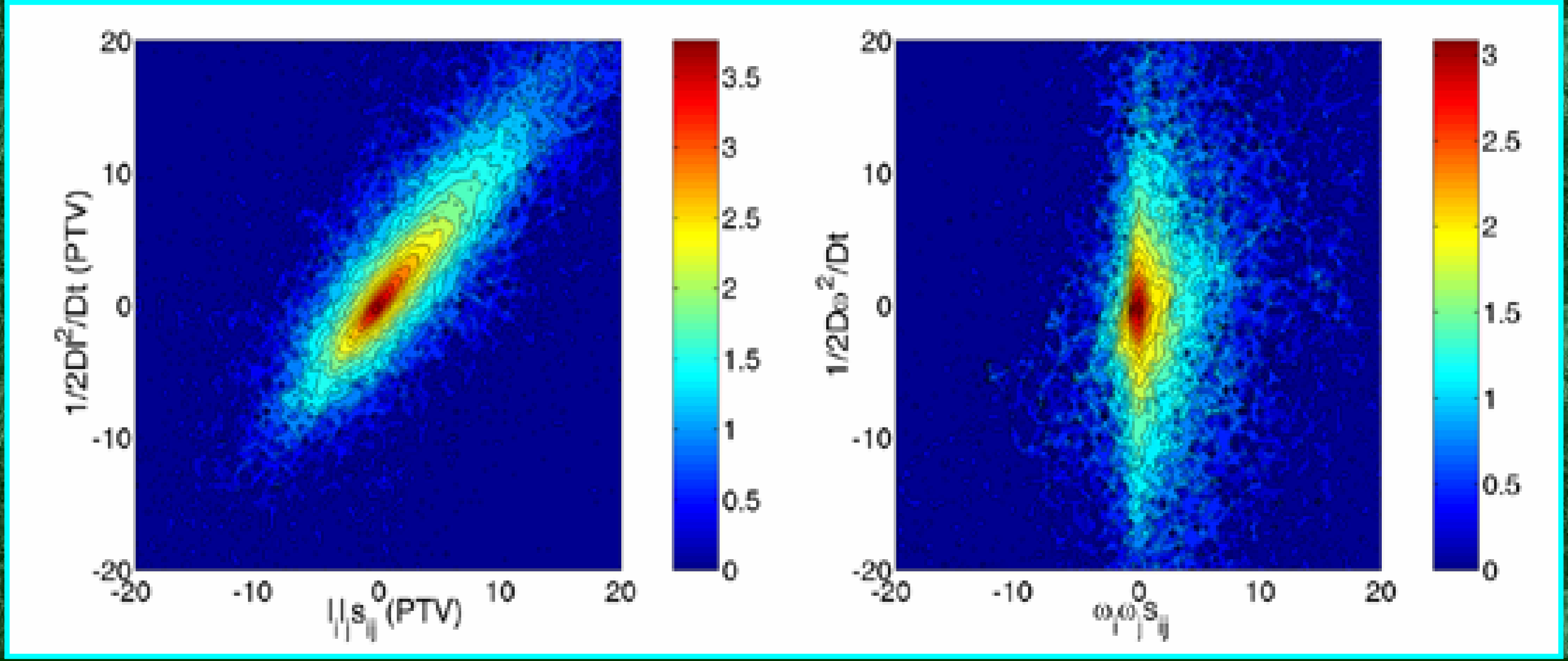
VORTICITY $\boldsymbol{\omega}$ VERSUS MATERIAL LINES $\mathbf{l}$

$$D\boldsymbol{\omega}/Dt = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} + \nu\,\Delta\boldsymbol{\omega}$$

$$D\mathbf{l}/Dt = (\mathbf{l}\cdot\nabla)\mathbf{u}$$

PRODUCTION OF $\mathbf{l}^2$ **PTV** PRODUCTION OF $\boldsymbol{\omega}^2$

Again **Nonlocality**



Vortex stretching versus stretching of material lines

A summary from Tsinober, 2001, p. 88

The equation for a material line element $\mathbf{l}$ is a linear one and the vector is passive, i.e. the fluid flow does not `know' anything whatsoever about the vector $\mathbf{l}$ (as any passive vector) does not exert any influence on the fluid flow. The material element is stretched (compressed) locally at an exponential rate proportional to the rate of strain along the direction of $\mathbf{l}$ since the strain is independent of $\mathbf{l}$.

On the contrary, the equation for vorticity is a nonlinear partial differential equation and the vorticity vector is an active one - it `reacts back' on the fluid flow. The strain does depend in a nonlocal manner on vorticity and vice versa, i.e. the rate of vortex stretching is a nonlocal quantity, whereas the rate of stretching of material lines is a local one. Therefore the rate of vortex stretching (compressing) is different from the exponential one and is unknown. There are 'fewer' vorticity lines than the material ones - at each point there is typically only one vortex line, but infinitely many material lines. This leads to essential differences in the statistical properties of the two fields.

In the absence of viscosity vortex lines are material lines, but they are special in the sense that they are not passive as all the other passive material lines. But the fact that vorticity is frozen in the inviscid flow field does not mean that vorticity behaves the same way as material lines, but the other way around: those material lines which coincide with vorticity behave like vorticity, because they are not passive anymore as are all the other material lines.

While a material element tends to be aligned with the largest (positive) eigenvector of the rate of strain tensor, vorticity tends to be aligned with the intermediate (mostly positive) eigenvector of the rate of strain tensor: its eigenframe rotates with an angular velocity of the order of vorticity

For a Gaussian isotropic velocity field the mean enstrophy generation vanishes identically, whereas the mean rate of stretching of material lines is essentially positive. The same is true of the mean rate of vortex stretching and for purely two-dimensional flows. This means that in turbulent flows the mean growth rate of material lines is larger than that of vorticity. The nature of vortex stretching process is dynamical and not a kinematic one as the stretching of material lines is.

Comparing vorticity with any passive vector (also with the same diffusivity as viscosity), the analogy is partial not just/only because the equation for vorticity is nonlinear, but also because in the case of vorticity the process is due to self -amplification of the field of velocity derivatives, whereas in case of a passive vector it is not.

Geniune Turbulence Versus Passive "Turbulence"

How analogous are the genuine and passive turbulence (if at all)? What are the main differences? Evolution of disturbances. What can be learned about genuine turbulence from its signature on the evolution of passive objects? What is the importance (if any) of statistical conservation laws in genuine turbulence (if such exist)?

Yet statistical properties of this so-called `passive scalar' turbulence are decoupled from those of the underlying velocity field (are they?)... The non-trivial statistical properties of scalar turn out to originate in the mixing process itself, rather than being inherited from the complexity of the turbulent velocity field (but this is just one part of the story). Study of passive scalar turbulence is therefore decoupled from the still intractable problem of calculating the velocity statistics, and so has yielded to mathematical analysis. ...the well established phenomenological parallels between the statistical description of mixing and fluid turbulence itself suggest that progress on the latter front may follow from a better understanding of turbulent mixing (really !). SHRAIMAN AND SIGGIA, 2000.

Passive contaminants are transported by turbulent motions in much the same way as momentum.... Momentum is not a passive contaminant; "mixing" of mean momentum relates to the dynamics of turbulence, not merely its kinematics. TENNEKES AND LUMLEY, 1972

The advection-diffusion equation, in conjunction with a velocity field model with turbulent characteristics (prescribed a priori), serves as a simplified prototype problem for developing theories for turbulence itself. MAJDA AND KRAMER, 1999

An important progress has been achieved in the last decade in understanding some simpler systems exhibiting behaviors similar to developed turbulence. These include the so-called weak or wave turbulence , the advection of passive scalar and vector fields by random velocities that mimic (do they? in what sense?) the turbulent ones, and, to certain extent, the so-called burgulence , the phenomena described by the Burgers equation.

GAWEDZKI 2002

Examples of passively advected quantities are the temperature or the impurity concentration in a fluid. Ideally one would be interested in the statistical properties of the advected field in the case where the underlying flow is turbulent. Significant progress has been achieved when the velocity field is taken random, with Gaussian statistics but decorrelated (white) in time. One mimics the important feature of turbulent flows by taking the velocities rough, i.e. only Hölder continuous, in space. For such an ensemble of velocities (called the Kraichnan model), it was possible to study the ensuing steady state of the advected fields both analytically and numerically. It appears to be a nonequilibrium state with nonzero flux of a conserved quantity, again in analogy to hydrodynamical turbulence. Moreover it exhibits intermittency in the form of anomalous scaling of moments of scalar differences in nearby points, the first (and so far only) nontrivial model where the anomalous scaling has been established analytically. GAWEDZKY 2002

MAJOR DIFFRENECS

The differences are more than essential: the evolution of passive objects is not related to the dynamics of turbulence in the sense that the dynamics of fluid motion does not enter in the problems in question - the velocity field is prescribed *a priory* in all problems on evolution of passive objects. Consequently the problems associated with the passive objects are linear; whereas genuine turbulence is a strongly nonlinear problem - nonlinearity is in the heart of turbulent flows and is underlying the main manifestations of the differences between genuine and passive turbulence.

Self-amplification of velocity derivatives.

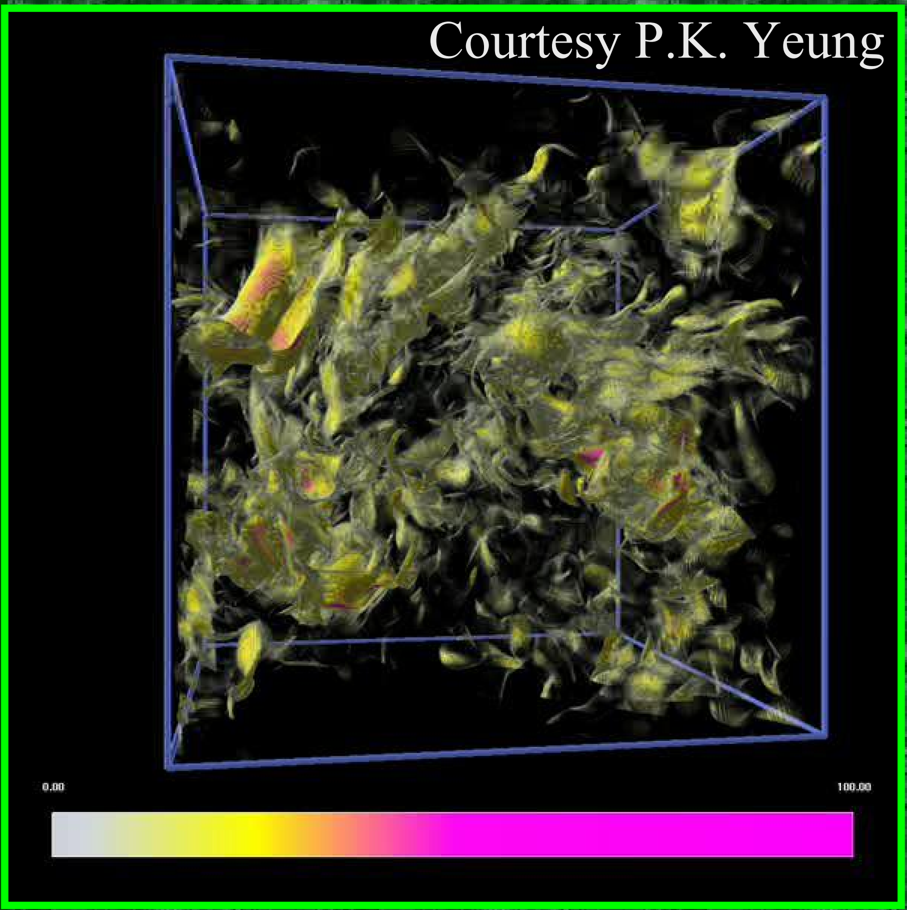
Nonlinearity of genuine turbulence is the reason for the self-amplification of the field of velocity derivatives, both vorticity and strain. In contrast there is no phenomenon of self-amplification in the evolution of passive objects (such as material lines, gradients of passive scalar and solenoidal passive vectors with finite diffusivity). We stress that the process of self-amplification of strain is a specific feature of the dynamics of genuine turbulence having no counterpart in the behavior of passive objects. In contrast, the process of self-amplification of vorticity, along with essential differences (We would like to stress again that vorticity is an active vector, since it `reacts back' on the velocity (and thereby on strain) field. This is not the case with passive objects - the process here is `one way': the velocity field does not `know' anything about the passive object), has common features with analogous processes in passive vectors; in both the main factor is their interaction with strain, whereas the production of strain is much more 'self'.

A related important difference is absence of pressure in case of passive objects.

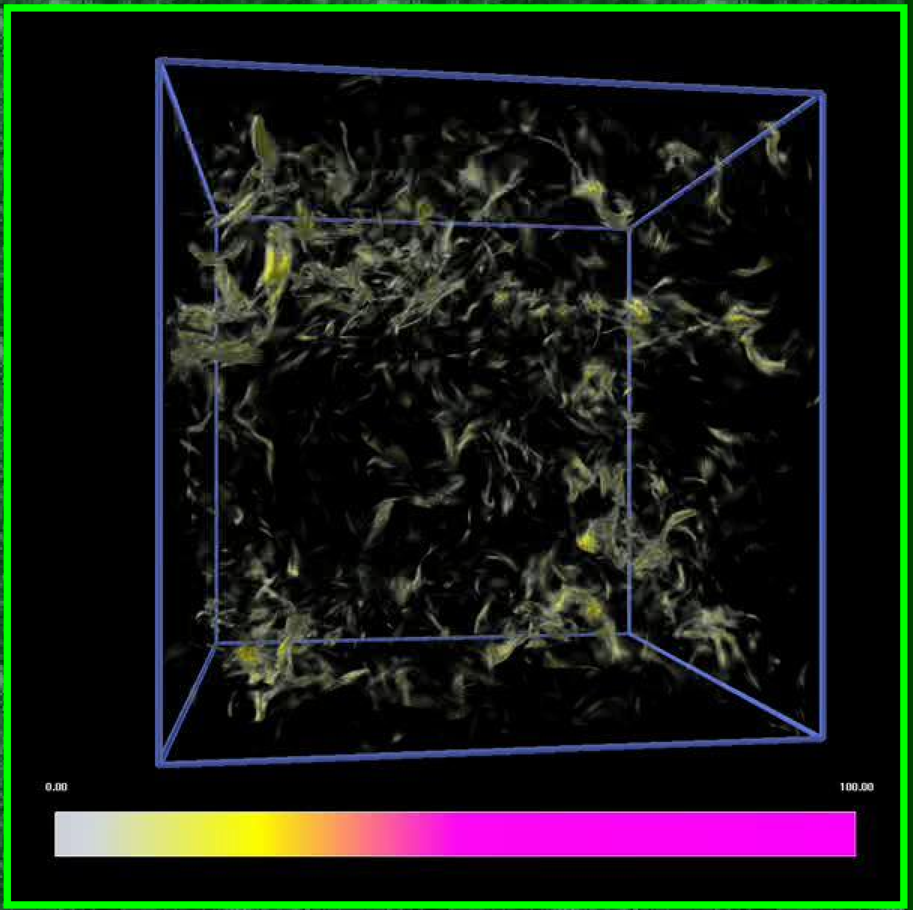
Differences in structure(s) Along with some common features the mechanisms of formation of structure(s) are essentially different for the passive objects and the dynamical variables. Among the reasons is the presence of Lagrangian chaos, which is manifested as rather complicated structure of passive objects even in very simple regular velocity fields (On the other hand, e.g. the ramp-cliff structures of a passive scalar are observed in pure Gaussian `structureless' random velocity field, just like those in a variety of real turbulent flows practically independently of the value of the Reynolds number). In other words the structure of passive objects in turbulent flows arises from two (essentially inseparable) contributions: one due to the Lagrangian chaos and the other due to the random nature of the velocity field itself (Therefore one cannot claim that statistical properties of this so-called `passive scalar' turbulence are decoupled from those of the underlying velocity field Shraiman Siggia2000) , since the non-trivial statistical properties of scalar turn out to originate not only in the mixing process itself but are inherited from the complexity of the turbulent velocity field as well. Study of passive scalar turbulence is therefore not decoupled from the still intractable problem of calculating the velocity statistics). Among other reasons are differences in sensitivity to initial (upstream) conditions (i.e. Lagrangian `memory'), 'symmetries', e.g. the velocity field may be locally isotropic, whereas the passive scalar may not be and some other (see references in TSINOBER 2001). A recent result, BAIG &CHERNYSHENKO 2005 for turbulent flow in a plane channel is an interesting addition to the list of these differences: although the vortical structure of the flow is the same, the scalar streak spacing varies by an order of magnitude depending on the mean profile of the scalar concentration. Moreover, passive scalar streaks were observed even in an artificial "structureless" flow field.

Differences in structure(s)

Passive scalar dissipation



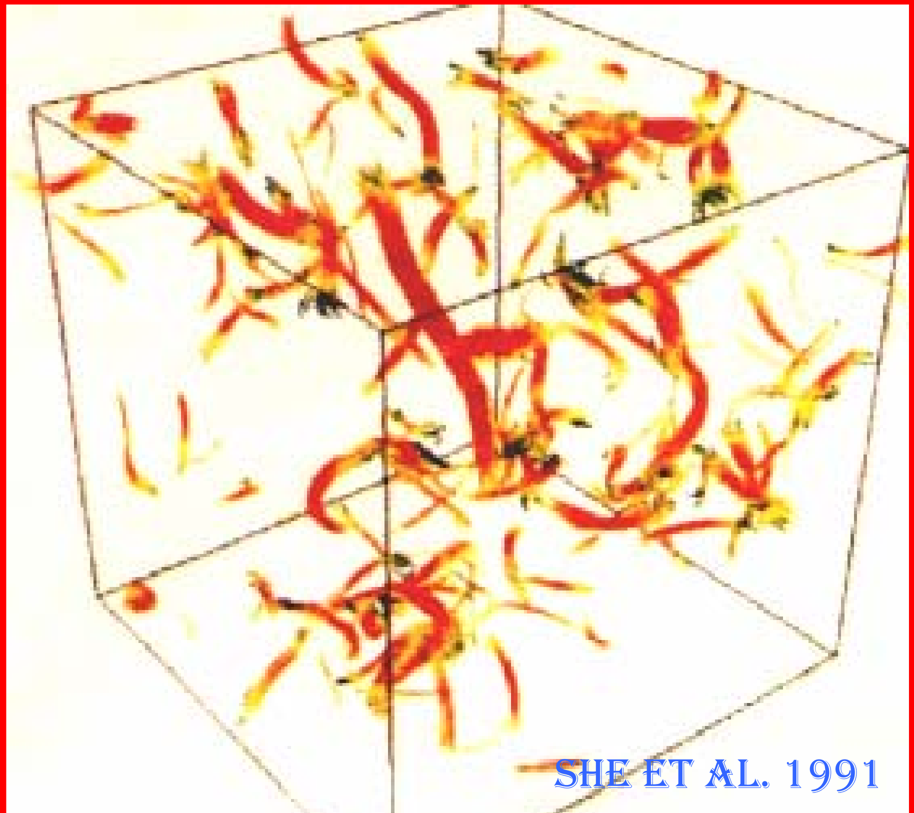
Dissipation of energy



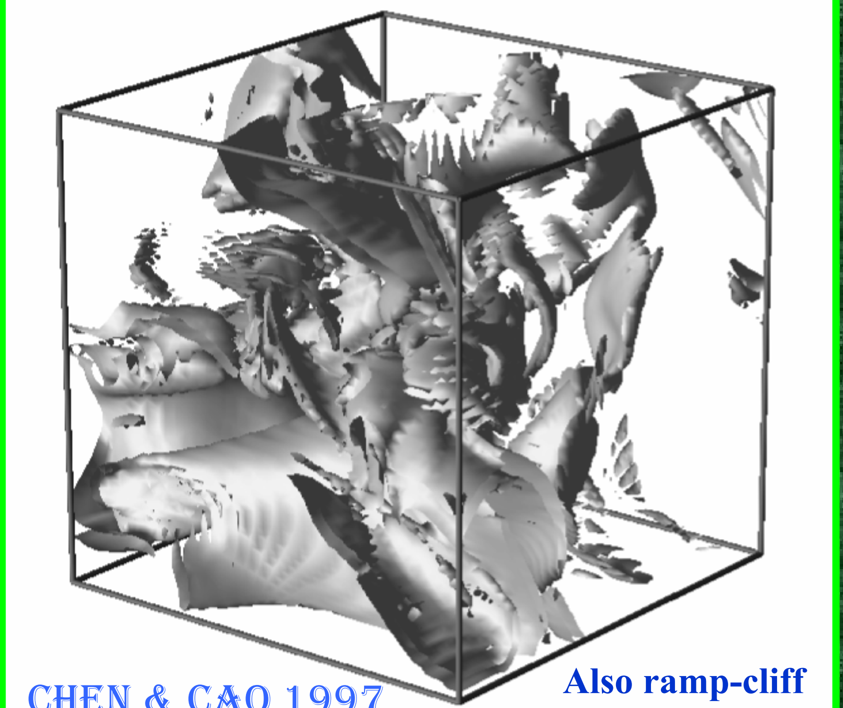
Differences in structure(s)

Vorticity

Gradient of passive scalar



Isosurfaces of enstrophy



CHEN & CAO 1997

FIG. 5. Isosurface of the scalar dissipation at the value $N = 10\langle N \rangle$. The resolution of the plot is 64^3 and the data shown represent the neighborhood of the overall maximum of scalar dissipation.

SAME FLOW - NOT THE SAME PATTERN

All frames (i.e. four different Lagrangian fields) correspond to the same (Eulerian)flow.

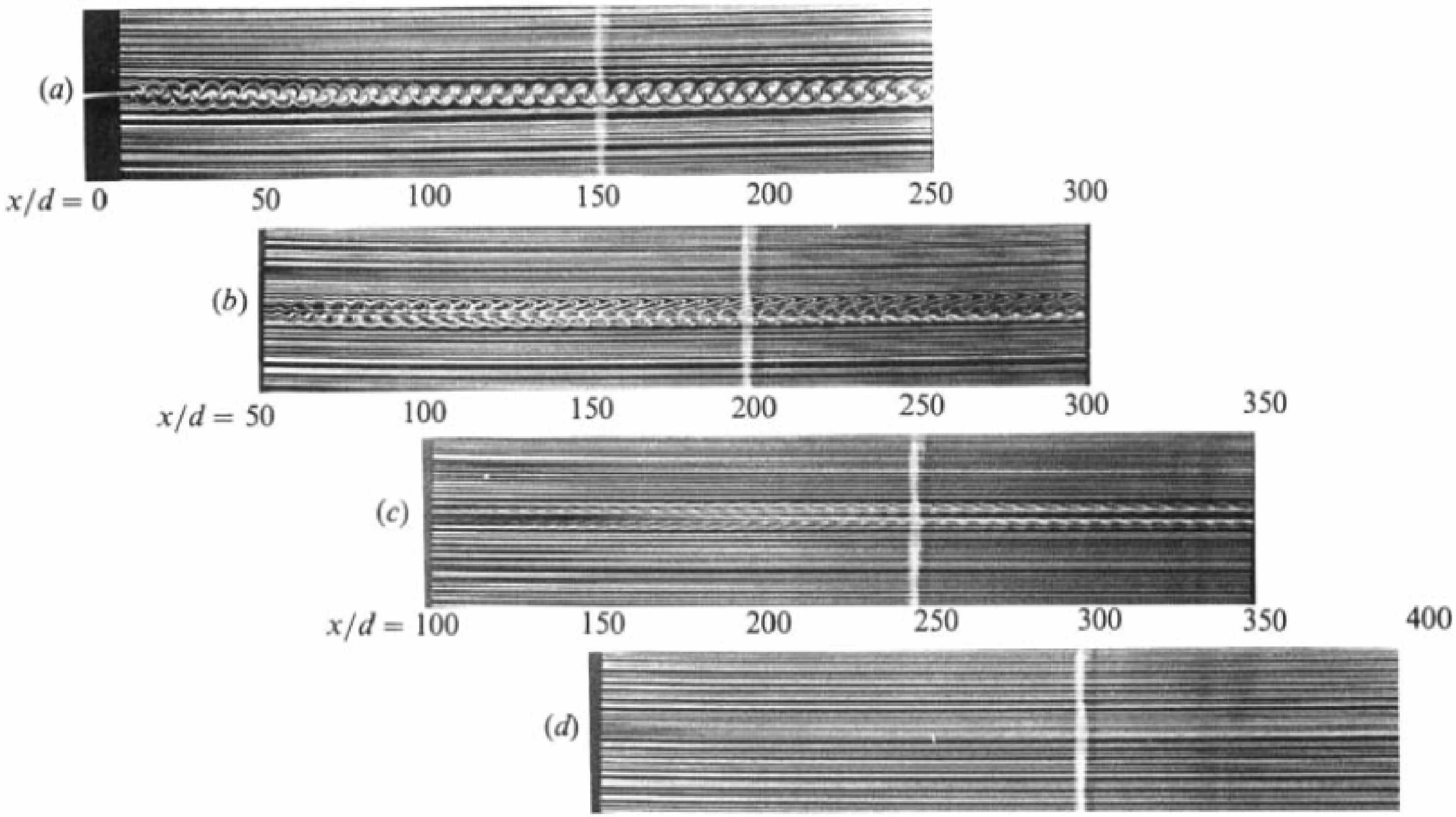


FIGURE 1. Circular-cylinder wake at $Re = 90$; smoke wire at (a) $x/d = 4$, (b) 50, (c) 100 and (d) 150.

Cimbala, J.M., Nagib, H. M and Roshko, A. (1988) Large structures in the far wakes of two-dimensional bluff bodies, J. Fluid Mech., **190**, 265--298.

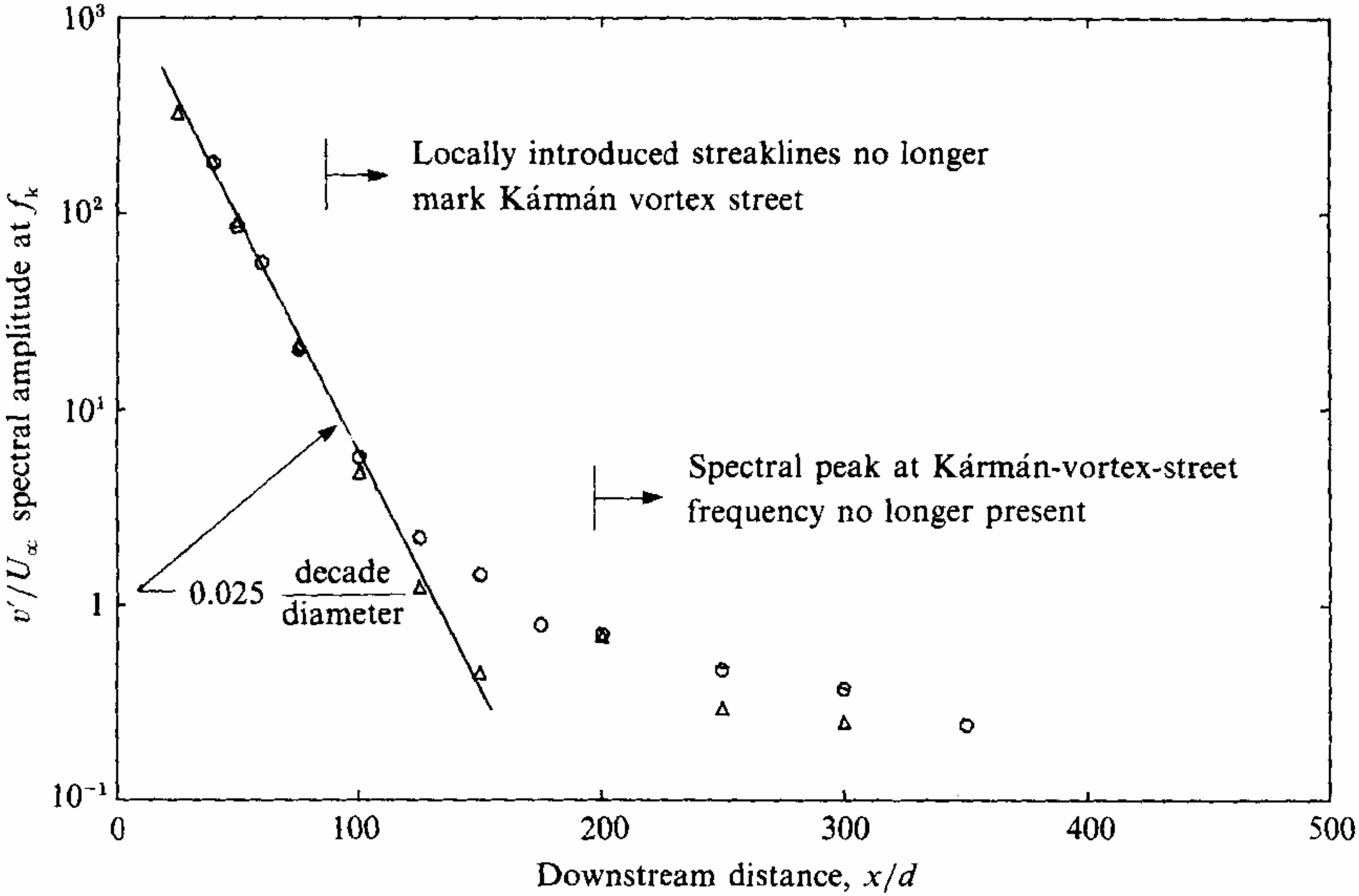


FIGURE 7. Exponential decay of Kármán vortex street; circular-cylinder wake at $Re = 140$ (○) and 150 (△).

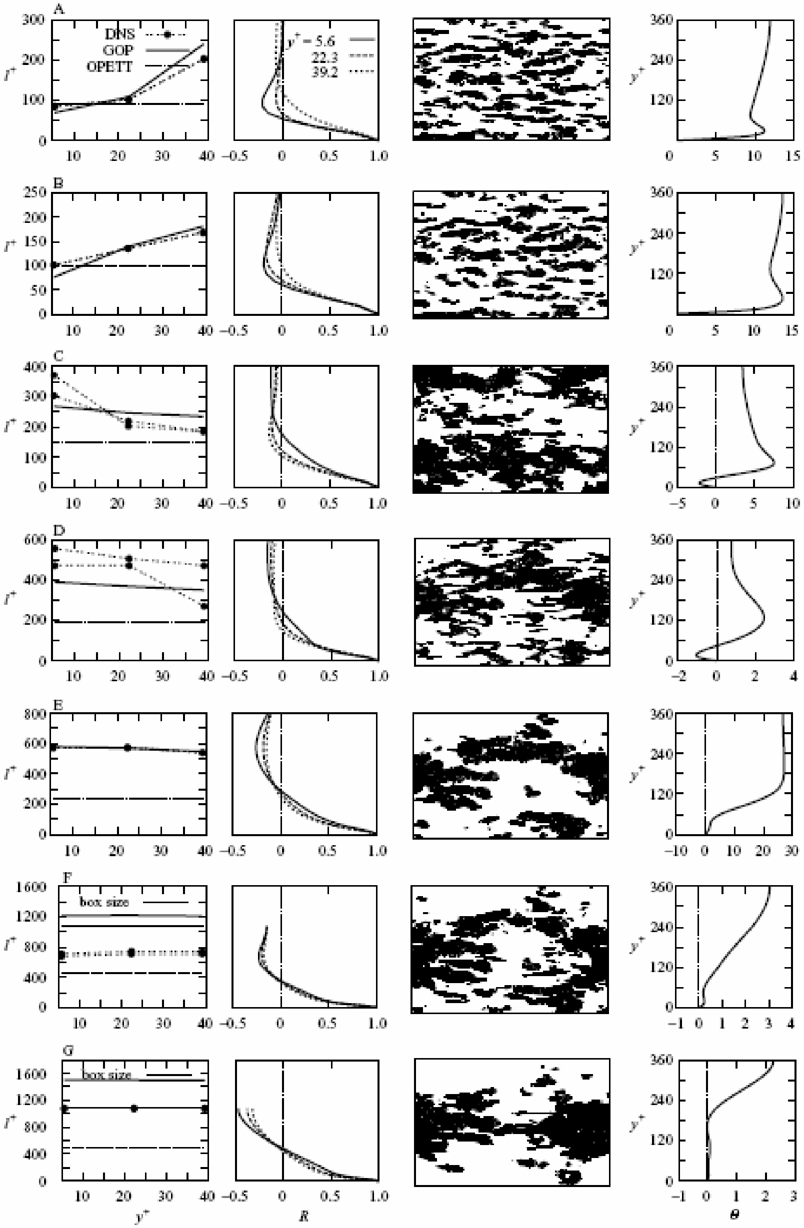
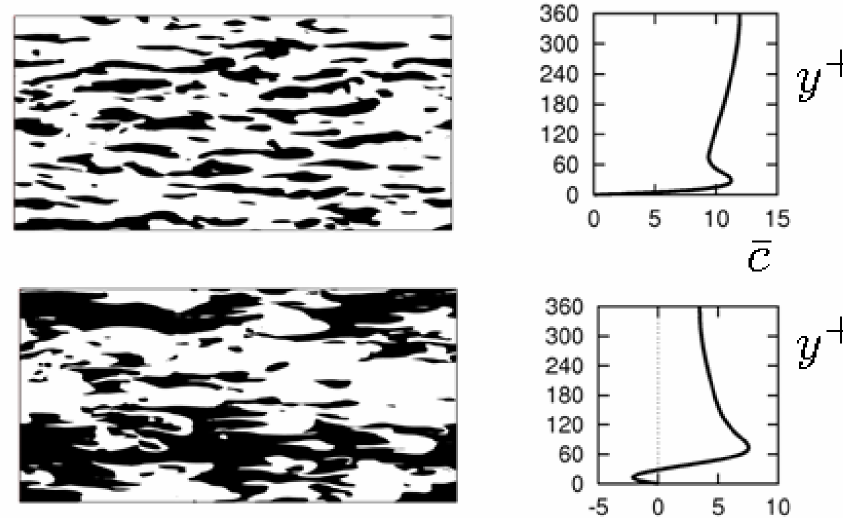


FIGURE 7. Instantaneous visualizations and relations between the streak spacing l^+, wall distance y^+, autocorrelation R, and mean scalar Θ for profiles A–G.

The same motion generates very different streaks



J. Fluid Mech. (2005), vol. 544, pp. 99–131.
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The mechanism of streak formation in near-wall turbulence

By S. I. CHERNYSHENKO AND M. F. BAIG

RDT-like processes/terms dominate the flow near the wall

Kolmogorov 4/5 law versus Yaglom 4/3 law

The Kolmogorov and the Yaglom laws are respectively

$$S_3(r) \equiv \langle(\Delta u_{||})^3\rangle = -(4/5)\,\varepsilon r$$

$$\langle\Delta u_{||}(\Delta\theta)^2\rangle = -\,(4/3)\,\varepsilon_\theta r$$

where $\Delta u_{||} \equiv [\mathbf{u}(\mathbf{x}+\mathbf{r})-\mathbf{u}(\mathbf{x})]\cdot\mathbf{r}/r$, $\Delta\theta = \theta(\mathbf{x}+\mathbf{r}) - \theta(\mathbf{x})$, ε - is the mean rate of dissipation of kinetic energy and $\varepsilon_\theta = \langle D\, \partial\theta/\partial x_i\, \partial\theta/\partial x_i\rangle$ - is the mean rate of dissipation of fluctuations of a passive scalar. The analogy between these two laws* though useful in some respects, e.g. ANTONIA ET AL 1997, is violated for a Gaussian velocity field.

*The 4/5 Kolmogorov law follows by isotropy from the the 4/3 law for the velocity field in the form $\langle\Delta u_{||}(\Delta \mathbf{u})^2\rangle = -(4/3)\,\varepsilon r$)

Namely, the **4/3 law remains valid for such (as any other random isotropic) velocity field**, whereas **the 4/5 law is not, because $S_3(r) \equiv 0$ for a Gaussian velocity field.** This difference is one of the manifestations of the dynamical nature of the Kolmogorov law as contrasted to the kinematical nature of the Yaglom law. It reflects the difference between **genuine turbulence as a dynamical phenomenon** and **‘passive' turbulence as a kinematical process**.

Vorticity versus passive vectors. Solenoidal vector fields with nonvanishing diffusivity

The usual comparison is based on looking at the equations for vorticity **ω** and the (solenoidal) passive vector, **B**, e.g. magnetic field in electrically conducting fluids, BATCHELOR1950

$$\partial \boldsymbol{\omega}/\partial t = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega}$$

$$\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Though a number of differences are known these differences are hidden when one looks at the equations for **ω** and **B**, which look quite \`similar' when $\nu = \eta$.

What is hardest to accept in Batchelor's discussion is the assumed simlarity between **B** *and* ω. LUNDQUIST, 1952

However, a more 'fair' comparison should be made between the velocity field, **u**, and the vector potential **A**, with $\mathbf{B} = \nabla \times \mathbf{A}$, TSINOBER & GALANTI 2003. Such a comparison allows to see immediately one of the basic differences between the fields **u** and **A** (apart of the first obeying nonlinear and the second linear equation) which is not seen from the above equations. Namely, the Euler equations conserve energy, since the scalar product of $\mathbf{u} \cdot (\boldsymbol{\omega} \times \mathbf{u}) \equiv 0$.

In contrast, (unless initially and thereby subsequently $\mathbf{u} \equiv \mathbf{A}$) the scalar product

$$\mathbf{A}\cdot(\mathbf{u}\times\mathbf{B}) \neq 0^{*}.$$

It is this term $\mathbf{A}\cdot(\mathbf{u}\times\mathbf{B}) \equiv -A_i A_k s_{ik} + \partial/\partial x_k\{\ldots\}$ which acts as a production term in the energy equation for $\mathbf{A}$. In other words the field $\mathbf{A}$ (and $\mathbf{B}$), is sustained by the strain, s_{ik} of the velocity field - in contrast to the field $\mathbf{u}$. This leads, in particular, to substantial differences in amplification of vorticity, $\boldsymbol{\omega}$ and magnetic field $\mathbf{B}$, e.g. in statistically stationary velocity field (both NSE and Gaussian) the enstrophy ω^2 saturates to some constant value, whereas the energy of magnetic field B^2 grows exponentially without limit (but there is much more, see below).

* the corresponding equation for the vector potential $\mathbf{A}$ has the form $\partial\mathbf{A}/\partial t+\mathbf{B}\times\mathbf{u} = \nabla p_A+\eta\nabla^2\mathbf{A}$

As in case of passive scalar an analogue of Kolmogorov 4/5 law* is valid for the vector **A** (see e.g. Gomez et al., 1999 and references therein)

$$\langle \Delta u_{||} (\Delta A)^2 \rangle = - 4/3 r \varepsilon_{\mathbf{A}}$$

where $\Delta u_{||} \equiv \Delta \mathbf{u} \cdot \mathbf{r}/r \equiv \{\mathbf{u}(x+\mathbf{r})-\mathbf{u}(\mathbf{x})\} \cdot \mathbf{r}/r$, $\Delta \mathbf{A}=\mathbf{A}(\mathbf{x}+\mathbf{r})-\mathbf{A}(\mathbf{x})$, and $\varepsilon_{\mathbf{A}}$ is the mean dissipation rate of the energy of **A**. Again the latter relation holds for any random isotropic velocity field including the Gaussian one.

*It is more convenient to use the 4/3 law for the velocity field in the form $\langle \Delta u||(\Delta u)^2 \rangle =-(4/3)\, \varepsilon r$, which turns into the 4/5 law by isotropy

Vorticity versus passive vectors with nonvanishing diffusivity
Evolution of disturbances

Important aspects of the essential difference between the evolution of fields **ω** and **B** arising from the nonlinearity of the equation of **ω** and linearity of the equation for **B** are revealed when one looks at how these fields amplify disturbances. The reason is that the equation for the disturbance of vorticity differ strongly from that for vorticity itself due to the nonlinearity of the equation for the undisturbed vorticity **ω**, whereas the equation for the evolution of the disturbance of **B** is the same as that for **B** itself due to the linearity of the equation for B. Consequently, the evolution of disturbances of the fields **ω** and **B** is drastically different. For example, in a statistically stationary velocity field the energy of the disturbance of **B** grows exponentially without limit (just like the energy of **B** itself), whereas the energy of vorticity disturbance grows much faster than that of B for some initial period until it saturates at a value which is of order of the enstrophy of the undisturbed flow.

It is noteworthy that much faster growth of the energy of disturbances of vorticityis observed during the very initial (linear in the disturbance) regime which is due to additional terms in the equation for the disturbance of vorticity, $\boldsymbol{\omega}$ which have no analogues in the case of passive vector $\mathbf{B}$. It is important to stress that these additional 'linear' terms arise due to the nonlinearity of the equations for the undisturbed vorticity. In this sense the essential differences between the evolution of the disturbances of vorticity $\boldsymbol{\omega}$ and the evolution of the disturbance of passive vector $\mathbf{B}$ with the same diffusivity can be seen as originating due to the nonlinear effects in genuine NSE turbulence even during the linear regime.

Looking at the evolution of the disturbance Δ^{u} of some flow realization $\mathbf{u}$ in a statistically steady state and similarly for other quantities.

Active: vorticity - Δ^{ω}, strain - Δ^{s};

Passive: magnetic field - Δ^{B}, its vector potential - Δ^{A}, passive scalar - Δ^{θ} and its gradient - Δ^{G}.

For more details see TSINOBER AND GALANTI 2003, *Phys. Fluids*, 15, 3514-3531.

The behavior of $\mathbf{\Delta}^u$ is governed by the equation, which is a direct consequence of NSE for $\mathbf{u}$ and $\mathbf{u}+\mathbf{\Delta}^u$

$$\frac{D\Delta_i^u}{Dt}=\frac{\partial}{\partial x_j}\left\{-\frac{1}{\rho}\Delta^p\delta_{ij}+2\nu\Delta_{ij}^s-\Delta_i^u\Delta_j^u\right\}-\Delta_j^u\frac{\partial u_i}{\partial x_j},$$

The corresponding equation for the energy of the disturbance, $e_{\Delta^u}=\frac{1}{2}\Delta_i^u\Delta_i^u$ is

$$\frac{De_{\Delta^u}}{Dt}=-\frac{\partial}{\partial x_j}\left\{\Delta_j^u e_{\Delta^u}+\frac{1}{\rho}\Delta^p\Delta_j^u-2\nu\Delta_i^u\Delta_{ij}^s\right\}$$

$$-2\nu\Delta_{ij}^s\Delta_{ij}^s\underline{-\Delta_i^u\Delta_j^u s_{ij}}.$$

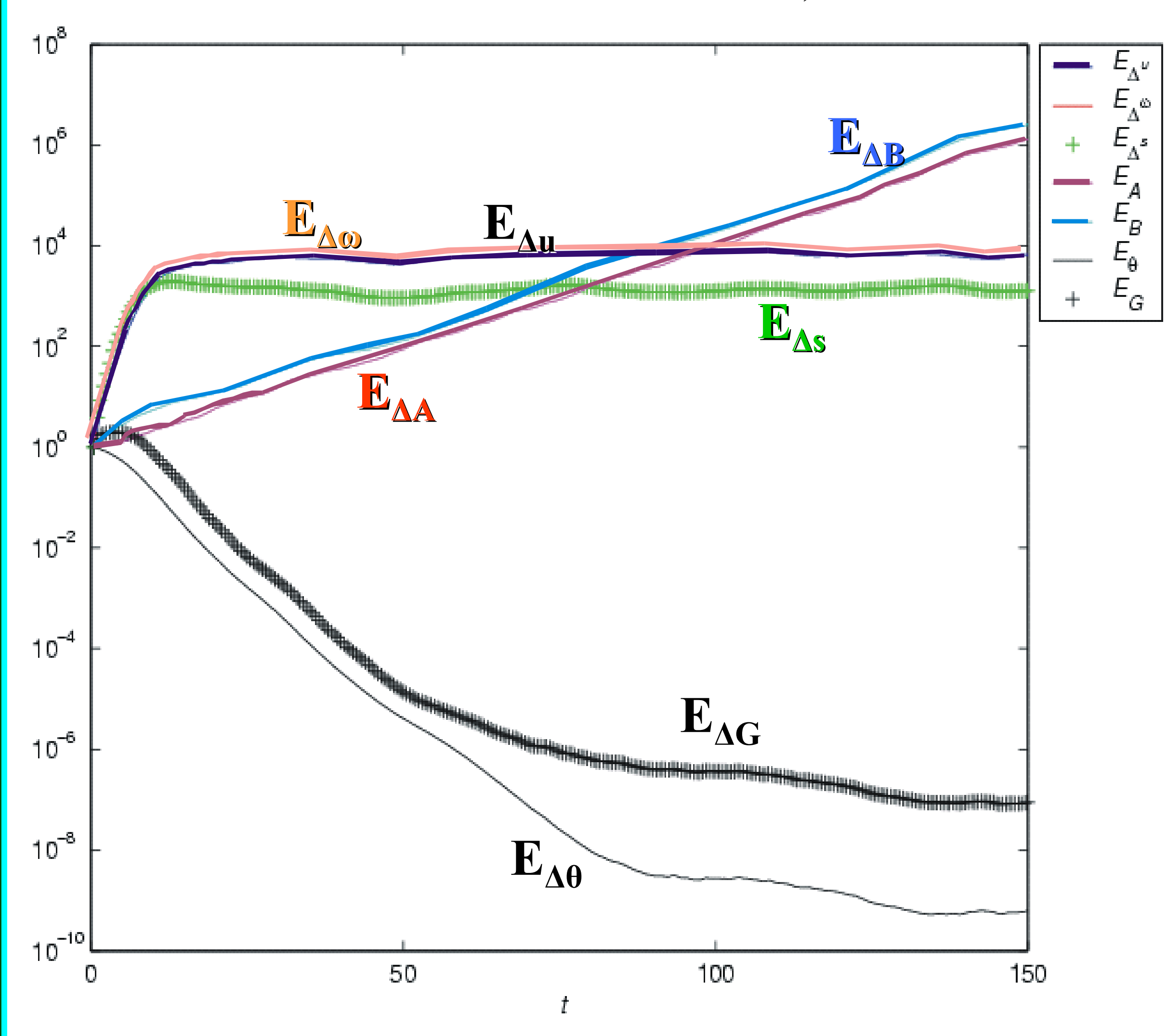
The equations for the disturbance of vorticity, Δ_i^ω, have the following form:[2]

$$\frac{D\Delta_i^\omega}{Dt} = \omega_j \Delta_{ij}^s + \Delta_j^\omega s_{ij} - \Delta_j^u \frac{\partial \omega_i}{\partial x_j} + \Delta_j^\omega \Delta_{ij}^s - \Delta_j^u \frac{\partial \Delta_i^\omega}{\partial x_j} + \nu \nabla^2 \Delta_i^\omega ,$$

$$\frac{De_{\Delta\omega}}{Dt} = \Delta_i^\omega \Delta_{ij}^s \omega_j + \Delta_i^\omega \Delta_j^\omega s_{ij} - \Delta_j^u \Delta_i^\omega \frac{\partial \omega_i}{\partial x_j} + \Delta_i^\omega \Delta_j^\omega \Delta_{ij}^s - \Delta_j^u \Delta_i^\omega \frac{\partial \Delta_i^\omega}{\partial x_j} + \nu \Delta_i^\omega \nabla^2 \Delta_i^\omega .$$

Note the additional linear in disturbance terms which arise due to the nonlinearity of the equations for the undisturbed vorticity and which have no analogues in the case of passive vector B. These additional terms are responsible for much faster growth of the energy of disturbances of vorticity during the very initial (linear in the disturbance) regime. In this sense the essential differences between the evolution of the disturbances of vorticity and the evolution of the disturbance of passive vector B with the same diffusivity can be seen as originating due to the nonlinear effects in genuine NSE turbulence even during the linear regime.

GROWTH OF ENERGY OF DISTURBANCES IN GENUINE $\{E_{\Delta u}, E_{\Delta\omega}, E_{\Delta s}\}$ AND PASSIVE $\{E_{\Delta A}, E_{\Delta B}, E_{\Delta\theta}, E_{\Delta G}\}$ TURBULENCE



Note the much faster growth of the energy of disturbances of active variables such as vorticity during the very initial (linear in the disturbance) regime and decay of disturbances associated with passive scalar

TSINOBER & GALANTI, 2003

Additional Issues

Scaling exponents and statsitically conserved quantities

There is a number of publications insisting in some sense on a kind of essential linearization of genuine turbulence problem when this concerns scaling exponents (mainly of structure functions) and the role of statistically conserved quantities.

The claims are summarized by arguing *that the mechanism leading to anomalous scaling in Navier-Stokes equations and other nonlinear models is identical to the one recently discovered for passively advected fields.* ANGHELUTA ET AL 2006

If this is really true it means that this is just one more aspect – as in RDT - which can be treated via a linear model which in *some* cases enables to handle *some* aspects of turbulent flows, but not their genuine nonlinear aspects*: One can thus speculate that the anomalous scaling for the genuine turbulence can also appear as a linear phenomenon in the following sense. Let us split the total velocity field into the two parts, the background field and the perturbation … linearize the original stochastic equation with respect to the latter, choose an appropriate statistics for the former … Then the small-scale perturbation field will show anomalous scaling behavior with nontrivial exponents, which can be calculated systematically within a kind of ε-expansion. model. In such a case the passive vector field can give the anomalous exponents for the NS velocity field exactly.* ANTONOV ET AL 2003

Similar statements are made in respect with so called statistically conserved quantities which have been discovered for passive objects, but not really for genuine NSE, see references in FALKOVICH AND SREENIVASAN 2006.

Analogy between genuine turbulence Lagrangian chaos

This analogy is closely related to those associated with the analogies between the genuine and passive turbulence in several respects. The main is that the former is a dynamical phenomenon (E-turbulent) whereas the latter is a kinematic one (L-turbulent, i.e. purely Lagrangian). The flow can be purely L-turbulent (i.e. E-laminar) at Re ~1 and Re << 1 (see exmples in TSINOBER 2001). This includes examples such as a number of mixing issues in flows in porous media, microdevices, and kinematic simulations of Lagrangian chaotic evolution (KS, turbulent-like motions). However if the flow is E-turbulent (i.e. Re >> 1) it is L-turbulent as well. An important consequence is that the structure and evolution of passive objects in genuine turbulent flows arises from **two** (essentially inseparable) contributions: one due to the Lagrangian chaos and the other due to the random nature of the (Eulerian) velocity field itself. Hence, one can expect adequate kinematic simulation of those properties which are insensitive (or weakly sensitive) to the differences between the genuine and synthetic velocity fields. An important counterexample is the difference between backwards and forwards relative dispersion (with the mean square separation following particle pairs backwards in time being twice as large as forwards) in genuine turbulence.

E-LAMINAR BUT L-TURBULENT

Since the equations describing the evolution of passive objects are linear, it may seem that there is no place for chaotic behaviour of passive objects if the velocity field is not random and is regular and fully laminar, because the chaotic behaviour appears/shows up in nonlinear systems. There is, however, no real contradiction or paradox. This apparent contradiction is resolved via looking at the the fluid flow in the Lagrangian setting in which the observation is made following the fluid particles wherever they move. Here the dependent variable is the position of a fluid particle, X(a,t), as a function of the particle label, a, (usually it's initial position, i.e. $a \equiv X(0)$) and time, t. The relation between the two ways of description is given by the following equation

$$\partial X(a,t))/\partial t = u[X(a,t)] \qquad \text{(E-L)}$$

i.e. the Lagrangian velocity field, $v(a,t) = \partial X(a,t))/\partial t$, is related to the Eulerian velocity field, u(x,t), as $V(a,t) \equiv u[X(a,t);t]$. Though the Eulerian velocity field, u(x;t) is not chaotic and is regular and laminar, the Lagrangian velocity field $v(a,t) \equiv u[X(a,t);t]$ is chaotic because X(a,t) is chaotic: the equation (E-L) is not integrable even for simplest laminar Euler fields with the exception of very simple flows such as unidirectional ones.

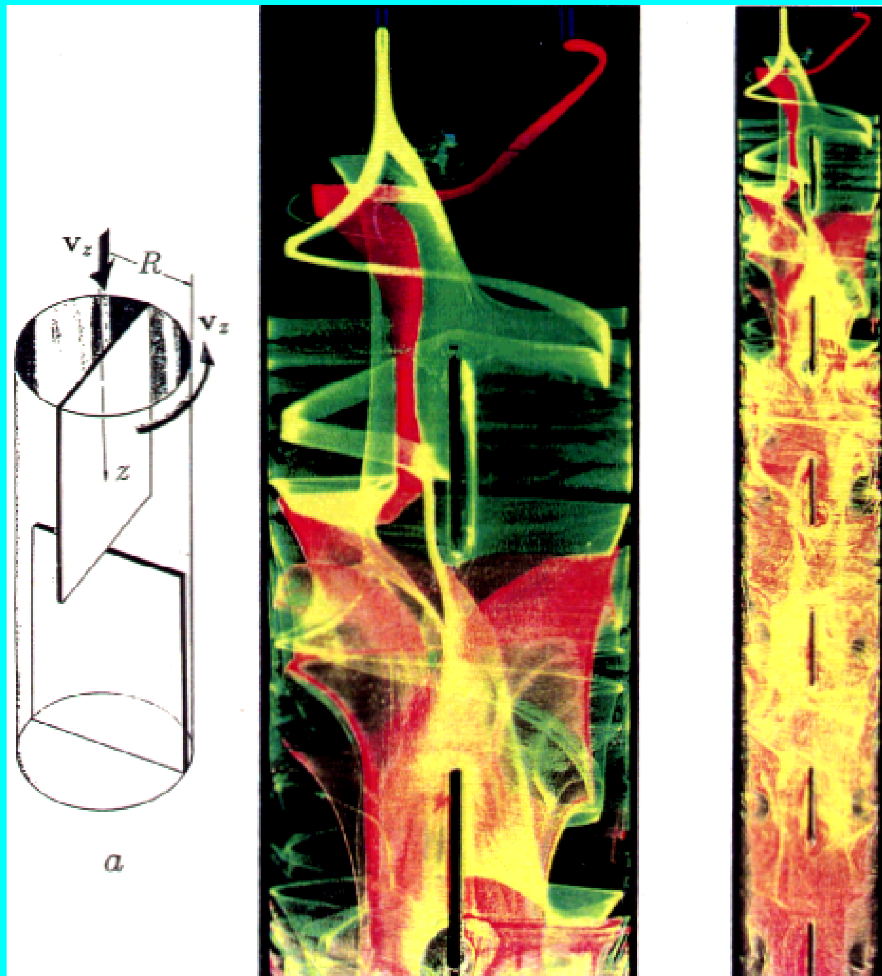
ON THE RELATION BETWEEN EULERIAN AND LAGRANGIAN FIELDS

 # Given the marker dispersion the problem is to determine the source(s) of agitation. In general, owing to chaotic advection, this inverse problem is impossible to solve AREF 1984

#...the possession of such relationship would imply that one had (in some sense) solved the general turbulence problem. Thus it seems arguable that such an aim, although natural, may be somewhat illusory McCOMB 1990

 # What one sees is real. The problem is interpretation

The relation between Eulerian and Lagrangian fields is a long-standing and most difficult problem. The general reason is because the Lagrangian field is an extremely complicated non-linear functional of the Eulerian field. This issue just as the whole theme of Lagrangian description of turbulent flows (not just kinematical chaos) will be addressed in several lectures later. Only few general notes are given here.



MIXING IN PMM, Re ~ 1 (!) KUSH & OTTINO (1992)

RELEVANT TO MICROFLUIDICS with Re ~ 0 (!);

Linked twist maps (LTMs), Bernoulli mixing…

The complexity and problematic aspects of the relation between the Lagrangian and Eulerian fields is seen in the example of Lagrangian (kinematic) chaos or Lagrangian turbulence (chaotic advection) with a priori prescribed and not random Eulerian velocity field (E-laminar). This is why Lagrangian description - being physiclly more transparent - is much more difficult than the Eulerian description. In such E-laminar but L-turbulent flows the Lagrangian statistics has no Eulerian counterpart, as in the flow shown at the left.

Indeed, though the Eulerian velocity field, $u(x;t)$ is not chaotic and is regular and laminar, the Lagrangian velocity field $v(a,t) \equiv u[X(a,t);t]$ is chaotic because $X(a,t)$ is chaotic. This shows that, in general, there does not exist a unique relation between Lagrangian and Eulerian statistical properties in genuine turbulent flows as was foreseen by CORRSIN 1959 : ***in general, there is no reason to expect that L_{ik}*** (the Lagrangian two point velocity correlation tensor) ***and E_{ik}*** (the Eulerian two point velocity correlation tensor) ***will be uniquely related***. In other words it may be meaningless to look for such a relation.

A list of a variety of other attempts to analogies

Turbulence is rent by **factionalism.** Traditional approaches in the field are under attack, and one hears intemperate statements against long time averaging, Reynolds decomposition, and so forth. Some of these are reminiscent of the Einstein–Heisenberg controversy over quantum mechanics, and smack of a mistrust of any statistical approach. **Coherent structures** people sound like The Emperor's new Clothes when they say that all turbulent flows consist primarily of coherent structures, in the face of visual evidence to the contrary. **Dynamical systems** theory people are sure that turbulence is chaos. Simulators have convinced many that we will be able to compute anything within a decade... **The card-carrying physicists** dismiss everything that has been done on turbulence from Osborne Reynolds until the last decade. **Cellular Automata** were hailed on their appearance as the answer to a maidens prayer, so far as turbulence was concerned .

LUMLEY 1990.

BURGULENCE

In order to keep the formalism as simple as possible, we shall, work here with the one-dimensional scalar analog (!!!) to the Navier-Stokes eugation proposed by Burgers[31]. In the method to be presented here, the true poroblem is replaced by models that lead, without approximaton, to closed equations for correlation functions and averaged Green's functions (p. 124). The treatment of Navier-Stokes equation for an incompressible fluid, which we shall discuss briefly, does not differ in essentials (p.143) KRAICHNAN, R.H. 1961, Dynamics of nonlinear stochastic systems, *J. Math Phys.*, 2(1), 124-148)

Mathematical analysis will deal with several basic models. The simplest one is the 1D Burgers equation with random forcing. It displays several basic features of turbulence…3D Navier-Stokes systems probably need completely new ideas. SINAI, YA.G. 1999 Mathematical Problems of Turbulence, *Physica,* A 263,565-566

Analogy between the Navier–Stokes equations and Maxwell's equations: application to turbulence. Screening.
Beyond the Navier–Stokes equations, e,g. analogy between Boltzmann kinetic theory of fluids and turbulence
Modeling nearly incompressible turbulence with minimum Fisher information.
Neural networks approach, the simulation and interpretation of free turbulence with a cognitive neural system
Variety of approaches from statistical physics/mechanics such as critical phenomena, Levy walks, Gibbsian hypothesis in turbulence, Tsalis nonextensive statistics, quantum kinetic models of turbulence
Polymer analogies
Stock market dynamics and turbulence: parallel analysis of fluctuation phenomena.
Dynamical systems, e.g. low dimensional description.

There are more but all with modest succes (if at all)

Perhaps the biggest fallacy about turbulence is that it can be reliably described (statistically) by a system of equations which is far easier to solve than the full time-dependent three-dimensional Navier-Stokes equations

BRADSHAW, 1994.

CONCLUDING

The essential differences between the genuine turbulence and its analogues (as those described above and many other not described) and the intricacy of the relation between them (e.g. between genuine and "passive turbulence") require caution in promoting analogies to far leading to grave misconceptions. On the other hand these very differences can be effectively used to gain more insight into the dynamics of real turbulence.

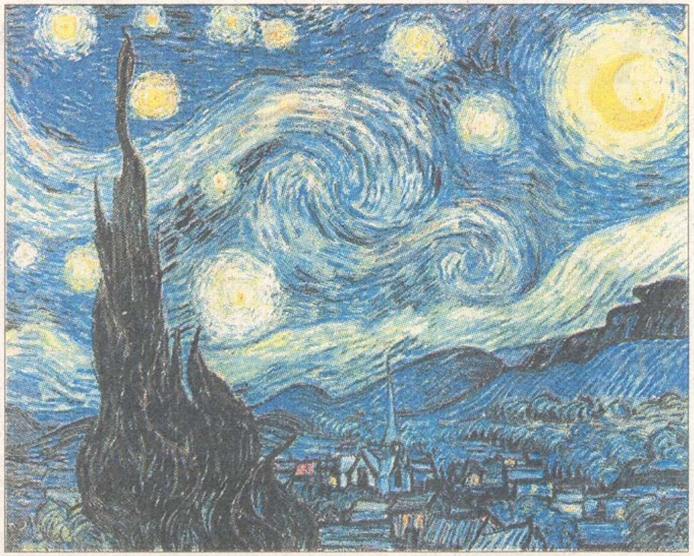
The above examples also serve as a warning that flow visualizations used for studying the structure of dynamical fields (velocity, vorticity, etc.) of turbulent flows may be quite misleading, making the question "what do we see?" extremely nontrivial. The general reason is that the passive objects may not `want' to follow the dynamical fields (velocity, vorticity, etc.) due to the intricacy of the relation between passive and active fields just like there is no one to one relation between the Lagrangian and Eulerian statistical properties in turbulent flows. As mentioned one of the reasons is the presence of Lagrangian chaos, which is manifested as rather complicated structure of passive objects even in very simple regular velocity fields. On the other hand the ramp-cliff structures of a passive scalar are observed in pure Gaussian `structureless' random velocity field just like those in a variety of real turbulent flows practically independently of the value of the Reynolds number TSINOBER 2001.

This does not mean that qualitative and even quantitative study of fluid motion by means of `color bands' (REYNOLDS1894) is impossible or necessarily erroneous. However, watching the dynamics of material 'colored bands' in a flow may not reveal the nature of the underlying motion, and even in the case of right qualitative observations the right result may come not necessarily for the right reasons. The famous verse by Richardson belongs to this kind of observation. On the other hand there are properties of passive objects which do depend on the details of the velocity field (TSINOBER 2001, TSINOBER & GALANTI 2003). Just these very properties can be effectively used to study the differences between the real turbulent flows and the artificial random fields, to gain more insight into the dynamics of real turbulence.

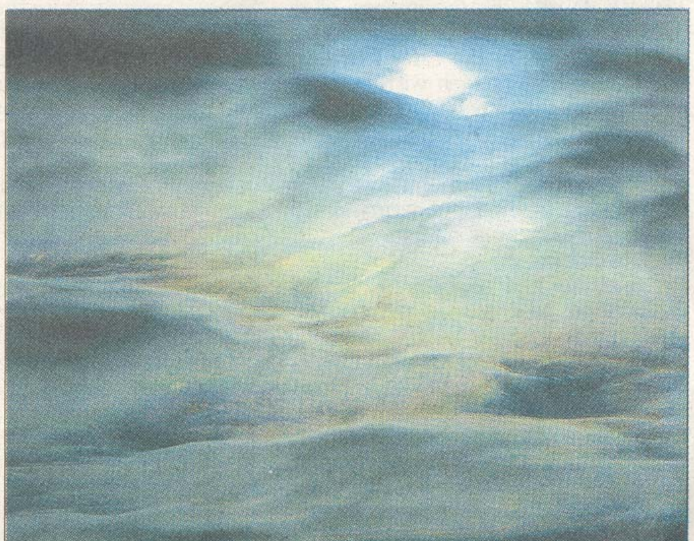
At present, however, the knowledge necessary for such a use is very far from being sufficient. With few exceptions it is even not clear what can be learnt about the dynamics of turbulence from studies of passive objects (scalars and vectors) in real and `synthetic' turbulence. This requires systematic comparative studies of both. An an attempt of such a comparative study was made by TSINOBER & GALANTI 2003. This is a relatively small part of a much broader field of comparative study of 'passive' turbulence reflecting the kinematical aspects and genuine turbulence representing also the dynamical processes.

A BIT OF FUN

2WC THE TIMES SATURDAY JULY 15 2006



The swirling clouds in Vincent van Gogh's *Starry Night* . . .



. . . exactly match the turbulence in real cloud patterns

Is there an analogy or should we believe our eyes ?