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On the recovery of surface wave by pressure transfer function

Cheng-Han Tsai[a,*], Min-Chih Huang[b], Fu-Ji Young[a], Yin-Chern Lin[a], Hsien-Wen Li[c]

[a]*Department of Oceanography, National Taiwan Ocean University, Keelung 202, Taiwan*

[b]*Department of Systems and Naval Mechatronic Engineering, National Cheng Kung University, Tainan 701, Taiwan*

[c]*Department of Civil Engineering, Ming Hsin University of Science & Technology, Hsin Feng, Hsin-Chu County 304, Taiwan*

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Abstract

Except the commonly selected pressure transfer function derived from the linear wave theory, a previous study on the pressure transfer function for recovering surface wave from underwater pressure transducer suggested that the pressure transfer function is a function of frequency parameter only. With careful analysis, this study showed that the pressure transfer function should include a transducer submergence parameter as that given by the linear theory. It was found that the previously suggested empirical formula should be restricted to measurements with the pressure transducer close to the surface; otherwise overestimation of wave height would result. Field measurements were carried out with an acoustic wave gauge and a synchronized pressure transducer located at various depths with submergence parameter close to 1 (near the sea floor). It was shown that the previous one-parameter empirical formula might overestimate the significant wave height by more than 30%. This study found that with deep-water wave bursts excluded, the transfer function based on the linear wave theory provided a fairly good estimation on the significant wave heights, with an average deviation of 3.6%.

Keywords: Frequency parameter; Submergence parameter; Acoustic wave gauge

* Corresponding author. Tel.: +886 2 2462 2192x6311; fax: +886 2 2462 1047.

E-mail address: chtsai@mail.ntou.edu.tw (C.-H. Tsai).

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1. Introduction

Underwater pressure transducers have long been used for wave measurements. There are many advantages for using them. First of all, wave gauges with pressure transducers are simple and less expensive to use in the field. Secondly, since, pressure gauges are deployed under water, they are less prone to be damaged by sea traffic, fishing activities (Wang et al., 1986) or deliberate vandalism. In order to convert the measured pressure data to the surface wave information, a simple pressure transfer function derived from the linear wave theory has long been widely used. However, numerous articles have also raised doubts on the accuracy of using the linear pressure transfer function for recovering surface waves from measurements using an underwater pressure transducer (Hom-ma et al., 1966; Cavaleri, 1980; Biesel, 1982; Wang et al., 1986). Along with these authors, Wang et al. (1986) and Nielsen (1989) also proposed methods to improve the accuracy of the surface wave recovery. On the other hand, Bishop and Donelan (1987) did an extensive laboratory experiment and showed that the surface wave height can be satisfactorily obtained by using the linear wave transfer function. Hence, the problem of adequacy of linear pressure transfer function remained open.

Kuo and Chiu (1994) suggested an one-parameter empirical formula for the pressure transfer function based on dimensional analysis and laboratory experiments. In the following sections, performance of the linear pressure transfer function and the one-parameter transfer function is re-examined based on field measurements using an ultrasonic wave gauge and a synchronized pressure transducer. Comparisons on the significant wave height as well as the water surface elevation in time domain were also presented.

2. Pressure transfer function

Dynamic pressure, p_d, beneath a two dimensional progressive wave, assuming a zero atmospheric pressure, can be expressed as

$$p_d(x,z,t) = \rho\frac{\partial\phi}{\partial t} - \frac{1}{2}\rho(u^2 + w^2) \tag{1}$$

in which t, time; x, horizontal coordinate; z, vertical coordinate measured positive upwards from the still water level; ρ, density of water; g, gravitational acceleration; ϕ, velocity potential; $u = -\partial\phi/\partial x$, horizontal water velocity; and $w = -\partial\phi/\partial z$, vertical water velocity.

From the linear wave theory, a monochromatic wave profile, η, and the corresponding velocity potential can be expressed as

$$\eta(x,t) = a\sin(kx - \omega t) \tag{2}$$

$$\phi(x,z,t) = \frac{ag}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\cos(kx - \omega t) \tag{3}$$

with

$$\omega^2 = gk \tanh kh \tag{4}$$

where a, wave amplitude; $\omega = 2\pi/T$, wave angular frequency; T, wave period; h, water depth; $k = 2\pi/L$, wave number; and L, wavelength. The linearized dynamic pressure, ignoring the non-linear terms given in Eq. (1), can then be expressed as

$$p_{\mathrm{d}}(x,z,t) = \rho g K_{p\mathrm{L}} \eta(x,t) = \rho g \frac{\cosh k(z+h)}{\cosh kh} \eta(x,t) \tag{5}$$

in which $K_{p\mathrm{L}}$ = pressure transfer function from linear wave theory. This result clearly indicates that at least three parameters (wave frequency, water depth and transducer location) are involved in the calculation of $K_{p\mathrm{L}}$.

For regular waves, Bergan et al. (1968) compared measured pressures with theoretical results of Stokes wave theories and found that discrepancies are far less when Stokes fifth-order wave theory is used instead of linear wave theory. Lee and Wang (1984) employed the perturbation technique up to the second order to deal with the weakly non-linear irregular waves and found that the non-linear effect is insignificant in the intermediate to deep-water waves since, the difference between the linear and non-linear transfer functions is small. However, it is universally understood that the non-linear correction is essential in shallow water or in surf zone. Kuo and Chiu (1994) also compared measured pressures with theoretical results of linear and Stokes third-order wave theories and found that the non-linear wave steepness effect is small for their experimental data. These seemingly incoherent results in fact only reflect the different wave conditions being studied. Chen (2000) has recently employed higher order Fourier wave theory to calculate the pressure transfer function under various wave conditions. These wave conditions cover a wide range of relative water depth from shallow water to deep water with different wave steepness. It was found that the pressure transfer function depends on both the frequency parameter, as defined by Kuo and Chiu (1994), and transducer submergence. A regression analysis was then used to obtain the empirical formulas of the non-linear transfer function with varying frequency parameter and different transducer submergence parameter.

An empirical correction factor, N, has often been introduced to account for the difference between measurement and linear wave theory as:

$$p_{\mathrm{d}}(x,z,t) = \rho g \frac{K_{p\mathrm{L}}}{N} \eta(x,t) \tag{6}$$

Forristall (1982) has pointed out potential reasons for the difference between measurement and linear wave theory can be attributed to not including second-order kinetic energy term or non-linear wave effect, and inappropriate analysis method. Bishop and Donelan (1987) have summarized previous results for N which included various laboratory data and field data. After carried out their own experiments, they concluded that linear wave theory is in fact adequate to calculate wave heights from subsurface pressure records. A well-designed pressure transducer system should give estimates of surface wave heights accurate to within 5%. They have indicated that previous results of substantial correction factor N probably suffered from one or more of the following reasons: inaccurate measurements of wave heights, instrument limitations (signal to noise ratio, calibration error, hydrodynamic noise, etc.) and inappropriate analysis methods (wave-by-wave method, spectral leakage).

Kuo and Chiu (1994) proposed an empirical formula for the pressure transfer function, K_{pE}, as:

$$K_{pE} = \exp(-0.905\frac{\omega^2|z|}{g} - 0.027); \quad 0.1 \le \frac{\omega^2|z|}{g} \le 5.0, \quad \frac{h}{L} \ge 0.07 \tag{7}$$

Their formula was based on laboratory data of intermediate and deep-water wave conditions.

Since, dimensional analysis would yield the functional relationship as

$$K_{pE} = f_1\left(\frac{\omega^2|z|}{g}, \frac{\omega^2 h}{g}, \frac{\omega^2 H}{g}\right) = f\left(\frac{\omega^2|z|}{g}, \frac{|z|}{h}, \frac{H}{L}\right) \tag{8}$$

in which H, wave height; $\omega^2|z|/g$, frequency parameter; $\omega^2 h/g$, depth parameter; H/L, wave steepness; $|z|/h$, transducer submergence parameter. Therefore, Eq. (7) was valid only for intermediate and deep-water wave conditions with small wave steepness and negligible transducer submergence effect.

In Kuo and Chiu's data (1994), the non-linear effect was found to be small since, the ratio of third-order pressure to linear order pressure was calculated to be less than 1.08. Kuo and Chiu (1994) also concluded that transducer submergence effect is negligible by saying that "transfer function remains unchanged as the water depth varies at the same frequency and the same submergence value". However, this is not correct if one looks carefully at Fig. 4 of Kuo and Chiu (1994). For instance, the transfer function at 0.5 Hz varies approximately from 0.55 to 0.75 when the transducer submergence parameter varies in the range of 0.625–1.0. Even the simplest form of pressure transfer function from linear wave theory, Eq. (5), indicates that it is strongly dependent on the submergence parameter. In fact, if one plots the variation of linear pressure transfer function versus frequency parameter with varying transducer submergence parameter, one would see a strong dependency of pressure transfer function on the submergence parameter. This is illustrated in Fig. 1, where the transducer submergence parameter varies from 0.2 to 1.0 while the frequency parameter lies in the range of 0–5.0. Since, the frequency parameter can be further expressed as $\omega^2|z|/g = \left(|z|/h\right)kh \tanh kh$, the pressure transfer function is in fact a function of both relative depth kh and transducer submergence parameter, $|z|/h$ (Fig. 1 of Lee and Wang (1984)). For comparison purpose, the empirical formula of Kuo and Chiu (1994) is now plotted in Fig. 1 along with several curves from linear wave theory and this empirical formula may be seen as an acceptable approximation to the curves with transducer submergence parameter smaller than 0.6. The regression formula of Chen (2000) corresponding to transducer submergence parameter 0.9, $K_{pN} = \exp(-0.711\omega^2|z|/g + 0.022)$, is also plotted in Fig. 1 and it lies somewhat in between the linear pressure transfer functions with transducer submergence parameters of 0.8 and 1.0.

For pressure transducer mounted near the sea floor as usually adopted in the field measurements (i.e. $|z|/h$ close to 1), the empirical approximation of Kuo and Chiu (1994) may thus give excess wave predictions when compared with linear wave theory. In their

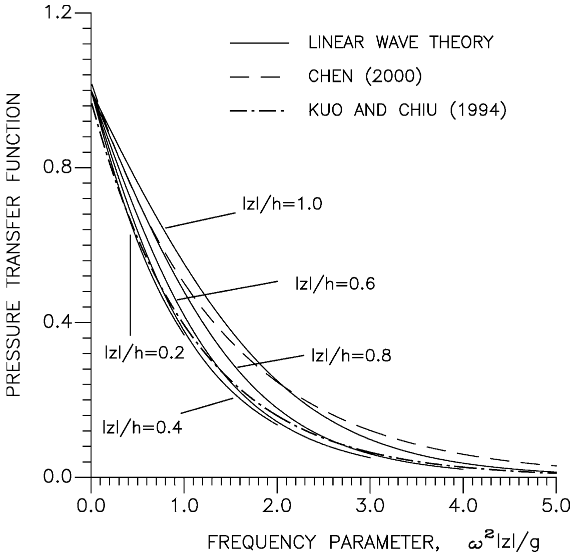

Fig. 1. Comparison of linear pressure transfer function (solid lines), empirical formula of Kuo and Chiu (1994) (dot-dashed line) and regression formula of Chen (2000).

comments to Kuo and Chiu (1994), Baquerizo and Losada (1995) have suggested that Kuo and Chiu re-examine the relationship between transfer function and the depth parameter, $\omega^2 h/g$, and that one should group the data in small range of depth parameter, or identically small range of submergence parameter, $|z|/h$. Baquerizo and Losada (1995) showed that for shallow water waves, the transfer function is also functions of the depth parameter. This point of view is justified when considering the results of linear wave theory and the preliminary results of non-linear wave theory (Fig. 1). Another problem relating to Eq. (7) is that there are very few data points in the upper frequency range to render the regression equation plausible. Since, excess high-frequency wave spectrum components may be introduced when converting from the pressure spectrum, one should be more cautious of using the empirical formula of Kuo and Chiu (1994).

3. Field tests

An ultrasonic wave gauge (model WH-103, manufactured by I.O. Technic Co. Ltd) was used for the field test. This instrument has an upward-looking ultrasonic acoustic transducer, a pressure transducer and an electromagnetic current meter. The acoustic transducer, equipped with a gimbaled mechanism, has a frequency of 200 kHz and a spreading angle of 3°. Using the ultrasonic acoustic transducer, the surface wave elevation can be fairly accurately measured. Since, this instrument also equipped with a synchronized pressure transducer, the reconstructed surface wave based on pressure signal can be compared to the surface wave elevations simultaneously measured by the acoustic wave gauge. The reconstructed surface wave elevations can be determined by using various different pressure transfer functions. For this, the dynamic pressure of the surface wave was first transformed into the frequency domain. The pressure transfer function was then applied, up to the deep-water wave limit, and with an inverse Fourier transformation the surface wave elevation was reconstructed. Pressure signal higher than the deep-water wave limit was not compensated. After the surface wave elevation was reconstructed, zero down-crossing wave height and period were calculated as the individual wave height and period. From that, significant wave height, mean of highest 1/3 waves (H_s), and mean zero-crossing wave period (T_z) of each measurement were determined from pressure data. Similarly, the significant wave height and mean zero-crossing wave period (T_0) from acoustic measurements were also calculated.

The wave measurements were carried out at four different locations on the North coast of Taiwan in water depths of 11, 18, 22 and 27 m with 605, 1365, 410 and 2715 measurements, respectively. The instrument was fixed in a steel frame and the sensors were 0.8 m above the sea floor. That is, the pressure transducer submergence parameter, $|z|/h$, was about 1. The data were recorded at a 2 Hz sampling rate, and each measurement lasted 20 min. Since, bubbles produced by breaking waves or ship traffics can easily interfere with the ultrasonic wave gauges, the measurements that were contaminated with this interference were discarded. The significant wave height of these measurements ranged as high as 4.8 m.

4. Results

Shown in Fig. 2 is the comparison of pressure transfer functions calculated from the linear theory, of Kuo and Chiu (1994) and inversely calculated from measured acoustic

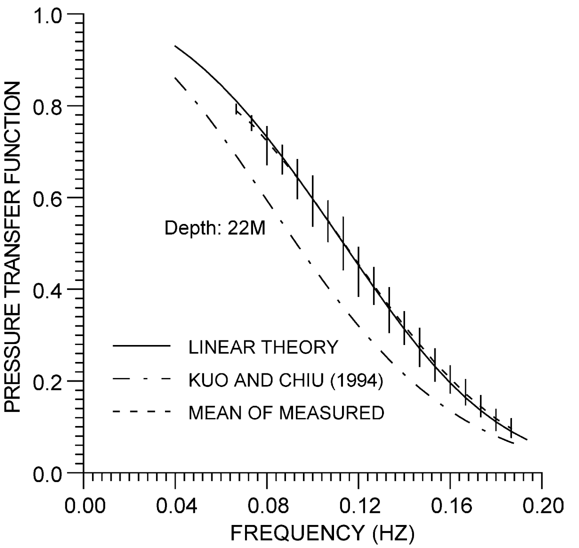

Fig. 2. Comparison of pressure transfer functions. The vertical line segments are the ranges of measurements.

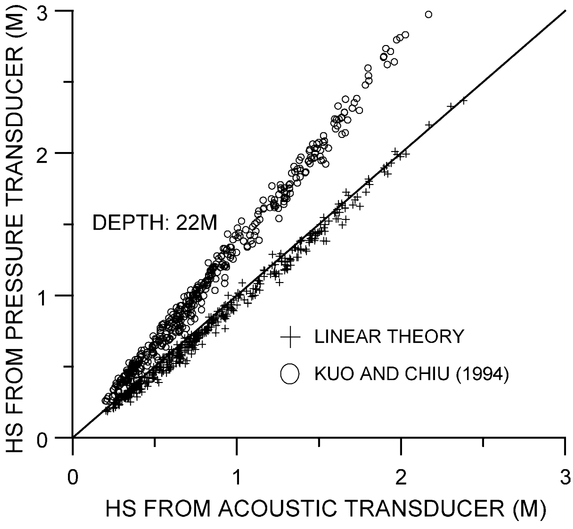

Fig. 3. Comparison of significant wave heights.

and pressure data for the water depth of 22 m. As can be seen, Kuo and Chiu's transfer function was consistently lower than that of measured values, while the linear transfer function was very close to the mean of the measured data. This indicates that using Kuo and Chiu's transfer function would over-predict wave height. From the same set of measurements, the significant wave heights H_s determined by the both transfer function was shown in Fig. 3. It did show that Kuo and Chiu's transfer function over predict wave heights.

Over all, the results show that H_s derived by using linear transfer function differed, on average, from that measured by the acoustic transducer by 3.3, 3.6, 6.9 and 18.4% for measurements conducted at depths 11, 18, 22 and 27 m, respectively (Table 1). The above percentages were the average over all valid measurements for a given depth and calculated by: $|H_s(\text{acoustic}) - H_s(\text{pressure})|/H_s(\text{acoustic})$. The data showed that in almost all cases the acoustic significant wave heights were higher than those obtained by linear transfer function (e.g. Fig. 3), i.e. linear transfer function somewhat underestimated the significant wave height. When the absolute deviations were not used, the average deviation percentages would be slightly lower, as deviation percentages for some measurements were negative. Clearly, one can see that the deviation percentage increased with the water depth. On the other hand, Kuo and Chiu's transfer function did significantly overestimate wave heights; the average differences were above 30% for all depths. Apparently,

Table 1
Mean deviation of H_s from that measured acoustically at various depths (measurement numbers in parentheses)

	11 m (605), %	18 m (1365), %	22 m (410), %	27 m (2715), %
Linear theory	3.3	3.6	6.9	18.4
Kuo and Chiu	31.2	34.2	31.1	34.1

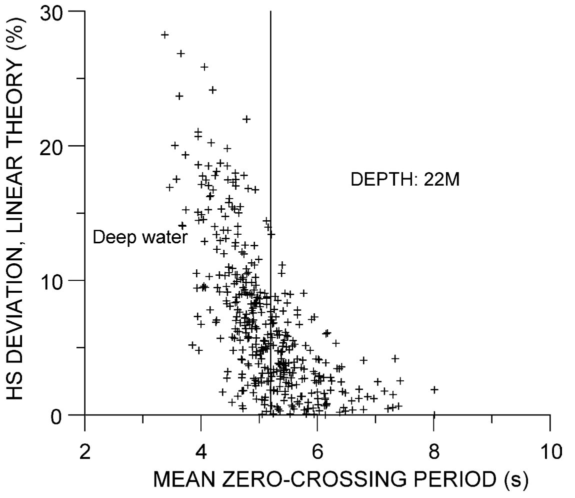

Fig. 4. Significant wave height deviation versus mean wave period. Points on the left of the vertical lines are from deep-water waves.

significant wave heights obtained by using linear transfer function were closer to that from the acoustic transducer than using Kuo and Chiu's empirical formula.

Since, the linear transfer function offered lower deviation for the significant wave height, the following discussion is focused on other aspects of the deviation of recovered waves using linear theory from the acoustic measurements. For example, Fig. 4 shows the significant wave height deviations from the acoustic measurements versus the mean zero-crossing period from acoustic data (T_0) for the measurements obtained in 22 m water. It can be seen that the deviation in significant wave height decreased with increasing period. It is known that deep-water waves cannot be measured correctly by using underwater pressure transducer. Accordingly, the surface wave information recovered from the pressure sensor would tend to have a high error rate, if the wave record contains a high percentage of deep-water waves, or has a low mean wave period. Fig. 4, in which the limiting wave period for the sensor depth is delineated, shows that most of the high percentage errors belongs to measurements with mean period smaller than the deep-water waves period. Then, it is logical to determine the significant wave height deviation by excluding measurements with mean period shorter than the deep-water wave period limit. Fig. 5 has the mean, maximum and minimum H_s deviation percentages for measurements with and without deep-water waves. It clearly shows that, with deep-water waves excluded, the mean errors of H_s using the linear transfer function were about 3.6% or less for all depths, and the maximum error among all measurements in each depth were between 10.5 and 13%. Furthermore, there were cases which the H_s derived by pressure were almost identical to that from the ultrasonic transducer. That is the minimal errors for all depths were near 0%.

One can also compare the error of surface wave elevation, reconstructed from the pressure sensor, using linear pressure transfer function. For this a parameter RSEP was defined

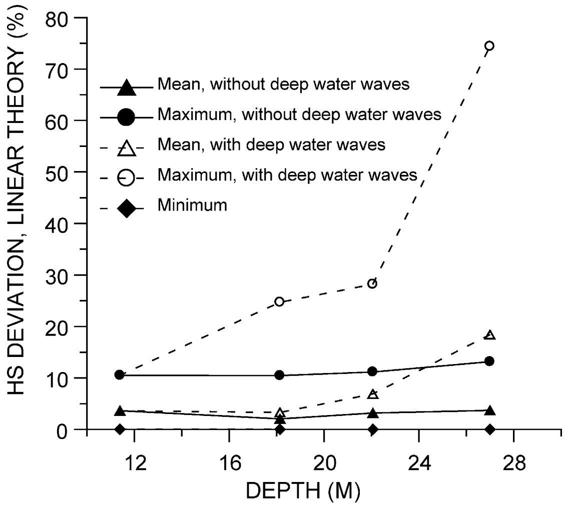

Fig. 5. Mean, maximum and minimum H_s deviation as a function of water depth with and without deep-water measurements.

$$\mathrm{RSEP} = \frac{1}{H_s}\sqrt{\frac{\sum_{i=1}^{n}(X_i - Y_i)^2}{n}} \tag{9}$$

where X_i is the surface wave elevation reconstructed from pressure, Y_i is the wave elevation acoustically measured, n is the number of data points in one measurement (2400 here) and H_s is the significant wave height of the corresponding measurement. RSEP is the root mean square error expressed as a percentage of the significant wave height. It was

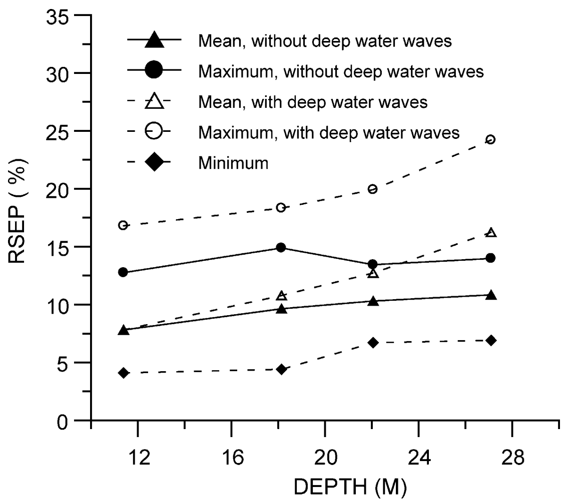

Fig. 6. Root mean square error of reconstructed surface elevation as a percentage of H_s for various water depths.

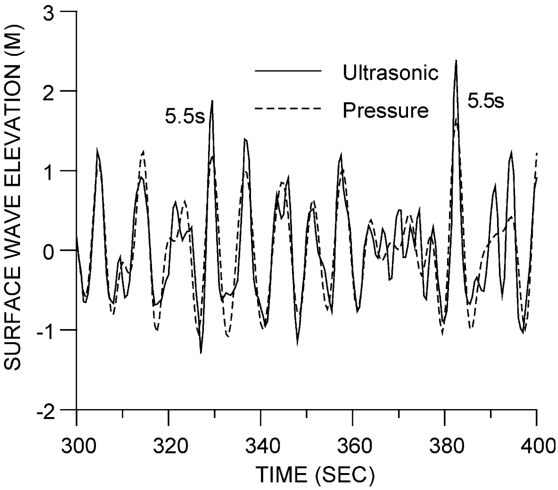

Fig. 7. Comparison of reconstructed surface wave with that obtained by ultrasonic wave gage. The measurement was obtained in 18 m deep water with a H_s of 3.1 m, mean zero-crossing period of 6.4 s, and RSEP 8.2%.

found that the mean values of RSEP over all measurements, with deep-water waves excluded, at a given depth were 7.8, 9.6, 10.3 and 10.8% for the respective depth of 11, 18, 22 and 27 m, while the maximum values of RSEP were 12.7, 14.9, 13.4 and 14%, respectively (Fig. 6). With deep-water waves included, the mean values of RSEP in wave elevation were between 7.8 and 16.2%. Again, the percentage error increased with depth.

From Fig. 6, one can see that using linear pressure transfer function, the reconstructed surface wave elevation still has considerable deviation from the measured one in some

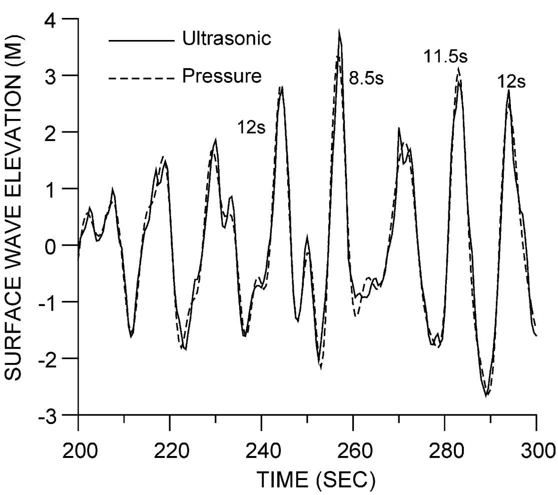

Fig. 8. Comparison of reconstructed surface wave with that obtained by ultrasonic wave gage. The measurement was obtained in 18 m deep water with a H_s 4.8 m, mean zero-crossing period of 10.6 s, and RSEP 4.4%.

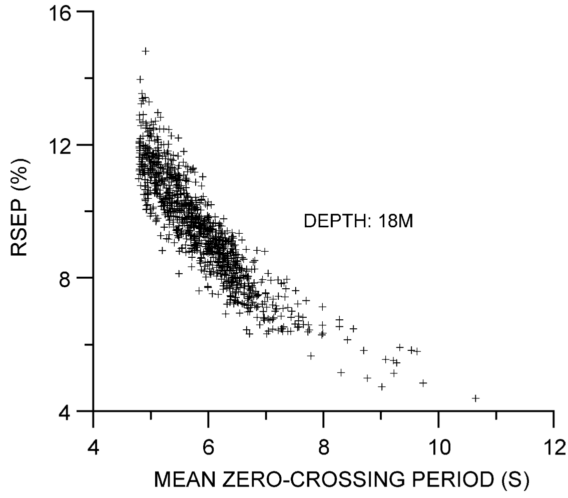

Fig. 9. Example of root mean square error of reconstructed wave surface as a percentage of significant wave height vs. mean wave period (deep-water wave measurements excluded).

cases. It is then constructive to compare wave records, which have high error percentage to that having low error. Figs. 7 and 8 are two measurements both obtained in 18 m depth but with different mean zero-crossing wave period. The former one has a significant wave height of 3.1 m, mean period of 6.4 s and a RSEP of 8.2%; while the latter one 4.8 m, 10.6 s and 4.4%, respectively. It can be seen from Fig. 7 that with a shorter period

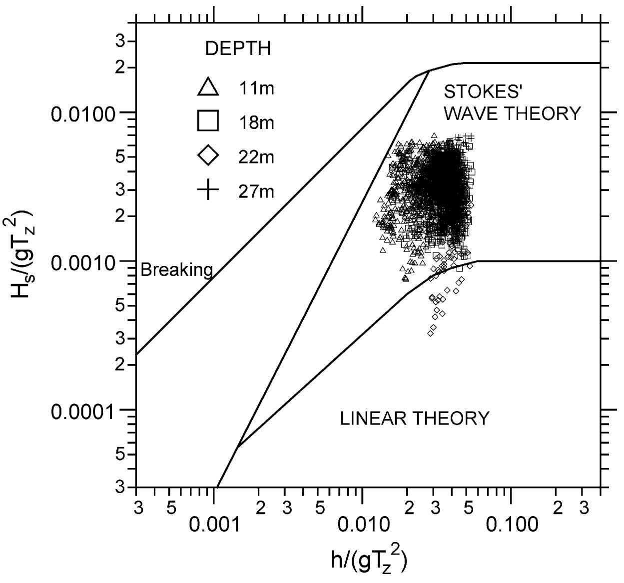

Fig. 10. Field measured waves in the regions of validity for wave theories according to Le Méhauté (1969).

the reconstructed surface waves clearly have a higher error percentage. Conversely, if the waves have longer period, the wave surface can be reconstructed rather nicely. As a matter of fact, if the RSEP is plotted against the mean zero-crossing period obtained from acoustic data, there is a strong correlation between the error percentage and wave period. Fig. 9 is an example to show this relationship. Measurements from all other depths have the same trend. Fundamentally underwater pressure sensor cannot pick up signals of short period waves well, due to the attenuation with water depth.

Finally, all the wave data obtained acoustically, excluding deep-water ones, were examined for its region of validity for various wave theories (Fig. 10). The range of $h/(gT_z^2)$ was between 0.013 and 0.055 and that for $H_s/(gT_z^2)$ was between 0.0003 and 0.007. In the figure, the H_s and T_z were determined from pressure data, rather than from acoustic measurements. According to Le Méhauté (1969), these waves were in the regions for Stokes wave theory and some in the linear theory region, and all waves were in the intermediate depth range.

5. Conclusions

Wave measurement using subsurface pressure transducer has been practiced since around 1947. Several problems and disadvantages associated with this indirect wave measurement method should be addressed and overcome in order to yield accurate and reliable results. These include finding an adequate pressure transfer function.

Kuo and Chiu (1994) proposed a pressure transfer function, which is solely a function of frequency parameter, $\omega^2|z|/g$. Dimensional analysis as well as comparison to linear pressure transfer function indicated that the transfer function should also be a function of transducer submergence parameter. Kuo and Chiu's empirical formula may be an acceptable approximation for the submergence parameter less than 0.6. In other words, the pressure transducer should be at least away from the sea floor at a height at least 40% of the water depth. Otherwise, the empirical formula would significantly overestimate wave height.

Theoretical transfer functions, whether they are linear or non-linear, with or without correction factor, have been previously demonstrated to be accurate to within an acceptable limit by comparing surface wave spectrum to that transformed from subsurface pressure gauge (Hom-ma et al., 1966; Bergan et al., 1968; Esteva and Harris, 1970; Lee and Wang, 1984). On the contrary, the empirical transfer function of Kuo and Chiu (1994), although simple and easy to use, was not properly demonstrated to be valid by such comparisons. This empirical formula is at best good for their experimental data of intermediate to deep-water wave conditions, small wave steepness effect and transducer not close to the seabed.

Field tests were carried out in coastal waters ranging from 11 to 27 m deep. An ultrasonic wave gauge equipped with a synchronized pressure transducer was deployed for wave measurements. It was found that using the Kuo and Chiu's transfer function, the significant wave heights were on average 30% higher than that acoustically measured, while linear transfer function produced better significant wave height estimates. Using the linear pressure transfer function, the deviation of estimated significant wave height from that obtained acoustically increased with the depth and decreased with increasing mean wave period. If deep-water wave measurements were excluded from the comparison, the mean and maximum deviations of significant height estimates for all depths were 3.6 and 13%, respectively. In almost all cases, the significant wave heights obtained by linear transfer function were lower than that by acoustic measurements. The mean deviation in the reconstructed wave elevation for all water depths were less than 11%, with deep-water waves excluded. Also the mean deviation of surface elevation as a percentage of the significant wave height was from 7.8 to 10.8%, which increased with increasing water depth. For a given depth, the surface elevation deviation percentage decreased with increasing mean zero-crossing wave period.

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