Observation and Analysis of Significant Height of Wind Waves Generated on the Surface of Currents*

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Abstract: When winds blow over the surface of sea where steady comparatively strong currents exist, energy fed from the winds to waves per one wave length is increased or decreased according as the waves travel against the currents or down the currents. Consequently, wave height of a single train of sine waves must be increased or decreased by a factor derived from the hydrodynamical equations, compared with the case of waves generating on a still water surface. The rates of increase or decrease of a significant height for waves with random phase generating on a surface of currents have been derived. The rates are found to be related to $U\cos\theta/v$ where $U\cos\theta$ is the component of the current velocity in the direction of waves and v the anemometer wind speed. Results of actual wave observations made by *Ryofu-maru* IGY*** cruise, June 1957 on the sea off the Sanriku district, Japan, show rough agreement with the above theory.

1. Introduction

When winds blow over the surface of sea where comparatively strong currents exist, waves generated by the winds may have somewhat different characters compared with the wind waves generated over a still water surface under the same wind conditions.

Visual wave observations made by the research ship *Ryofu-maru* off the Sanriku district, Japan, during 1957 as a part of work planned by IGY, are valuable to examine the above mentioned wave characteristics influenced by the Kuroshio currents. When winds blow in the opposite direction of currents, the height of generated waves is larger than the predicted wave height,**** and white caps are observed more frequently. However, waves generated by winds blowing downstream in

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the direction of currents, are flat and the sea surface is smooth.

Recently, the author (TOMINAGA, 1964a) has theoretically derived the amplifying factor or damping factor of wave height variation due to currents, assuming the waves are developed by the resonance mechanism proposed by PHILLIPS (1957) and by the sheltering mechanism proposed by MILES (1960). It was evident that the height of single sine waves generated by the resonance mechanism on the surface of currents is increased (or decreased) by the factor

$$f(G) = \frac{2}{1-4G+(1-2G)\sqrt{1-4G}}, \qquad (1)$$

where $G=2\omega U\cos\theta/gT$,***** U being the current velocity, θ the angle between direction of currents and waves, T the period of waves, and g the acceleration of gravity; in the case of the sheltering mechanism, the above factor is expressed by

$$\frac{2}{1-4G+(1-2G)\sqrt{1-4G}}$$
$$\exp\left[\frac{1-3G-(1-G)\sqrt{1-4G}}{2G^3\sqrt{1-4G}}\gamma\frac{4\pi^2}{gT^2}x\right], \qquad (2)$$

where $\gamma=2\pi\zeta c_1/gT$, c_1 being the wave velocity referred to the fixed space, ζ the rate of mean energy increase introduced by MILES (1960),

and x the fetch of winds. If the duration of winds t is expressed by the relation

$$x \fallingdotseq (c_g - U \cos \theta) t,$$

where c_g is the group velocity of waves referred to the current, then the exponential factor of (2) is replaced by $e^{A(G)\zeta \omega t}$ where

$$A(G) = \frac{1}{G \sqrt{1-4G}} \cdot \frac{1-3G-(1-G)\sqrt{1-4G}}{1-2G-\sqrt{1-4G}}$$
$$\cdot \frac{1-4G+\sqrt{1-4G}}{4}. \qquad (3)$$

When winds blow over the surface of currents, theoretical energy spectra of wind-generated waves can be obtained from the Neumann's wave spectrum and the amplifying factor (1) and (2). The results are summarized in the following three formulae (TOMINAGA (1964b), (5.2)):

$$S_1(\omega) = C\rho\pi^3 g^3 \frac{e^{-2\left(\frac{g}{v\omega}\right)^2}}{\omega^6} f^2(G), \quad \omega < \omega_s,$$

$$S_2(\omega) = C\rho\pi^3 g^3 \frac{e^{-2\left(\frac{g}{v\omega}\right)^2}}{\omega^6} f^2(G) e^{2\nu t}, \quad \omega_s < \omega < \omega_b,$$

$$S_3(\omega) = C\rho\pi^3 g^3 \left(\frac{1-2G+\sqrt{1-4G}}{2}\right)^2 \omega^{-6},$$
$$\omega_b < \omega < \omega_c, \qquad (4)$$

where ω is the angular frequency of waves, ρ the density of water, C a constant and

$$\nu = \left[A(G)\zeta - \frac{\zeta_0}{2} \right] \omega,$$

ζ_0 being the value of ζ when $U=0$.

The waves are developed by the resonance

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* The steepness of waves is assumed to be always less than 1/7 when ω is larger than ω_b, then ω_b is obtained by the formula

$$\frac{H'}{\lambda} = \frac{2H'}{1-2G_b + \sqrt{1-4G_b}} \cdot \frac{\omega_b^2}{2\pi g} = \frac{1}{7}$$

or

$$f(G_b) \frac{2}{1-2G_b+\sqrt{1-4G_b}} e^{\nu_b t} e^{-\left(\frac{g}{v\omega_b}\right)^2} = 1,$$

where subscript b refers to the value when the waves begin to break. And ω_c is obtained by $G_c = \omega_c U/g = 1/4$.

mechanism when the frequency ω is less than certain value ω_s and by the sheltering mechanism when ω is larger than ω_s, where ω_s can be obtained by

$$\frac{g}{\omega_s} \cdot \frac{1-2G_s + \sqrt{1-4G_s}}{2} = v, \qquad (5)$$

v being the wind speed at anemometer level (about 10 m height), and $G_s = \omega_s U/g$. If $\cos \theta > 0$, the waves of frequency larger than ω_b^* break partly and those of frequency just equal to ω_s break completely, but if $\cos \theta < 0$, waves do not break no matter how ω may be small.

The root mean square of wave height or $\sqrt{\bar{\eta}^2}$ can be defined by

$$\sqrt{\bar{\eta}^2} = \left[\int_0^{\omega_c} S(\omega) d\omega \right]^{\frac{1}{2}}$$
$$= \left[\int_0^{\omega_s} S_1(\omega) d\omega + \int_{\omega_s}^{\omega_b} S_2(\omega) d\omega \right.$$
$$\left. + \int_{\omega_b}^{\omega_c} S_3(\omega) d\omega \right]^{\frac{1}{2}}, \qquad (6)$$

therefore, the root mean square of wave height or significant wave height $H_{\frac{1}{3}}$ is increased (or decreased) by the factor α compared with the height of waves generated over the surface of a still water; α is given by

$$\alpha = \left[\frac{\int_0^{\omega_c} S(\omega) d\omega}{\int_0^{\infty} S_0(\omega) d\omega} \right]^{\frac{1}{2}}, \qquad (7)$$

where $S_0(\omega)$ is the Neumann's spectrum given by

$$S_0(\omega) = C\rho\pi^3 g^3 \omega^{-6} e^{-\left(\frac{g}{v\omega}\right)^2}. \qquad (8)$$

The value of α against $U\cos\theta/v$ for the

Table 1. Values of α against U/v for various wind speed and durations.

U/v	-0.1	0.05	0.1	0.15
$v=10$ m/s, $t=10$ hrs	0.70	1.40	1.51	1.67
$v=10$ m/s, $t=20$ hrs	0.67	1.44	1.59	1.71
$v=15$ m/s, $t=25$ hrs	0.68	1.34	1.55	1.72

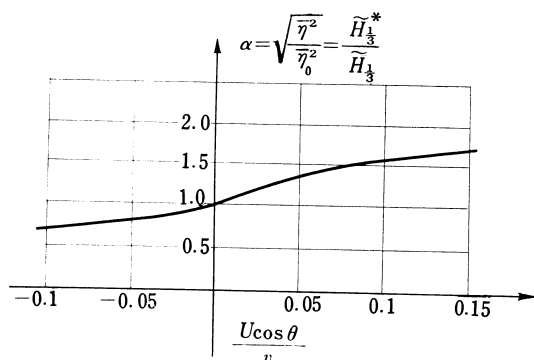given wind speed and durations are shown in Table 1 and Fig. 1.

Fig. 1. Amplifying factor α represented by a solid curve against $\dfrac{U\cos\theta}{v}$ for wind speed v =10 m/s and duration $t=10$ hrs.

2. Analysis of actual wave height observations

To show the validity of the present theoretical results, the informations of wind-generated waves obtained from the visual observations made by members of the research ship *Ryofumaru* during the period of IGY (1957) were used. The observational area is located about 1,000 km distant off the Sanriku district, east of Japan, where sometimes comparatively strong and steady currents have been observed. When winds blow for sufficiently long time upstream or downstream in the direction of currents, then the heights of wind-generated waves are larger (or smaller) than the heights predicted by the PNJ method.

Examples of such situations are given below and analysed by the theory described in Section 1.

Case 1. A depression was approaching from the south-west toward the area where wave observations were made (small hatched area illustrated in the Fig. 2). The anemometer wind speed observed in this area amounted to 13 knots at 18h, 20 knots at 21h on 17th June and 25 knots at 0h on 18th, wind direction

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* The observed wave height is considered to be the average height of one third heighest waves or significant wave height.

being always about 130° from the north. At 0h on 18th, (point P in the annexed figure) wind waves with period of 2 sec and height of 3 m* were coming from 160° direction, and currents of 1.6 knots speed were flowing toward 143° direction. Average wind speed v during these 6 hours was about 20 knots, then
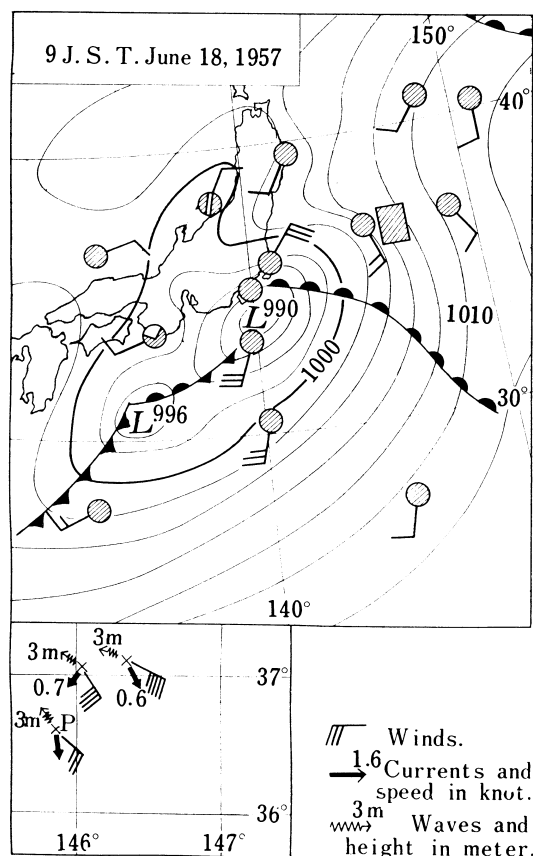

Fig. 2. Surface weather chart. Winds, waves and currents in the area considered (hatched rectangle) are illustrated at the lower left corner.

$U\cos\theta=1.6\cos(160°-143°)$ knots$=1.5$ knots,
$\dfrac{U\cos\theta}{v}=\dfrac{1.5}{20}=0.075$.

The significant wave height predicted by the PNJ method is about 2.1 m for the wind speed of 20 knots. However the amplifying factor α of about 1.4 against $U\cos\theta/v=0.075$ being used (Fig. 1), the significant wave height must be

$$2.1\,\mathrm{m}\times1.4=2.9\mathrm{m}\ ,$$

agreeing with actual observation.

Case 2. In this case, an anticyclone (1018 mb) was present and remained stationary for a few days to the north of the area where
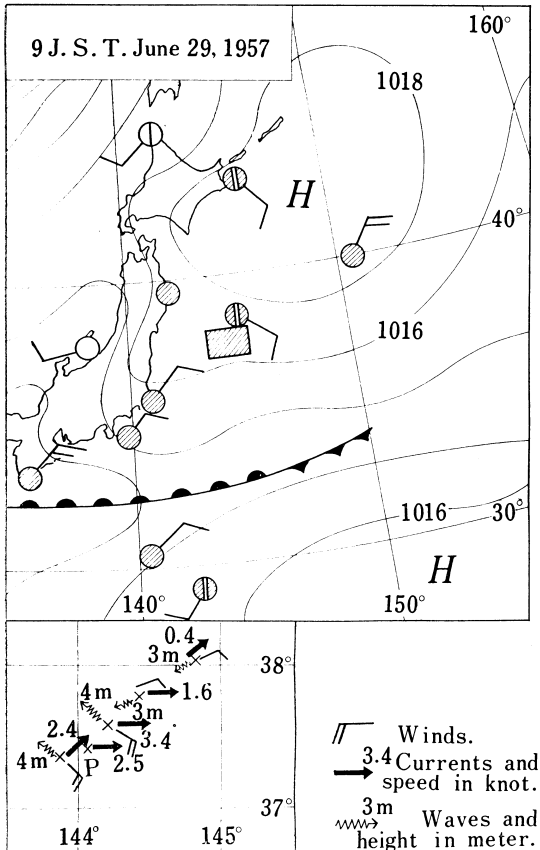

Fig. 3. Surface weather chart. Winds, waves and currents in the area considered (hatched rectangle) are illustrated at the lower left corner.

observations were made. On 29th June, winds were blowing for nearly ten hours from 110° to 130° with 18 knots speed on the average. The observed significant wave height was about 3 m to 4 m, and wind waves of 4 sec period were travelling from 110° direction on the surface of sea where currents with about 3 knots speed and direction of 90° from the north were observed. Therefore,

$$U\cos\theta=3.0\times\cos\,(110°-90°)\ \mathrm{knots}=2.8\ \mathrm{knots},$$

and

$$\frac{U\cos\theta}{v}=\frac{2.8}{18}=0.16\ .$$

The significant wave height predicted by the PNJ method is about 2 m. If this is increased by the factor $\alpha=1.7$ (Fig. 1 for $U\cos\theta/v=0.16$) by the opposing current, the height becomes $2\,\mathrm{m}\times1.7=3.4\mathrm{m}$, roughly in agreement with the observation.

Case 3. In the area near the point P illustrated in Fig. 4, comparatively strong currents (average speed is 3.5 knots) were flowing to the south-east. At 21h on 19th June, the period and significant height of wind waves were 3 sec and 3 m respectively, and those were travelling downstream almost in the same
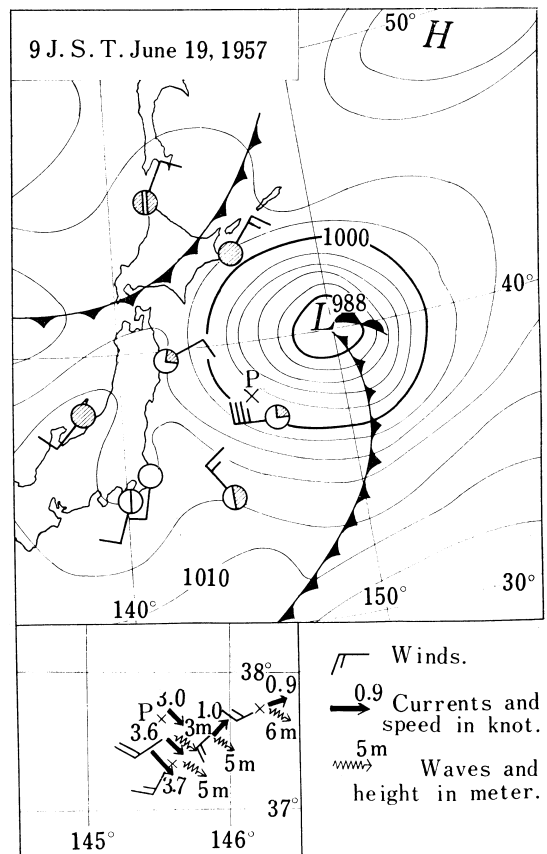

Fig. 4. Surface weather chart. Winds, waves and currents in the area considered (near the point P) are illustrated at the lower left corner.

direction of the currents.

The angle θ between the direction of waves and currents was around 20°, therefore we have

$$U \cos \theta = 3.5 \times \cos 20° \text{ (knots)} = 3.3 \text{ knots} .$$

The strong cyclone passed over the northern part of this area toward the north-east since the evening of 18th June, and the waves generated by west winds with speed of about 30 knots in the morning on 19th, had been remaining until 21h on 19th in the same area, despite the wind direction had changed to SSW (blowing almost normal to the direction of waves) since 15h of the same day. Significant wave height predicted by the PNJ method is about 6 m for wind speed of 30 knots, and actually the wave height of 6 m was observed before the noon of 19th in the eastern part of the area considered, where weak currents were flowing in the normal direction of waves. Near the point P where the waves were travelling downstream in the direction of strong currents, the wave height decreased by the factor $\alpha = 0.7$ which we can get from Fig. 1, against

$$-\frac{U \cos \theta}{v} = -\frac{3.3}{30} = -0.11 .$$

Therefore, the predicted wave height must be

$$6 \text{ m} \times 0.7 = 4.2 \text{ m} ,$$

which is slightly larger than the observed value but is in rough agreement with it, if we admit the fact that the waves may diminish their height by the decrease in the wind speed and by the change in the wind direction.

Case 4. A moderate depression with pressure of 1003 mb at its centre passed over the southern part of the sea area near the point P illustrated in Fig. 5 on 22nd June. The wind speed in this area was between 10 knots and 15 knots, the wind direction was always 50° for more than 15 hours. A strong current with speed of 2.6 knots to 3.3 knots in the

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* This is the average wind speed from 18th on 22nd to 0h on 23rd June, in the area considered.
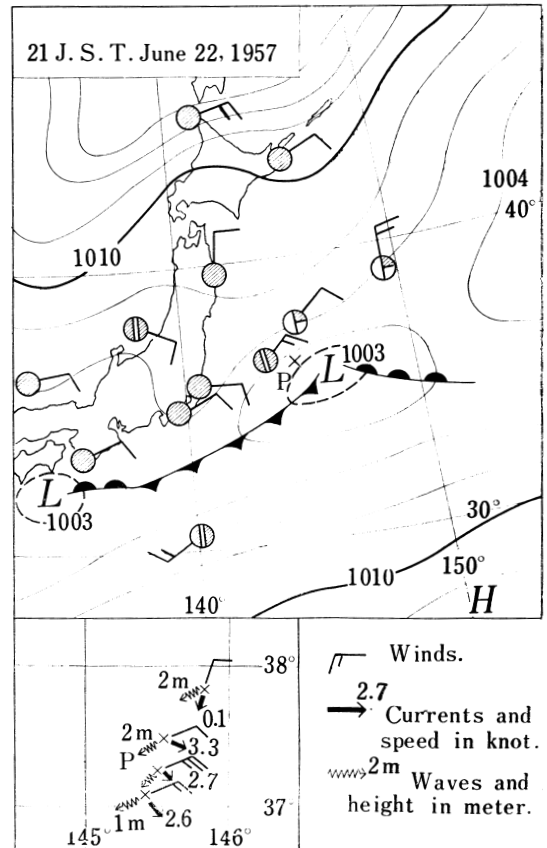

Fig. 5. Surface weather chart. Winds, waves and currents in the area considered (near the point P) are illustrated at the lower left corner.

direction of about 90° was also observed near the point P, therefore, the surface water was streaming almost perpendicular to the direction of waves. The effect of currents here on wave motion may be very slight in spite of the large speed of currents.

The predicted significant wave height by means of the PNJ method becomes 1.3 m for wind speeds of 16 knots;* on the other hand, the observed wave height was 1 m at 21 h on 22nd (wind speed was 17 knots) and 2 m at 0h on 23rd (wind speed was 21 knots). Therefore, the rough accordance of both values, observed and predicted, shows the effect of currents on wave heights is very slight.

Case 5. As illustrated in Fig. 6, a travelling anticyclone had been covering the area con-

sidered, since the previous day (9th June), and on 10th June winds were not so strong in the present area. It is estimated that the waves with period of 3 sec and height of 2~3 m may originate in the western part of the depression of 1006 mb which is located about 600 km to the north from the area considered.

Meteorological and sea conditions at the points A, B and C in Fig. 6 are tabulated in Table 2.

Table 2. Meteorological and sea conditions observed at A, B, and C illustrated in Fig. 6.

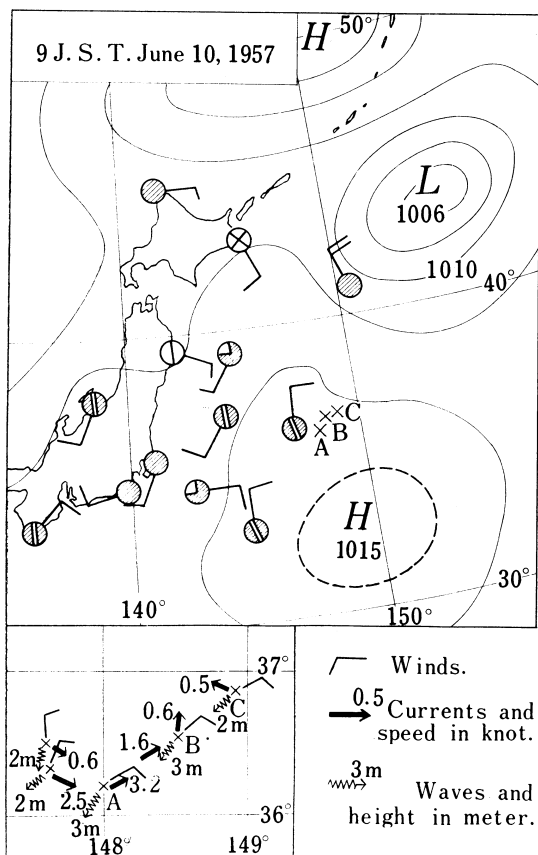| Position		A	B	C
Time		9h	12h	15h
Winds	{direction	260°	230°	250°
	{speed	5 kts	4 kts	11 kts
Waves	{direction	50°	50°	50°
	{period	2 sec	2 sec	3 sec
	{height	3 m	3 m	2 m
Current	{direction	73°	4°	297°
	{speed	3.2 kts	0.6 kt	0.5 kt

Fig. 6. Surface weather chart. Winds, waves and currents in the area considered are illustrated at the lower left corner.

Waves with a period of 3 sec would likely have originated from the generating area with velocity of 17 km/hour and would travel the distance of 600 km for about 35 hours and would have reached the position near C at 9h on 10th. The significant height of waves es-

timated by the PNJ method must be around 2.5 m (or less than 2.5 m) near C if we assume the wind speed in the generating area be about 20 kts, and considering the decay effect due to lateral angular spreading of waves. Such waves were increased in their height after entering into the region (B to A) where currents flowing against waves were gradually increased in speed in the direction of waves. LONGUET-HIGGINS (1962) has given a theory of wave amplification when waves travelled against currents whose speed increased upstream, and has given the following relation,

$$\frac{a}{a_0} = \left[\frac{c(c_0 - 2U_0)}{c(c - 2U)} \right]^{\frac{1}{2}} \tag{9}$$

and

$$\frac{c}{c_0} = \frac{1}{2(1-\gamma)} \left\{ 1 + \sqrt{1 - \frac{4(1-\gamma)U}{c_0}} \right\}, \tag{10}$$

where $\gamma = \dfrac{U_0}{c_0}$, a is amplitude of waves, c and U are phase velocity of waves referred to the water and current velocity respectively, and the subscript 0 refers to the origin, and U must be replaced by $-U$ when waves are travelling downstream.

In the case now considered, the following values are adopted:

At C: $U_0 = -0.1$ m/s (Component of current velocity in the direction of waves)

$$c_0 = \frac{gT}{2\pi} = \frac{9.8 \times 3}{2\pi} = 4.7 \text{ m/s},$$

$$\gamma = \frac{U_0}{c_0} = 0.021 \ .$$

Near A: $\quad U = 1.0 \, \text{m/s}$,

$$c = \frac{4.7}{2 \times 1.021}$$
$$\cdot \left\{ 1 + \sqrt{1 - \frac{4 \times 1.021 \times 1.0}{4.7}} \right\}$$
$$= 3.14 \, \text{m/s} \ .$$

Therefore, from the equation (9) we have

$$\frac{a}{a_0} = 1.24 \ ,$$

then the significant wave height must be

$$2.5 \times 1.24 = 3.1 \, \text{m}$$

near the point A, roughly in agreement with the observed height.

3. Conclusion

Precise visual observations of height of waves generated by winds on the sea with currents show changes in wave height, compared with the waves generated by the same weather conditions on the surface of a still water. This fact can be explained by the considerations that the energy of waves fed by winds per one wave length accumulates or not, according as the waves travel against the currents or not.

Predicted wave height for certain weather conditions must be modified in the above-mentioned case.

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