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Local Balance in the Air-Sea Boundary Processes

III. On the Spectrum of Wind Waves*

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Abstract: A combination of the three-second power law, presented in part I for wind waves of simple spectrum, and the similarity of the spectral form of wind waves, leads to a new concept on the energy spectrum of wind waves. It is well substantiated by data from a wind-wave tunnel experiment.

In the gravity wave range, the gross form of the high frequency side of the spectrum is proportional to $g u_* \sigma^{-4}$, where g represents the acceleration of gravity, u_* the friction velocity, σ the angular frequency, and the factor of proportionality is 2.0×10^{-2}. The wind waves grow in such a way that the spectrum slides up, keeping its similar form, along the line of the gross form, on the logarithmic diagram of the spectral density, ϕ, versus σ. Also, the terminal value of ϕ, at the peak frequency of the fully developed sea, is along a line of the gradient of $g^2 \sigma^{-5}$.

The fine structure of the spectrum from the wind-wave tunnel experiment shows a characteristic form oscillating around the σ^{-4}-line. The excess of the energy density concentrates around the peak frequency and the second- and the third-order harmonics, and the deficit occurs in the middle of these frequencies. This form of the fine structure is always similar in the gravity wave range, in purely controlled conditions such as in a wind-wave tunnel. Moving averages of these spectra tend very close to the form proportional to σ^{-5}.

As the wave number becomes large, the effect of surface tension is incorporated, and the σ^{-4}-line in the gravity wave range gradually continues to a $\sigma^{-8/3}$-line in the capillary wave range, in accordance with the wind-wave tunnel data. Likewise, the σ^{-5}-line gradually continues to a $\sigma^{-7/3}$-line.

Also, through a discussion on these results, is suggested the existence of a kind of general similarity in the structure of wind wave field.

1. Introduction

Although the wind waves are a kind of water waves, they are special phenomena, in a sense that their interactions with the surface skin flows, caused by the surface friction by the wind, are very strong. The wind waves thus have a character of turbulence, or strong nonlinearity, and it will be essentially impossible to treat the growth of individual waves in a deterministic manner.

In parts I and II of the present series of the articles, growth equations for wind waves were presented based on a new conception. Namely, the above-mentioned turbulence in the wind wave field was eliminated by conceptional averaging operation, and the significant wave was used as the abstract representation of the wind wave field. This was based on the fact that the energy of the wind waves concentrates at the spectral peak frequency, which corresponds approximately to the significant wave. The increase of the wave energy was treated as the rate of work done to the wave by the momentum transferred from the wind to the water, and the growth equations were formulated by the use of the restrictions that water waves have, namely, the relations between the dimension of the wave and the wave current, and between the wave momentum and the wave energy.

In the course of the derivation of the growth equations, the three-second power law for wind

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waves of simple spectrum was proposed, concerning the dimensionless period and the height of the significant wave, and was substantiated by some data.

In this article, the three-second power law is extended to discuss the spectral form of pure wind waves, and a new concept of the spectral form is presented including the capillary-gravity wave range. The concept is examined by the use of data from wave records in a wind-wave tunnel, and also some discussion is given concerning the comparison of the new concept of the spectral form, with the existing concepts.

2. Form of energy spectra of wind waves

The same notations are followed from parts I and II, for dimensional and dimensionless variables. Since the spectral form of wind waves is considered in the present article, the angular frequency, σ, or the wave number, κ, and the energy spectrum density, ϕ, are incorporated to our system of the variables. The spectral peak frequency, σ_p, may now be substituted approximately for the significant wave period, T, with the relation,

$$\sigma_p=\frac{2\pi}{T} \tag{2.1}$$

The significant wave height, H, is expressed by ϕ by the use of

$$\int_0^\infty \phi d\sigma=\frac{\overline{a^2}}{2}=\frac{H^2}{2(2.83)^2}=\frac{H^2}{16} \tag{2.2}$$

In part I, the three-second power law for wind waves of simple spectrum was derived in the form

$$H^*=BT^{*3/2} \tag{2.3}$$

where $H^*\equiv gH/u_*^2$ and $T^*\equiv gT/u_*$, and B was the 1st universal constant having the value of

$$B=6.2\times10^{-2} \tag{2.4}$$

Equation (2.3) may be written in a dimensional form as

$$H^2=B^2gu_*T^3 \tag{2.5}$$

Now the fourth concept is introduced:

The 4th concept: As to the form of the energy spectrum of pure wind waves, there is a similarity, namely, if the spectral density, ϕ, is normalized by its peak value ϕ_p, and the angular frequency, σ, by the spectral peak frequency σ_p, the spectrum has the same form in the gravity wave range.

It will be shown later in section 3 that this concept is in good accordance with the experimental fact in the wind-wave tunnel.

From this concept the following derivation may be possible. The ϕ is normalized by ϕ_p, namely,

$$\frac{\phi}{\phi_p}=\phi' \tag{2.6}$$

The σ is normalized by σ_p, namely,

$$\frac{\sigma}{\sigma_p}=\sigma' \tag{2.7}$$

Then the 4th concept demands that

$$\int_0^\infty \phi' d\sigma'=\frac{1}{\phi_p\sigma_p}\int_0^\infty \phi d\sigma=\text{constant}=A \tag{2.8}$$

From Equations (2.8) and (2.2), it follows that

$$H^2=16A\phi_p\sigma_p \tag{2.9}$$

From Equations (2.5) and (2.9), together with Equation (2.1), the form of ϕ_p is given by

$$\phi_p=\alpha_p g u_* \sigma_p^{-4},\ \alpha_p=\frac{\pi^3B^2}{2A} \tag{2.10}$$

Since the theoretical form of the spectrum is not known at this stage, A is an indeterminate coefficient. From actual data, which will be presented in the next section, the coefficient α_p in Equation (2.10) may be assigned empirically as

$$\alpha_p = 1.1\,\pi^3 B^2 = 0.13 \qquad (2.11)$$

Namely, the empirical value of A is to be 0.45. Now the conclusion is that wind waves, in the gravity wave range, grow in such a way that the energy spectrum slides up along the straight line of Equation (2.10) on a $\log\phi$-$\log\sigma$ diagram, keeping its similar form, with the peak point on the straight line. This situation may be compared to an electric train climbing up a hill with the top of the pantograph on an aerial line.

As will be shown in the next section, the fact is that the high frequency side of the spectrum, within the gravity wave range, tends to a straight line parallel to the line of Equation (2.10). If we regard the new straight line as a gross form of the spectrum, it may be expressed empirically by

$$\phi_g = \alpha_g g u_* \sigma^{-4},\ \alpha_g = \frac{1}{6}\pi^3 B^2 = 2.0\times 10^{-2} \qquad (2.12)$$

the subscript g representing the gross form.

From Equations (3.26) and (3.27) in part I, namely from

$$K = T_1^{*-2} = 2.16\times 10^{-5} \qquad (2.13)$$

the spectral peak frequency of the fully developed sea, σ_1, is given as a function of u_* by

$$\sigma_1 = \frac{2\pi}{T_1} = \frac{2\pi g K^{1/2}}{u_*} = \frac{0.029g}{u_*} \qquad (2.14)$$

where K is the second universal constant introduced in part I. The subscript 1 stands for the fully developed sea. Elimination of u_* from Equations (2.10) and (2.14) is possible since σ_1 is a special case of σ_p. Then, also by the use of Equation (2.11), the form of $\phi(\sigma_1)$ or ϕ_1, namely, the contour of the ϕ_p for the fully developed sea, is given by

$$\phi_1(\sigma_1) = \alpha_1 g^2 \sigma_1^{-5} \qquad (2.15)$$

where α_1 is another universal constant having the form of

$$\alpha_1 = 2.2\,\pi^4 B^2 K^{1/2} \qquad (2.16)$$

Equations (2.10), (2.12) and (2.15) may be expressed by the wave number, κ, where

$$\kappa = \frac{\sigma^2}{g} \qquad (2.17)$$

The spectral density $\phi_\kappa(\kappa)$ on the κ-space should be expressed by $\phi(\sigma)$ multiplied by $d\sigma/d\kappa$, by the relation

$$\phi_\kappa(\kappa)d\kappa = \phi(\sigma)\frac{d\sigma}{d\kappa}d\kappa \qquad (2.18)$$

Namely, it follows that

$$\phi_{\kappa p} = \frac{1}{2}\alpha_p g^{-1/2} u_* \kappa_p^{-5/2} \qquad (2.19)$$

$$\phi_{\kappa g} = \frac{1}{2}\alpha_g g^{-1/2} u_* \kappa^{-5/2} \qquad (2.20)$$

and

$$\phi_{\kappa 1}(\kappa_1) = \frac{1}{2}\alpha_1 \kappa_1^{-3} \qquad (2.21)$$

respectively.

For the range where the surface tension plays a significant role, the acceleration of gravity, g, in all equations hitherto used may be replaced by

$$g_* \equiv g + \frac{S\kappa^2}{\rho_w} \qquad (2.22)$$

from the infinitesimal wave theory, where S represents the surface tension, and ρ_w the density of water. Applying this extension, Equations (2.10), (2.12) and (2.15), or, (2.19), (2.20) and (2.21) become

$$\phi_p = \alpha_p g_* u_* \sigma_p^{-4} \qquad (2.23)$$

$$\phi_g = \alpha_g g_* u_* \sigma^{-4} \qquad (2.24)$$

and

$$\phi_1(\sigma_1) = \alpha_1 g_*^2 \sigma_1^{-5} \qquad (2.25)$$

or

$$\phi_{\kappa p}=\frac{1}{2}\alpha_p g_*^{-1/2}u_*\kappa_p^{-5/2} \qquad (2.26)$$

$$\phi_{\kappa g}=\frac{1}{2}\alpha_g g_*^{-1/2}u_*\kappa^{-5/2} \qquad (2.27)$$

and

$$\phi_{\kappa 1}(\kappa_1)=\frac{1}{2}\alpha_1\kappa_1^{-3} \qquad (2.28)$$

respectively. Since g_* contains κ's, in order to express Equations (2.23), (2.24) and (2.25) purely as functions of σ's, the solution of the cubic equation

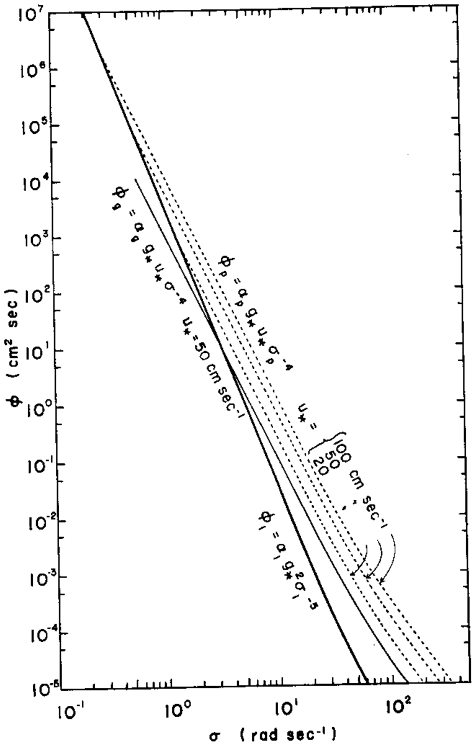

Fig. 1. Peak values of the energy spectrum of wind waves grow along the dotted lines, expressed by Equation (2.23). The thick line is the terminal of Equation (2.23), and is expressed by Equation (2.25) for the fully developed sea. The gross form of the spectrum, expressed by Equation (2.24), which is proportional to u_*, is shown by the thin line.

$$\frac{S}{\rho_w}\kappa^3+g\kappa-\sigma^2=0 \qquad (2.29)$$

is further used. The solution is given by

$$\kappa=\frac{\sigma_m^2}{2g}f(\sigma^*) \qquad (2.30)$$

where

$$f(\sigma^*)=\left\{\sigma^{*2}+\left(\sigma^{*4}+\frac{1}{27}\right)^{1/2}\right\}^{1/3}+\left\{\sigma^{*2}-\left(\sigma^{*4}+\frac{1}{27}\right)^{1/2}\right\}^{1/3} \qquad (2.31)$$

and where

$$\sigma^*=\frac{\sigma}{\sigma_m},\quad \sigma_m=\sqrt{2g}\left(\frac{g\rho_w}{S}\right)^{1/4} \qquad (2.32)$$

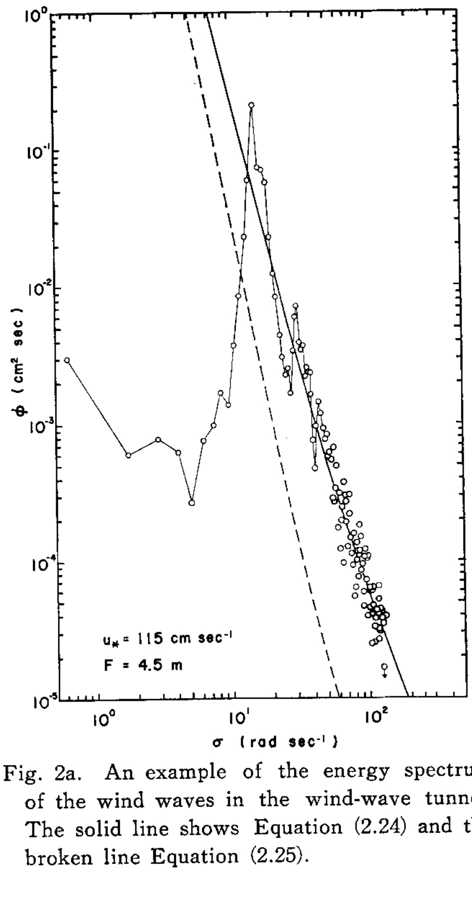

Fig. 2a. An example of the energy spectrum of the wind waves in the wind-wave tunnel. The solid line shows Equation (2.24) and the broken line Equation (2.25).

The σ_m is the value of σ where the second term of the right hand side of Equation (2.22) is equal to the first term.

NEUMAN and PIERSON (1966) described an alternative form of Equations (2.30) and (2.31). It seems to have been obtained through a cubic equation of L/L_m. The difference is that in Equation (2.30) the f-function, which is much simpler, appears in the numerator, and σ_m^2 instead of σ^2 appears outside the f-function.

For the capillary wave range where

$$g \ll \frac{S\kappa^2}{\rho_w} \qquad (2.33)$$

Equations (2.26), (2.27) and (2.28) are reduced to

$$\phi_{\kappa p} = \frac{1}{2}\alpha_p\left(\frac{\rho_w}{S}\right)^{1/2} u_* \kappa_p^{-7/2} \qquad (2.34)$$

$$\phi_{\kappa g} = \frac{1}{2}\alpha_g\left(\frac{\rho_w}{S}\right)^{1/2} u_* \kappa^{-7/2} \qquad (2.35)$$

and

$$\phi_{\kappa 1}(\kappa_1) = \frac{1}{2}\alpha_1 \kappa_1^{-3} \qquad (2.36)$$

These equations are expressed in terms of σ's by

$$\phi_p = \alpha_p\left(\frac{S}{\rho_w}\right)^{1/3} u_* \sigma_p^{-8/3} \qquad (2.37)$$

$$\phi_g = \alpha_g\left(\frac{S}{\rho_w}\right)^{1/3} u_* \sigma^{-8/3} \qquad (2.38)$$

and

$$\phi_1(\sigma_1) = \alpha_1\left(\frac{S}{\rho_w}\right)^{2/3} \sigma_1^{-7/3} \qquad (2.39)$$

In Figure 1 are shown Equation (2.23) for $u_* = 100$, 50 and 20 cm sec^{-1} by dotted lines, Equation (2.24) for $u_* = 50$ cm sec^{-1} by a thin line, and Equation (2.25) by a thick line, on a

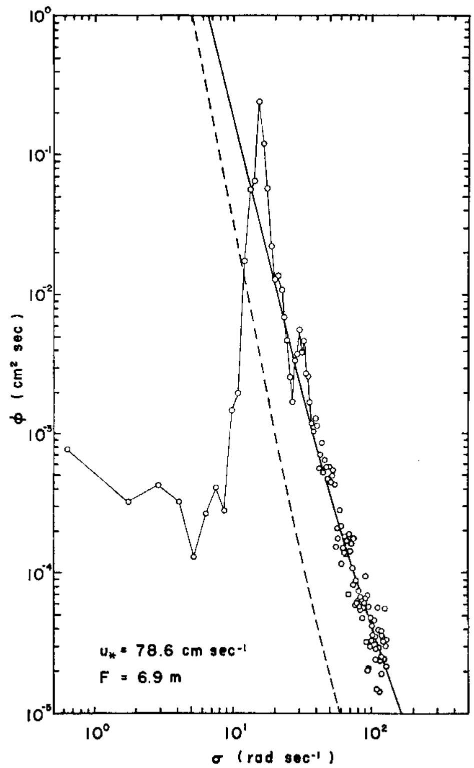

Fig. 2b. The same with 2a.

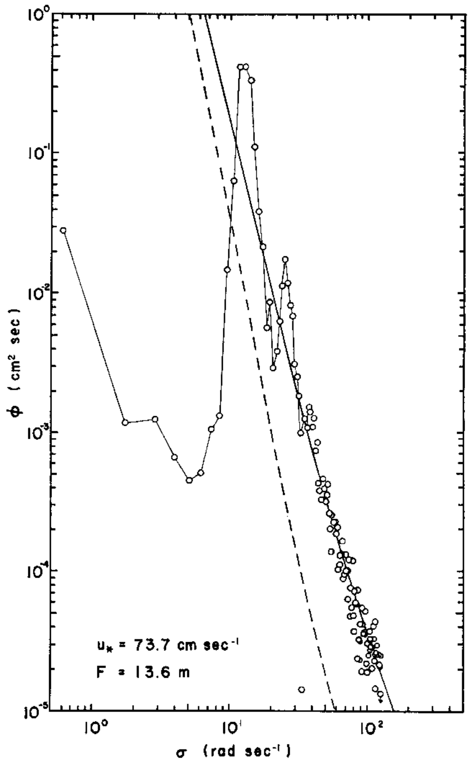

Fig. 2c. The same with 2a.

log ϕ–log σ diagram. Lines for various values of u_* may be drawn by vertical displacement of the dotted and the thin lines. The only point is that the dotted lines must stop at the thick line. It is noted that the dotted and the thin lines have a gradient of σ^{-4}, or in κ-space, of $\kappa^{-5/2}$, and the thick line σ^{-5} or κ^{-3}, in the gravity wave range, and the gradients gradually change to $\sigma^{-8/3}$ or $\kappa^{-7/2}$ and to $\sigma^{-7/3}$ or κ^{-3}, respectively, in the capillary wave range.

3. Comparison with wind-wave tunnel data and discussion

In this section, the derivation of the energy spectrum of wind waves in the preceding section is compared with some data from a wind-wave tunnel experiment performed by the author (TOBA, 1961). The main data concerning the characteristic waves was shown also in Table 1 of part I. The data of the spectrum computations are twenty-one wave records digitized at 0.025 second intervals for an average of 40 seconds (average record 1,600 points).

1. *Gross form*

In Figures 2a, 2b and 2c are shown three examples of the spectra. Equation (2.24) is entered in the figures as thin lines. Although the spectrum has an oscillating character, as will be discussed later, the gross form is in excellent agreement with the line including the capillary wave range. For comparison, the line of Equation (2.25) for σ_1, which has a gradient of σ^{-5} in the gravity wave range, is also entered as broken lines.

Although not shown explicitly by figures, the level of the gross form of the spectrum actually shifts with the change in u_*, just as predicted by Equation (2.23) or (2.24).

In Figure 3 are shown two examples of the deviation of the actual spectra from the gross form of the spectrum expressed by Equation (2.24). The ordinate indicates the ratio of the actual spectra to the Equation (2.24), and the abscissa is σ normalized by σ_p. It is seen that, althougn oscillating, the points lie around the value of unity.

PHILLIPS (1958) presented the form of

$$\phi(\sigma)=\alpha_5 g^2\sigma^{-5} \qquad (3.1)$$

as the spectrum of wind-generated waves. It was the result of a dimensional analysis in which the effect of u_* was excluded. The form including u_* is more general. However, a dimensional analysis cannot give a unique form, if u_* is included. The form given by Equation (2.23) or (2.24) has been obtained by invoking the three-second power law for wind waves for simple spectrum presented in part I, combined with the assumption of similarity adopted as the 4th concept in the present article.

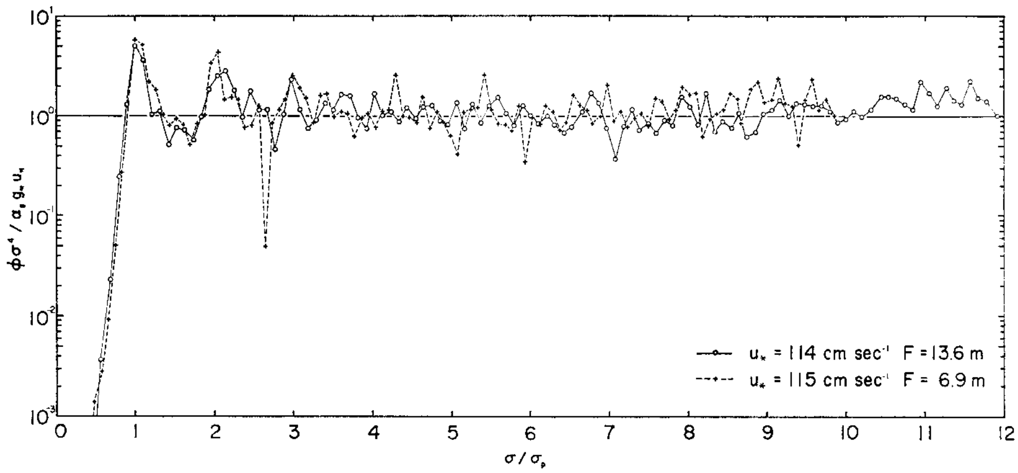

Fig. 3. Two examples showing the deviation of the actual spectra from the gross form expressed by Equation (2.24).

MITSUYASU (1968) and LONGUET-HIGGINS (1969) discussed that the factor α_5 in Equation (3.1) was to be regarded as a function of F and u_*, or of $F^* \equiv gF/u_*^2$. However, Phillips' spectral form was obtained by an assumption that the effect of u_* and F did not enter the spectral form, and their results presumably came out from a forced application of the Phillips' form. Consequently, it is considered that it means the contradiction contained in Phillips' form. In fact, by a close inspection of Figure 4.8 of PHILLIPS (1966), one may recognize that the σ^{-5}-line is a summation of individual data which have gradients near to σ^{-4}.

The contour of the peak frequencies for fully developed sea means the elimination of u_*. In this case, the form naturally becomes Equation (2.25) which does not contain u_*, nor of course F.

PIERSON and STACY (1972) proposed a concept of the spectral form consisting of five different frequency ranges. The ranges are (1) the gravity wave-gravity equilibrium range, (2) the isotropic turbulence range, or the Kitaĭgorodskiĭ range, (3) the connecting range due to Leykin and Rosenberg, (4) the capillary range, and (5) the viscous cutoff range, respectively. The first range is the Phillips' equilibrium gravity-wave range expressed by Equation (3.1) with $\alpha_5 = 8.1 \times 10^{-3}$. Since the spectrum in the capillary range has values one order or so higher than the extrapolation of the first range, they adopted the two ranges, the second and the third, as the connection between the first and the capillary ranges. The Kitaĭgorodskiĭ range was first proposed by KITAĬGORODSKIĬ (1961), as a hypothetical range where the frequency is somewhat above the equilibrium range, the small-scale turbulent motions are such that they are affected only negligibly by the gravitational force, and the surface tension and the molecular viscosity do not affect the phenomena. There is no evidence, however, that such a hypothetical range really exists. It is only incidental that the Kitaĭgorodskiĭ range has the form similar to Equation (2.10). It is natural to consider that the spectral form through the whole range should be derived from a unified physical principle. The description in the preceding section was along this line.

As to the capillary range, PIERSON and STACY (1972) reported that there was no equilibrium range, and that the spectrum is here a function of wind speed, or, of u_*, and is strongly variable according to local wind or gust. It seems very natural since already in the gravity wave range, the spectrum is a function of u_* as expressed by our equations. According to PIERSON and STACY, the u_*-dependence in the capillary range is a little different from linearity. It is considered that this is a second order problem, which the breaking of waves must be responsible for.

2. *Fine structure*

Now we turn to the fine structure of the spectrum. It is seen from Figures 2 and 3 that an excess of the energy concentrates at the peak frequency, and the spectra oscillate with σ, although the gross form of the spectra is expressed by Equation (2.24). As seen from Figure 3, the first trough is located at near $\sigma/\sigma_p = 1.5$, the second peak at near 2.0, thus the oscillation of the spectra seems to show the presence of the harmonics which damps with the order. For twenty-one wind wave spectra from the wind wave tunnel, average values of the ratios of the frequencies of the peaks and the troughs of the harmonics to that of the first peak σ_p have been estimated and listed in Table 1. Strictly, the peaks and the troughs here mean the points of maximum deviation from the Equation (2.24). They are very close to 1.5, 2.0, 2.5 and 3.0, respectively. This evidence of the existence of the harmonics seems to correspond to what MITSUYASU (1969) reported.

As to the excess energy that concentrates at

Table 1. Average values of the normalized peak- and trough-frequencies of the harmonics in the twenty-one wind wave spectra from the wind wave tunnel. The subscripts u_1, p_2, u_2 and p_3 represent the first trough, the second peak, the second trough, and the third peak, respectively.

$\frac{\sigma_{u1}}{\sigma_p}$	$\frac{\sigma_{p2}}{\sigma_p}$	$\frac{\sigma_{u2}}{\sigma_p}$	$\frac{\sigma_{p3}}{\sigma_p}$
1.58	2.03	2.42	3.04

the first peak frequency, there is no systematic variation in the ratio between the peak value of ϕ and the value given by Equation (2.24) at the peak frequency. The ratio is, in an average, 6.7. The same tendency is seen in the peaks and troughs of the harmonics, although the ratio becomes small.

These characteristics of the fine structure of the wind wave spectra lead to the following conclusion. The structure of the excess and the deficit of the energy on the spectra is always similar for the wind waves of pure conditions, irrespective of u_* and F. This is the basis of the 4th concept. The energy spectrum of wind waves grows as if it slides up along the σ^{-4}-line, with the above described fine structure holding the same figure, in agreement with the inference. In Figure 4 are shown three examples of the spectra in the gravity wave range. The ordinate represents $\phi/\alpha_g g u_* \sigma_p^{-4}$, and the abscissa σ/σ_p. In Figure 4, the similarity in the fine structure of the spectra is clearly recognized. The straight lines along the points have gradients of σ^{10} for the low frequency sides, and σ^{-10} for the high frequency sides of the oscillations, and the gross form is just on the σ^{-4}-line through the point (10^0-10^0).

If one observes the value of ϕ at a fixed σ in the growing stage, the value should be oscillating. This is nothing but the phenomenon of overshoot and undershoot reported by BARNETT and WILKERSON (1967), SUTHERLAND

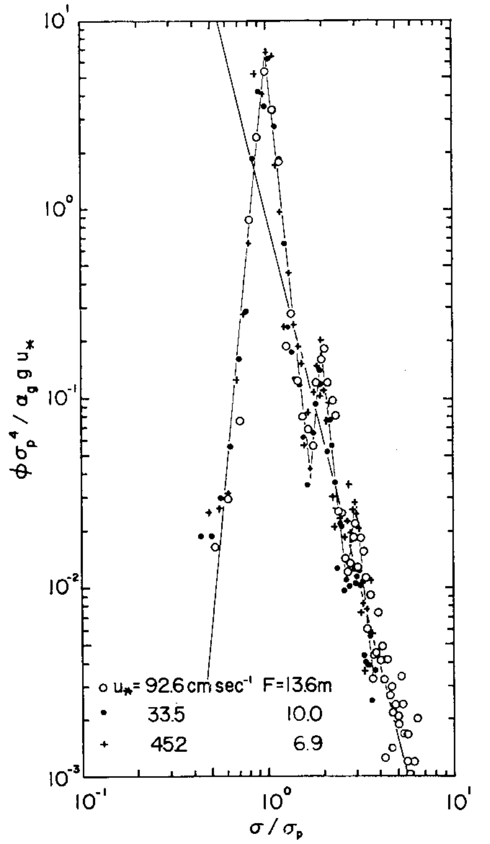

Fig. 4. Three examples of wind wave spectra in the gravity wave range in a normalized form. The similarity in the fine structure is clearly seen. The straight line is the σ^{-4}-line through the point (10°-10°).

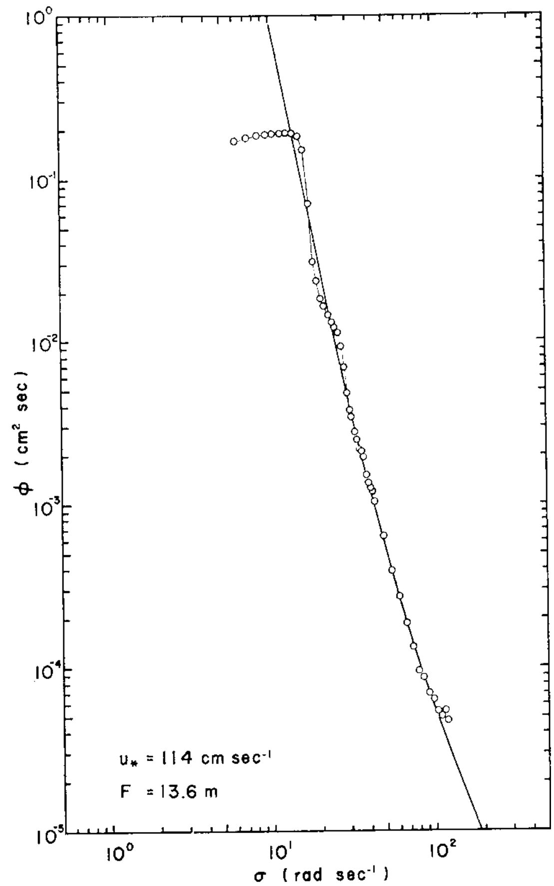

Fig. 5. An example of the 11-point moving average of the spectrum. The data is taken from Fig. 3. The line entered is Equation (3.2): $\phi_a(\sigma)=0.105\, g_*^2\sigma^{-5}$.

(1968), BARNETT and SUTHERLAND (1968), MITSUYASU (1968, 1969), TAIRA (1972) and others.

3. *Moving average*

It should be noted here that, if we draw a line through the first, the second and the third peaks in Figure 4, the line has a gradient of σ^{-5} approximately. In the actual ocean, the wind is always changing. Consequently, the above described fine structure of the wave spectra becomes obscured. In these cases, if we perform some averaging operation of the spectra, the apparent gross form will tend to approach the σ^{-5}-line, since the above described fine structure is on the logarithmic diagram.

Actually, in Figure 5 is shown the 11-point moving average of one of the data of Figure 3. The line entered is

$$\phi_a(\sigma)=\alpha_a g_*^2\sigma^{-5},\ \alpha_a=0.105 \qquad (3.2)$$

Equation (3.2) has a similar form with Equation (3.1). But evidently the value of α_a should be a function of u_* and F. The above value of α_a approximates the value given by MITSUYASU (1973) as a function of gF/u_*^2. But, it is noted that Equation (3.2) is only an empirical form of the moving average.

Since the high frequency part more quickly responds to the change in the local wind, the spectra observed in the sea, where the wind is always changing, will show fluctuation in the gradient, ranging from σ^{-4} to, say, σ^{-6}.

4. *Suggestion of more general similarity*

What is the above described very curious feature of the fine structure? One may consider non-linear interactions of the component waves. This line of study will be helpful in the understanding of the problem, on the one hand. However, on the other hand, although the spectral analysis is performed on the concept that the wind waves are regarded as a linear combination of small amplitudes of component waves of various frequencies, the wind waves are phenomena including strong non-linearity, as already mentioned in the introduction. Consequently, the problem may be of very different origin. The fine structure may be a manifestation of the shape of wind waves, especially of the primary waves that constitute the peak frequency.

Now it will be worthwhile reviewing the line of inference in the present article. The 4th concept of the similarity on the shape of the energy spectrum of wind waves stems from the experimental fact. The 4th concept is then combined with the three-second power law, which was presented in part I, and which was equivalent to the 2nd concept (revised in part II). The both concepts are very simple in nature. The combination leads to the conclusion of the sliding-up growth of the spectrum along the line of Equation (2.23). It is in accordance with the experiment.

There is no need, from this line of inference, for the gross form to coincide with the line along which the spectrum grows. But the fact is the case. It is interpreted suggesting the existence of some statistical hydrodynamical ground to be found in the system of wind waves, namely a kind of general similarity in the wind wave field itself. It may be analogous to the inertial subrange in the field of turbulence. Thus suppose tentatively that the three-second power law, which holds for the significant wave or for waves of the peak frequency, also holds for higher frequencies. This assumption stands on the idea that the above described fine structure is a manifestation of the shape of the primary waves of the peak frequency, and that there is a similarity in the distribution of the energy of waves of the whole frequency range. This assumption might require a similar form with Equation (2.5):

$$H_\sigma^2=B^2gu_*T_\sigma^3 \qquad (3.3)$$

where H_σ and T_σ are not the significant wave height and the significant wave period, but values significant for the frequency of σ. H_σ does not stand for the energy of the component waves. Analogously to Equation (2.2) we may put

$$c\int_\sigma^\infty \phi d\sigma=H_\sigma^2 \qquad (3.4)$$

and also

$$T_\sigma = \frac{2\pi}{\sigma} \qquad (3.5)$$

where c is a certain constant. With Equations (3.4) and (3.5), Equation (3.3) is expressed as

$$\int_\sigma^\infty \phi d\sigma = \frac{(2\pi)^3}{c} B^2 g u_* \sigma^{-3} \qquad (3.6)$$

Differentiating this, it follows

$$\phi = \frac{2}{c} \pi^3 B^2 g u_* \sigma^{-4} \qquad (3.7)$$

arriving at the same form with (2.10). Comparing this with Equation (2.11), it follows that

$$c = 1.8 \qquad (3.8)$$

For σ_p, from a comparison with Equation (2.2), it follows that

$$\frac{H^2}{16} \simeq \frac{H_\sigma^2}{1.8} \text{ or } \frac{H_\sigma}{H} \simeq \frac{1}{3} \qquad (3.9)$$

Consequently, the value of H_σ for σ_p is one third of the significant wave height.

Lastly, it is noted that the results of the present article are interpreted as giving a further substantiation to the three-second power law. It is equivalent to say that, in the growth process of wind waves, conditions expressed by the 2nd concept do hold as the first-order problem.

4. Relation between energy spectrum and significant wave

In Figure 6 is shown the comparison between the significant wave period, T, which was used in part I, and the corresponding period, T_p, which is obtained from the peak frequency, σ_p, by the relation

$$T_p = 2\pi/\sigma_p \qquad (4.1)$$

In Equation (2.1), T_p was equated to T. The substantiation has now been given. Strictly, the peak frequency, and the frequency at which

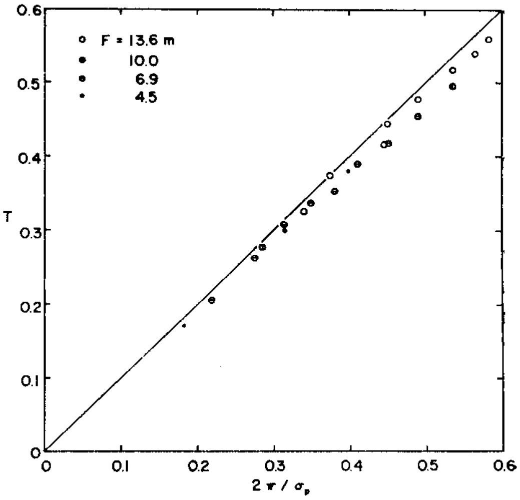

Fig. 6. A comparison between the significant wave period, T, and the corresponding period obtained from the peak frequency, σ_p.

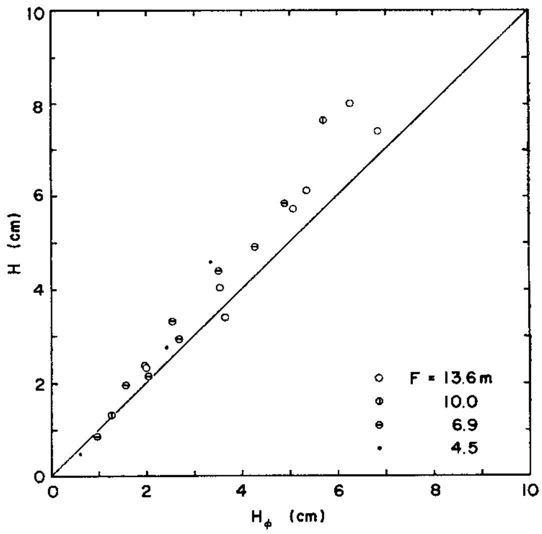

Fig. 7. A comparison between the significant wave height, H, and the corresponding wave height $H\phi$ estimated from the energy spectrum by the use of Equations (4.3) and (4.4).

the spectral density has the maximum deviation from the line of the gross form, do not always coincide with each other, when the peak of the spectrum is a little flat. In the estimation of T_p here, the frequency of the maximum deviation is used as σ_p. The relation between T and T_p, for the data, is approximately expressed by

$$T = 0.95\,T_p \tag{4.2}$$

Also, significant wave height is estimated from the energy spectrum by

$$H_\phi = 2.83\sqrt{2\overline{\eta^2}} \tag{4.3}$$

where

$$\overline{\eta^2} = \int_0^\infty \phi\, d\sigma \tag{4.4}$$

and η represents water level displacement. In Figure 7 is shown the comparison between H_ϕ and H which was used in part I. The relation is approximately expressed by

$$H = 1.15\,H_\phi \tag{4.5}$$

If we use H_ϕ and T_p instead of H and T, the three-second power law is expressed, for the present case, by

$$H_\phi^* = B_s T_p^{*3/2}, \quad B_s = 5.0\times10^{-2} \tag{4.6}$$

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海面境界過程における局所平衡

III. 風波のスペクトルについて

鳥　羽　良　明

要旨: 前報Iにおいて提出された単純なスペクトルの風波に関する3/2乗則と，風波のスペクトル形の相似性とを組合わせて，風波のエネルギースペクトルに関する新しい概念に到達した．それは，風洞水槽実験の資料とよく一致するものである．

重力波領域では，ピークより高周波側のスペクトルの全体としての形は $gu_*\sigma^{-4}$ に比例している．ここで，g は重力加速度，u_* は摩擦速度，σ は角周波数で，比例係数は 2.0×10^{-2} である．風波は，そのスペクトルが，スペクトル密度 ϕ と σ との対数グラフ上で，同じ形を保ったまま，スペクトルの全体としての形の線にそってスライドしていく形で発達する．また，じゅうぶん発達した風波の，ピーク周波数における ϕ の値（終点の値）は，$g^2\sigma^{-5}$ の勾配の線にそっている．

風洞水槽実験によるスペクトルの微細構造は，σ^{-4} 線のまわりに振動する特徴的な形を示す．エネルギー密度の過剰分がピーク周波数とその高調波の付近に集中し，不足がそれらの周波数の中間に起こる．重力波領域におけるこの微細構造の形は風洞水槽におけるような純粋に制御された条件のもとでは，常に相似である．これらのスペクトルの移動平均をとると，σ^{-5} に比例する形に非常に近くなる．

波数が大きくなると表面張力の効果が入ってきて，重力波領域における σ^{-4} 線は，連続的に，表面張力波領域における $\sigma^{-8/3}$ 線へとつながる．これは，風洞水槽の資料と一致している．同様に，σ^{-5} 線は連続的に $\sigma^{-7/3}$ 線へとつながる．

また，これらの結果の議論を通して，風波の場の構造における一種の一般的な相似性の存在が示唆される．