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Local Balance in the Air-Sea Boundary Processes

II. Partition of Wind Stress to Waves and Current*

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Abstract: In the course of the new treatment of the growth process of wind waves presented in part I of the present series of the articles, there was a point where the wave energy and wave momentum were not related correctly. This point has been revised with critical argument, and at the same time, the form of the ratio r, between the wind stress that directly enter the wind waves and the total wind stress, has been derived analytically. The growth equation, under the condition that the wind stress is constant, is still the same with that derived in part I, with the exception that the ratio r is given analytically.

A comparison between the ratio r obtained analytically and that estimated empirically in part I, raises a problem to be studied about the wave current of the actual wind waves.

1. Introduction

In part I of the present series of the articles, new treatment on the problem of the air-sea boundary processes was presented, and especially, the growth process of wind waves was discussed. The aim of the present series of the new treatment is not to see the problem as of the growth process of wind waves only, but to grasp the structure of the system as a whole, in which the momentum of the wind enters the water to change to the momenta of wind waves and drift current, and the work done by the wind stress to the water becomes mechanical energies of wind waves, turbulence in water, and drift current.

If we consider the whole system, the budgets of the momentum and energy should be balanced locally, and the way of acquisition of the momentum and energy by wind waves and current, and the transfer of wave momentum to the current by the collapse of wind waves, should be determined by the local conditions of the wind and wave fields. This was the concept of local balance.

As long as the wind is blowing, the wind waves have character of randomness or turbulence in contrast to inviscid water waves, and there are skin flows and the breaking of waves. Consequently, the effect of strong non-linearity makes rigorous analytical treatment very difficult. So we proceeded from the standpoint of seeing the whole structure macroscopically, in a form which enclosed the non-linear effect, leaving out for the moment the problems of the interactions among different spectral components of waves, processes of the spectral formation, the structure and the role of skin flows, the wind structure above the water surface, and so forth.

In the course of the derivation of the growth equation of wind waves, however, there was an incorrect point that the equations were inconsistent with the relation $M=E/C$, where M was the wave momentum, E the wave energy and C the phase velocity. In the present article, this point will be discussed more strictly, and the revisions of part I will be presented. At the same time, the ratio r between the portion of the wind stress that directly enters the wind waves, τ_w, and the total wind stress, τ, will be derived analytically. It was estimated only empirically in part I. Thus the growth equation including the form of r will be derived. It will be shown, however, that the form of the growth equation for constant wind stresses remains the same as that of part I.

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2. Rate of work done to wind waves

The same notations for variables are used as those in part I. In dimensionless forms, the independent variables are t^* and F^*, the parameter giving the external conditions is u_*^*, the dependent variables are T^*, H^* and r, and S^* $(=S^3/g\rho^3\nu^4)$ is a parameter constructed by physical constants. If we put aside the effect of the surface tension, S^* may be disregarded. In the course of the following treatment, the number of the dependent variables will be diminished by the use of relations inherent in water surface waves, and H^* and r will be expressed by T^*.

The total wind stress which acts on water surface, or the rate of transfer of horizontal momentum from the air to the water, is τ, and the wave current, or the average velocity of horizontal transport of surface water particles due to the orbital motion of waves, is u_0. Since wind waves are eventually generated by the wind stress τ, the rate of work done by the wind stress to wind waves, or the time rate of increase of the average wave energy per unit horizontal area should be τu_0. In part I, it was unproperly related by $r\tau u_0$. It should be noted here that in this expression of τu_0, τ includes all the wind stresses exerted on the water surface irrespective of the mechanism by which the momentum transfer occurs from the wind to the water, and in any case u_0 indicates the average horizontal velocity which *the wind waves* possess at the surface. Since the momentum is a vector quantity, the wave current, u_0, must be caused by the momentum transfer, τ_w. The wind stress τ can do the work to wind waves by the existence of u_0. The expression of τu_0 is thus an abstract expression.

We approximate wind waves by Stokes wave. Then the wave has a restriction that the wave current at the surface has the form of

$$u_0=\kappa^2 a^2 C=\frac{\pi^3 H^2}{gT^3} \tag{2.1}$$

and the dimensionless rate of increase of the wave energy is expressed, using the dimensionless variables, by

$$\frac{\tau u_0}{\rho g \nu}=\frac{\pi^3 u_*^* H^{*2}}{T^{*3}} \tag{2.2}$$

Originally, this rate should be determined by the only one variable expressing the external conditions, u_*^*. This forms the 2nd concept:

The 2nd concept (revised): The rate of work done by the wind stress to wind waves, namely, the time rate of increase of the average wave energy per unit horizontal area, is proportional to u_*^*.

The three-second power law for wind waves of simple spectrum, or

$$H^*=BT^{*3/2} \tag{2.3}$$

immediately follows. The physical meaning of the three-second power law is that the number of the dependent variables has been diminished by one by the restriction expressed by Equation (2.1), which is inherent in water waves. The value of the 1st universal constant, B, and three lemmas described in part I are not affected by this revision.

It should be pointed out that the value of B was determined by the relation of H and T expressed by Equation (2.1). Consequently, the value of u_0 given by lemma I,

$$u_0=\pi^3 B^2 u_* \tag{2.4}$$

is that of Stokes wave. It may somewhat be different from the value for real wind waves, since the effect of viscosity is neglected. This point will be discussed in section 6.

3. Partition of wind stress to waves and current—Analytical form of r

In Stokes wave, the wave momentum, M, is related to the wave energy, E, by

$$M=\frac{E}{C} \tag{3.1}$$

where C is the phase velocity. Using this relation, the number of the dependent variables may again be decreased by one. Namely, r will here be given by T^*, which will then be the only one dependent variable.

Equation (3.1) leads to the relation between the increments of E and M:

$$\delta E = C\delta M + M\delta C \tag{3.2}$$

We are now concerned with the way of acquisition of the wave energy and the wave momentum, and we may put aside for the moment the problem of dissipation of the wave energy, since the dissipation occurs after they enter the waves. We consider the changes of E and M caused by the wind stress. From the consideration of the previous section,

$$\frac{\delta E}{\delta t} = \tau u_0 \tag{3.3}$$

Then, by the definition,

$$\frac{\delta M}{\delta t} = \tau_w \tag{3.4}$$

Equation (3.2) then becomes

$$\tau u_0 = \tau_w C + M\frac{\delta C}{\delta t} \tag{3.5}$$

and it follows that

$$r = \frac{\tau_w}{\tau} = \frac{u_0}{C} - \frac{M}{\tau C}\frac{\delta C}{\delta t} \tag{3.6}$$

Transforming this into a dimensionless form, and by the use of Equation (2.4), namely, lemma I of the three-second power law, we obtain

$$r = \frac{2\pi^4 B^2}{T^*} - \frac{1}{2R}\frac{\delta}{\delta t^*}(T^{*2}) \tag{3.7}$$

with

$$R = \frac{8\rho}{\pi \rho_w B^2}$$

In order to obtain another expression for the second term of the right hand side, we transform the equation

$$\frac{\delta M}{\delta t} = \tau\frac{u_0}{C} - \frac{M}{C}\frac{\delta C}{\delta t} \tag{3.8}$$

into a dimensionless form to obtain

$$\frac{\delta}{\delta t^*}(u_*{}^* T^{*2}) + \frac{1}{2}u_*{}^*\frac{\delta}{\delta t^*}(T^{*2}) = 2\pi^4 B^2 R\frac{u_*{}^*}{T} \tag{3.9}$$

When $u_*{}^*$ is constant, it is reduced to

$$\frac{\delta}{\delta t^*}(T^{*2}) = \frac{4}{3}\pi^4 B^2 R\frac{1}{T^*} \tag{3.10}$$

and Equation (3.7) is reduced to

$$r = \frac{4}{3}\pi^4 B^2\frac{1}{T^*} = \frac{0.50}{T^*} \simeq \frac{0.0032}{C/U} \tag{3.11}$$

According to lemma I, u_0 is proportional to u^*, and consequently, τu_0 is constant if u_* is constant. That r is inversely proportional to T^* for constant $u_*{}^*$ stems from this. The zero value of T^* forms a singular point, since there is no wave and no u_0 then. An initial wavelet should be generated, by some disturbance, for its further growth.

4. Dissipation of wave energy

The dissipation of wave energy was treated in section 3.3 of part I. However, the same line of the revision with section 2 should be applied to it. Consequently, the rate of dissipation of the wave energy should be expressed by a function of u_*L/ν, or $u_*{}^*T^{*2}$ instead of ru_*L/ν, or $ru_*{}^*T^{*2}$. The energy that remains the wind waves is then expressed by

$$\frac{dE}{dt} = \tau u_0(1 - KT^{*2}) = \pi^3 B^2 \rho g \nu u_*{}^*(1 - KT^{*2}) \tag{4.1}$$

The value of the 2nd universal constant, K, is unaltered. The equation for the growth of wave momentum will be treated in the next section. The 3rd concept should be described as:

The 3rd concept (revised): The rate of dissipation of the energy of wind waves is proportional to the dimensionless quantity u_*L/ν (or $u_*{}^*T^{*2}$), and the factor K is a constant.

It is noted here that the revision of the 3rd concept fully supports the formerly presented idea by TOBA and KUNISHI (1970) that the breaking of wind waves is governed by u^*L/ν.

5. The growth equation of wind waves

In section 3 was treated the relation between the wave energy and the wave momentum in the process of their entering the waves. In this section, the actual growth process of wind waves is considered. Consequently, we now use Equation (4.1) as the time rate of change in E, instead of Equation (3.3), and then, basing on Equation (3.1) or (3.2), seek the expression for the time rate of change in M.

From Equations (3.2) and (4.1), the next equation follows,

$$\frac{dM}{dt}=\tau(1-KT^{*2})\frac{u_0}{C}-\frac{M}{C}\frac{dC}{dt} \quad (5.1)$$

Transforming this into a dimensionless form, we obtain

$$\frac{d}{dt^*}(u_*^*T^{*2})+\frac{1}{2}u_*^*\frac{d}{dt^*}(T^{*2})=\frac{3}{2}rRu_*^*(1-KT^{*2}) \quad (5.2)$$

In the case that u_*^* is constant, it is reduced to

$$\frac{d}{dt^*}(T^{*2})=rR(1-KT^{*2}) \quad (5.3)$$

Equation (5.2) has a slightly different form from Equation (3.29) of part I, but Equation (5.3) is exactly the same with Equation (3.30) of part I. The most noticeable change is that the form of r is now given by Equation (3.11), whereas it was only known empirically in part I.

The fetch equations are obtained, just as in part I, as

$$\frac{d}{dF^*}(u_*^*T^{*2})+\frac{1}{2}u_*^*\frac{d}{dF^*}(T^{*2})=6\pi rRu_*^*(1-KT^{*2})T^{*-1} \quad (5.4)$$

and, for constant u_*^*, as

$$\frac{d}{dF^*}(T^{*2})=4\pi rR(1-KT^{*2})T^{*-1} \quad (5.5)$$

which is the same with Equation (3.34) in part I.

6. Comparison with wave data and discussion

In Table 1 is shown a comparison between the predicted values of T by Equations (3.11) and (5.5), and the observed values in the wind-wave tunnel shown in part I. The initial values are taken at $F=6.9$ m. The agreement is again fairly good. However, some systematic tendensy of the deviation seems to exist.

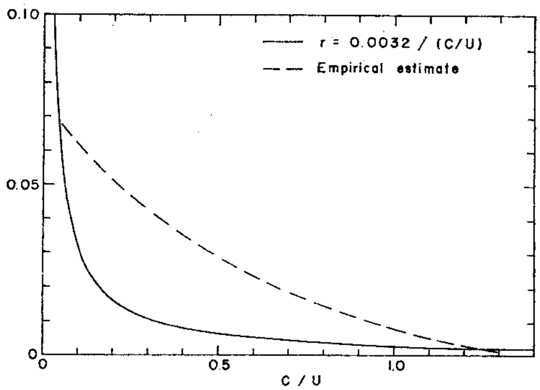

Fig. 1. Value of r given by Equation (3.11), and empirical estimate of r by the use of Wilson's 1965 empirical wave formula.

Table 1. Comparison between values of wave period, T, observed in a wind-wave tunnel (TOBA, 1961), and predicted by the Equations (3.11) and (5.5).

Mean values of u_* (cm/sec)	T (sec)				
	$F=6.9$ m	$F=10.0$ m		$F=13.6$ m	
	Observed	Observed	Predicted	Observed	Predicted
35.4	0.206	0.277	0.283	0.326	0.327
41.2	.264	.320	.323	.374	.364
48.7	.308	.352	.362	.416	.404
57.6	.337	.386	.396	.443	.441
67.3	.353	.416	.421	.478	.472
84.3	.390	.445	.468	.518	.526
101.1	.418	.482	.508	.540	.572
113.6	.455	.496	.545	.560	.611

In Figure 1 is shown Equation (3.11), together with the value of r estimated in part I by putting r as the only unknown in Equation (5.5), and by the use of Wilson's 1965 empirical wave formula. At the zero value of C/U, the theoretical value of r has a singular point as mentioned in section 3. For very small values of C/U, a region follows where the surface tension plays a large part. The form of r for this region should be derived separately. In the region of C/U of 0.036 through 0.058, in which the above values in the wind-wave tunnel fall, the both values of r in Figure 1 coincide incidentally. In the main region of C/U, say, from 0.15 to 1.0, the value of r by Equation (3.11) is smaller than the empirical value by a factor of three to four. This raises a very important problem.

In deriving Equation (3.11), we used Equation (2.4), or lemma I. As already noted in section 2, u_0 by Equation (2.4) gives the value of Stokes wave. However, the actual wind waves will have much larger values by the effect of molecular viscosity. In Equations (3.3) and (4.1), we should have used the actual value of the wave current, $\varepsilon_1 u_0$ ($\varepsilon_1>1$). Also, Equation (3.1) should have been replaced by

$$\varepsilon_2 M=\frac{E}{C} \qquad (6.1)$$

Then, Equation (3.6) should be replaced by

$$r=\frac{\varepsilon_1}{\varepsilon_2}\frac{u_0}{C}-\frac{M}{\tau C}\frac{\partial C}{\partial t}-\frac{M}{\varepsilon_2\tau}\frac{\partial\varepsilon_2}{\partial t} \qquad (6.2)$$

and if the third term on the right hand side is neglected compared to the second term, Equation (3.11) is replaced by

$$r=\left(\frac{3}{2}\frac{\varepsilon_1}{\varepsilon_2}-\frac{1}{2}\right)\frac{4}{3}\pi^4B^2\frac{1}{T^*} \qquad (6.3)$$

The factor ε_1 may also contain an adjustment of the effect of the fluctuations of τ and u_0. Namely, in our treatment, we have used the term $\bar{\tau}\bar{u}_0$ only and neglected the term $\overline{\tau' u_0'}$, where the bars represent mean values, and the primes fluctuation.

If the factor $\left(\frac{3}{2}\frac{\varepsilon_1}{\varepsilon_2}-\frac{1}{2}\right)$ in Equation (6.3) has a value between 3 and 4, that is, if the factor $\varepsilon_1/\varepsilon_2$ has a value between 2.4 and 3, the both values of r coinside in the main region of C/U. It is now considered that the points of principal problem have been focused on the mass transport of the actual wind waves.

It should be mentioned here that an approximate expression derived in part I for the empirical value of r:

$$r=0.075\exp(-0.012\,T^*) \qquad (6.4)$$

or

$$r=0.075\exp(-1.9\,C/U) \qquad (6.5)$$

still has significance from the practical point of view of wave forecasting.

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海面境界過程における局所平衡

II. 風の応力の波と流れへの分配

鳥　羽　良　明

要旨: この論文のIで提出した風波の発達過程の新しい取扱いの中で，波のエネルギーと波の運動量との関係が正しくない点があった．この点を修正しながら精密に議論したところ，海にかかる風の応力全体の中で直接風波に入る部分の割合 r の形が理論的に導かれた．風の応力が変化しない条件下では，発達方程式はやはりIで導かれたものと同じであり，r が解析的に与えられている点だけが異なっている．

理論的に得られた r 値とIで経験的に得られた r 値との比較は，現実の風波の質量輸送に関して，研究されるべき問題を提起している．