

Local Balance in the Air-Sea Boundary Processes

I. On the Growth Process of Wind Waves*

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Abstract: A new growth equation for wind waves of simple spectrum is presented upon three basic concepts. The period and the wave height of significant waves in dimensionless forms, which are considered to correspond to the peak frequency and the energy level, respectively, are used as representative quantities of wind waves. One of the three basic concepts is the concept of local balance, and the other two concern the acquisition of wave energy and the dissipation of wave energy, respectively. It is shown from some actual data that the equation, together with two universal constants concerning the acquisition and the dissipation of wave energy ($B=6.2\times 10^{-2}$ and $K=2.16\times 10^{-5}$, respectively), is applied universally to wide ranges of wind waves from those in a wind-wave tunnel to fully developed sea in the open ocean.

“The three-second power law for wind waves of simple spectrum”, and a few relations as the lemmas, are derived, such that the mean surface transport due to the orbital motion of wind waves is always proportional to the friction velocity in wind, and that the steepness is inversely proportional to the root of the wave age. It is also derived that the portion of wind stress which directly enters the wind waves decreases exponentially with increasing wave age and is 7.5% of the total wind stress for very young waves.

Also, equations are presented as to the increase of momentum of drift current, and as to the supply of turbulent energy by wind waves into the upper ocean.

1. Introduction

The sea surface is a boundary between two fluids, the air and the sea, and so, unique physical processes are taking place there. The momentum of the air enters the sea at the sea surface, to become the momentum of wind waves and drift current, and the work done to the sea water becomes the mechanical energy of the wind waves, turbulence and the drift current. The momentum and the energy thus wander about among the waves, turbulence and current, and are transported downward by turbulence, convection and internal waves, interacting with the field of density stratification in the upper ocean.

Since the wind waves make themselves conspicuous as a phenomenon at the sea surface,

they have been a long subject of investigation as wind waves themselves among many researchers. During a quarter century since SVERDRUP and MUNK (1947), much data and knowledge have been accumulated about empirical characteristics of significant waves, as well as about spectral structure of wind waves. The basic problems of the air-sea boundary processes, however, including the physical processes of the generation and growth of wind waves, has been in a chaotic state.

In the present article, a new treatment is presented, mostly about the growth process of wind waves, which is very independent of the existing concepts. It is an attempt from the view point of macroscopic treatment, without entering deep into detailed mechanisms such as the structure of the air flow over the sea surface, and as the interactions among the spectral components of the waves.

Several simple relations obtained in the course

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of the derivation, as a whole, seem also to present some implications concerning the dynamical processes of the growth of wind waves.

2. Variables concerning the growth of wind waves

2.1 Dimensional variables

It is known, since PHILLIPS (1958), that wind waves, under simple conditions free of swells, have the equilibrium range where the energy spectrum density, $\phi(\sigma)$, as a function of the angular frequency, σ , has a form approximately expressed by

$$\phi(\sigma) = \alpha g^2 \sigma^{-5} \quad (2.1)$$

If cases where the wind waves have a complicated spectrum including swells are left out of consideration, the characteristics of the wind wave field will be represented by the peak frequency and the energy level. To express the wave field by use of the two parameters is convertible physically to the use of the period, $T_{\frac{1}{3}}$, and the wave height, $H_{\frac{1}{3}}$, of significant waves, that have been used since SVERDRUP and MUNK (1947).

Further, as is evident from equation (2.1), the main part of the wave energy is concentrated at the peak frequency. The purpose of the present article is to see the general feature of the acquisition and dissipation of the momentum and mechanical energy of wind waves, and the two parameters, $T_{\frac{1}{3}}$ and $H_{\frac{1}{3}}$, are adopted as the only two variables representing the wind wave field, and the subscript of 1/3 will be omitted hereafter.

The above mentioned two variables belong to the dependent variables. Among the independent variables, which determine the state of wind waves, are the wind stress, or the total rate of transfer of momentum from the air to the sea, τ , the fetch, F , and the duration, t .

The wind stress τ may be represented by the friction velocity, u_* , defined by

$$u_* = \sqrt{\frac{\tau}{\rho_a}} \quad (2.2)$$

where ρ_a is the density of the air. The u_* is

related to the wind speed at 10-m level, U_{10} , by the equation

$$u_* = \gamma_{10} U_{10} \quad (2.3)$$

where $\gamma_{10}(=C_{D10})$ is the friction coefficient.

The value of γ_{10} was studied formerly as a function of U_{10} , but TOBA and KUNISHI (1970) showed that the value is affected largely by the state of water surface, especially, that the value increases with the increasing rate of breaking of wind waves. The breaking of wind waves bears a great significance in the present article, and it is not appropriate here to give a functional form of γ_{10} *a priori*. Since it is the rate of transfer of momentum from the air to the sea that is essential to the growth process of wind waves, u_* is adopted as the independent variable, without entering into the details of the problem of γ_{10} . The change of the value of γ_{10} should be discussed separately.

In this context, mention should be made about the roughness length, z_0 , which is a parameter commonly used in the treatment of the air-sea boundary processes. In the neutral stratification, the wind profile is usually expressed by

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{z}{z_0} \quad (2.4)$$

consequently, it follows that

$$\gamma_{10} = \left(\frac{1}{k} \ln \frac{z_{10}}{z_0} \right)^{-1} \quad (2.5)$$

where k is the von Kármán constant. It is evident from equation (2.5) that z_0 and γ_{10} are convertible to each other, and it is not necessary in this article to adopt z_0 as an independent variable. In the following, the subscript 10 will be omitted in γ_{10} and U_{10} .

As is well known, the wind stress causes drift current as well as wind waves. As long as the air-sea boundary processes are treated, surface current should enter the system. In order to make the matter simple and to abstract the essentials clearly, it is assumed here that there is no current independent of the wind. In this case, the horizontal movement of water particles at the surface still increases

as the wind continues to blow. Now we separate the average horizontal movement of the surface water particles due to the orbital motion of wind waves, or the wave current, from the rest of the average horizontal movement, and denote the former u_0 . When we put our eyes on waves themselves, the rest of the surface current may be regarded as the horizontal sliding of the coordinate system. This is the use of the concept of relativity. Strictly speaking, if the wind speed is taken relatively to the sliding coordinate system, the velocity of which is much smaller than the absolute wind speed, the surface current other than u_0 may be left out of consideration.

There is, however, another important factor that must enter the treatment of the wind wave-current system, in place of the disregarded surface current. It is the proportion, r , of the wind stress which directly enters the wind waves, τ_w , to the total wind stress, τ . The rest of the wind stress, τ_c , directly enters the current other than u_0 . Namely,

$$r = \frac{\tau_w}{\tau}, \quad \tau = \tau_w + \tau_c \quad (2.6)$$

Besides, there are some dimensional physical constants that concern the phenomena of wind waves, and that are constant in the air-sea boundary on the earth. They are the acceleration of gravity, g , the surface tension of sea water, S , the density of the air and sea water, ρ_a and ρ_w , and the kinematic viscosity of the air and sea water, ν_a and ν_w . But, ρ_a , ρ_w , ν_a and ν_w have the same dimensions, respectively, and so we have equations $\rho_a/\rho_w = (\text{const.})_1$, and $\nu_a/\nu_w = (\text{const.})_2$. Consequently, we may adopt either of them, respectively. In the following, ρ_a and ν_a are adopted, and the subscript a will be omitted.

In conclusion, it is sufficient to consider ten variables: independent variables of u_* (or τ , originally), F , and t , dependent variables of T , H and r (or τ_w , originally), and constant physical parameters of g , S , ρ and ν .

2.2 Dimensionless variables

From the above-mentioned ten dimensional variables, we may construct seven dimension-

less variables to treat the growth processes of wind waves. They are:

$$\frac{u_*^3}{g\nu} \equiv u_*^* \quad (2.7)$$

$$\frac{gF}{u_*^2} = \frac{gF}{\tau^2 U^2} \equiv F^* \quad (2.8)$$

$$\frac{gt}{u_*} = \frac{gt}{\tau U} \equiv t^* \quad (2.9)$$

$$\frac{gT}{u_*} \equiv T^* \quad (2.10)$$

$$\frac{gH}{u_*^2} \equiv H^* \quad (2.11)$$

$$\frac{\tau_w}{\tau} \equiv r \quad (2.12)$$

and

$$\frac{S^3}{g\rho^3\nu^3} \equiv S^* \quad (2.13)$$

Although u_*^* expresses the wind stress, more strictly it represents the work done by the wind stress to the water, since it contains u_*^3 . The F^* and t^* represent the fetch and duration, respectively, normalized by the use of u_* . These three are independent variables.

Since the wave age β is related to T^* by

$$\beta = \frac{C}{U} = \frac{\tau}{2\pi} T^* \quad (2.14)$$

where C is the phase velocity of significant waves, T^* has approximately the same meaning with β . Namely, it represents, firstly, the relative relation between C and U as the physical situation of the local conditions, and secondly, the present situation of wind waves relative to the equilibrium state for a given u_* —the wave age. The H^* also represents the present situation of wind waves, but since the wave energy is proportional to H^2 , it is an expression of the present energy level of wind waves. The steepness, δ , is expressed by H^* and T^* by

$$\delta = \frac{H}{L} = \frac{2\pi H^*}{T^{*2}} \quad (2.15)$$

where L is the wave length.

The last variable S^* is a constant as far as the air-sea boundary on the earth is concerned. The first five variables all contain g and no S . If these variables are multiplied by S^* , variables containing S instead of g will be obtained. Namely, S^* is used for the conversion from the effect of g to the effect of S as the main restoring force. For wind waves of the range of smaller frequencies where the effect of g is much greater than that of S , it will be sufficient to study relationship among three dependent variables of T^* , H^* , r and three independent variables of u_* , F^* and t^* .

3. Growth equation of wind waves

3.1 Concept of local balance

The present treatment of the growth of wind waves is based upon three basic concepts. The first is the concept of the local balance:

The 1st concept: Physical processes of the transfer of momentum and mechanical energy from the air to the sea are determined locally.

This concept postulates that the growth of wind waves is predicted by an integration with respect to the fetch and duration. Consequently, F^* and t^* become variables only for the integration and u_* becomes the only parameter having the meaning of the external condition.

About the ways of the acquisition of wave momentum and wave energy, and of their dissipation by turbulence, two further concepts will follow.

3.2 Acquisition of wave energy—The three-second power law for wind waves of simple spectrum

The wind stress which acts on wind waves, is $r\tau$, and the average velocity of horizontal transport of surface water particles due to the orbital motion of wind waves is u_0 . Consequently, the rate of work done by the wind stress to wind waves, or the time rate of increase of the average wave energy per unit horizontal area is $r\tau u_0$.

In Stokes' wave, that is an irrotational wave of finite amplitude, u_0 is given to second order by

$$u_0 = \kappa^2 a^2 C \quad (3.1)$$

where a is the wave amplitude, and κ the wave number expressed by

$$\kappa = \frac{2\pi}{L} = \frac{\sigma^2}{g} = \frac{g}{C^2}$$

The phase velocity C is related to the period, T , by

$$C = \frac{gT}{2\pi}$$

If the mean square amplitude $\bar{a}^2$ is used as a^2 , equation (3.1) is reduced to

$$u_0 = \frac{\pi^3 H^2}{g T^3} \quad (3.2)$$

since according to LONGUET-HIGGINS (1952)

$$\bar{a}^2 = \frac{H^2}{(2.83)^2} = \frac{H^2}{8}$$

Consequently, it follows that

$$r\tau u_0 = r \frac{u_*^* H^{*2}}{T^{*3}} = \pi^3 \rho g \nu \quad (3.3)$$

and a dimensionless quantity that represents the rate of acquisition of wave energy may be written as

$$r \frac{u_*^* H^{*2}}{T^{*3}}$$

Now the second important concept is presented:

The 2nd concept: The rate of work done by the wind stress to wind waves, namely, the time rate of increase of the average wave energy per unit horizontal area, depends only on r and u_^* , and, as the simplest case, it is proportional to ru_*^* .*

This may be a kind of hypothesis for the present. This statement is expressed by

$$r \frac{u_*^* H^{*2}}{T^{*3}} = B^2 r u_*^* \quad (3.4)$$

where B is a universal constant. Substituting equation (3.4) into equation (3.3), it follows that

$$\tau u_0 = \pi^3 B^2 \rho g \nu r u_*^* \quad (3.5)$$

and the following relation is immediately obtained from equation (3.4):

$$H^* = B T^{*\frac{3}{2}} \quad (3.6)$$

This is named *the three-second power law for wind waves of simple spectrum*. Since the value of γ does not largely change, an alternative expression is given as

$$\frac{gH}{U^2} = B' \left(\frac{C}{U} \right)^{\frac{3}{2}}, \quad B' = (2\pi)^{\frac{1}{2}} \gamma^{\frac{1}{2}} B \quad (3.7)$$

Using well known empirical formulas for equilibrium wind waves:

$$\frac{gH_1}{U^2} = 0.30 \quad (3.8)$$

and

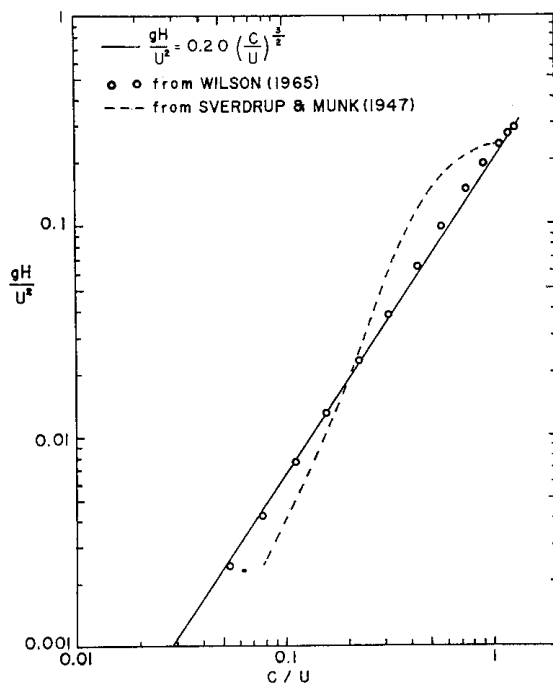


Fig. 1. Reduction from Wilson's 1965 empirical formulas shows a close correspondence to the three-second power law for wind waves of simple spectrum, with the constant value of $B' = 0.20$.

$$\frac{C_1}{U} = \frac{gT_1}{2\pi U} = 1.37 \quad (3.9)$$

where subscript 1 represents values for equilibrium waves, the value of B' in equation (3.7) is determined to be

$$B' = 0.19$$

If an approximate value of γ of 0.040 is used, the value of B is determined to be

$$B = 0.059$$

WILSON (1965) proposed some empirical formulas for wind waves in the growth stage, from much observational data, such as

$$\frac{gH}{U^2} = 0.30 \left[1 - \left\{ 1 + 0.004 \left(\frac{gF}{U^2} \right)^{\frac{1}{2}} \right\}^{-2} \right] \quad (3.10)$$

and

$$\frac{gT}{2\pi U} = 1.37 \left[1 - \left\{ 1 + 0.008 \left(\frac{gF}{U^2} \right)^{\frac{1}{2}} \right\}^{-5} \right] \quad (3.11)$$

which were an improvement of the fetch graph of SVERDRUP and MUNK (1947). Eliminating gF/U^2 from equations (3.10) and (3.11), values of gH/U^2 are plotted against C/U in Figure 1. Now it is considered that the three-second power law has been substantiated from actual data of wind waves. From the points plotted,

$$B' = 0.20 \quad (3.12)$$

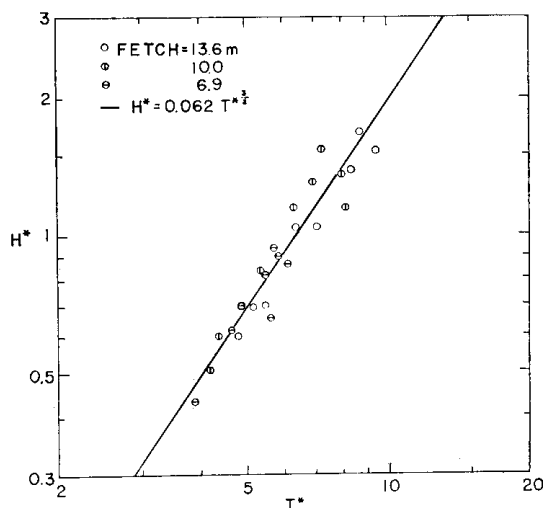


Fig. 2. Wind-wave tunnel data by TOBA (1961) gives further substantiation for the three-second power law.

and for $r=0.040$,

$$B=0.063 \quad (3.13)$$

is obtained. The straight line shows it. In Figure 1 is also shown curves that are obtained by eliminating gF/U^2 from the old fetch graph of SVERDRUP and MUNK (1947).

MITSUYASU, *et al.* (1971) proposed the following formulas, on the basis of the data obtained in a wave tank and at fetches smaller than 20 km in Hakata Bay:

$$\frac{gH}{U^2} = 2.15 \times 10^{-3} \left(\frac{gF}{U^2} \right)^{0.504}$$

and

$$\frac{gT}{2\pi U} = 5.07 \times 10^{-2} \left(\frac{gF}{U} \right)^{0.330}$$

The form of the formulas also gives support to the three-second power law, and provides

almost the same value of B' .

In Table 1 is shown data of Toba's 1961 wind-wave tunnel experiment. In Figure 2 are plotted values of H^* against T^* from Table 1, where values of H are estimated from mean wave height, $\bar{H}$, by

$$H=1.60 \bar{H} \quad (3.14)$$

The value of B in this case is directly determined to be

$$B=0.062 \quad (3.15)$$

In Figure 3 are plotted values of gH/U^2 against C/U from data obtained at Shirahama Oceanographic Tower Station by TOBA, *et al.* (1971). The straight line indicates $B'=0.20$. STEWART (1961) discussed the momentum flux from the air to wind waves, by the use of collected wave data by GROEN and DORRESTEIN (1958). From

Table 1. Toba's 1961 wind wave data.

F (m)	$\bar{U}^\dagger$ (m/sec)	U_{10} (m/sec)	u_* (cm/sec)	$10^3 \gamma_{10}^2$	T (sec)	$\bar{H}$ (cm)	$\alpha^{\dagger\dagger}$
6.9	5.1	8.81	35.9	1.66	0.206	0.54	0.00
	6.1	10.7	45.2	1.78	.264	1.22	.00
	7.5	12.2	49.3	1.63	.308	1.34	.00
	8.7	13.9	56.5	1.66	.337	1.83	.00
	9.7	15.5	63.2	1.67	.353	2.07	.01
	10.8	17.9	78.6	1.92	.390	2.75	.04
	11.6	19.8	88.1	1.98	.418	3.06	.12
	12.1	23.3	114.9	2.44	.455	3.64	.22
10.0	5.1	8.35	33.5	1.61	0.277	0.82	0.00
	6.1	9.86	39.4	1.60	.320	1.35	.00
	7.5	11.9	47.8	1.61	.352	2.24	.00
	8.7	13.9	54.8	1.57	.386	2.51	.01
	9.7	16.0	64.8	1.64	.416	3.09	.07
	10.8	18.6	81.7	1.93	.445	3.56	.16
	11.6	23.1	113.0	2.39	.482	4.12	.26
	12.1	23.1	111.6	2.33	.496	4.77	.38
13.6	5.1	8.79	36.7	1.74	0.326	1.45	0.00
	6.1	9.91	38.9	1.54	.374	1.48	.00
	7.5	12.1	48.9	1.63	.416	2.12	.01
	8.7	14.6	61.5	1.78	.443	2.52	.06
	9.7	16.8	73.7	1.93	.478	3.57	.15
	10.8	20.0	92.6	2.14	.518	3.82	.22
	11.6	22.1	102.2	2.14	.540	4.62	.32
	12.1	23.8	114.3	2.29	.560	5.00	.38

† Mean wind speed in the tunnel section.

†† Rate of breaking crests at fixed fetches.

Temperature was around 15°C.

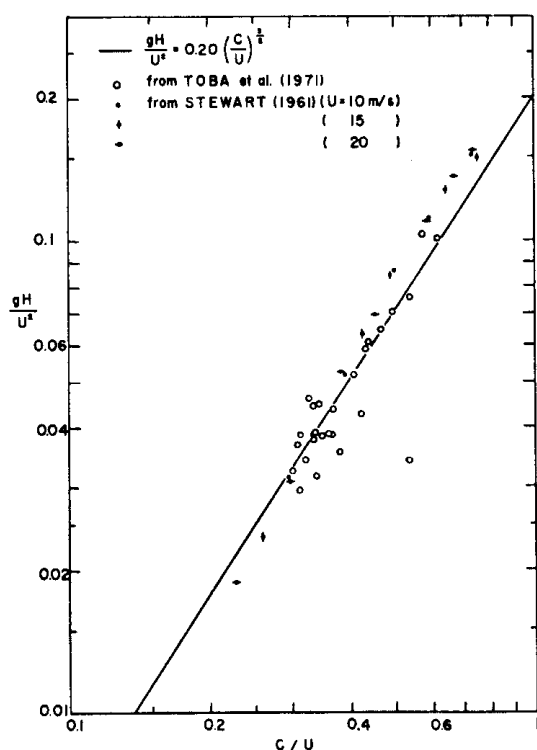


Fig. 3. Wind wave data by TOBA *et al.* (1971) and by STEWART (1961) also give substantiation for the three-second power law.

Stewart's Table 1, values of gH/U^2 are plotted also in Figure 3 against C/U . It is considered, from Figures 2 and 3, that the three-second power law has been further substantiated, and that the remarkable consistency of the value of B shows that it is a universal constant.

In Figure 3, Stewart's data seems to show a slight systematic deviation from the straight line. This may be attributed to the problem of γ , and this point will be discussed separately.

Now, when u_* changes with time, it is required, from the concept of the local balance, that the wind waves adapt themselves to the new local conditions in order to fulfil the three-second power law.

Further, as a result of the three-second power law, a few lemmas are obtained. Firstly, from equation (3.5), Lemma I follows.

Lemma I: The u_0 is uniquely related to u_* by

$$u_0 = \pi^3 B^2 u_* = 0.12 u_* \quad (3.16)$$

namely, the mean surface transport due to the orbital motion of wind waves is always proportional to the friction velocity in the wind.

Dimensional analysis cannot enter deep into mechanical processes of the acquisition of wave energy. This is a subject belonging to a different way of study. Nevertheless, it seems that Lemma I has the following implication for the mechanical processes.

When the wind begins to blow on the still surface of water, a skin flow arises just as KUNISHI (1963) observed in a wind-wave tunnel. But, since the skin flow will have a large velocity shear near the surface, the flow will not last stably, and turbulence will arise. When turbulence arises at the water surface, it makes some undulations of the water surface, and the undulations propagate as water waves, under the restoring forces of the gravitational force and the surface tension. After some water waves develop, a difference of the skin flow occurs between the fore side and the back side of the waves, and a mass of water converges at the crest, causing the growth of the waves. In other words, the wind waves grow by feeding on the skin flow. This does not say anything about the magnitude of the surface flow other than that included in the orbital motion, but it at least postulates that the way of acquisition of the wave energy has some restriction expressed in the form of equation (3.16).

Although Stoke's wave is an inviscid and irrotational wave model, the above-mentioned implication of the skin-flow feeding waves does not conflict with the irrotational wave model, if we consider that it is the part of the converged mass, namely, the difference of the skin flows, that is concerned with the growth of wind waves. Further, the actual wind waves may be viscous and rotational. But, since characteristics of the actual wind waves may well be approximated by Stoke's waves, it may be considered that the use of equation (3.1) for u_0 leads at least to an approximately correct result. Also, it may be considered that the value of B includes some correction for the Stoke's wave approximation by equation (3.1).

Lemma II: The wave steepness, δ , is expressed

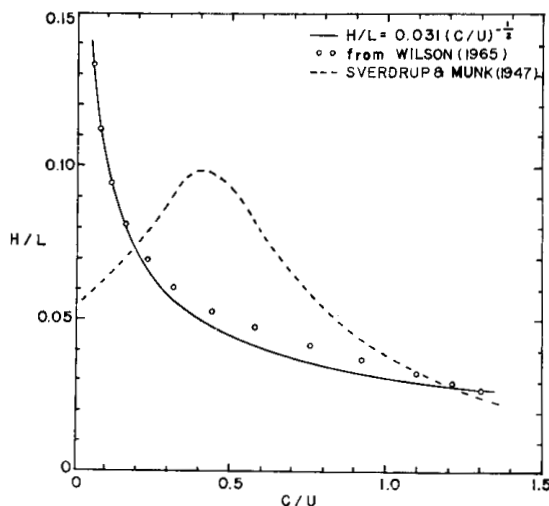


Fig. 4. Relation between the wave age and the steepness.

by the following equations :

$$\delta = \frac{H}{L} = 2\pi B T^{*-1/2} \quad (3.17)$$

or

$$\delta = (2\pi\gamma)^{1/2} B\beta^{-1/2} \quad (3.18)$$

These equations are obtained immediately from equations (2.15) and (2.14). This formulation will not hold for small waves on which the surface tension primarily acts, since u_0 has then a different form.

In Figure 4 is shown equation (3.18), assuming that $\gamma=0.040$. In Figure 4 are also entered a curve by SVERDRUP and MUNK (1947), and values obtained from the before-mentioned Wilson's 1965 formula.

Lemma III: Between two Reynolds numbers, u_*H/ν and u_*L/ν , there is the relationship :

$$\frac{u_*L}{\nu} = \frac{1}{2\pi B} T^{*1/2} \frac{u_*H}{\nu} \quad (3.19)$$

This is an immediate result of Lemma II, but is presented here since KUNISHI (1963) found that there was a universal relationship between u_*L/ν and u_*H/ν in waves in the wind-wave tunnel. Except in initial stages of the generation of wind waves, Kunishi's relationship actually agrees with equation (3.19). The reason that KUNISHI reported as if there was

a universal relationship between u_*L/ν and u_*H/ν , lies in the fact that the range of the value of T^* was very small in the wind-wave tunnel. For initial stages of the generation of wind waves, another formulation should be sought which includes the effect of surface tension.

It should be mentioned here that u_*L/ν and u_*H/ν may be expressed by u_*^* and T^* as

$$\frac{u_*L}{\nu} = \frac{1}{2\pi} u_*^* T^{*2} \quad (3.20)$$

and

$$\frac{u_*H}{\nu} = u_*^* H^* = B u_*^* T^{*1/2} \quad (3.21)$$

3.3 Dissipation of wave energy

The work done by the wind stress to wind waves, namely, the acquisition of wave energy was expressed by equation (3.5). Consequently, if there is no dissipation of the wave energy, wind waves will grow continually as long as ru_*^* exists. Actually, there is the dissipation of wave energy, and, sooner or later, the equilibrium is reached beyond which wind waves no longer grow. The expression of the dissipation process is the content of this section.

In an extreme expression, the wind waves is originally the turbulence at the water surface, but the turbulence that has the restriction of "at the water surface." Namely, the disturbance at the water surface must propagate under the restoring forces of the gravitational force and the surface tension. In other words, the wind waves are "the turbulence conditioned by characteristics of propagating organized wave motion." Thus wind waves have two aspects of the organized wave motion and the turbulence. Especially, smaller waves act as turbulence for larger waves, and the organized wave motion is perpetually being destroyed by this turbulence. Larger scales of turbulence manifest themselves as the breaking of wind waves, and the smallest scale of turbulence will be the molecular viscosity. Consequently, the energy of organized wave motion cascades down through smaller turbulence, and is dissipated by molecular viscosity at the end. The rate of energy given to the wind waves is τu_0 per

unit horizontal area as was already shown. The rate of dissipation of the energy by molecular viscosity has a form of

$$\int_0^Z \mu \bar{D} dz$$

where $\bar{D}$ has a form of the squared space derivative of velocity, or $(\partial u / \partial z)^2$, etc., and Z is the limit of the depth where the dissipation occurs. Dividing this term by ρ , and expressing it dimensionally by the use of the relating physical quantities: r , u_* , L and ν , we obtain $\nu u_*^2 / L$. In the same way, dividing $r\tau u_0$ by ρ , the energy input is expressed by ru_*^3 . Consequently, the dimensionless quantity, that is relevant to the dissipation of the wave energy, is ru_*L/ν or $ru_*^*T^{*2}$. Namely, the rate of the dissipation of wave energy must be expressed as a function of $ru_*^*T^{*2}$.

As to this number, two further reasonings are considered. Firstly, u_*L/ν is proportional to the momentum itself that the wind waves presently possess. Namely, the average wave momentum per water column of unit horizontal area, M , is expressed by

$$\begin{aligned} M &= \frac{E}{C} = \frac{1}{2C} \rho_w g \bar{a}^2 \\ &= \frac{\pi \rho_w H^2}{8T} \end{aligned} \quad (3.22)$$

where E represents the wave energy. Consequently, a dimensionless quantity representing the wave momentum, M^* , is given by

$$\begin{aligned} M^* &= \frac{H^2}{\nu T} = \frac{u_*^* H^{*2}}{T^*} = B^2 u_*^* T^{*2} \\ &= 2\pi B^2 \frac{u_* L}{\nu} \end{aligned} \quad (3.23)$$

Secondly, u_* is proportional to the current due to the organized orbital motion of wind waves, from Lemma I, and $L/2$ is the depth where this current is confined. Namely, u_*L/ν is a Reynolds number representing the strength of turbulence of the wave field. So, ru_*L/ν may be considered as a Reynolds number which is constructed by ru_*^* , the wind stress working on wind waves, and T^* , the quantity represent-

ing the local wave field.

Considering the conditions that the rate of dissipation of the wave energy is zero for $L=0$, and that it is equal to the rate of acquisition of wave energy from the wind when the equilibrium waves are reached, the rate of energy that remains in the wind waves may be expressed by

$$\begin{aligned} \frac{dE}{dt} &= r\tau u_0(1 - KT^{*2}) \\ &= r\pi^3 B^2 \rho g \nu u_*^*(1 - KT^{*2}) \end{aligned} \quad (3.24)$$

where K is a constant and is given for the equilibrium waves by

$$\begin{aligned} r\tau u_0 &= r\pi^3 B^2 \rho g \nu u_*^* \\ &= r\pi^3 B^2 \rho g \nu u_*^* K T_1^{*2} \end{aligned} \quad (3.25)$$

Namely, K is determined to be

$$K = T_1^{*-2} \quad (3.26)$$

Assuming $r=0.040$, the value is obtained from equation (3.8) as

$$K = 2.16 \times 10^{-5} \quad (3.27)$$

This is the second universal constant for wind waves.

By dividing equation (3.24) by u_0 , the rate of momentum that remains in the wind waves is expressed by

$$\begin{aligned} \frac{dM}{dt} &= r\tau(1 - KT^{*2}) \\ &= r\rho u_*^2(1 - KT^{*2}) \end{aligned} \quad (3.28)$$

The above result on the dissipation of the wave energy may be expressed as follows:

*The 3rd concept: The rate of dissipation of the energy of wind waves, or the rate of transfer of the wave momentum to current, is proportional to the dimensionless quantity ru_*L/ν or $ru_*^*T^{*2}$, and the factor K appearing there is a constant.*

One aspect of the substantiation of the 3rd concept will be given in the succeeding part of the present series of the articles, by the use of data of the breaking of wind waves.

3.4 The growth equation of wind waves

Equation (3.28) may be transformed to a dimensionless form to obtain the growth equation of wind waves:

$$\frac{d}{dt^*}(u_*^* T^{*2}) = r u_*^* R(1 - K T^{*2})$$

$$R = \frac{8\rho}{\pi \rho_w B^2} \quad (3.29)$$

If u_*^* is constant during the change of t^* , equation (3.29) is reduced to

$$\frac{d}{dt^*}(T^{*2}) = r R(1 - K T^{*2}) \quad (3.30)$$

Any state of the wind waves may be taken as the initial condition at that time.

The transformation from equation (3.29) of "the duration equation" to "the fetch equation" is performed by replacing dt by dF/C_g , where C_g is the group velocity of waves:

$$C_g = \frac{C}{2} = \frac{gT}{4\pi} \quad (3.31)$$

Namely,

$$\frac{d}{dt^*} \rightarrow \frac{1}{4\pi} T^* \frac{d}{dF^*} \quad (3.32)$$

Consequently, the fetch equation is

$$\frac{d}{dF^*}(u_*^* T^{*2}) = 4\pi r u_*^* R(1 - K T^{*2}) T^{*-1} \quad (3.33)$$

If u_*^* is constant, it is reduced to

$$\frac{d}{dF^*}(T^{*2}) = 4\pi r R(1 - K T^{*2}) T^{*-1} \quad (3.34)$$

3.5 Values of r

Assuming that Wilson's empirical formula of equation (3.11) approximates actual growth processes of wind waves, and substituting it into equation (3.34), values of r are empirically obtained, as a function of gF/U^2 , as follows:

$$r = \frac{r^2}{2\pi R} \frac{C_3 \left(\frac{gF}{U^2} \right)^{-\frac{2}{3}} \left\{ 1 + C_2 \left(\frac{gF}{U^2} \right)^{\frac{1}{3}} \right\}^{-6}}{C_1^{-2} \left[1 - \left\{ 1 + C_2 \left(\frac{gF}{U^2} \right)^{\frac{1}{3}} \right\}^{-5} \right]^2 - K} \quad (3.35)$$

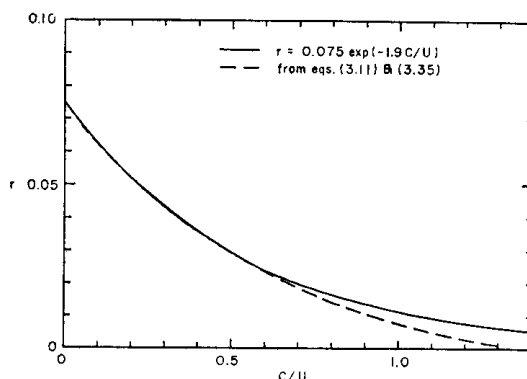


Fig. 5. Values of r as a function of C/U (see the context).

where

$$C_1 = \frac{2\pi}{\gamma} \times 1.37$$

$$C_2 = 0.008$$

and

$$C_3 = \frac{5}{3} C_1 C_2$$

Eliminating gF/U^2 from equations (3.35) and (3.11), values of r are shown by the broken line in Figure 5 as a function of C/U . The full line in Figure 5 is equation (3.36), which fits the broken line for smaller values of C/U :

$$r = 0.075 \exp(-1.9 C/U) \quad (3.36)$$

For $\gamma = 0.040$, it follows that

$$r = 0.075 \exp(-0.012 T^*) \quad (3.37)$$

The reliability of this form of r depends on the empirical formula of equation (3.11). The true form of r should be sought by a study of physical processes. However, it may be said that the rate of momentum which enters the wind waves is 7.5% for small C/U , and that it decreases nearly exponentially with the increasing C/U .

This seems to support also the before-mentioned notion of the skin-flow feeding waves, since the effect of mass convergence may cease for waves of which C is comparable to U .

The skin flow, that causes the mass convergence at the crest of large waves, may not

Table 2. Comparison between values of wave period, T , observed in a wind-wave tunnel (TOBA, 1961), and predicted by equations (3.34) and (3.37).

Mean values of u_* (cm/sec)	T (sec)				
	$F=6.9$ m	$F=10.0$ m		$F=13.6$ m	
	Observed	Observed	Predicted	Observed	Predicted
35.4	0.206	0.277	0.277	0.326	0.328
41.2	.264	.320	.320	.374	.367
48.7	.308	.352	.358	.416	.403
57.6	.337	.386	.388	.443	.431
67.3	.353	.416	.408	.478	.456
84.3	.390	.445	.447	.518	.498
101.1	.418	.482	.478	.540	.530
113.6	.455	.496	.512	.560	.565

always be the laminar skin flow, but may be some kind of larger scales of skin flow such as that caused by the form drag of small wavelets which make the surface of large waves rough, and also may include the maser effect proposed by LONGUET-HIGGINS (1969).

In the second column of Table 2 are shown values of T at $F=6.9$ m from Toba's 1961 wave data in the wind-wave tunnel from Table 1. Taking these values as initial conditions, the growth equation of wind waves, equation (3.34), has been integrated, for the average u_* values of each wind class, using r values of equation (3.37), and $B=0.062$, to predict values of T at $F=10.0$ m and $F=13.6$ m. The obtained values are shown in Table 2 together with the observed values. The agreement of the predicted values with the observed ones is remarkable, and it seems to show that the above treatment, including values of the two universal constants and of r , holds universally from the waves in a wind-wave tunnel to those in the open sea.

4. Development of drift current and production of turbulence

As already mentioned, the momentum of the air enters the water as the wind stress to become the momentum of the wind waves and the drift current, while the work done by the wind stress to the water becomes the mechanical energy of wind waves, turbulence and drift current. The momentum is a vector quantity while the energy is a scalar quantity. Consequently, turbulent motion, or a random motion, has the kinetic energy, but no average

momentum. We will have two systems of equations for the momentum and the energy.

In the total wind stress, τ , the part that once enters the waves is $r\tau$, so the rest of the part, $(1-r)\tau$, directly enters the drift current. As the wind waves grow, $r\tau KT^{*2}$ transfers from the waves to the current by turbulence. Namely, the rate of increase of the vertically integrated current momentum per unit horizontal area, M_c , is expressed by

$$\frac{dM_c}{dt} = \tau \{1 - r(1 - KT^{*2})\} \quad (4.1)$$

The rate of increase of the kinetic energy is not expressed in a simple way, since the vertical distribution of the drift current must be taken into account. However, one may write down the rate of supply of the part of turbulent energy, that is produced by the dissipation of energy of organized wave motion, E_t , as follows:

$$\begin{aligned} \frac{dE_t}{dt} &= r\tau u_0 K T^{*2} \\ &= r\pi^3 B^2 \rho g \nu K u_*^* T^{*2} \\ &= 2\pi^4 B^2 \rho g \nu K r u_* L / \nu \end{aligned} \quad (4.2)$$

As is evident from the before-mentioned discussion, it is proportional to $ru_* L / \nu$.

Further treatment about the compound system of wind waves, turbulence and drift current will be given separately.

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海面境界過程における局所平衡

I. 風波の発達過程について

鳥 羽 良 明

要旨 単純なスペクトルの風波に対して、3つの基本的概念の上に、新しい発達方程式が提出された。波の代表量としては、ピーク周波数とエネルギーレベルとに対応するものとしての、無次元形の有義波の周期と波高とを用いている。3つの基本的概念の1つは、局所平衡の概念であり、他の2つは、それぞれ、風波のエネルギーの獲得、および、風波のエネルギーの逸散もしくは風波の運動量の流れへの移行に関するものである。この方程式は、風波のエネルギーの獲得と逸散に関する2つの普遍定数（それぞれ、 $B=6.2 \times 10^{-2}$ と $K=2.16 \times 10^{-5}$ ）とともに、風洞水槽の中の波から、外洋の十分発達した風波に至る広範囲の風波に普遍的にあてはまるようである。

また、「単純なスペクトルの風波に関する3/2乗則」、および、その系として、風波の軌道運動による表面の平均流速は常に風の摩擦速度に比例すること、重力波の範囲で波形勾配は波令の平方根に逆比例することなど、一連の関係が導かれた。また、風の海面応力のうち直接風波に入る部分の割合は、波令に対してほぼ指数函数的に減少し、波令の若い波に対しては、7.5%であることも導かれた。

また、吹走流の運動量の増加、および、風波による乱れのエネルギーの上層海洋への供給に関する方程式も示された。