INSTITUTE OF PHYSICS PUBLISHING WAVES IN RANDOM MEDIA

Waves Random Media **14** (2004) 97–108 PII: S0959-7174(04)63641-X

Wave scattering in a multiscale random inhomogeneous medium

Mikhail V Tinin[1]

Miryang National University, 1025-1, Naei-dong, Miryang, Gyeongsangnam-do, 627-702, Korea

E-mail: mtinin@mnu.ac.kr

Received 18 May 2003
Published 10 December 2003
Online at stacks.iop.org/WRM/14/97 (DOI: 10.1088/0959-7174/14/2/001)

Abstract
In this paper, using the Fock method of the fifth parameter and weighted Fourier-transform with respect to the coordinates of the source and observer, an integral representation is obtained for the wave field in a randomly inhomogeneous medium without invoking the assumption about small-angle propagation. Random trajectory variations to a first approximation are taken into account in calculating the partial wave phase (the expression under the integral sign). The expressions for the field in a medium with different-scale irregularities and for the scintillation index, obtained using this integral representation, are compared with known results. The good agreement with results from the theory of single scattering in a medium with background irregularities, and with investigations of the scintillation index made in terms of Rytov's method and path integrals, indicates that it is possible to use the approach developed in this study to describe the effects of simultaneous influence of different-scale irregularities.

1. Introduction

A variety of approximate methods of solving the wave equation are extensively used to describe wave propagation and scattering in inhomogeneous media. This is because rigorous solutions of the wave equation in inhomogeneous media are quite few. Such solutions exist mainly for some stratified inhomogeneous media and are commonly used to analyse some characteristic properties of wave propagation, and the validity range of approximate methods. An especially large role is played by the approximate methods in investigating the wave propagation in random inhomogeneous media. When the wave problem is solved using approximate methods, the presence of a particular small parameter in the problem is used. In some methods, such a parameter is represented by the relative deviation of dielectric permittivity of the propagation medium from an 'undisturbed' one. In other methods, the small parameter is represented by the ratio of some characteristic scales of the problem. The most widely used technique of the former

[1] Permanent address: Irkutsk State University, b. Gagarina-20, Irkutsk, 664003, Russia.

type is the perturbation method (Born approximation), which is extensively used in the single-scattering theory. The method of the latter type may be exemplified by the ray approximation, based on using the smallness of the characteristic field scale (the wavelength and the Fresnel zone) when compared with the irregularity size. Within the small-angle approximation, the wave equation reduces to a parabolic equation. In this case, using the method of smooth perturbations [1] it is possible to find the solution taking into account both the diffraction effects (as in the Born approximation) and the random refraction in an inhomogeneous medium caused by multiple scattering (as in the ray approximation). However, this method neglects strong fluctuations caused by multipathing. It is a relatively easy matter to take into account the random multipathing that appears behind the bounded layer with irregularities, by replacing it by a certain phase screen. For investigating the wave propagation in an extended random inhomogeneous medium, it is customary to use asymptotic methods to seek the solution of the parabolic equation for statistical field moments [2–6]. However, for developing techniques for optimal processing of signals in communication, navigation and diagnostic systems, it is often necessary to know not the field moments but the spatial and temporal field structure. The objective of this paper is to develop a method for describing the field of the wave propagating in a multiscale inhomogeneous medium.

In [7], for investigating the propagation of radio waves in a two-dimensional inhomogeneous medium within the small-angle approximation, the parabolic equation is solved using a double (over the coordinates of the source and observer) weighted Fourier transform. It is shown that the expression thus obtained for the wave field is consistent with results from the method of smooth perturbations, the phase screen method, and the ray approximation; that is, the resulting expression takes into account the Fresnel diffraction effects and strong intensity fluctuations. In [8], the same problem is solved without assuming the small-angle propagation; for this purpose, Fock's method of the fifth parameter is used to reduce the initial wave equation to a parabolic equation which is solved by the method of mixed weighted Fourier transform. It is shown that within the small-angle approximation, results obtained change to the findings from [7], and for weak fluctuations they change to the Born approximation. However, although the results from [8] transform to corresponding expressions for large-scale and small-scale irregularities, both irregularities are not considered simultaneously here.

Taking into account different-scale irregularities simultaneously is possible in the case of a sufficiently rapidly decreasing irregularity spectrum. In this case the small-scale part of the spectrum can be taken into account in terms of perturbation theory if the medium with a large-scale irregularity is taken as an 'undisturbed' medium [9]. But the question about the solution of the unperturbed part of the problem in the case of strong phase fluctuations remains open, because the usual ray optics is inapplicable in conditions where random caustics and strong intensity fluctuations appear.

In this paper we develop the results of [8] to obtain an integral representation for the field of the wave propagating in an inhomogeneous medium in which large-scale and small-scale irregularities are present at the same time. For this purpose, in section 2 we find the integral representation for the wave field in an inhomogeneous medium, as is done in [8] using the fifth-parameter Fock's method and the method of mixed integral representations. Unlike in [8], here we take into account the variations of the trajectory along which the eikonal equation is integrated. Next, in section 3, we compare the resulting equation with known results. Section 3.1 shows a good agreement of our results with those from the hybrid approach for the propagation of the wave in an inhomogeneous medium with different scales. Section 3.2 compares the results from the present approach and from the method of path integrals derived from investigation the scintillation index. Concluding remarks are made in section 4.

2. Integral representation for the field of a point source in an inhomogeneous medium

Within the limits of a scalar approximation, a wave equation for the field $U(\mathbf{r}, \mathbf{r}_0)$ of a harmonic point source located at the point $\mathbf{r}_0$ in an inhomogeneous medium has the form

$$\Delta U(\mathbf{r}, \mathbf{r}_0) + k^2\varepsilon(\mathbf{r})U(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0), \tag{1}$$

where $k = \omega/c$ is the wavenumber, c is the velocity of light in a free space, ω is angular frequency and $\varepsilon(\mathbf{r}) = 1 + \tilde{\varepsilon}(\mathbf{r})$ is dielectric permittivity of the medium.

Following Fock's method of the fifth parameter (see, for example, [10]), we represent the solution of (1) as an integral over the parameter τ:

$$U(\mathbf{r}, \mathbf{r}_0) = -\frac{\mathrm{i}}{2k}\int_0^\infty U_\tau(\tau, \mathbf{r}, \mathbf{r}_0)\,\mathrm{d}\tau. \tag{2}$$

It is easy to see that $U_\tau(\tau, \mathbf{r}, \mathbf{r}_0)$ satisfies the parabolic equation

$$2\mathrm{i}k\frac{\partial U_\tau}{\partial \tau} + \Delta U_\tau + k^2\varepsilon(\mathbf{r})U_\tau = 0 \tag{3}$$

with the initial condition

$$U_\tau(0, \mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0). \tag{4}$$

In a homogeneous medium where $\varepsilon(\mathbf{r}) \equiv 1$, equation (3) takes the form

$$2\mathrm{i}k\frac{\partial U_{\tau 0}}{\partial \tau} + \Delta U_{\tau 0} + k^2 U_{\tau 0} = 0. \tag{5}$$

It is an easy matter to find the solution of (5) with the condition (4):

$$U_{\tau 0}(\tau, \mathbf{r}, \mathbf{r}_0) = \left(\frac{k}{2\pi\tau}\right)^{3/2}\exp\left\{\mathrm{i}k\left[\frac{\tau}{2} + \frac{(\mathbf{r} - \mathbf{r}_0)^2}{2\tau}\right] - \mathrm{i}\frac{3\pi}{4}\right\}. \tag{6}$$

To solve (3) and (4), we use the method of weighted Fourier transform [7]. To do this, we introduce an auxiliary function

$$V(\tau, \mathbf{r}, \mathbf{r}_2, \mathbf{r}_0, \mathbf{r}_{02}) = U_\tau(\tau, \mathbf{r}, \mathbf{r}_0)U^*_{\tau 0}(\tau, \mathbf{r}_2, \mathbf{r}_{02}), \tag{7}$$

where $U^*_{\tau 0}(\tau, \mathbf{r}_2, \mathbf{r}_{02})$ is the complex conjugate of the field at the point $\mathbf{r} = \mathbf{r}_2$ of the point source located at the point $\mathbf{r}_{02}$ in a homogeneous medium, i.e. $U^*_{\tau 0}(\tau, \mathbf{r}_2, \mathbf{r}_{02})$ satisfies the equation

$$2\mathrm{i}k\frac{\partial U^*_{\tau 0}}{\partial \tau} - \Delta U^*_{\tau 0} - k^2 U^*_{\tau 0} = 0, \tag{8}$$

$$U^*_{\tau 0}(0, \mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0). \tag{9}$$

If we introduce the new variables

$$\mathbf{r}_+ = \tfrac{1}{2}(\mathbf{r} + \mathbf{r}_2), \qquad \mathbf{r}_{0+} = \tfrac{1}{2}(\mathbf{r}_0 + \mathbf{r}_{02}), \qquad \mathbf{r}_- = \mathbf{r} - \mathbf{r}_2, \qquad \mathbf{r}_{0-} = \mathbf{r}_0 - \mathbf{r}_{02}, \tag{10}$$

then from (3), (4) and (8), (9) it is possible to obtain the following equation for (7):

$$\frac{\partial V}{\partial \tau} = \frac{\mathrm{i}}{k}\frac{\partial^2 V}{\partial \mathbf{r}_+\partial \mathbf{r}_-} + \frac{\mathrm{i}k}{2}\left[\varepsilon\left(\mathbf{r}_+ + \frac{\mathbf{r}_-}{2}\right) - 1\right]V, \tag{11}$$

$$V(0, \mathbf{r}_+, \mathbf{r}_-, \mathbf{r}_{0+}, \mathbf{r}_{0-}) = \delta(\mathbf{r}_+ - \mathbf{r}_{0+})\delta(\mathbf{r}_- - \mathbf{r}_{0-}). \tag{12}$$

To solve (11), we pass from $V(\tau, \mathbf{r}_+, \mathbf{r}_-, \mathbf{r}_{0+}, \mathbf{r}_{0-})$ to the Fourier transform with respect to $\mathbf{r}_{0+}$, which is similar to using the interference integral method [6, 11, 12] and the Fourier transform with respect to $\mathbf{r}_-$, which is similar to the approach of the Maslov method [6, 11, 12]. As a result, we obtain

$$V(\tau, \mathbf{r}_+, \mathbf{r}_-, \mathbf{r}_{0+}, \mathbf{r}_{0-}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{V}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})\exp[\mathrm{i}k(\mathbf{r}_- \cdot \mathbf{p} - \mathbf{r}_{0+} \cdot \mathbf{s})]\,\mathrm{d}^2s\,\mathrm{d}^2p, \tag{13}$$

where $\bar{V}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})$ is the solution of the equation

$$\frac{\partial}{\partial \tau}\bar{V} = -\mathbf{p}\cdot\frac{\partial}{\partial \mathbf{r}_+}\bar{V} + \frac{\mathrm{i}k}{2}\tilde{\varepsilon}\left(\mathbf{r}_+ - \frac{1}{2\mathrm{i}k}\frac{\partial}{\partial \mathbf{p}}\right)\bar{V}, \tag{14}$$

$$\bar{V}(0, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \left(\frac{k}{2\pi}\right)^6 \exp[-\mathrm{i}k(\mathbf{p}\cdot\mathbf{r}_{0-} - \mathbf{s}\cdot\mathbf{r}_+)]. \tag{15}$$

We seek the solution of the problem of (14) and (15) in the form of a Debye series

$$\bar{V}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \sum_{m=0}^{\infty}\frac{A^{(m)}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})}{(\mathrm{i}k)^m}\exp[\mathrm{i}k\varphi(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})]. \tag{16}$$

On substituting (16) into (14) and equalizing terms with identical powers, we obtain the eikonal equation for $\Phi(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})$

$$\frac{\partial\varphi}{\partial\tau} + \mathbf{p}\cdot\frac{\partial\varphi}{\partial\mathbf{r}_+} - \frac{1}{2}\tilde{\varepsilon}\left(\mathbf{r}_+ - \frac{1}{2}\frac{\partial\varphi}{\partial\mathbf{p}}\right) = 0, \tag{17}$$

$$\varphi(0, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \mathbf{s}\cdot\mathbf{r}_+ - \mathbf{p}\cdot\mathbf{r}_{0-}, \tag{18}$$

and a system of recurrent transfer equations for $A^{(m)}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})$. Next, we confine ourselves to the first term in (16), for which $A(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = A^{(0)}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-})$ satisfies the equation

$$\frac{\partial A}{\partial\tau} + \mathbf{p}\cdot\frac{\partial A}{\partial\mathbf{r}_+} + \frac{A}{4}\frac{\partial^2}{\partial\mathbf{p}\partial\mathbf{r}_+}\tilde{\varepsilon}\left(\mathbf{r}_+ - \frac{1}{2}\frac{\partial\varphi}{\partial\mathbf{p}}\right) + \frac{\partial A}{\partial\mathbf{p}}\cdot\frac{\partial}{\partial\mathbf{r}_+}\tilde{\varepsilon}\left(\mathbf{r}_+ - \frac{1}{2}\frac{\partial\varphi}{\partial\mathbf{p}}\right) = 0 \tag{19}$$

$$A(0, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \left(\frac{k}{2\pi}\right)^6. \tag{20}$$

With the known solution for equation (17), it is easy to find the solution to the first-order linear equation (19).

Solving (17) by the method of characteristics, we obtain

$$\varphi(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \bar{\varphi}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) + \tilde{\varphi}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}), \tag{21}$$

where

$$\bar{\varphi}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = -\mathbf{s}\cdot\mathbf{p}\tau + \mathbf{s}\cdot\mathbf{r}_+ - \mathbf{p}\cdot\mathbf{r}_{0-}, \tag{22}$$

$$\tilde{\varphi}(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \frac{1}{2}\int_0^{\tau}\tilde{\varepsilon}[\bar{\mathbf{r}}(\tau') + \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau')]\,\mathrm{d}\tau' - \frac{1}{2}\int_0^{\tau}\tilde{\mathbf{r}}_-(\tau')\cdot\frac{\partial}{\partial\mathbf{r}}\tilde{\varepsilon}[\bar{\mathbf{r}}(\tau') + \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau')]\,\mathrm{d}\tau'. \tag{23}$$

Here

$$\bar{\mathbf{r}}(\tau') = \mathbf{p}(\tau' - \tau) + \mathbf{r}_+ + (\mathbf{r}_{0-} + \mathbf{s}\tau')/2 \tag{24}$$

and $\tilde{\mathbf{r}}_+(\tau')$, $\tilde{\mathbf{r}}_-(\tau')$ are the solutions of the same system of ray equations

$$\frac{\mathrm{d}\tilde{\mathbf{r}}_{+,-}}{\mathrm{d}\tau'} = \tilde{\mathbf{p}}_{+,-} \qquad \frac{\mathrm{d}\tilde{\mathbf{p}}_{+,-}}{\mathrm{d}\tau'} = \frac{1}{4}\frac{\partial}{\partial\mathbf{r}}\tilde{\varepsilon}[\bar{\mathbf{r}}(\tau') + \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau')] \tag{25}$$

but with different boundary conditions:

$$\tilde{\mathbf{r}}_+(\tau' = \tau) = \tilde{\mathbf{p}}_+(\tau' = \tau) = 0, \tag{26}$$

$$\tilde{\mathbf{r}}_-(\tau' = 0) = \tilde{\mathbf{p}}_-(\tau' = 0) = 0. \tag{27}$$

It is assumed that the medium is weakly inhomogeneous, that is, the permittivity variance σ_ε^2 is small. We can then solve equations (19), (20) and (25)–(27) using the perturbation method to give

$$\tilde{\mathbf{r}}_+(\tau') \simeq -\frac{1}{4}\int_{\tau'}^{\tau}(\tau' - \tau'')\frac{\partial}{\partial \mathbf{r}}\tilde{\varepsilon}[\bar{\mathbf{r}}(\tau'')]\,\mathrm{d}\tau'' + \mathrm{O}(\tilde{\varepsilon}^2), \tag{28}$$

$$\tilde{\mathbf{r}}_-(\tau') \simeq \frac{1}{4}\int_{0}^{\tau'}(\tau' - \tau'')\frac{\partial}{\partial \mathbf{r}}\tilde{\varepsilon}[\bar{\mathbf{r}}(\tau'')]\,\mathrm{d}\tau'' + \mathrm{O}(\tilde{\varepsilon}^2), \tag{29}$$

$$A(\tau, \mathbf{r}_+, \mathbf{p}, \mathbf{s}, \mathbf{r}_{0-}) = \left(\frac{k}{2\pi}\right)^6 + \mathrm{O}(\tilde{\varepsilon}^2). \tag{30}$$

From this, in view of (16), (21)–(30), it is easy to find the solution to equation (14) and then from (13), (7) and (6) to find $U_\tau(\tau, \mathbf{r}, \mathbf{r}_0)$; next, substituting $\mathbf{r}_2 - \mathbf{r}_{02} = -(\mathbf{r} + \mathbf{r}_0)$, $\mathbf{s} \to 2\mathbf{s}$, $\mathbf{p} \to -\mathbf{p}$ and $U_\tau(\tau, \mathbf{r}, \mathbf{r}_0)$ into (2) we obtain the following expression for the field:

$$U = C\int_0^\infty \tau^{3/2}\,\mathrm{d}\tau \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^3 s\,\mathrm{d}^3 p \exp\left\{\mathrm{i}k\left[\frac{\tau}{2} + \frac{(\mathbf{r}+\mathbf{r}_0)^2}{2\tau} + 2(\mathbf{s}\cdot\mathbf{p}\tau - \mathbf{s}\cdot\mathbf{r}_0 - \mathbf{p}\cdot\mathbf{r}) + \tilde{\varphi}(\tau, \mathbf{s}, \mathbf{p})\right]\right\}. \tag{31}$$

Here $C = \frac{4}{k}(\frac{k}{2\pi})^{9/2}\exp(\frac{\mathrm{i}3\pi}{4})$, and $\tilde{\varphi}(\tau, \mathbf{s}, \mathbf{p})$ is determined by (23), (28), (29) with $\bar{\mathbf{r}}(\tau') = \mathbf{p}(\tau - \tau') + \mathbf{s}\tau'$.

The resulting expression (31) is similar to the expression obtained in [8]. The only difference is that in [8], for calculating the eikonal, the integral is taken along a straight unperturbed ray, whereas in the case under consideration it is taken along the ray trajectory in an inhomogeneous medium. This difference leads to the fact that, as will be shown below, unlike the results from [8] that make it possible to describe the effects of both small-scale and large-scale irregularities, the expression (31) can be used to take into account these effects at the same time. Note that this case uses two systems of rays satisfying the same equation, but with the initial conditions at different points. A single system of rays could be used. But then we would obtain boundary conditions of the mixed type that make the solution of the trajectory problem difficult.

3. The relation of the resulting solution to known results

To obtain the integral representation (31), a number of approximations were used: (i) only the first term is left in the Debye expansion (16); and (ii) perturbation theory was used to determine the eikonal and amplitude of this term. In asymptotic methods, because of the great complexity of the subsequent approximations, the validity range of an approximate solution is usually not determined through a comparison with them. The validity range is normally determined through a comparison with results from other methods for which the validity range is known. Here we avail ourselves of just such an approach and compare our results with those from the hybrid approach [9], and with results of an asymptotic integration of path integrals.

3.1. The scattering of fluctuating waves from small-scale irregularities

When trajectory variations are small in comparison with characteristic scales of irregularities, these variations can be neglected in (23) to obtain the expressions for the field investigated in [8], where it was pointed out that these expressions change to results of the Born approximation for small-scale irregularities and ray approximation for large-scale irregularities. In a multiscale inhomogeneous medium, situations are possible where trajectory variations are of the same order of magnitude as or larger than the irregularity scales from the high-frequency part of the spectrum. In this case we perform an analysis of the solution (31) by representing, as is done in [9], the random irregularity field $\tilde{\varepsilon}(\mathbf{r})$ as the sum of two random uncorrelated fields:

$$\tilde{\varepsilon}(\mathbf{r}) = \tilde{\varepsilon}_1(\mathbf{r}) + \tilde{\varepsilon}_2(\mathbf{r}). \tag{32}$$

Here $\tilde{\varepsilon}_1(\mathbf{r})$ is the large-scale field, and $\tilde{\varepsilon}_2(\mathbf{r})$ the small-scale field.

Then

$$\tilde{\varphi}(\tau, \mathbf{s}, \mathbf{p}) = \tilde{\varphi}_1(\tau, \mathbf{s}, \mathbf{p}) + \tilde{\varphi}_2(\tau, \mathbf{s}, \mathbf{p}), \tag{33}$$

where

$$\tilde{\varphi}_i = \frac{1}{2}\int_0^\tau \left\{ \tilde{\varepsilon}_i[\bar{\mathbf{r}}(\tau') + \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau')] - \tilde{\mathbf{r}}_-(\tau') \cdot \frac{\partial}{\partial \mathbf{r}} \tilde{\varepsilon}_i[\bar{\mathbf{r}}(\tau') + \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau')] \right\} \mathrm{d}\tau' \\ i = 1, 2. \tag{34}$$

Taking into account the rapid decrease of the irregularity spectrum, it is possible to choose such a separation of (32) where

$$k\tilde{\varphi}_2(\tau, \mathbf{s}, \mathbf{p}) \ll 1 \tag{35}$$

at least in the region of importance for integrating in (31). On substituting $\exp\{\mathrm{i}k\tilde{\varphi}_2\} \approx 1 + \mathrm{i}k\tilde{\varphi}_2$ into (31), we obtain

$$U(\mathbf{r}, \mathbf{r}_0) = U_1(\mathbf{r}, \mathbf{r}_0) + U_2(\mathbf{r}, \mathbf{r}_0) + \mathrm{O}(\tilde{\varepsilon}_2^2), \tag{36}$$

where

$$U_1 = C\int_0^\infty \tau^{3/2}\,\mathrm{d}\tau \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^3 s\,\mathrm{d}^3 p \exp\left\{\mathrm{i}k\left[\frac{\tau}{2} + \frac{(\mathbf{r}+\mathbf{r}_0)^2}{2\tau} + 2(\mathbf{s}\cdot\mathbf{p}\tau - \mathbf{s}\cdot\mathbf{r}_0 - \mathbf{p}\cdot\mathbf{r})\right]\right\} \\ \times \exp\{\mathrm{i}k[\tilde{\varphi}_1(\tau, \mathbf{s}, \mathbf{p})]\}, \tag{37}$$

$$U_2(\mathbf{r}, \mathbf{r}_0) = C\int_0^\infty \tau^{3/2}\,\mathrm{d}\tau \int_0^\tau \mathrm{d}\tau' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^3 s\,\mathrm{d}^3 p\, \tilde{\varepsilon}_2[\bar{\mathbf{r}}(\tau') + \rho(\tau')] \\ \times \exp\left\{\mathrm{i}k\left[\frac{\tau}{2} + \frac{(\mathbf{r}+\mathbf{r}_0)^2}{2\tau} + 2(\mathbf{s}\cdot\mathbf{p}\tau - \mathbf{s}\cdot\mathbf{r}_0 - \mathbf{p}\cdot\mathbf{r}) + \tilde{\varphi}_1(\tau, \mathbf{s}, \mathbf{p})\right]\right\}. \tag{38}$$

Here

$$\tilde{\varphi}_1(\tau, \mathbf{s}, \mathbf{p}) \approx \tfrac{1}{2}\int_0^\tau \{\tilde{\varepsilon}_1[\bar{\mathbf{r}}(\tau')]\}\,\mathrm{d}\tau', \tag{39}$$

$$\rho(\tau') = \tilde{\mathbf{r}}_+(\tau') + \tilde{\mathbf{r}}_-(\tau') = \frac{1}{4}\int_0^\tau |\tau' - \tau''| \frac{\partial}{\partial \mathbf{r}} \tilde{\varepsilon}_1[\bar{\mathbf{r}}(\tau'')]\,\mathrm{d}\tau'', \tag{40}$$

and it is taken into consideration that the main contribution to the ray fluctuations is made by the large-scale part of the irregularity spectrum. The field component $U_1(\mathbf{r}, \mathbf{r}_0)$ is related to large-scale irregularities only, and is responsible for the appearance of random multipathing and random caustics. Small-scale irregularities govern only $U_2(\mathbf{r}, \mathbf{r}_0)$ that describes the scattering by these irregularities of the wave propagating in a medium with large-scale irregularities. If in the neighbourhood of a small-scale irregularity there are no random caustics for sources located at the points of radiation and observation, then, with the typical scale of a large-scale (background) irregularity larger than the largest section of the Fresnel volume [12], it is possible to obtain the following expression through an asymptotic integration of (38) using the method of stationary phase:

$$U_2(\mathbf{r}, \mathbf{r}_0) \approx -k^2 \int_V \mathrm{d}^3 r'\, \tilde{\varepsilon}_2(\mathbf{r}') U_{go}(\mathbf{r}, \mathbf{r}') U_{go}(\mathbf{r}', \mathbf{r}_0), \tag{41}$$

where $U_{go}(\mathbf{r}, \mathbf{r}')$ is the Green function for a medium with large-scale irregularities. The expression (41) coincides with the formula obtained in [9] by the hybrid method. As pointed out in [9], in the absence of large-scale irregularities, the expression (41) changes into a usual Bragg approximation and is its generalization to the case of an inhomogeneous background. Since the expression (41) describes the enhancement of backscattering [9], it follows that formula (31) can also be used in investigating the enhancement of backscattering.

3.2. Intensity fluctuations

Let us consider the potential of our approach for describing the wave propagation in a random inhomogeneous medium. For this purpose, it is necessary to use formula (31) to determine the statistical moments for the field of the wave propagating in a random inhomogeneous multiscale medium, and to compare them with existing results. Of these results, the most interesting findings were derived from investigating the behaviour of intensity fluctuations [1], as they show up diffraction effects over small distances, the appearance of random multipathing and random caustics over intermediate distances, and a normalization of the field and saturation of fluctuations over large distances. Since the published results mainly refer to small-angle propagation, we shall use here the small-angle approximation of formula (31). To do this, we choose a coordinate system $\mathbf{r} = \{z, \mathbf{r}_\perp\}$ in which the axis z passes through the points of the source and observation, and introduce new notations for the integration variables in (31): $\mathbf{s} = \{s_z, \mathbf{s}_\perp\}$, $\mathbf{p} = \{p_z, \mathbf{p}_\perp\}$. Taking into account $kl_\varepsilon \gg 1$ (l_ε is the irregularity size) and the smallness of deviation of the trajectory from the axis z (that is, when $|\mathbf{s}_\perp| \ll 1$, $|\mathbf{p}_\perp| \ll 1$), the integrals over s_z, p_z and τ in (31) are evaluated by the method of stationary phase to give a small-angle approximation

$$U(\mathbf{r}, \mathbf{r_0}) \approx C_2 \int\!\!\int \mathrm{d}^2 s_\perp\, \mathrm{d}^2 p_\perp \exp\Bigg\{\mathrm{i}k\Bigg[|z - z_0| + 2(\mathbf{s}_\perp \cdot \mathbf{p}_\perp (z - z_0) - \mathbf{s}_\perp \cdot \mathbf{r}_{0\perp} - \mathbf{p}_\perp \cdot \mathbf{r}_\perp) + \tfrac{1}{2}\int_{z_0}^{z} \tilde{\varepsilon}[\bar{\mathbf{r}}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z') + \boldsymbol{\rho}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z'), z']\,\mathrm{d}z'\Bigg]\Bigg\}(z - z_0). \tag{42}$$

Here

$$\bar{\mathbf{r}}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z') = \mathbf{p}_\perp(z - z') + \mathbf{s}_\perp(z' - z_0), \tag{43}$$

$$\boldsymbol{\rho}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z') = \frac{1}{4}\int_{z_0}^{z} |z' - z''| \frac{\partial}{\partial \mathbf{r}_\perp} \tilde{\varepsilon}[\bar{\mathbf{r}}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z''), z'']\,\mathrm{d}z'', \tag{44}$$

$$C_2 = \frac{2k^2}{(2\pi)^3} \exp\left\{\mathrm{i}\frac{3\pi}{4}\right\}. \tag{45}$$

As is known [1], intensity fluctuations are described by the scintillation index

$$\beta^2 = \frac{\langle |U|^4 \rangle}{\langle |U|^2 \rangle^2} - 1. \tag{46}$$

In calculating the moments using (42), we again use the hybrid approach [9]. To do this, as is done in the previous section, we represent the random irregularity field $\tilde{\varepsilon}(\mathbf{r})$ as the sum (32).

Using (42), for the intensity squared we obtain

$$I^2 = |U|^4 = |C_2|^4 (z - z_0)^4 \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mathrm{d}^2 s_{\perp 1} \cdots \mathrm{d}^2 s_{\perp 4}\, \mathrm{d}^2 p_{\perp 1} \cdots \mathrm{d}^2 p_{\perp 4} \times \exp\left\{\mathrm{i}k\left[2\sum_{m=1}^{4} (-1)^{m+1} \mathbf{s}_{\perp m} \cdot \mathbf{p}_{\perp m}(z - z_0) + \tilde{\varphi}_{1+} + \tilde{\varphi}_{2+}\right]\right\}, \tag{47}$$

where

$$\tilde{\varphi}_{1+} = \sum_{m=1}^{4} \frac{(-1)^{m+1}}{2} \int_{z_0}^{z} \tilde{\varepsilon}_1[\bar{\mathbf{r}}_\perp(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z'), z']\, dz', \tag{48}$$

$$\tilde{\varphi}_{2+} = \sum_{m=1}^{4} \frac{(-1)^{m+1}}{2} \int_{z_0}^{z} \tilde{\varepsilon}_2[\bar{\mathbf{r}}_\perp(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z') + \boldsymbol{\rho}_{\perp 1}(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z'), z']\, dz', \tag{49}$$

$$\boldsymbol{\rho}_{\perp 1}(\mathbf{p}_\perp, \mathbf{s}_\perp, z') = \frac{1}{4} \int_{z_0}^{z} |z' - z''| \frac{\partial}{\partial \mathbf{r}_\perp} \tilde{\varepsilon}_1[\bar{\mathbf{r}}_\perp(\mathbf{p}_\perp, \mathbf{s}_\perp, z'), z'']\, dz''.$$

Here we again take into consideration that the main contribution to the ray fluctuations is made by the large-scale part of the irregularity spectrum, and trajectory variations can be of the same order of magnitude as the irregularity scales from the high-frequency part of the spectrum.

We assume that a separation of the irregularity of the medium (32) was carried out in accordance with the applicability of the condition

$$k\tilde{\varphi}_{2+} \ll 1. \tag{50}$$

Then

$$\exp\{ik\tilde{\varphi}_{2+}\} \approx 1 + ik\tilde{\varphi}_{2+} - \frac{k^2}{2}\tilde{\varphi}_{2+}^2. \tag{51}$$

We now substitute (51) into (47) and average the resulting expression over the realizations of fast (small-scale) irregularities $\tilde{\varepsilon}_2$. As a result, we obtain

$$\langle I^2 \rangle|_2 \approx I_1^2 + I_2^2, \tag{52}$$

where

$$I_1^2 = |C_2|^4 (z - z_0)^4 \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d^2 s_{\perp 1} \cdots d^2 s_{\perp 4}\, d^2 p_{\perp 1} \cdots d^2 p_{\perp 4} \times \exp\left\{ik\left[2\sum_{m=1}^{4} (-1)^{m+1} \mathbf{s}_{\perp m} \cdot \mathbf{p}_{\perp m}(z - z_0) + \tilde{\varphi}_{1+}\right]\right\}, \tag{53}$$

$$I_2^2 = -\frac{k^2 |C_2|^4}{2} (z - z_0)^4 \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d^2 s_{\perp 1} \cdots d^2 s_{\perp 4}\, d^2 p_{\perp 1} \cdots d^2 p_{\perp 4} \times \exp\left\{ik\left[2\sum_{m=1}^{4} (-1)^{m+1} \mathbf{s}_{\perp m} \cdot \mathbf{p}_{\perp m}(z - z_0) + \tilde{\varphi}_{1+}\right]\right\} \langle \tilde{\varphi}_{2+}^2 \rangle|_2. \tag{54}$$

Here the notation $\langle\ \rangle|_2$ defines a condition averaging over the realizations $\tilde{\varepsilon}_2(\mathbf{r})$ at a fixed $\tilde{\varepsilon}_1(\mathbf{r})$.

To calculate $\langle \tilde{\varphi}_{2+}^2 \rangle|_2$ we use [13] the following representation:

$$\tilde{\varepsilon}_2[\rho, z'] = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2\chi\, d^2 r\, \tilde{\varepsilon}_2[\mathbf{r}, z'] \exp\{i\boldsymbol{\chi} \cdot (\mathbf{r} - \rho)\}. \tag{55}$$

Taking into account (55), the statistical inhomogeneity and the delta correlation in z of the random field $\tilde{\varepsilon}_2(\mathbf{r})$, we obtain

$$\begin{aligned} \langle \tilde{\varphi}_{2+}^2 \rangle|_2 = \frac{2}{(4\pi)^2} \int_{z_0}^{z} dz_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^2\chi\, d^2 r \exp\{i\boldsymbol{\chi} \cdot \mathbf{r}\}\{2A_2(\mathbf{r}) \\ - A_2(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)] \\ + A_2(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_3 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_3)] \\ - A_2(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_4 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_4)] \\ - A_2(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_3 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_3)] \\ + A_2(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_4 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_4)] \\ - A_2(\bar{\mathbf{r}}_3 - \bar{\mathbf{r}}_4 + \mathbf{r}) \exp[-i\boldsymbol{\chi} \cdot (\boldsymbol{\rho}_3 - \boldsymbol{\rho}_4)]\}, \end{aligned} \tag{56}$$

where

$$
\begin{aligned}
&\bar{\mathbf{r}}_m = \bar{\mathbf{r}}_\perp(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z_1),\\
&\boldsymbol{\rho}_m = \boldsymbol{\rho}_{\perp 1}(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z_1) = \frac{1}{4}\int_{z_0}^{z} |z_1 - z'| \frac{\partial}{\partial \mathbf{r}_\perp}\tilde{\varepsilon}_1[\bar{\mathbf{r}}_\perp(\mathbf{p}_{\perp m}, \mathbf{s}_{\perp m}, z'), z']\,\mathrm{d}z',\\
&A_2(\boldsymbol{\rho}) = \int_{-\infty}^{\infty} \Psi_{\varepsilon 2}(\boldsymbol{\rho}, \xi)\,\mathrm{d}\xi = 2\pi\int_{-\infty}^{\infty} \mathrm{d}^2\kappa\,\Phi_{\varepsilon 2}(\boldsymbol{\kappa}, 0)\exp(\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho}).
\end{aligned}
\tag{57}
$$

$\Psi_{\varepsilon 2}(\mathbf{r}_1 - \mathbf{r}_2) = \langle \tilde{\varepsilon}_2(\mathbf{r_1})\tilde{\varepsilon}_2(\mathbf{r_2})\rangle$ and $\Phi_{\varepsilon 2}(\boldsymbol{\kappa}, \kappa_z)$ are the correlation function and the spectrum of the small-scale component of dielectric permittivity, respectively.

Then we substitute (56) into (54) and note that for a smooth irregularity $\tilde{\varepsilon}_1(\mathbf{r})$ it is possible to make use of an approximate relation

$$
\begin{aligned}
&\pm\frac{\boldsymbol{\chi}}{4k}\cdot\int_{z_0}^{z} |z_1 - z'|\frac{\partial}{\partial \bar{\mathbf{r}}_m}\tilde{\varepsilon}_1(\bar{\mathbf{r}}_m, z')\,\mathrm{d}z' + \frac{1}{2}\int_{z_0}^{z}\tilde{\varepsilon}_1(\bar{\mathbf{r}}_m, z')\,\mathrm{d}z'\\
&\qquad\approx \frac{1}{2}\int_{z_0}^{z}\tilde{\varepsilon}_1\left[\pm\frac{\boldsymbol{\chi}}{2k}|z - z'| + \bar{\mathbf{r}}_m, z'\right]\mathrm{d}z'.
\end{aligned}
\tag{58}
$$

Now, by averaging (in view of (58)) the intensity-squared (52) over the realizations of large-scale irregularities $\tilde{\varepsilon}_1(\mathbf{r})$, it is possible to obtain the formula for $\langle |U|^4\rangle$.

As would be expected, an averaging of the intensity in the same manner shows that the mean intensity coincides with the undisturbed intensity. As a result, for the scintillation index (46) we obtain

$$
\begin{aligned}
\beta^2 \simeq\ & \beta_1^2 - (\beta_1^2 - 1)\frac{k^2 d}{2}A_2(0)\\
&+ 2\pi k^2\int_{-\infty}^{\infty}\mathrm{d}^2\kappa\int_{z_0}^{z}\mathrm{d}z_1\,\Phi_2(\boldsymbol{\kappa}, 0)\left[1 - \cos\left(\frac{\kappa^2(z_1 - z_0)(z - z_1)}{k(z - z_0)}\right)\right]\\
&\times\exp\left\{-\frac{\pi k^2}{2}\int_{z_0}^{z_1} H_1\left[\frac{\boldsymbol{\kappa}(z - z_1)(z' - z_0)}{k(z - z_0)}\right]\mathrm{d}z'\right.\\
&\left.-\frac{\pi k^2}{2}\int_{z_1}^{z} H_1\left[\frac{\boldsymbol{\kappa}(z - z')(z_1 - z_0)}{k(z - z_0)}\right]\mathrm{d}z'\right\}.
\end{aligned}
\tag{59}
$$

Here

$$
A_1(\boldsymbol{\rho}) = \int_{-\infty}^{\infty}\Psi_{\varepsilon 1}(\boldsymbol{\rho}, z')\,\mathrm{d}z' = 2\pi\int_{-\infty}^{\infty}\mathrm{d}^2\kappa\;\Phi_{\varepsilon 1}(\boldsymbol{\kappa}, 0)\exp(\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho}), \tag{60}
$$

$$
H_1(\boldsymbol{\rho}) = [A_1(0) - A_1(\boldsymbol{\rho})]/\pi = 2\int_{-\infty}^{\infty}\mathrm{d}^2\kappa\;\Phi_{\varepsilon 1}(\boldsymbol{\kappa}, 0)[1 - \exp(\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho})]. \tag{61}
$$

$\Psi_{\varepsilon 1}(\boldsymbol{\rho}, z')$ and $\Phi_{\varepsilon 1}(\boldsymbol{\kappa}, \kappa_z)$ are the correlation function and the spectrum for large-scale irregularities, respectively; β_1^2 is the scintillation index in a medium with large-scale irregularities:

$$
\begin{aligned}
\beta_1^2 = \frac{k^4}{(2\pi)^8}&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}^2 v_1\,\mathrm{d}^2 u_1\,\mathrm{d}^2 v_2\,\mathrm{d}^2 u_2\\
&\times\ \exp\left\{2\mathrm{i}k(z - z_0)(\mathbf{v_1}\cdot\mathbf{u_2} + \mathbf{v_2}\cdot\mathbf{u_1}) - \frac{k^2\pi}{2}B_1\right\} - 1,
\end{aligned}
\tag{62}
$$

where

$$
\begin{aligned}
B_1 = \tfrac{1}{2}\int_{z_0}^{z}&\{2H_1[\mathbf{u}_2(z - z') + \mathbf{u}_1(z' - z_0)] + 2H_1[\mathbf{v}_2(z - z') + \mathbf{v}_1(z' - z_0)]\\
&- H_1[(\mathbf{v}_2 + \mathbf{u}_2)(z - z') + (\mathbf{u}_1 + \mathbf{v}_1)(z' - z_0)]\\
&-H_1[(\mathbf{v}_2 - \mathbf{u}_2)(z - z') + (\mathbf{u}_1 - \mathbf{v}_1)(z' - z_0)]\}\,\mathrm{d}z'.
\end{aligned}
\tag{63}
$$

The scintillation index (62) in a medium with large-scale irregularities can be investigated using known [6, 14] asymptotic methods. It should be noted that separating the irregularities into small-scale and large-scale irregularities depends not only on the irregularity size but also on a number of other parameters of the problem. In particular, in the case of propagation in a statistically inhomogeneous medium within small distances from the source, fluctuation phases are small, and all irregularities satisfy the condition (50) and can be categorized as 'small-scale'. In this case $\beta_1^2 = 0$, and for the scintillation index we obtain the expression

$$\beta^2 \simeq 2\pi k^2 \int_{-\infty}^{\infty} \mathrm{d}^2\kappa \int_{z_0}^{z} \mathrm{d}z_1 \Phi_\varepsilon(\boldsymbol{\kappa}, 0)\left[1 - \cos\left(\frac{\kappa^2(z_1 - z_0)(z - z_1)}{k(z - z_0)}\right)\right], \quad (64)$$

where

$$\boldsymbol{\Phi}_\varepsilon(\boldsymbol{\kappa}, \kappa_z) = \boldsymbol{\Phi}_{\varepsilon 1}(\boldsymbol{\kappa}, \kappa_z) + \boldsymbol{\Phi}_{\varepsilon 2}(\boldsymbol{\kappa}, \kappa_z). \quad (65)$$

This same expression is also obtained from analysing the intensity fluctuations by Rytov's method [1].

When the distance from the source is increased, there appears a part of the irregularities that does not satisfy condition (50). Here one must use formula (59). Then, at a large distance the 'large-scale' component of the scintillation index β_1^2 tends rapidly to unity, and we obtain

$$\begin{aligned}\beta^2 \simeq 1 + 2\pi k^2 \int_{-\infty}^{\infty} \mathrm{d}^2\kappa \int_{z_0}^{z} \mathrm{d}z_1 \Phi_{\varepsilon 2}(\boldsymbol{\kappa}, 0)\left[1 - \cos\left(\frac{\kappa^2(z_1 - z_0)(z - z_1)}{k(z - z_0)}\right)\right] \\ \times \exp\left\{-\frac{\pi k^2}{2}\int_{z_0}^{z_1} H_1\left[\frac{\boldsymbol{\kappa}(z - z_1)(z' - z_0)}{k(z - z_0)}\right]\mathrm{d}z' \right. \\ \left. - \frac{\pi k^2}{2}\int_{z_1}^{z} H_1\left[\frac{\boldsymbol{\kappa}(z - z')(z_1 - z_0)}{k(z - z_0)}\right]\mathrm{d}z'\right\}. \quad (66)\end{aligned}$$

Thus the scintillation index in the region of saturation of fluctuations is determined here (see also [15]) by the diffraction of the wave transmitted through large-scale irregularities, from small-scale irregularities. Noteworthy is the similarity of formula (66) with the formula obtained in [16] for intensity fluctuations in a turbulent medium as a result of an asymptotic calculation of expressions obtained using Feynman path integrals. This is no surprise, as a turbulent medium is characterized by the influence of different-scale irregularities on the wave propagation. Formula (66) transfers to the results reported in [16] if in the expression under the integral sign in (66), in view of the inequality (50), H_1 is replaced by

$$H = H_1 + H_2 = 2\int_{-\infty}^{\infty} \mathrm{d}^2\kappa\ \Phi_\varepsilon(\boldsymbol{\kappa}, 0)[1 - \exp(\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho})] \quad (67)$$

and if it is taken into consideration that by virtue of the filtering character of the term $1 - \cos[\kappa^2(z_1 - z_0)(z - z_1)/(k(z - z_0))]$ in the spectrum, low-frequency (large-scale) components disappear and, hence, the following replacement can be done: $\Phi_{\varepsilon 2}(\boldsymbol{\kappa}, 0) \to \Phi_\varepsilon(\boldsymbol{\kappa}, 0)$.

4. Conclusion

In this paper we have developed a new method for solving the wave equation. In doing so, we used the method of the Fock fifth parameter to reduce the wave equation to a parabolic equation. After that, using a double weighted Fourier transform we found the solution of the parabolic equation. The resulting parabolic equation is solved using a weighted Fourier transform relative to the source and observer coordinates. Furthermore, the well-known Debye procedure is used to solve the equation for partial waves (of the expression under the integral sign). The resulting eikonal and transport equations are solved by the perturbation method.

In this case, unlike the other asymptotic integral representations, the integration of the eikonal equation is performed not along the unperturbed trajectory but along a perturbed (random) trajectory that is calculated to a first approximation of perturbation theory.

To determine the validity range of the resulting integral representation, we have compared the results of this representation under different conditions with existing results. We have shown the transfer to results of the hybrid approach generalizing the Born approximation. Also for this purpose, the scintillation index was investigated within the small-angle approximation. We have shown its transition to the results of Rytov's method over small distances, and to the results of an asymptotic evaluation of path integrals over large distances.

Thus the combination of an integral representation with perturbation ray theory of the first approximation provides a means of investigating the effects of the simultaneous presence of different-scale irregularities in the propagation medium. Large-scale irregularities lead to strong intensity fluctuations while small-scale irregularities lead to diffraction effects of the Fresnel type and to the scattering into large angles, including backscattering. A simultaneous account of the effects of different-scale irregularities opens up new possibilities for investigating the enhancement of intensity fluctuations in the case of backscattering, incoherent scatter, the meteor propagation of radio waves, and other mechanisms of radio wave propagation in conditions of a substantial influence of different-scale irregularities.

Acknowledgments

This work was done with partial financial support under Russian Ministry of Education grant No. E02-3.5-197. I am grateful to Professor Yu A Kravtsov for useful discussions.

References

[1] Rytov S M, Kravtsov Yu A and Tatarskii V I 1989 Introduction to statistical radiophysics *Wave Propagation Through Random Media* vol 4 (New York: Springer)
[2] Whitman A M and Beran M J 1985 Two-scale solution for atmospheric scintillation *J. Opt. Soc. Am.* A **2** 2133–43
[3] Frankenthal S, Whitman A M and Beran M J 1984 Two-scale solution for the intensity fluctuations in strong scattering *J. Opt. Soc. Am.* A **6** 585–94
[4] Mazar R, Gozani J and Tur M 1985 Two-scale solution for the intensity fluctuations of two-frequency wave propagation in a random medium *J. Opt. Soc. Am.* A **2** 2152–60
[5] Mazar R, Kodner L and Samelsohn G 1997 Modeling of high-frequency wave propagation with application to the double-passage imaging in random media *J. Opt. Soc. Am.* A **14** 2809–19
[6] Tinin M V 1998 Quasi-geometrical optics methods in the theory of wave propagation through a randomly inhomogeneous layer *Waves Random Media* **8** 329–38
[7] Kravtsov Yu A and Tinin M V 2000 Representation of a wave field in a randomly inhomogeneous medium in the form of the double-weighted Fourier transform *Radio Sci.* **35** 1315–22
[8] Tinin M V, Kolesnik S N and Kravtsov Yu A 2002 Integral representation for the field of the wave propagating in a medium with random inhomogeneities of different scales *Proc. SPIE* **4678** 605–9
[9] Vinogradov A G and Kravtsov Yu A 1973 Hybrid method for the field fluctuations calculation in a medium with inhomogeneities of large and small scales *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **16** 1055–63
Vinogradov A G and Kravtsov Yu A 1973 *Radiophys. Quantum Electron.* **16** N7 (Engl. Transl.)
[10] Pavlenko Yu G, Afinogenov Yu A and Gonzales P 1988 Calculation of Greenfunction of Helmholtz equation by the method of fifth parameter *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **31** 1507–12
Pavlenko Yu G, Afinogenov Yu A and Gonzales P 1988 *Radiophys. Quantum Electron.* **31** N12 (Engl. Transl.)
[11] Kravtsov Yu A and Orlov Yu I 1983 Caustics catastrophes and waves fields *Sov. Phys.—Usp.* **26** 1038–58
[12] Kravtsov Yu A and Orlov Yu I 1997 *Caustics Catastrophes and Wave Fields* (Berlin: Springer)
[13] Tatarskii V I 1971 *The Effect of a Turbulent Atmosphere on Wave Propagation* (Springfield, VA: National Technical Information Service)

[14] Yakushkin I G 1974 Intensity fluctuations of the plane wave field behind the chaotic phase screen *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **17** 1350–55

[15] Yakushkin I G 1985 Intensity fluctuations at small-angle scattering of the wave fields *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **28** 535–65

[16] Wang G and Dashen R 1993 Intensity moments for waves in random media: three-order standard asymptotic calculation *J. Opt. Soc. Am.* A **10** 1226–32