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Flexural strength equation for sea ice

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Abstract

The measured flexural strengths of freshwater ice and sea ice have been compiled with a view towards correlating the measured results. Two thousand, four hundred and ninety-five experimentally measured data points from nineteen investigators have been used. This correlation has been done as input for a new system being developed to quickly characterize sea ice properties. The results indicate a very good correlation ($r^2=0.77$) between the flexural strength (σ_f) and the brine volume (ν_b) with a functional form $\sigma_f = 1.76\ e^{-5.88\sqrt{\nu_b}}$ where the flexural strength is in MPa, and the brine volume is expressed as a brine volume fraction. The value of 1.76 MPa for zero brine volume is in excellent agreement with the average value (1.73 MPa) measured for freshwater ice.

1. Introduction

The National Research Council of Canada is working with the Canadian Coast Guard to develop a method to quickly and accurately determine the physical properties of sea ice. In many situations, it is important to be able to do this. For example, during high-cost field projects involving ships in ice, an understanding of the strength and thickness of the ice are essential in order to reliably assess the ships performance in ice. To date, data collection of the ice properties has been slow and cumbersome. In many instances, due to logistics problems, local polar bear activity or inclement weather, ice data has not been collected at all. From an operational point of view, it is important to be able to collect accurate ice data, in sufficient quantity, in as short a time as possible. Current techniques prohibit this, and new technology must be developed in this area.

In this program two systems are being developed. One system will allow the direct measurement of the ice thickness, temperature, salinity and density, as well as the snow thickness, temperature and density. A second system will use a calibrated video camera to record the ice thickness on a continuous basis. Neither system requires on-ice access. These systems are being developed by Canpolar Inc. of Toronto in collaboration with A.R. Engineering of Calgary. Although the physical properties of the ice are of interest in themselves, it would be much more useful to be able to relate them to the mechanical properties of the ice. For this, a correlation must be found between the physical properties and the mechanical strength. Recently, Timco and Frederking (1990) developed a model which relates the physical properties of sea ice to the uni-axial compressive strength of the ice. In this model, the compressive strength of the ice sheet can be determined if the physical properties of ice tem-

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perature, salinity and density are known. For a ship in ice, however, the ice usually fails in bending so the flexural or bending strength is of great importance. Thus, there is incentive to try to develop a correlation between the physical properties and the flexural strength of the ice. There have been several brine volume–flexural strength relationships derived for sea ice. These relationships, however, have been based on relatively few data points spread over a small range of test conditions. Thus, a comprehensive relationship is required which takes into account all factors. This is described in this paper.

2. Flexural behaviour of ice

The flexural behaviour of ice has important implications in several problems in ice engineering. Information on the flexural properties of ice sheets has direct application to the assessment of an icebreaker's performance, ice ride-up and pile-up, ridge-building, the ice forces on structures with inclined faces, and general bearing capacity problems. Because of its importance, the flexural strength of sea ice has been measured by several investigators. The measured strength values range from 0.2 MPa to 3.0 MPa for freshwater ice, and from 0.1 MPa to 1.5 MPa for sea ice. This wide range is a result of the wide variability of the parameters which influence the flexural strength of ice.

Sea ice is a complex material which is composed of solid ice, brine, air and, depending upon the temperature, solid salts. When the ice grows, it traps some of the salt which is present in the sea water. Although the amount that is trapped is affected by several factors, typically the ice has a salinity of 4 to 6 parts per thousand (‰) salt. This is lower than the salinity of the sea water which is typically 35‰. The ice grows with different types of grain structure including columnar (congelation ice), granular, discontinuous columnar and frazil. The brine, air and solid salts are usually trapped at sub-grain boundaries between mostly pure ice lattice. There is almost always some salinity variation in the ice sheet with depth. Also, a temperature gradient exists with the upper surface temperature close to the ambient air temperature, and the lower surface temperature at the freezing point (usually $-1.8\,^{\circ}C$ for sea ice). Because there are a number of salts in the ice, the phase relationship with temperature is complex. All of these factors make the understanding and characterization of sea ice difficult. Considerable insight has been gained in understanding this, however, and the reader is referred to Cox and Weeks (1988) for a detailed discussion of the profile properties of sea ice. In the present analysis, these profile properties are not always explicitly taken into account, primarily due to a lack of information supplied by the various researchers.

The flexural strength of ice is not a basic material property. The test for flexural strength creates non-uniform stress fields in the ice and assumptions are required about the material behaviour in order to interpret the test results. Thus, the flexural strength is generally regarded as an *index* test. Because of its importance and use in ice engineering problems, however, several investigators have measured this property.

In measuring the flexural strength, two different approaches have been used: cantilever beam tests, and simple-beam tests. For the *in situ cantilever* test, the ice is cut to form three sides of a beam with the fourth side uncut and connected to the floating ice sheet. An increasing vertical load is applied to the free end of the beam until it breaks at the root of the beam. This test has the advantage of being relatively easy to perform on a large beam, and of maintaining the temperature gradient in the ice sheet. Usually the test results are analyzed in terms of simple elastic beam theory. For the *simple beam* test, the beam is completely cut free of the ice sheet and loaded at three (or four) equidistant points such that the centre load is parallel to but opposed to the load at the ends of the beam. More often this test is performed in a laboratory on smaller samples of ice cut from the ice cover. There have been, however, a few tests using the simple beam approach on full-thickness ice beams. The IAHR Committee on Ice Problems has published some guidelines for correctly determining the flexural strength of ice (Schwarz et al., 1981).

The strength of ice may depend on a large number of parameters including the temperature, the loading direction on the ice, the ice grain structure, the grain size, the test type (cantilever or simple beam), the loading rate, beam size and, for sea ice, the ice salinity and brine volume. This large number of parameters may make a correlation of the flexural strength of sea ice to the physical properties difficult. In this analysis the following restrictions apply:

- *Temperature* (T). For freshwater ice, the data measured on ice with a temperature equal to or less than $-4.5\,^{\circ}\mathrm{C}$ have been used. For temperatures higher than this, there was a great deal of scatter with some tests giving very low strengths. These low strengths are probably the result of "candling" where melting occurs along the grain boundaries through absorption of solar radiation. When this occurs, the ice strength can be almost zero even though the ice still has considerable thickness. For sea ice, all measured temperatures were used. For the case where there was a temperature gradient in the ice, the average temperature of the beam was used.
- *Loading direction.* A loading direction parallel to the vertical direction in the original ice sheet was used. This direction corresponds to the loading direction of flexural breaking by an icebreaking vessel, or an inclined offshore structure. This loading direction has been used in almost all tests reported to date. In some tests the ice was loaded with the top in tension (downward loading on the beam) whereas in other tests, the bottom of the ice was put in tension (upward loading). It is known that for freshwater ice, this does influence the strength (Timco and Frederking, 1982; Gow and Ueda, 1984). The difference has been attributed to a difference in grain size. In almost all reported tests, however, no information is given on the grain size so a separation of the data for various grain sizes is not possible. Thus, no distinction was made for top or bottom tension tests. Measurements by Weeks and Anderson (1958) on young sea ice do not indicate any large difference between pull-up and push-down tests.
- *Grain structure.* No distinction was made amongst the various grain structures of the ice, since, in most cases, the grain structure was not reported.
- *Grain size.* As discussed above, little information was supplied on the grain size of tested specimens, so this factor could not be taken into account.
- *Test type.* Both cantilever and simple beam approaches have been used to test the flexural strength for both freshwater ice and sea ice. In this analysis, a distinction is always made in terms of these test types.
- *Loading rate.* Very few investigators report the loading rate for their tests. Typically, the time-of-loading for tests of this type are on the order of 0.5 s to 30 s. With this loading time it is generally considered that there is not a strong functional dependence of the loading rate on the flexural strength of ice. This was investigated here and, for the limited number of tests with this information, no rate dependence was observed.
- *Beam size.* There has been considerable discussion in the literature on the influence of beam size on the strength of ice. Parsons et al. (1992) have shown that there is not a large size-effect for flexural strength of sea ice. Thus, all beam sizes were considered, with, however, a distinction made between large and small beams. In this study, a small beam was considered to be one in which the cross-sectional area of the failure plane was less than 100 cm^2.
- *Ice salinity* (S). For sea ice, the salinity is usually expressed as the fraction by weight of the salts contained in a unit mass (see e.g. Pounder, 1965). It is usually quoted as a ratio of g per kg of sea water, that is, in parts per thousand (‰). In sea ice there is usually a salinity variation with depth in the ice sheet. This depth dependence of the salinity changes throughout the winter as the salt within the ice migrates downward through the ice. There can be a salinity variation even within a small sample. However, as a first approximation it was assumed that the bulk salinity is uniform within the sample. Thus, in this analysis, the average salinity of the beam was used as a representative value for the salinity.
- *Brine volume* (ν_b). Historically the strength of the ice has been related to the brine volume of the ice. There is a good reason for this. It is gen-

erally assumed that as the brine (or liquid) volume in the ice increases, the strength should decrease since there is less "solid ice" that has to be broken (see Weeks and Assur, 1967). The brine volume of the ice is related to the temperature (T) of the ice, the salinity (S) of the ice and the types of salts present. For sea ice, the brine volume can be determined from the Frankenstein and Garner (1967) Equation:

$$\nu_b = S\left(\frac{49.185}{|T|} + 0.532\right) \tag{1}$$

where $-0.5°C \geqslant T \geqslant -22.9°C$; or, from the Cox and Weeks (1982) Equation:

$$\nu_b = \rho S / F_1(T) \tag{2}$$

where ρ is the bulk ice density, and

$$F_1(T) = -4.732 - 22.45T - 0.6397T^2 - 0.01074T^3$$

for $-2°C \geqslant T \geqslant -22.9°C$, and

$$F_1(T) = 9899 + 1309T + 55.27T^2 + 0.716T^3$$

for $-22.9°C \geqslant T \geqslant -30°C$. Although the latter is more accurate, the former provides a reasonable estimate of the brine volume. The brine volume is usually quoted in terms of the volume in parts per thousand, similar to the salinity. Alternatively, it can be expressed as a volume fraction. (For example, a brine volume of 20‰ is equivalent to a brine volume fraction of 0.020).

It should be noted that the previous correlation by Timco and Frederking (1990) on the compressive strength of the ice related the strength to the *total* porosity (i.e. brine porosity plus air porosity) of the ice. In order to determine the total porosity, an accurate measurement of the ice density must be performed. The density was measured in only a few studies reporting flexural strength, so the correlation to total porosity could not be done accurately. This avenue should be explored further if new data becomes available.

3. Data sources

The reports of flexural strength measurements on ice are scattered throughout the literature. In many instances, the flexural strength was measured as part of a larger test program. This makes compiling all tests rather difficult. For the present analysis, the available literature was surveyed and the results of 5 investigators (Frankenstein, 1959, 1961; Lavrov, 1969; Timco and Frederking, 1982; Gow et al., 1988) representing 1556 measurements were selected for tests performed on freshwater ice. For sea ice, 14 investigators (Butkovich, 1956; Weeks and Anderson, 1958; Butkovich, 1959; Brown, 1963; Tabata, 1964; Dykins, 1968, 1971; Airaksinen, 1974; Tabata et al., 1975; Vaudrey, 1977; Frederking and Hausler, 1978; Saeki et al., 1981; Timco and Frederking, 1983; Williams et al., 1992) representing 939 measurements on sea ice were selected. Emphasis was placed on those studies where the data was listed in tabular form (as opposed to graphical plots). The data used for this analysis do not represent all reported measurements. However, this data source, representing about 2500 tests, was felt to be sufficient to give a good representative indication of the flexural behaviour of ice. It should be noted that the measured values for sea ice were obtained from ice world-wide, including the Canadian and Alaskan Beaufort Sea, Labrador, Baffin Island, Greenland, Gulf of Bothnia, Japan and the Antarctic.

Tables 1 and 2 list the data sources for both the freshwater ice and the sea ice results. The tables list information on the name of the investigators, the location and date of the tests, the test type, beam size and ice type (laboratory or field ice). Each investigation is indicated by a unique symbol. These symbols are used to indicate the investigator in each of the graphs. Note that some of the small scale data from the Naval Civil Engineering Laboratory (NCEL) of Port Hueneme, California are presented as the average of a large number of tests. These are indicated in the figures with a number which represents the number of tests. Also, the data from Weeks and Anderson (1958) represent, on average, six individual tests. In determining the best-fit through the data, each of these data points was weighted accordingly.

Table 1
Data sources for tests on freshwater ice

Investigator	Symbol	No. of data points	Test date	Location	Test type	Beam size	Ice type
Frankenstein (1959)	○	541	1955–56	Lake Anne Portage Lake Garrison Res. Lake Bemidji	Cant Simp	Large Small	Field
Frankenstein (1961)	▽	274	1958	Chassell Bay Keweenaw Bay	Cant Simp	Large	Field
Lavrov (1969)	□ ■	3 18	1960–61	Lake Ladoga Laboratory	Simp Cant Simp	Large Large Small	Field Lab.
Timco and Frederking (1982)	⧓	43	1981	Laboratory	Cant Simp	Small	Lab.
Gow et al. (1988)	⧗	677	1983	Laboratory	Cant Simp	Large Small	Lab.

4. Results for freshwater ice

Tests of the flexural strength of freshwater ice were compiled to serve as a baseline for the sea ice values. That is, the freshwater ice strength represents tests performed on ice with no liquid brine inclusions in it. For the present purpose, it is assumed to represent the "zero brine volume" data. Tests conducted in both the laboratory and field are used. No distinction is made between grain structure or grain size. For freshwater ice it is known that there is a significant difference in the strength result depending on the test technique. In general, flexural strengths determined using the cantilever beam approach are lower than those determined using the simple beam approach. This is attributed to a stress concentration at the root of the beam in a cantilever test (see Svec et al., 1985). This stress concentration is very prevalent for freshwater ice, which is very brittle, but has little effect on sea ice beams since they are more ductile (Timco, 1985). For this reason, the tests on freshwater ice make a distinction between the two test types.

It would seem reasonable to look at the flexural strength of ice as a function of the temperature. Figs. 1 and 2 show the temperature dependence of the flexural strength as a function of temperature for the simple beam and cantilever beam test approach. There are several things to note. First, there is a distinct difference between the results for the two test types. This is further illustrated in Fig. 3 which compares the probability density of the flexural strength, for tests conducted at temperatures less than $-4.5\,^{\circ}$C. Second, there is a very large range of scatter. Scatter of this type is, however, very characteristic of strength tests performed on brittle materials. For brittle materials, measured strength values can be up to three times higher, or one-third lower than the measured average value. Third, there is no indication of a strong temperature influence on the strength of the ice. This may be excepted, however, given the large range of scatter of the data.

The tests results using the simple beam approach have been used as a representative value of sea ice with no brine volume, since it is assumed that there is no stress concentration effect with this data. All data for temperatures below $-4.5\,^{\circ}$C have been averaged to give a value of 1.73 ± 0.25 MPa. In this paper, this range of the

Table 2
Data sources for tests on sea ice

Investigator	Symbol	No. of data points	Test date	Location	Test type	Beam size	Ice type
Butkovich (1956)	*	44	1956	Labrador	Simp	Small	Field
Weeks and Anderson (1958)	⧗	208	1958	Greenland, Labrador	Cant	Large	Field
Butkovich (1959)	×	37	1957	Greenland	Simp	Small	Field
Brown (1963)	■	23	1958–61	Alaska, Greenland	Cant	Large	Field
Tabata et al. (1964)	•	22	1964	Japan	Cant	Large	Field
Dykins (1968)	●	7	1968	Antarctica	Cant	Large	Field
	○	12		Antarctica	Simp	Large	Field
	◯	28		Antarctica	Simp	Small	Field
	◯	37		NCEL	Simp	Small	Lab.
Dykins (1971)	+	214	1971	Antarctica	Simp	Small	Field
Airaksinen (1974)	▼	11	1972	Baffin Island	Cant	Large	Field
Tabata et al. (1975)	▲	17	–	Bothnia	Cant	Large	Field
Vaudry (1977)	■	9	1977	Alaska	Cant	Large	Field
	□	10		Alaska	Simp	Large	Field
	□	25		Alaska	Simp	Small	Field
	□	116		NCEL	Simp	Small	Lab.
Frederking and Hausler (1978)	⧓	11	1977	Spitzbergen	Cant	Large	Field
Saeki et al. (1981)	♦	5	1981	Japan	Cant	Large	Field
	◇	10			Simp	Large	Field
Timco and Frederking (1983)	⧖	21	1982	Beaufort Sea	Simp	Small	Field
Williams et al. (1992)	△	12	1992	Antarctic	Simp	Large	Field
		60			Simp	Small	Field

average value plus and minus one standard deviation is indicated by a solid bar on the zero brine volume axis of the appropriate plots (Figs. 5–7).

5. Results for sea ice

For sea ice, a distinction was initially made between the test type and the size of the beam. Initial plots were made of the flexural strength versus the loading rate and the area of the failure plane. There was no apparent functional dependence between these factors. Thus, a simple approach was taken, and all experimental data were included in the analysis.

Fig. 4 shows the temperature dependence of the flexural strength for all tests. The scatter, although large, does not appear to be as large as that for the freshwater ice test results. There appears to be a general increase in strength with decreasing temperature. For sea ice there are two factors which contribute to the temperature dependence of the ice: the temperature dependence

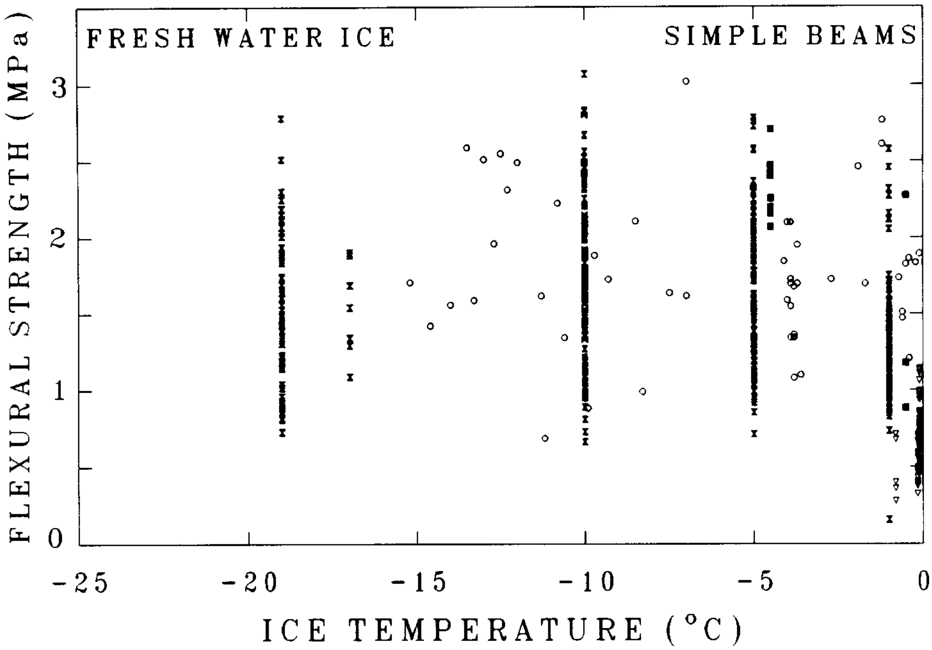

Fig. 1. Flexural strength versus the ice temperature for freshwater ice measured using the simple beam approach.

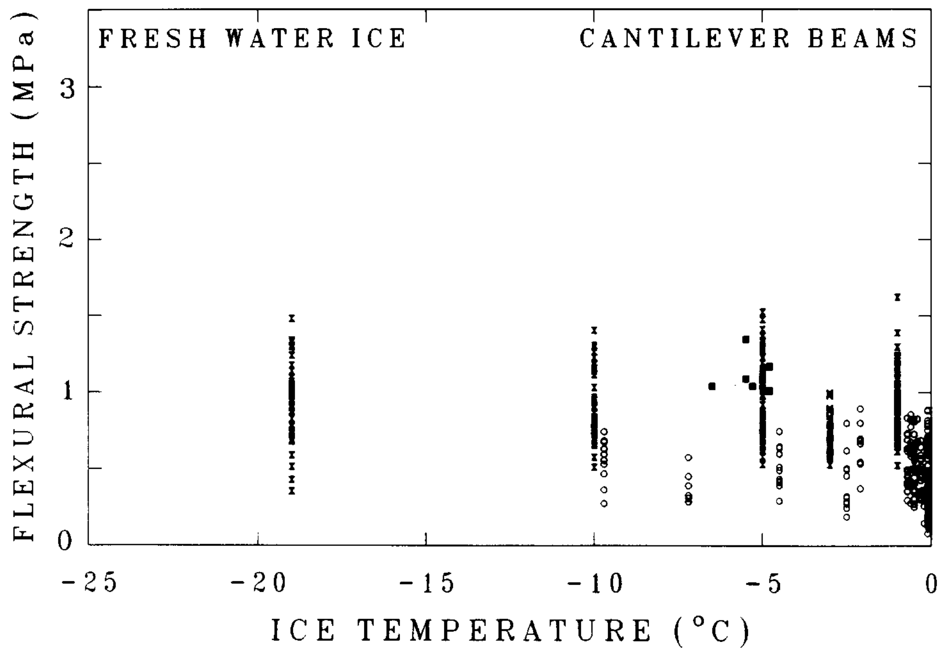

Fig. 2. Flexural strength versus the ice temperature for freshwater ice measured using the cantilever beam approach.

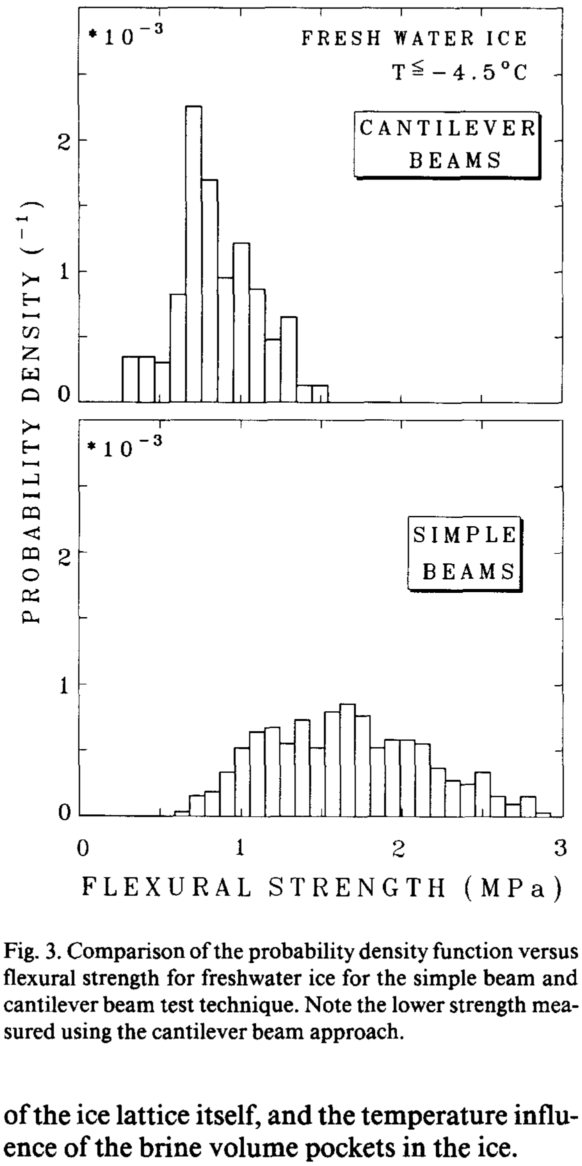

Fig. 3. Comparison of the probability density function versus flexural strength for freshwater ice for the simple beam and cantilever beam test technique. Note the lower strength measured using the cantilever beam approach.

of the ice lattice itself, and the temperature influence of the brine volume pockets in the ice.

Figs. 5 and 6 show the flexural strength plotted versus the square root of the brine volume of the ice for large beams and small beams respectively. In these plots, the brine volume is expressed as a brine volume fraction. Both plots contain data from simple beam and cantilever tests, with no apparent difference between the two measurement techniques. In both plots there is a definite decrease in the strength with increasing brine volume. Moreover, there appears to be good agreement between the two plots showing the insensitivity to the beam size. It should be mentioned that there has been some discussion in the literature on a size dependence between large beams and small beams. Figs. 5 and 6 show that, *on average*, there is a difference between the strength values measured on small and large beams. However, this difference is largely a reflection of the differences in brine volume between the two test types. In general, the small beam tests have been performed in a laboratory with relatively cold temperatures; thus, the ice has a relatively low brine volume. Large beams, on the other hand, are usually tested in situ when the air (and ice) temperatures are relatively high, resulting in a high brine volume in the ice. Thus, the lower strength values in this case are a reflection of this higher brine volume, and not the beam size.

Fig. 7 shows a compilation of all flexural tests performed on sea ice. Although there is some scatter, there is a pronounced general trend. A least squares exponential fit to the data gives:

$$\sigma_f = 1.76e^{-5.88\sqrt{\nu_b}} \qquad (3)$$

where the flexural strength is in MPa, and the brine volume is expressed as a brine volume fraction. The correlation coefficient (r^2) for this curve is 0.77 which is quite high given the natural variability in sea ice itself. It is interesting to note that the general scatter increase with decreasing brine volume. This is a reflection of the fact that, at low brine volumes the ice is much more brittle. The range of scatter is approaching that seen for freshwater ice (Figs. 1 and 2). It should be noted that the strength value for zero brine volume agrees remarkably well with the average value determined from tests on freshwater ice. This fact, plus the very large number of data points used in this analysis indicates that this equation is a very good representation of the brine volume dependence of sea ice.

6. Flexural strength variations during the winter

It is possible to use this equation and approach to extend this work to predict the flexural strength

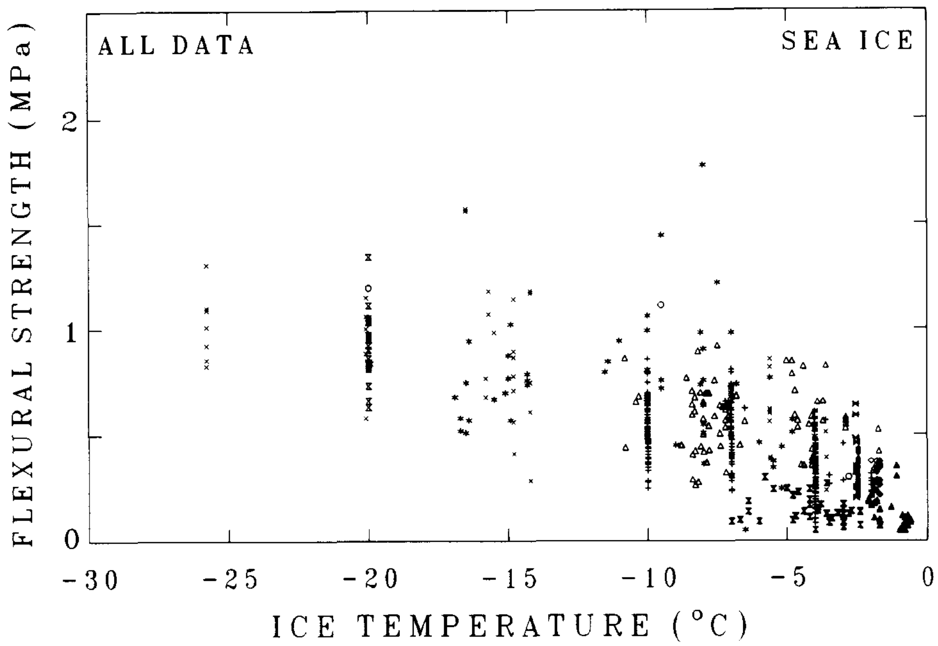

Fig. 4. Flexural strength versus temperature for all data for sea ice.

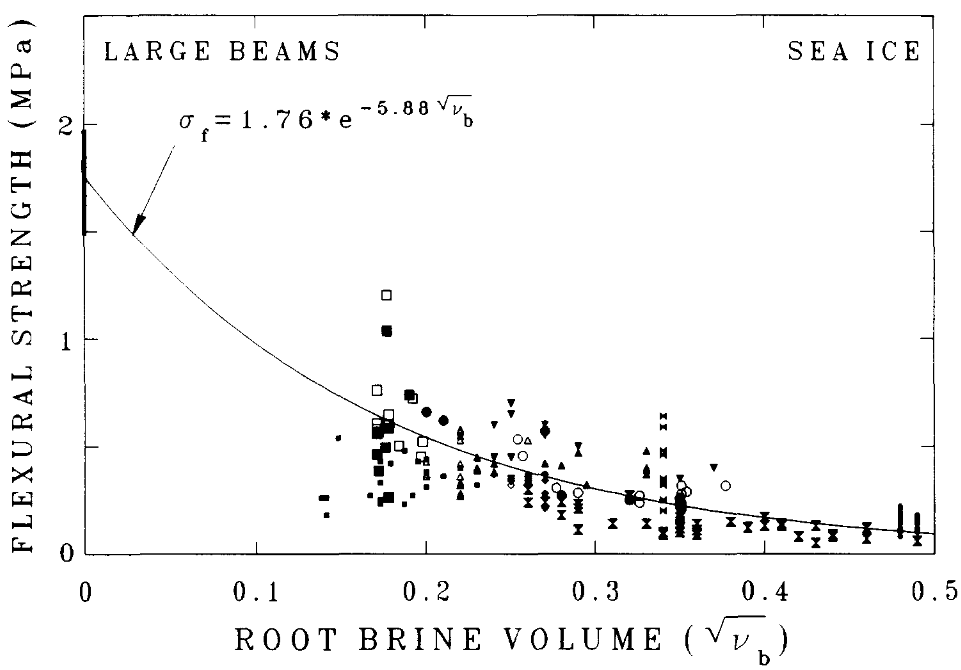

Fig. 5. Flexural strength versus the square root of the brine volume for sea ice for tests with large beams. The tests were conducted using the simple beam (open symbols) and cantilever beam (solid symbols) approach. For sea ice there does not appear to be a strong influence of test type.

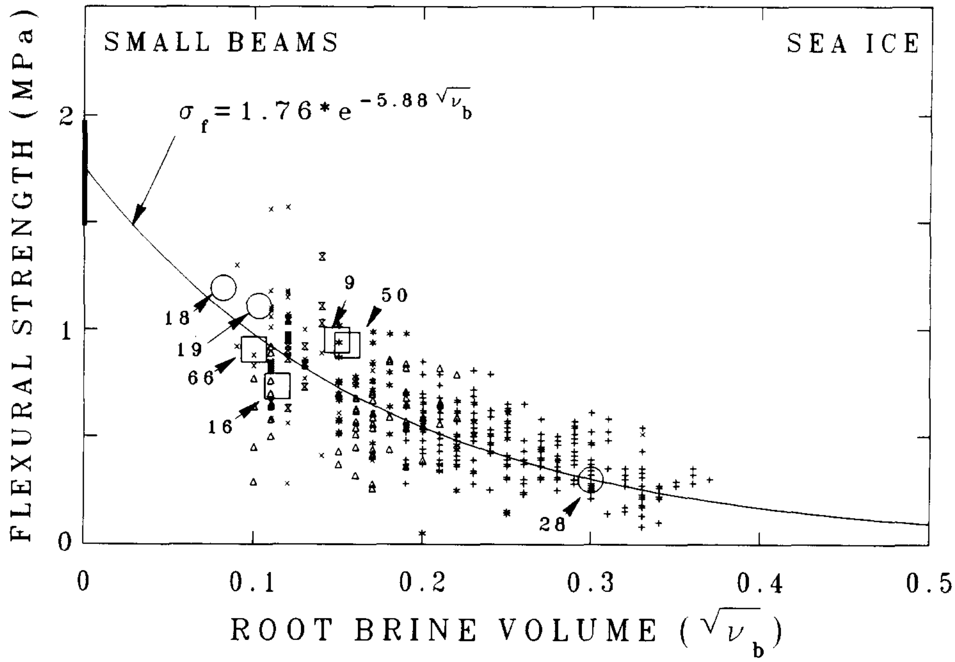

Fig. 6. Flexural strength versus the square root of the brine volume for sea ice for tests with small beams. The tests were conducted mostly using the simple beam approach.

of sea ice throughout the winter season. Timco and Frederking (1990, 1991) extended their model for the compressive strength of an ice sheet to predict the uni-axial compressive strength for sea ice for the whole Arctic region throughout an average and an extreme winter. To do this, they determined relationships for the properties of ice temperature, salinity and density, and related them to the meterological information for the Arctic. The original paper should be consulted for full details. From this, it is possible to use this information to calculate the brine volume of the ice throughout the winter. Knowing this and using Eq. (3), the flexural strength can be determined.

Timco and Frederking (1990) found that, following early work by Cox and Weeks (1974), the salinity of an ice cover can be related to the thickness by:

$$S = 13.4 - 17.4h \quad \text{for } h \leqslant 0.34 \text{ m}$$
$$S = 8.0 - 1.62h \quad \text{for } h >= 0.34 \text{ m} \qquad (4)$$

where h is the thickness of the ice. This approach implies that there is no salinity variation with depth, in agreement with the previous assumption in this paper. Thus, in order to determine the average salinity of the ice, only the thickness is required. Further, they found that, on average, the upper surface temperature of the ice sheet could be related to the air temperature by:

$$T_s = T_a \quad \text{for } -2 \geqslant T_a \geqslant -10°\text{C}$$
$$T_s = 0.6T_a - 4 \quad \text{for } T_a < -10°\text{C} \qquad (5)$$

where T_s is the surface ice temperature, and T_a is the air temperature. To calculate the average brine volume in the ice sheet it is necessary to know the average ice temperature (T_{av}). Since the temperature at the bottom of the ice sheet is always at -1.8 °C for sea ice, then

$$T_{av} = \frac{T_s + (-1.8)}{2} \qquad (6)$$

With Eqs. (4) and (6), the flexural strength can be determined knowing only the ice thickness and air temperature.

Timco and Frederking (1991) presented information on the ice thickness and air tempera-

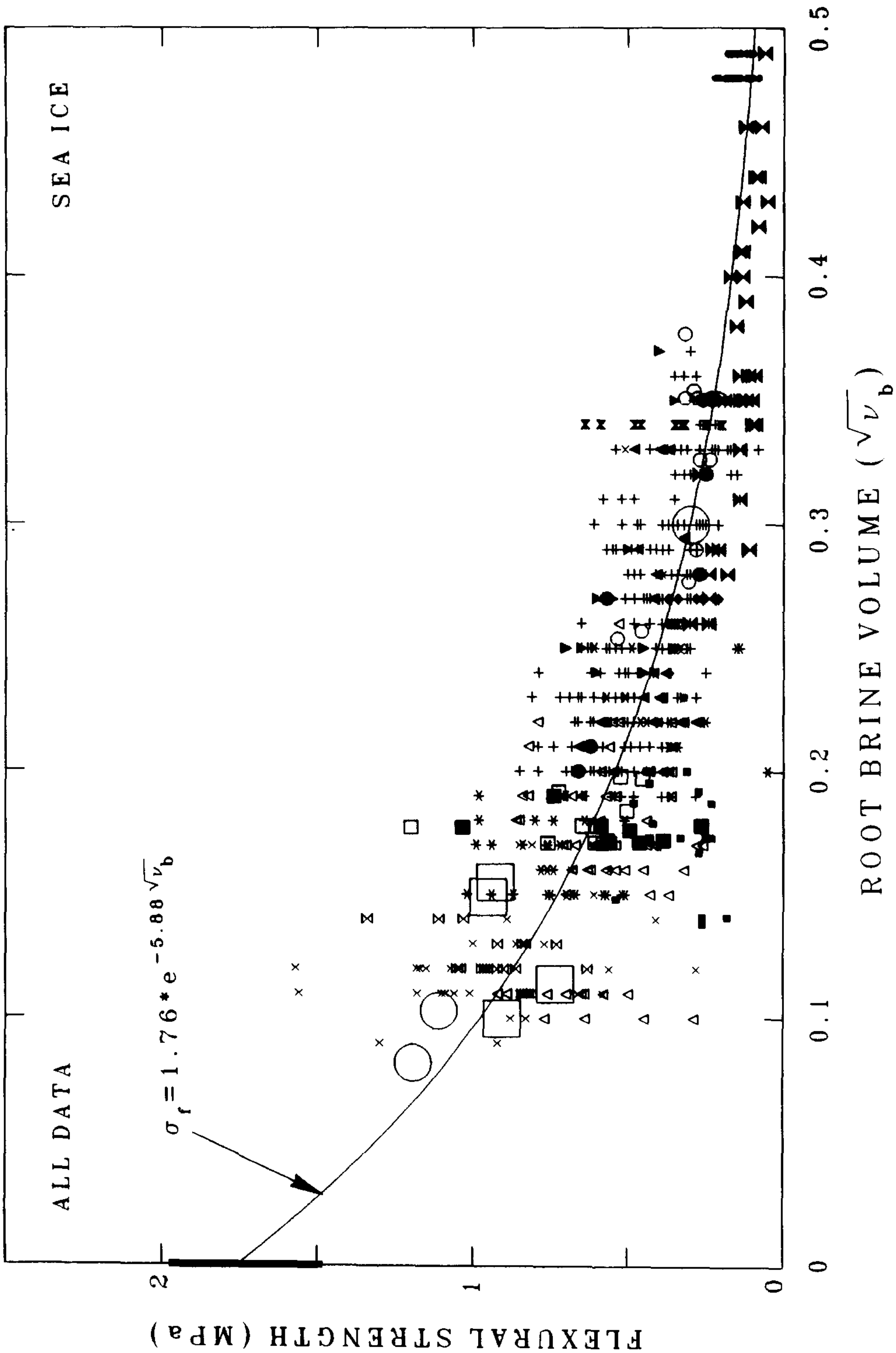

Fig. 7. Flexural strength versus the square root of the brine volume for all tests on sea ice. Although there is some scatter, there is good correlation amongst the tests from all of the fourteen different investigators. The curve is the best-fit exponential through the data.

ture variation for the Canadian Beaufort Sea. From this information it is possible to determine the average brine volume in the ice throughout the winter. With this and Eq. (3), the strength variation can be determined.

Fig. 8 shows the variation of the flexural strength of sea ice throughout the winter in the Beaufort Sea region of Canada. The corresponding plots of the compressive strength are also shown for comparison in Fig. 9. From these figures it can be seen that the flexural strength is quite low in the early winter when the ice is thin (and highly saline) and the temperatures are still relatively warm. As the winter progresses, the ice gets thicker and colder and the strength increases. Peak strength values are, on average, 0.6 to 0.7 MPa. In the spring, the air temperature rises and the strength decreases.

In contrast to the flexural strength, the compressive strength is very dependent on the rate of loading. The flexural strength values are quite low, compared to the compressive strength. It can be seen that the flexural strengths are comparable to the compressive strength with the later at a strain rate ($\dot{\epsilon}$) of 10^{-7} s^{-1}.

It is possible to estimate the strain rate of various types of interactions of a ship and ice and a structure subjected to the forces of a moving ice

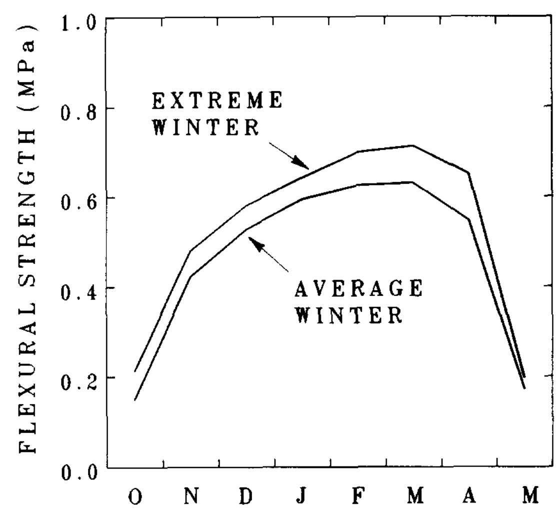

Fig. 8. Flexural strength versus the time of year (from October (O) to May (M)) for Beaufort Sea ice sheets for both an average and an extreme winter.

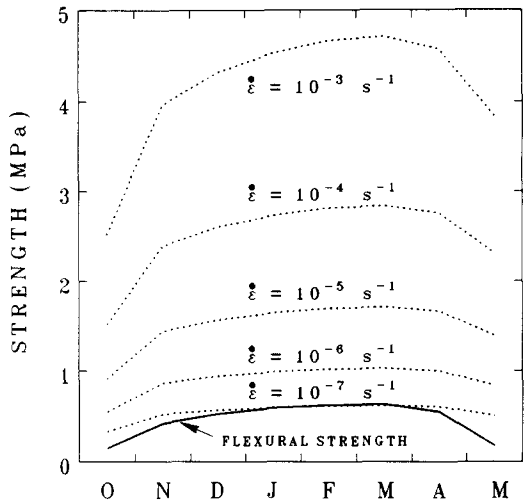

Fig. 9. Comparison of the compressive strength (dotted lines) and flexural strength (solid line) during an average winter. Note that the compressive strength is highly strain rate ($\dot{\epsilon}$) dependent.

cover. For example, a common estimate of the strain rate of an interaction process in ice relates the strain rate ($\dot{\epsilon}$) to the relative speed of the ice (v) and the width (D) of the structure by:

$$\dot{\epsilon}=\frac{v}{2D} \tag{7}$$

If an icebreaking vessel with a beam of say 20 m is moving through an ice cover at a speed of 2 m s^{-1}, the estimated strain rate would be 5×10^{-2} s^{-1}; that is, at a very high rate in the brittle range of loading. For a stationary structure in the Arctic offshore region, the strain rate would be lower. If the structure width is 100 m, and the ice moves past it at a speed of 0.1 m s^{-1}, the estimated strain rate would be 5×10^{-4} s^{-1}. For the ship in ice, the compressive strength determined at a rate 10^{-3} s^{-1} can be used to represent the loading in the brittle range. For both these situations, the calculated ratio of the compressive strength to flexural strength is shown in Fig. 10. With the very low flexural strengths in the early and later parts of the winter, the calculated ratio is quite high. During the mid-winter months, the ratio is on the order of 7 for the ship in ice, and on the order of 6 for the structure subjected to the forces of a moving ice cover. Ratios

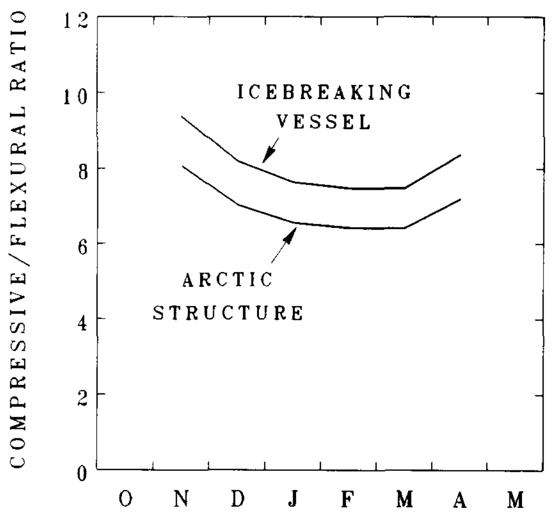

Fig. 10. The ratio of the compressive strength to flexural strength during the winter months for an icebreaking vessel and an Arctic structure in ice. See the text for details of the assumptions made in the calculation.

of this order are characteristic of brittle materials.

This high value for the ratio of the compressive strength to flexural strength has important implications in the design of a ship or structure subjected to ice conditions. It is well known that an offshore structure which breaks the ice in flexure has significantly lower loads than a similar structure which fails the ice by crushing. Fig. 10 clearly explains why this is the case. Similarly, an icebreaking vessel which has a bow which allows any ice crushing would be much more inefficient than one which breaks the ice only in flexure.

7. Conclusions

The present approach is very simplistic, yet it appears to offer quite reasonable results and a very practical approach to understanding and using the flexural properties of ice. The analysis shows that for sea ice, factors such as the loading rate, sample size and test type have little influence on the measured flexural strength. Further, the analysis gives a very good correlation of the flexural strength of sea ice with the brine volume. The functional form of this relationship is exponential with a zero brine value equal to the average strength for freshwater ice. The equation is based on the correlation of 939 flexural strength tests on sea ice. The brine volume of the ice can be determined knowing the temperature, salinity and density of the ice using Eq. (1) or (2). The new test apparatus being developed to measure these physical properties should provide this information. Then the derived relationship (Eq. 3) can be used to determine the flexural strength of the ice. Further, the Timco and Frederking (1990) equations can be used to determine the compressive strength of the ice. In combination, this system should provide very useful information on the properties and characteristics of the ice.

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