On the Meandering and Dispersion of a Plume of Floating Particles Caused by Langmuir Circulation and a Mean Current

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ABSTRACT

Simple analytical models are devised to describe the dispersion of a plume of buoyant material from a fixed source on the sea surface under the action of both mean current and the spatially variable flows induced by Langmuir circulation. A "free" plume meanders at source and later becomes broken into bands lying in the convergence regions produced in the circulation pattern. A model is used to compare free plume dispersion and that in which the floating material is constrained to lie in a band along the water surface, as described in a recent study by Thorpe and Curé.

1. Introduction

A pattern with streaks aligned approximately in the wind direction is often visible when dye or oil is released as a plume or patch on the sea surface (e.g., Katz et al. 1965). We describe here simple models of the meandering and dispersal of floating material resulting from the advection of an idealized steady array of Langmuir cells past a fixed source in a tidal stream, which explains some of the observed features. Dispersion produced by Langmuir circulation has been discussed by Csanady (1973, 1974), Faller and Auer (1988), and more recently by Thorpe and Curé (1994) and Thorpe et al. (1994). In the last two references numerical models have been used in association with real data derived from upward pointing side-scan observations of subsurface bubble clouds drawn into linear bands by the Langmuir circulation (Thorpe 1984) to estimate the associated dispersion. Thorpe and Curé describe the dispersion that would occur if the particles are artificially constrained to lie in a line along the direction of a narrow sonar beam, as if the particles were continuously released into a narrow "chicken wire cage" consisting of two parallel and vertical wire meshes extending across the water surface, through which the water could flow unimpeded but beyond which the floating particles are prevented from moving, although they are free to move in (i.e., along) the cage. The data from the sonar provides information about the movement of bubble bands in the "cage" from which dispersion is estimated. In contrast, Thorpe et al. model "free" dispersion, including downwind motion and advection by a mean current.

The idealized models described below elucidate the differences between dispersion in the two cases. These also help explain the differences that occur when the wind and current are aligned or, as is more common, are in different directions. In reality the transient, variable, and nonlinear structure of Langmuir circulation, described by Faller and Auer (1988) and Thorpe (1992) and seen in side-scan sonograph images (e.g., Zedel and Farmer 1991; Thorpe et al. 1994), will contribute further to meandering and dispersion.

2. Free dispersion by steady circulation

The advection of a regular and steady Langmuir circulation pattern over a fixed source will cause the plume of buoyant released material to meander near the source and to be distorted even when the field of motion is steady.

We suppose that the motion induced at the sea surface can be represented by components of current, $u_c = u \sin kx$, directed in the x direction normal to the axes of the Langmuir cells, representing convergence, and $v_c = v_0 - v \cos kx$, directed in the y direction parallel to the cell axes, representing the more rapid downwind advection in the windrows (Kenney 1977; Weller and Price 1988). Typical measured values of u and v are 1–20 cm s^{-1}, probably depending on wind speed and its time history, fetch, surface heat flux, and surface waves. The position $x = 0$ is selected to lie on a line of divergence at the surface (midway between windrows), and k is the wavenumber of the cells so that $2\pi/k$ represents the distance between neighboring windrows. The component u is positive at small pos-

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itive kx, negative at small negative kx, and consistent with the choice of origin in a zone of divergence, while v_c is a maximum for positive v_0 and v at $kx = \pm(2n + 1)\pi$, $n = 0, 1, 2, \cdots$, the location of convergence, representing the relative downwind motion observed in the windrows.

Suppose now that this pattern of motion is advected by a steady current (e.g., a tidal current) with components U and V past a fixed source from which a continuous stream of floating particles is released. The subsequent motion of a particle free to move with the fluid in a frame of reference (X, Y) fixed in space at the release location is then given by

$$\frac{dX}{dt} = U + u\sin kx$$
$$\frac{dY}{dt} = V + v_0 - v\cos kx\,, \tag{1}$$

where now $x = X - Ut$; t is the time since release began, and we have assumed that the cell divergence line $x = 0$ passed through the release point $X = Y = 0$ at $t = 0$. The positions X and Y are functions of t and of t_0, the time at which the particle in question was released at $X = Y = 0$. We can integrate

$$\frac{dx}{dt} = \frac{dX}{dt} - U = u\sin kx \tag{2}$$

to obtain

$$kx = 2\tan^{-1}\left\{-\tan\left(\frac{kUt_0}{2}\right)\exp[ku(t - t_0)]\right\} \tag{3}$$

and

$$X = Ut - 2k^{-1}\tan^{-1}\left\{\tan\left(\frac{kUt_0}{2}\right)\exp[ku(t - t_0)]\right\}. \tag{4}$$

Similarly,

$$Y = (V + v_0 - v)(t - t_0) + \frac{v}{kU}\ln\left\{\cos^2\left(\frac{kUt_0}{2}\right) + \exp[2ku(t - t_0)]\sin^2\frac{kUt_0}{2}\right\}. \tag{5}$$

If axes X and Y are chosen parallel to $y = x\cot\alpha$ and $y = -x\tan\alpha$, respectively, to represent the sonar beam directions, and the mean flow is V directed at an angle β to the X direction (as shown in Fig. 1), a more general solution can be found given by

$$\begin{aligned} X_1 = {} & \left(\cos\beta + \frac{v_0}{V}\cos\alpha - \frac{w_1}{V}\cos\phi_1\right)(T - T_0) + \frac{w_1}{u}T_0\sin\phi_1\sin(\alpha - \beta) \\ & + \frac{w_1}{u}\left(\cos\phi_1\ln\left\{\cos^2\frac{(T_0\sin(\alpha - \beta))}{2} + \exp\left[\frac{2u}{V}(T - T_0)\right]\sin^2\frac{(T\sin(\alpha - \beta))}{2}\right\}\right. \\ & \left. - 2\sin\phi_1\tan^{-1}\left\{\exp\left[\frac{u}{V}(T - T_0)\right]\tan\left[\frac{T_0\sin(\alpha - \beta)}{2}\right]\right\}\right), \end{aligned} \tag{6}$$

$$\begin{aligned} Y_1 = {} & \left(\sin\beta + \frac{v_0}{V}\sin\alpha - \frac{w_2}{V}\sin\phi_2\right)(T - T_0) + \frac{w_2}{u}T_0\cos\phi_2\sin(\alpha - \beta) \\ & + \frac{w_2}{u}\left(\sin\phi_2\ln\left\{\cos^2\left[\frac{T_0\sin(\alpha - \beta)}{2}\right] + \exp\left[\frac{2u}{V}(T - T_0)\right]\sin^2\left[\frac{T\sin(\alpha - \beta)}{2}\right]\right\}\right. \\ & \left. + 2\cos\phi_2\tan^{-1}\left\{\exp\left[\frac{u}{V}(T - T_0)\right]\tan\left[\frac{T_0\sin(\alpha - \beta)}{2}\right]\right\}\right), \end{aligned} \tag{7}$$

where $(X_1, Y_1) = (kX, kY)$ and X, Y describe the position of the particle at time $t = T/kV$ following release at time $t_0 = T_0/kV$, and

$$w_1 = (u^2\sin^2\alpha + v^2\cos^2\alpha)^{1/2},$$
$$w_2 = (u^2\cos^2\alpha + v^2\sin^2\alpha)^{1/2},$$

$$\phi_1 = \tan^{-1}\left(\frac{u}{v}\tan\alpha\right),$$
$$\phi_2 = \tan^{-1}\left(\frac{v}{u}\tan\alpha\right). \tag{8}$$

As an illustration of the nature of dispersion, the circles in Fig. 2a show the positions of particles released

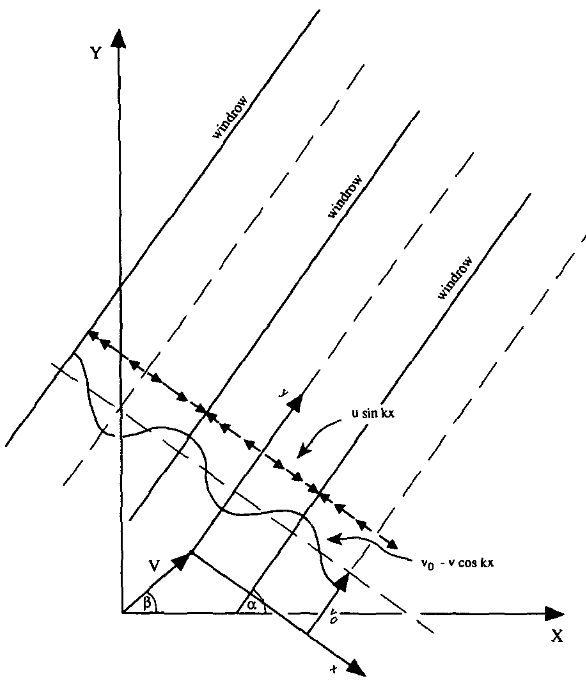

FIG. 1. The coordinates and currents. A Langmuir circulation pattern oriented at angle α to the X axis is advected in a mean flow V in direction β. In a frame of reference (x, y) carried with the circulation pattern, the Langmuir-induced flow at the surface has components $(u \sin kx,\ v_0 - v \cos kx)$ as sketched.

at equal increments of time $0.1\pi kV/\sin\alpha$ into a flow in which $u = v = v_0 = 0.1V$, $\beta = 0$, and $\alpha = 30$ deg. The instantaneous positions of the convergence lines associated with the windrows at the time illustrated ($t = 10\pi kV/\sin\alpha$) are marked by straight lines. The following may be seen:

(i) There is a spacial (and temporal; compare Figs. 2a,b) meandering pattern close to the source at $X_1 = Y_1 = 0$. This may account for the meandering seen in the diesel plume shown in Fig. 3.

(ii) Far from the source the particles become aligned in the windrows, and the length of the particle lines in the windrows increases with distance from the source. An example of this effect is shown in Fig. 4.

(iii) Particles, released into the flow as the divergence line between windrows passes the source, remain in this position and are carried by the flow V in the X direction. Since $v = v_0$, the Langmuir circulation flow is zero at the divergence line.

(iv) The centroid of the particles does not move in the direction of the mean surface flow $(V + v_0 \cos\alpha,\ v_0 \sin\alpha)$. Particles in the windrows are advected at an additional velocity $(v \cos\alpha,\ v \sin\alpha)$, and so, for example, drift cards released on the surface would move neither at the speed of the current averaged over the water surface nor in the direction of the average current.

(v) In each windrow the particles that are moved farthest in the direction of motion are clustered closest

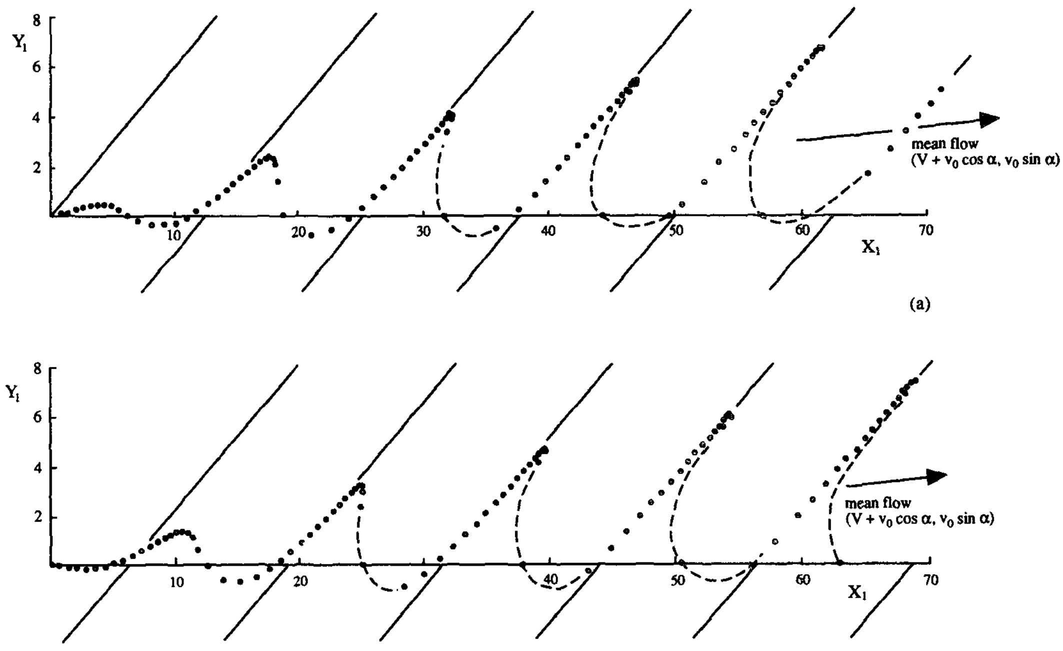

FIG. 2. The displacements of floating particles released at equal time intervals $0.1\pi/kV \sin\alpha$ into a flow with $\alpha = 30$ deg, $u/V = 0.1$, and $u = v = v_0$. The axes are X_1, $Y_1 = k(X, Y)$. (a) Position of particles at a time when a windrow (continuous line) passes the release point $X_1 = Y_1 = 0$; (b) the pattern at a time $\pi/kV \sin\alpha$ later when the divergence line of the circulation pattern passes the release point.

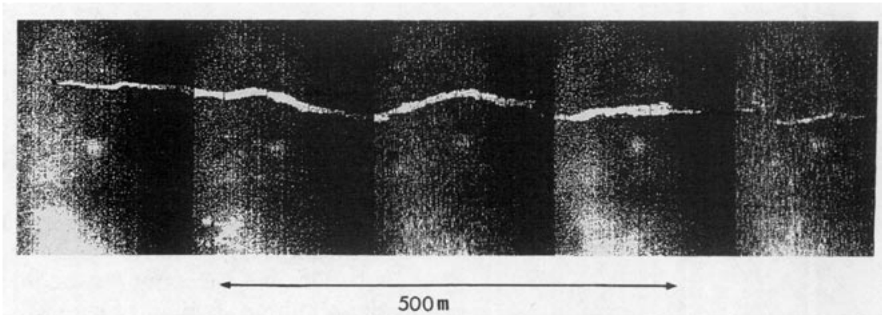

FIG. 3. A plume of floating diesel oil in the southern North Sea, meandering as it disperses from a moored source to the left of the picture. The figure is produced from a series of infrared images taken from an overflying aircraft. The water depth is about 45 m, and the water column is well mixed. The tidal current is 0.6 m s^{-1} in direction 220 deg and the wind speed is 6 m s^{-1} in direction 130–135 deg. The evolution into a linear, wind-aligned pattern can be seen at the right of the picture.

together as they approach the stagnation line for the component of velocity normal to the windrows.

(vi) The timescale for the particles to reach the windrow is given by $2\pi/ku$. In this time, the Langmuir circulation pattern is carried a distance $X_1 = kX = 2\pi V/u \approx 60$, consistent with Fig. 2.

(vii) The advection of the Langmuir circulation pattern causes a lateral spreading of particles (Fig. 4). The consequent dispersion of particles in the direction normal to the mean flow in a period of timescale $t_1 \approx (\pi/2ku)$ (i.e., until, on average, the particles reach the windrows) is independent of V and α and occurs at a rate with a diffusion coefficient proportional to uk^{-1}. Once the majority of particles have reached a windrow (e.g., for X_1 greater than about 60 in Fig. 2), the dispersion is diminished, at least until the pattern of Langmuir circulation breaks up or cells amalgamate, as eventually happens in reality (Thorpe 1992; Thorpe et al. 1994).

3. "Constrained" dispersion by "steady" circulation

Solutions for the dispersion of floating particles can also be found when the particles are restricted to lie within a two-sided "chicken wire" cage extending along the sonar beam (Thorpe and Curé 1994), a cage through which the flow passes freely but within which the particles remain. Their position is determined in terms of the change in their position in the beam Y, as a function of time t, and their time of release t_0.

In general the distances of the particles from the release point differ from those found in section 2 when

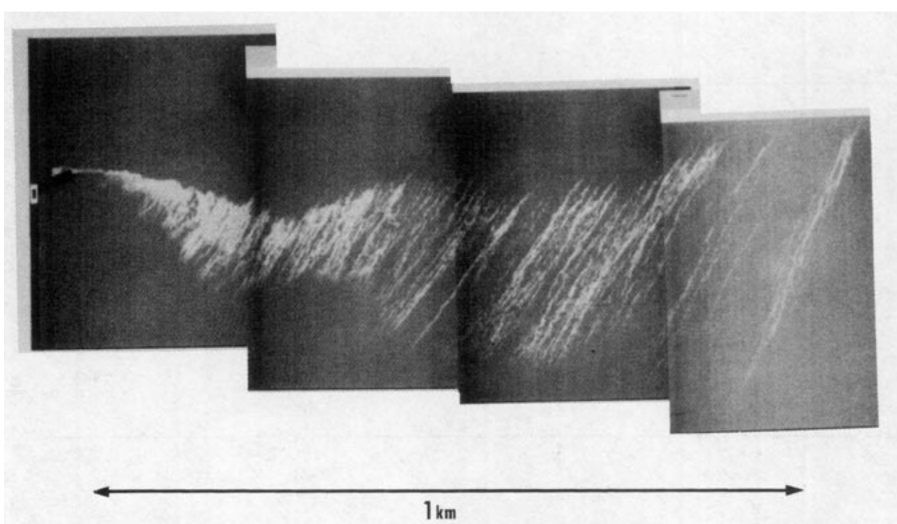

FIG. 4. Dispersion of Kuwait crude oil from a fixed source at the left of the image in the same area as Fig. 3. This is a composite infrared image obtained from an overflying aircraft. The effect of Langmuir circulation on the dispersion results in the banded structure that develops as the plume is advected by the tidal stream. The wind speed was 10 m s^{-1}. The image shows evidence of large-scale meandering with a wavelength of some 500 m, possibly caused by local topography.

they were free to move with the flow. An extreme case but one that serves nicely to illustrate the difference between the two cases is when the windrow convergence lines are aligned with the beam (or cage) direction, the mean current V is directed normal to the beam and when, for simplicity, $v_0 = 0$ so that there is no mean flow along the beam. The particles constrained to lie in the beam are carried by the component of current v, which varies sinusoidally as the Langmuir circulation pattern is advected through the beam, and their motion is therefore also sinusoidal; they remain close to the release point. If, however, the particles are free as in section 2, they will tend to congregate in windrows and be carried by the motion there at speed v in a direction parallel to the beam. Their distance in the beam direction increases and is approximately $v(t - t_0)$ for large t.

The equation for the displacement Y of a particle released at time t_0 in the "chicken wire cage" is given by

$$Y_1 = \frac{\alpha}{\cos\alpha} - T\tan\alpha + \frac{2}{\cos\alpha}\tan^{-1}\Gamma, \qquad (9)$$

where

$$\Gamma = q\tan\left(\frac{\sin\alpha}{2}\left(1 + \frac{v_0}{V}\cos\alpha\right)q(T - T_0) + \tan^{-1}\left\{\frac{\tan}{q}\left[(T_0\sin\alpha - r) + r\right]\right\}\right) - r, \text{ if } r < 0,$$

$$\Gamma = \left\{\frac{\sin\alpha}{2}\left(1 + \frac{v_0}{V}\cos\alpha\right)(T - T_0) + \left[1 + \tan\frac{(T_0\sin\alpha - \alpha)}{2}\right]^{-1}\right\}^{-1} - 1, \text{ if } r = 0,$$

or

$$\Gamma = r + q\frac{(1 - p)}{1 + p}, \text{ if } 1 > r > 0,$$

with

$$p = \left\{\left[r + q - \tan\frac{(T_0\sin\alpha - \alpha)}{2}\right]\cdot\left[q - r + \tan\frac{(T_0\sin\alpha - \alpha)}{2}\right]^{-1}\exp\left[q\sin\alpha\left(1 + \frac{v_0}{V}\cos\alpha\right)(T - T_0)\right]\right\}.$$

Here $Y_1 = kY$, $T = kVt$, $T_0 = kVt_0$ as before, $r = u/(V\tan\alpha + v_0\sin\alpha)$, and $q = (1 - r^2)^{1/2}$. We have taken $u = v$, $r < 1$, and $\beta = 0$.

As an example, Fig. 5 shows the displacement Y_1 of particles released at equal intervals of time free to follow the mean flow (circles) or constrained to lie within the beam (crosses) when $\alpha = 30$ deg, $v_0 = v = u = 0.1V$. The timescale $T' = kV\sin\alpha t/2\pi$. Free particles undergo more rapid dispersion.

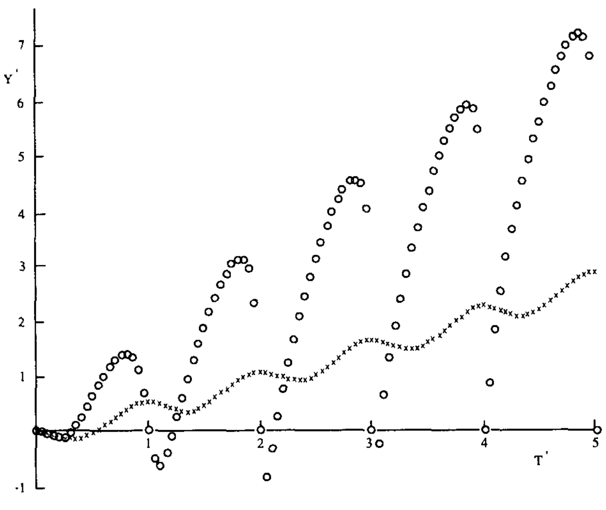

FIG. 5. The $Y_1 = kY$ displacement of floating particles at times $T' = kVt\sin\alpha/2\pi$ after release into a flow with $\alpha = 30$ deg, $u/V = 0.1$, and $u = v = v_0$. The circles mark the displacement of particles free to advect in the flow, while the crosses mark the displacement of particles restricted to remain on the Y axis in a chicken wire cage.

4. Conclusions

We have shown how very simple models may give some insight into the spread of floating material from a source fixed at the sea surface. Langmuir circulation affects dispersion at scales of 2 m to 1 km (Thorpe et al. 1994) and, as Fig. 4 demonstrates, may affect the local distribution and concentration of buoyant material in a plume originating from a fixed point.

Our simplistic choice of velocity field taken to represent the surface motions induced by Langmuir circulation contains the main elements, but not the details, of the real flow. There is some evidence, for example, that Langmuir cells have intensified flows close to the windrows, with stronger downward jets and weaker upward return flows (e.g., see Leibovich 1983, Fig. 1a); the surface flows will be correspondingly distorted. Although these may alter the time history of particle motions, they will not change the substance of our conclusions.

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