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Excitation of microseisms: views from the normal-mode approach

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SUMMARY

Non-linear interaction of ocean waves is a widely accepted mechanism of microseism excitation for frequencies approximately between 0.05 and 0.5 Hz. Longuet-Higgins published the most influential paper on this subject in 1950 and his main contributions can be summarized in two points; the first is on the double-frequency mechanism and the second is on the dependency of excitation on ocean depth. The two results were derived for different media, the first point for an incompressible liquid (ocean) and the second point for a compressible liquid layer over an elastic half-space. The two features naturally come out in the normal-mode formulae. We note, however, that the use of the Longuet-Higgins formula, that showed efficiency of excitation based on ocean depth, is not suitable for interpreting land-based observations because his formula was for displacement at ocean bottom. Reevaluation of this dependency for a land-based observation shows similar results for the depth of maximum excitation, but the sharpness of this peak is reduced significantly. Some recent observational studies tried to identify the source area of excitation by using this ocean-depth dependency, but such a result needs to be revised to a larger source region according to our results. We confirm that the maximum excitation of microseisms exists and occurs when the ocean depth is about 2.7 km. It is thus not surprising to find an efficient pelagic source for microseisms, although existence of the wave–wave interaction in the source region is essential.

Key words: Non-linear differential equations; Surface waves and free oscillations; Theoretical seismology; Wave propagation.

1 INTRODUCTION

Seismic noise makes up a significant portion of seismograms and has been regarded as unwanted noise until about 10 yr ago. However, recent developments in seismology have shown that useful information about the Earth structure can be obtained from its analysis (e.g. Campillo & Paul 2003; Sabra *et al.* 2005; Shapiro *et al.* 2005). Its source of excitation is not well understood, however, and sources may vary depending on the frequency band of seismic noise. We do know, however, that the most dominant seismic noise, the microseisms for frequencies approximately between 0.05 and 0.5 Hz, is caused by ocean waves.

Longuet-Higgins (1950) has been the most influential paper on the theory of microseism excitation by ocean waves. We examine the same problems in this study by applying a seismic normal-mode theory that has been developed after his paper (Gilbert 1970; Dahlen & Tromp 1998). This was partly done by Tanimoto (2007) which showed that the original pressure term by Longuet-Higgins (1950), the famous double-frequency pressure term, naturally arises in the normal-mode analysis. We will extend this analysis to a laterally heterogeneous Earth by the JWKB approximation and examine the other aspect of the results in Longuet-Higgins (1950), namely on the influence of ocean depths for the excitation of microseisms. It is essential to extend the theory to a laterally heterogeneous Earth because the excitation source is in the ocean and most observations are on land.

Recent numerical attempts to estimate microseism amplitudes tend to use the original Longuet-Higgins' formula, thus using a formula for observation at ocean bottom (e.g. Kedar *et al.* 2008; Ardhuin *et al.* 2011; Stutzmann *et al.* 2012), which possibly lead to incorrect results. Evaluation of this incorrectness requires numerical analysis, but modelling microseisms using a formula for an ocean-bottom observation cannot be correct for land-based observations.

Longuet-Higgins (1950) stated clearly that the analysis that led to his famous double-frequency pressure formula was valid in an incompressible liquid but the assumption of incompressibility was not justified for commonly observed microseisms. He extended the theory to a compressible fluid layer over an elastic half-space and showed that the excitation becomes a function of ocean depth. This result is important as the excitation becomes the maximum for a particular ocean depth. It has been used recently by Kedar *et al.* (2008) to argue for a pelagic microseism source in the North Atlantic Ocean and a claim for a relatively focused source. We will evaluate how land-based observations could differ from those results.

The aims of this paper are twofold: one to extend the formulae to a laterally heterogenous Earth so that we can evaluate the land-based observations and the other to examine the ocean-depth dependence of efficiency of microseism excitation. We will summarize our approach in Section 2, and describe Longuet-Higgins' derivation on the optimum ocean depth and compare the differences by numerical evaluation in Section 3. Conclusion will be in Section 4.

2 NORMAL-MODE APPROACH AND JWKB FORMULA

In this section, we summarize our normal-mode formulae for seismic noise excitation. They are based on the formulae in Tanimoto (2007) which was for a laterally homogeneous Earth and will be extended to a laterally heterogeneous Earth by the JWKB approximation. This will be the basis for numerical evaluation in Section 3.

2.1 Normal-mode formula in spherical coordinate

In Tanimoto (2007), it was shown that the displacement wavefield generated by interactions among ocean waves is given, in the Cartesian coordinate, by

$$\mathbf{u}(t) = -\frac{L_x L_y}{(2\pi)^2}\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}\, R(k) \int_{-\infty}^{t} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}' \times \frac{\sin\omega(t-\tau)}{\omega}\rho\omega'^2 U(0)\cos(2\omega'\tau)a(\mathbf{k}')a(-\mathbf{k}'), \quad (1)$$

where $\mathbf{u}(t)$ is the displacement vector, L_xL_y is the source area with x and y defining the horizontal plane, $\mathbf{R}(\mathbf{k})$ is an eigenfunction vector of Rayleigh waves with horizontal wavenumber $\mathbf{k} = (k_x, k_y)$ and is given by

$$\mathbf{R}(k) = \begin{bmatrix} U(z) \\ V(z)\frac{\mathrm{i}k_x}{k} \\ V(z)\frac{\mathrm{i}k_y}{k} \end{bmatrix} e^{\mathrm{i}(k_x x + \mathrm{i}k_y y)}, \quad (2)$$

where the order is z, x and y components. The integration with respect to τ exists in (1) because the excitation source is continuous in time. The variables $\mathbf{k}'$ and ω' are the wavenumber and the angular frequency of ocean waves and they are distinguished from those of Rayleigh waves, ω and $\mathbf{k}$, by the primes. $a(\mathbf{k}')$ is the surface amplitude of ocean waves with wavenumber $\mathbf{k}'$ and $a(\mathbf{k}')a(-\mathbf{k}')$ means multiplication of amplitude of waves that propagate in the opposite directions. ρ is density in the ocean and dispersion relation for ocean waves, $\omega'^2 = gk' \tanh(k'd)$, relates ω' and $\mathbf{k}'$ ($k' = |\mathbf{k}'|$) where d is the ocean depth. The excitation source is at $x = y = z = 0$ at the ocean surface. Strictly speaking, the source is within the depth range of ocean waves which is typically 100 m (ocean waves at period about 14 s), but considering the wavelength of excited seismic waves, it can be treated to be at the surface. Since the excitation source is essentially a single force at the surface of the ocean, the integrand contains $U(0)$ which is the surface value of Rayleigh wave vertical eigenfunction.

In the formal normal-mode derivation, there emerges a horizontal force term in the analysis (Tanimoto 2007, 2010; Webb 2007, 2009) but it is much smaller than the vertical force term in the microseism frequency band (approximately 0.05–0.5 Hz) and thus is not considered in this paper.

In eq. (1), the Cartesian coordinate was used. We will convert the above formula to a spherical one in this paper, as microseisms are observed in a broad area and sphericity may not be ignored. In terms of formula, only a few changes need to be made in the above formula. First, the integral over $\mathbf{k}$,

$$\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}\, \mathbf{R}(\mathbf{k}) \quad (3)$$

will be replaced by a summation over normal modes

$$\sum_p \mathbf{s}_p(\mathbf{x}), \quad (4)$$

where $\mathbf{s}_p$ is an eigenfunction vector for the pth spheroidal mode. It has the form (e.g. Dahlen & Tromp 1998)

$$\mathbf{s}_p(\mathbf{x}) = \left(U_n(r)Y_l^m,\ \frac{V_n(r)}{\sqrt{l(l+1)}}\frac{\partial Y_l^m}{\partial\theta},\ \frac{V_n(r)}{\sqrt{l(l+1)}}\frac{\partial Y_l^m}{\sin\theta\,\partial\phi} \right), \quad (5)$$

where the mode number p specifies a set of (n, l, m); n is the overtone number and Y_l^m is spherical harmonics with angular degree l and angular order m.

Hereafter, we restrict to a vertical component for simplicity. Eq. (1) becomes

$$u_z(t) = -L_x L_y \sum_p U_p(r)U_p(r')Y_l^m(\theta,\phi)Y_l^m(\theta',\phi') \times \int_{-\infty}^{t} \mathrm{d}\tau \frac{\sin\omega_p(t-\tau)}{\omega_p} \int \mathrm{d}\mathbf{k}'\rho\omega'^2\cos(2\omega'\tau)a(\mathbf{k}')a(-\mathbf{k}'), \quad (6)$$

where (r, θ, ϕ) is the location of seismograph and (r', θ', ϕ') is the source location.

The pressure generated by ocean waves was given by (Tanimoto 2007)

$$p(t) = \int \mathrm{d}\mathbf{k}'\rho\omega'^2\cos(2\omega t)\, a(\mathbf{k}')a(-\mathbf{k}'), \quad (7)$$

which is equivalent to the Longuet-Higgins' (1950) pressure formula (in our notation). We introduce the single force (source) $f(t)$ by absorbing $-L_xL_y$ in it, assuming the source area is small.

$$f(t) \equiv -L_x L_y p(t). \quad (8)$$

If we denote the Fourier spectra for this force by $F(\omega)$, we can write

$$F(\omega) = -L_x L_y \int \mathrm{d}\mathbf{k}' \frac{\mathrm{i}\rho\omega\omega'^2}{4\omega'^2 - \omega^2} a(\mathbf{k}')a(-\mathbf{k}'), \quad (9)$$

where we used the fact that Fourier transformation of $\cos(2\omega' t)$ is $\mathrm{i}\omega/(4\omega'^2 - \omega^2)$.

Using the fact that time domain convolution becomes a simple multiplication in the Fourier domain and the Fourier spectra of $\sin(\omega_p t)/\omega_p$ is $1/(\omega_p{}^2 - \omega^2)$, we can write the Fourier spectra of $u_z(t)$, $u_z(\omega)$, as

$$u_z(\omega) = \sum_p \frac{F(\omega)}{\omega_p^2 - \omega^2} U_p(r)U_p(r')Y_l^m(\theta,\phi)Y_l^m(\theta',\phi'), \quad (10)$$

where the primed location (r', θ', ϕ') is the source location and (r, θ, ϕ) is the receiver location.

This is the normal-mode formula for microseisms in a layered laterally homogeneous Earth.

2.2 JWKB formula

Typical microseism observations are made by seismographs on land. Since the excitation is in the ocean, seismic waves naturally go through laterally varying structure. We will obtain the JWKB formula that will be an extension of the formula (10). The derivation was already summarized by Dahlen & Tromp (1998) and we will simply follow their procedure. The main parts on this topic in Dahlen & Tromp (1998) were originally published in Tromp & Dahlen (1992a,b, 1993).

For a laterally homogeneous Earth, Dahlen & Tromp (1998) showed that the Green's tensor in the frequency domain can be written

$$\mathbf{G}(\mathbf{x}, \mathbf{x}'; \omega) = \sum_p \frac{1}{\omega_p^2 - \omega^2} \mathbf{s}_p(\mathbf{x})\mathbf{s}_p(\mathbf{x}'). \tag{11}$$

For a vertical displacement due to a vertical force (source), we can write

$$G_{zz}(\omega) = \sum_p \frac{1}{\omega_p^2 - \omega^2} U_p(r)U_p(r')Y_l^m(\theta, \phi)Y_l^m(\theta', \phi'), \tag{12}$$

which is essentially the same with (10) except for the source term $F(\omega)$.

This eq. (12) is for a laterally homogeneous Earth. Extending it to a laterally smoothly varying medium and restricting to a propagating wave, Dahlen & Tromp (1998) derived the JWKB formula

$$G_{zz}(\mathbf{x}, \mathbf{x}') = \sum_{\text{rays}} \frac{U(r)U'(r')}{\sqrt{8\pi k_R(r)S_R(\mathbf{x}, \mathbf{x}')}} \exp^{\mathrm{i}(\int_S^R \mathbf{k_R} \mathrm{d}l - M\frac{\pi}{2} + \frac{\pi}{4})}, \tag{13}$$

where $\mathbf{k}_\mathrm{R}$ is the wavenumber vector along a propagation path ($k_R = |\mathbf{k}_\mathrm{R}|$) and $S_R(\mathbf{x}, \mathbf{x}')$ is the geometrical spreading factor for each ray. We now distinguish eigenfunctions at the source (U') and at the receiver (U) by using the prime because Earth structure differs laterally. This formula is for an elastic case. The effects of attenuation requires addition of an exponential term to this formula. The summation indicates contributions from multipaths and higher modes. M is the Maslov index, counting the number of caustics along a ray path.

As the difference between (10) and (12) is only due to the term $F(\omega)$, we can immediately write down our solution for vertical displacement spectra

$$u_z(\omega) = F(\omega) \sum_{\text{rays}} \frac{U(r)U'(r')}{\sqrt{8\pi k_R(r)S_R(\mathbf{x}, \mathbf{x}')}} \exp^{\mathrm{i}(\int_S^R \mathbf{k_R} \mathrm{d}l - M\frac{\pi}{2} + \frac{\pi}{4})}. \tag{14}$$

This is the JWKB formula for a microseism source in the frequency domain.

3 OCEAN DEPTH DEPENDENCE

Now we focus on the ocean depth dependency of microseism excitation. We summarize the Longuet-Higgins' result first and then discuss our normal-mode formula.

3.1 Longuet-Higgins' approach

By analysing the case of a compressible liquid layer (ocean) over an elastic half-space, Longuet-Higgins (1950) showed that the efficiency of microseism excitation varies with ocean depth. In the context of this paper, an important point is that he used a displacement formula at ocean bottom. This makes the use of his formula unsuitable for the interpretation of many microseism observations as most seismographs are on land.

Longuet-Higgins (1950) derived a formula for the ocean-bottom displacement for a medium with a homogeneous liquid layer over an elastic half-space. The excitation source was a point source at the surface of the liquid layer. The displacement at ocean bottom was expressed as (eq. 178 in Longuet-Higgins, 1950, but using our notation for angular frequency and wavenumber)

$$W(\omega, r)\mathrm{e}^{\mathrm{i}\omega t} = -\frac{1}{2\pi} \int_0^\infty \frac{J_0(kr)k\mathrm{d}k}{\rho_2\omega^2 G(\omega, k)} \mathrm{e}^{\mathrm{i}\omega t}, \tag{15}$$

where J_0 is the Bessel function of zeroth order, k is the wavenumber, r is the horizontal distance, ρ_2 is the density of the elastic (lower) half-space and $G(\omega, k)$ is the characteristic function for the medium. The zeroes of $G(\omega, k)$ determine the dispersion relation for surface waves. He refers to them as Stoneley waves instead of surface waves. In current literatures, Stoneley modes often refer to modes that are trapped at a discontinuity but in his derivation the modes contain all surface wave modes as the function G is complete for the medium that had a homogeneous liquid layer over an elastic homogeneous half-space.

From this formula, he derived the next formula (eq. 186 in Longuet-Higgins 1950) for the spectral amplitude which has been quoted often

$$\bar{W} = \frac{\omega^{1/2}}{\rho_2\beta_2^{5/2}(2\pi r)^{1/2}} \left[\sum_{m=1}^{N} c_m^2\right]^{1/2}, \tag{16}$$

which contains contributions from fundamental modes to the $(N-1)$th higher mode. Residue contributions for surface wave poles, yield the terms c_i ($i = 1, 2, \ldots, m$). From the variations of c_i ($i = 1, 2, \ldots$) with ocean depth, he pointed out that the excitation of microseisms vary with ocean depth. For the fundamental-mode Rayleigh waves, his relation for the most efficient ocean depth is

$$d = 0.85\frac{\beta_2}{\omega}. \tag{17}$$

If $\beta_2 = 3.2$ (km s^{-1}), for example, the maximum excitation of secondary microseisms (0.15 Hz) occur for an ocean depth of 2.9 (km). For the next three higher modes, he showed that 0.85 has to be replaced by 2.7, 4.1 and 6.3, respectively, although for most observations, the fundamental modes are most relevant to the discussion as the excitation source is shallow.

3.2 Numerical comparison

Eq. (14) shows that the main difference between an ocean-bottom observation and an on-land observation will arise from the difference for $U(r)U'(r')$. We now suppose that the radius at the ocean surface and at the receiver on land are both $r = R$. If the ocean depth is d, the Longuet-Higgins' formula corresponds to the case when this term is $U'(R)U'(R-d)$. We assume that the ocean-bottom site for observation has the same oceanic structure with the source location. For a land-based observation site, this term becomes $U'(R)U(R)$.

In this section, we numerically compare these terms. We will compute the eigenfunctions of oceanic structure with different ocean depths from 0 to 10 km. Beneath the ocean, we keep the same solid Earth structure with PREM (Preliminary Reference Earth Model; Dziewonski & Anderson 1981). For a structure on land, we use the average structure for an SCEC CVM model (Southern California Earthquake Center Community Velocity Model 3.0; Kohler *et al.* 2003).

Fig. 1 shows a comparison of the two terms at 0.15 Hz, the dominant frequency of the secondary microseisms. For different thicknesses of the oceanic layer, we computed Rayleigh wave eigenfrequencies and eigenfunctions, and numerically evaluated the two terms. Both curves were normalized by the maximum of each curve.

We note the two main differences; the first is that the values for observation at ocean bottom (solid) show a much sharper peak, as a

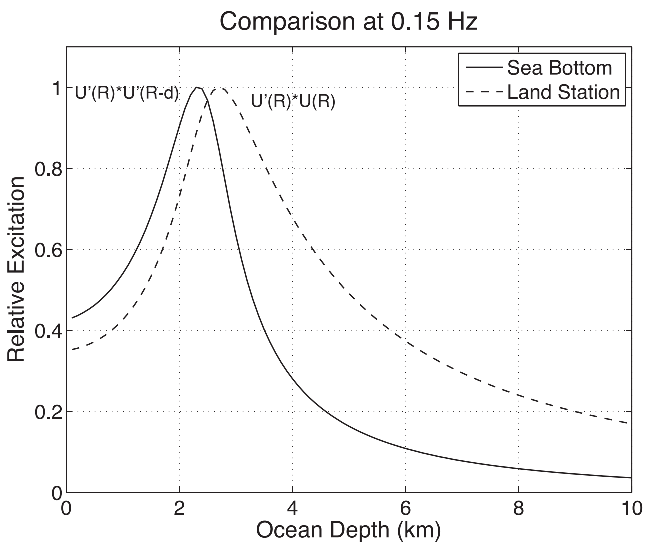

Figure 1. Comparison of displacements generated at ocean bottom and on land. Most microseism observations are on land, thus the dash curve should apply. Longuet-Higgins' formula (1950) was for displacement at ocean bottom. In this case, the peak (as a function of ocean depth) becomes very sharp.

function of ocean depth, than the values for a land-based observation (dash). The second is that the ocean depth for the maximum peak differs slightly. The peak for the ocean-bottom displacement is at 2.3 km while the peak for the land-based observation is at 2.7 km.

The reason for these differences can come from in the values for $U'(R-d)$, the ocean-bottom values for a vertical eigenfunctions, and the values for $U(R)$, the surface eigenfunction values of a receiver side on land. The difference between them is caused by the existence of an oceanic (liquid) layer on top which shows up in the shape of $U'(R-d)$. Fig. 2 shows the variations of $U'(R-d)$ at four different frequencies 0.05, 0.10, 0.15 and 0.20 Hz. This figure shows how the ocean-bottom values $U'(R-d)$ change with different thickness of oceans. The values of $U'(R-d)$ are 1 for shallow oceans but they sharply decrease beyond a certain ocean depth. In the case of 0.15 Hz (red), the values of $U'(R-d)$ change sharply beyond the ocean depth 2 km. The eigenfunction values at an on-land site, $U(R)$, do not have such features. This sharp change contributes to the two features in Fig. 1 described earlier; first, a peak in Fig. 1 becomes sharper because the right side of the peak drops off due to this feature in $U'(R-d)$. And second, a slight change of the peak depth towards a shallower depth (2.3 from 2.7 km) occurs, because of the same rapid fall on the deeper ocean side.

These results indicate that a land-based observation of microseisms should show dependence on ocean depths but the effects are much more milder than indicated by the Longuet-Higgins' formula. It follows that an identification of a source area, using this information on ocean depths, becomes much broader than that indicated by Kedar *et al.* (2008). However, the location of the maximum excitation should not be affected very much.

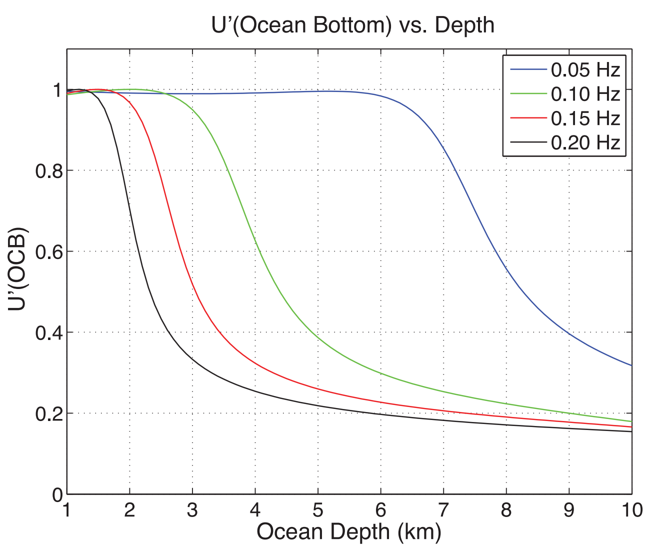

Figure 2. The reason for the sharpness in peak in Fig. 1 is in the term $U(R-d)$, the vertical eigenfunction at ocean bottom. The values at four frequencies are shown. All show rapid drop-off beyond a certain depth. For 0.15 Hz, a sharp drop-off occurs when the ocean depth exceeds about 2 km. Multiplication of this function will make the peak sharper and also move the peak to smaller ocean depth (Fig. 1, solid). There is no such feature in the values $U(R)$, the surface value of eigenfucntion for a continental structure.

3.3 The most efficient ocean depth for excitation

While our formula differs from the Longuet-Higgins' formula, our results confirm an important aspect of his results. That is on the existence of most efficient ocean depth for the excitation of microseisms. In Fig. 3, we show the depth dependence of the term $U'(R)U(R)$ at four frequencies. They are for land-based observations. The curve for 0.15 Hz (red) is the same with the dash line in Fig. 1. The peak for 0.15 Hz is found when the ocean depth is 2.7 km. While this value could change for different solid Earth structures, this result suggests that the most efficient excitation occurs when the depth of ocean is around 2–3 km.

This statement only applies to the microseisms at 0.15 Hz, the dominant microseisms in the Earth. For different frequencies, the optimum ocean depths for the excitation vary; they are 2.0 km for 0.20 Hz, 4.0 km for 0.10 Hz and 7.7 km for 0.05 Hz (Fig. 3 and Table 1). In general, the optimum ocean depth becomes larger for lower frequency noise.

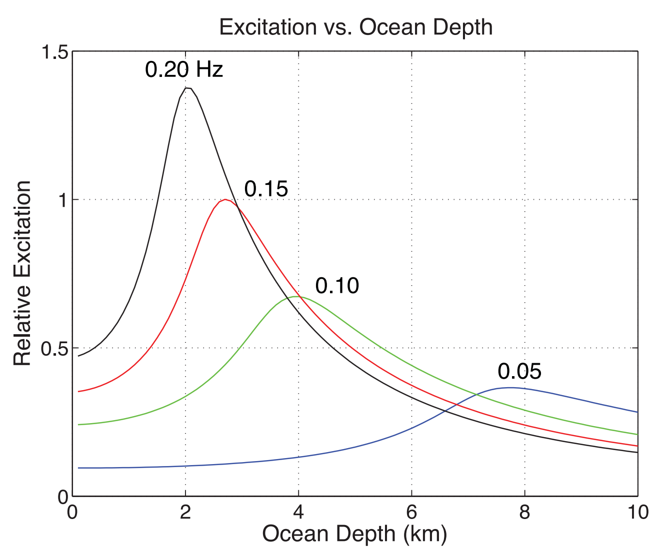

Figure 3. Ocean depth effects by our formula at four frequencies. It is important that there is an optimum ocean depth for microseism excitation. At 0.15 Hz, the peak emerges at 2.7 km (of ocean depth) for a PREM-like solid structure. For different frequencies, the peak ocean depth varies. The widths of the peaks are much broader than what were predicted by the Longuet-Higgins' formula.

Table 1. Ocean depths of the most efficient excitation of microseisms at four frequencies. The maximum amplitudes occur at 0.15 Hz. Our results were numerically obtained. Longuet-Higgins' results were computed using his formula $h = 0.85\beta_2/\omega$ using $\beta_2 = 3.2$ (km s^{-1}).

Frequency (Hz)	Our estimates (km)	Longuet-Higgins formula (km)
0.05	7.7	8.7
0.10	4.0	4.3
0.15	2.7	2.9
0.20	2.0	2.2

Numerical predictions for the most efficient ocean depths do not differ very much from Longuet-Higgins (1950) values. Table 1 compares the values for the most efficient ocean depths from the eigenfunctions and the values predicted by the formula $d\,0.85\beta_2/\omega$ (0.85 is from Longuet-Higgins 1950). β_2 is the *S*-wave velocity of the underlying solid half-space. We used $\beta_2 = 3.2$ (km s^{-1}) for Table 1 which was the *S*-wave velocity at the top of the solid layer in our model. The difference for 0.05 Hz is larger than other cases but within the main microseism frequency range (0.10–0.20 Hz), the differences are only about 0.2–0.3 km. They basically confirm Longuet-Higgins' results although his results were derived for a very simple structure.

4 CONCLUSION

By deriving an independent normal-mode theory for the excitation of microseisms, we evaluated the main conclusions by Longuet-Higgins (1950). The importance of wave-wave interactions and the double frequency mechanism naturally emerge in the normal mode theory.

Excitation of microseisms varies with ocean depth. The maximum excitation occurs for an ocean depth of 2.7 km for a PREM-like solid Earth structure, basically supporting Longuet-Higgins' (1950) conclusion derived for a simple layer over a half-space medium. Differences are found in the width of the peak or the sensitivity of excitation to ocean depths. These differences arose from the fact that Longuet-Higgins (1950) used a formula for displacement at the ocean bottom. For land-based seismograph data, depth dependence is much less sharper than his results indicated. An estimate for a source area, based on the Longuet-Higgins' formula (1950), needs some revision and the source area should become larger than previous estimates. Overall locations of the source area, however, are not likely to be affected very much.

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