

## RESEARCH ARTICLE

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## Spectral form and source term balance of short gravity wind waves

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## Key Points:

- We investigated the characteristics of short gravity wind waves
- The observed wave number spectra showed the spectral power laws
- The nonlinear dissipation term significantly improved the model results

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**Abstract** We investigated the spectral structure and source term balance of short gravity waves, based on *in situ* observations of wave number spectra retrieved by air-sea interaction spar (ASIS) buoys. The behaviors of wave number spectra up to 10 rad/m (the gravity wave regime) were analyzed for a wide range of wind and wave conditions. The observed wave number spectra showed the spectral power laws described by Toba (1973) and Phillips (1958) in addition to the characteristic nodal point at  $\sim 10$  rad/m where spectral energy becomes constant over the entire wind speed range. We also improved the third-generation wave model using the nonlinear dissipation term. The wave model reproduced the spectral form in the higher wave number domain. In the equilibrium range, nonlinear transfer played a major role in maintaining equilibrium conditions. On the other hand, in the saturation range, which starts at the upper limit of the equilibrium range, the nonlinear transfer tended to be out of balance with other source terms, and the dissipation term was in balance with wind input.

## 1. Introduction

Ocean surface waves, from capillary waves to swells, vary greatly on a wide range of temporal and spatial scales. Whereas the contribution of small-scale surface waves to the total wave energy is negligible, they contribute significantly to aerodynamic roughness and air-sea interaction processes, such as momentum transfer and gas exchange between the atmosphere and ocean. The modeling and analysis of microwave backscatter captured by scatterometers is significantly dependent on knowledge of the spectral characteristics of short gravity waves and gravity-capillary waves. Therefore, a better understanding of the spectral structure of small-scale ocean surface waves is crucial.

The spectral shape and wind speed dependence of spectral energy vary with wave number. The spectral structure in the equilibrium range has been addressed in a number of studies for wave numbers ranging up to  $\sim 1$  rad/m (6.3 m in wave length), or roughly three times the typical peak frequency. Nowadays, a spectral form of  $F(k) = \alpha_4 u_* g^{-1/2} k^{-5/2}$  (where  $F(k)$  is the omnidirectional surface elevation spectrum in the wave number domain,  $k$  is the wave number,  $g$  is the gravitational acceleration, and  $u_*$  is the friction velocity) in this range is commonly seen as robust in the ocean [e.g., Toba, 1973; Donelan et al., 1985; Resio et al., 2004]. Banner et al. [1989] analyzed the wave number spectra ranging from 4 to 31 rad/m, which are much shorter than the spectral components in the equilibrium range. They concluded that  $u_*$  dependence of the spectral energy might be neglected, and the wave number spectrum can be represented by  $F(k) = \alpha_5 k^{-3}$  to the lowest-order approximation. This form was originally proposed by the pioneering work of Phillips [1958], and Forristall [1981] also described a transition of the spectral power law from  $k^{-5/2}$  to  $k^{-3}$ . The backscattered power of L-band microwaves (wave number: 27 rad/m, wavelength: 0.24 m) shows weak dependence on surface wind [e.g., Isoguchi and Shimada, 2009]. Evidence consistently suggests that the spectral density in the intermediate range ( $\sim 5$ –30 rad/m) barely responds to the surface wind, because the backscattered power is basically proportional to the spectral density of the Bragg waves.

In the present study, we investigated the characteristics of the spectral structure for short gravity waves. In particular, we were interested in furthering the understanding of the dynamic balance of three source terms: wind input, nonlinear spectral transfer, and dissipation. We used observations from two campaigns in which wind velocities and surface elevations were measured by air-sea interaction spar (ASIS) buoys

[Graber *et al.*, 2000]. We investigated the behavior of observed wave number spectra up to 10 rad/m (the gravity wave regime) for a wide range of wind and wave conditions. We also conducted wave hindcasts to validate the applicability of the 3rd generation wave model and to discuss spectral forms and source term balances in the short gravity wave regime. It should be noted that the wave number range larger than the gravity-capillary wave regime ( $\sim 10^2 - 10^3$  rad/m) is beyond the scope of the present study.

The remainder of the paper is organized as follows. Section 2 describes the *in situ* data sets of wind and surface waves analyzed in this study. The model configurations of wave hindcasts are described in section 3. In Section 4, we show the observational and model results in terms of wave number spectra in short gravity wave regimes. Then we discuss the spectral form and source term balance using hindcast experiments in section 5. Finally, a summary is provided in section 6.

## 2. In Situ Data and Analysis Method

Traditionally many studies of surface gravity waves have relied on frequency spectra estimated from time series of surface elevation. However, as reported by Hwang and Wang [2004] and elsewhere, the Doppler frequency shift of short waves being convected by surface current (and longer waves) can cause serious problems in the interpretation of short wave spectra. For this reason, the study of short waves is best carried out with wave number spectra, calculated from spatial data. These data include those collected from stereophotography [Banner *et al.*, 1989], sparse spatial arrays [Donelan *et al.*, 1985] or radar backscatter [e.g., Hwang and Plant, 2010]. Such data are difficult to obtain, hence are relatively rare. Here we apply the wavelet directional method (WDM) proposed by Donelan *et al.* [1996] to get wave number spectra directly from the time series of surface elevations from wave staff arrays. The WDM provides the instantaneous local wave number vector and the corresponding wave energy with the assumption that at most one wave of a given scale propagates through the array at a given time. Following Donelan *et al.* [1996], we used the Morlet wavelet, with eight voices. Mean wave number spectra were estimated by binning the averages of the wavelet spectrum at each instance in time. Donelan *et al.* [1996] showed that frequency-directional spectra calculated by the wavelet method compared well to those calculated by traditional Fourier analysis and the Maximum Likelihood Method. Here we focus on 1-D (nondirectional) wave number spectra calculated by WDM.

We used wind and wave data from the FETCH (flux, état de la mer et télédétection en Conditions de fetch variable) experiment [Hauser *et al.*, 2003] and the DOGEE (Deep Ocean Gas Exchange Experiment) [Brooks *et al.*, 2009]. The details of the wind and wave data retrieved in the two field campaigns are described below.

### 2.1. The FETCH Data Sets

The FETCH experiment took place from March to April (Year Day YD: 77–99) 1998, in the Gulf of Lion, Mediterranean Sea. During the FETCH campaign, an ASIS buoy [Graber *et al.*, 2000] was deployed at 42.98°N, 04.25°E (roughly 50 km SSW of the Rhone delta, at a depth of 100 m) to measure the temporal evolution of turbulent momentum fluxes (or friction velocity) and the surface gravity waves. The study site is well known for frequent strong northerly events [e.g., Sampe and Xie, 2007], known as Mistral. During the observation period, the surface wind field was dominated by Mistral events from YD 78.8 to 81.2, and from YD 83 to 84.5 [Drennan *et al.*, 2003]. The temporal changes in surface elevations were obtained from an array of six capacitance wave staffs arranged in a centered pentagon of radius 0.927 m. The sampling rate was 12 Hz and wind and wave parameters were calculated from data segments of 28.5 min. A full description of the data collection and processing is provided by Drennan *et al.* [2003]. We analyzed a total of 857 wind and wave data sets. The highest wave number analyzed was 1 rad/m (wave length: 6.3m).

### 2.2. DOGEE Data Sets

Wind and wave data were also collected during DOGEE (Cruise D320) from June 22 to July 11 (YD: 173–192) 2007 in the northeast Atlantic. An ASIS buoy was configured for Lagrangian measurements (i.e., free drifting) and deployed at 43.753°N, 18.116°W, at a depth of  $\sim 4000$  m. The wave number spectra were obtained from an array of four capacitance wave staffs placed in a square with sides of 0.2 m. The array was on an outrigger 2.6 m away from the ASIS buoy to eliminate possible contamination due to the main 0.2 m diameter spars of the buoy. Data were sampled continuously at 20 Hz and wind and wave parameters were calculated from data segments of 30 min. The phase differences detected by wave staffs were used to provide

**Table 1.** Design of Hindcast Experiments for FETCH and DOGEE

Hindcast Experiments	Computational Domain	Horizontal Resolution	Computational Period
FETCH	37°N–44°N, 0°–13°E (Western Mediterranean)	0.05°	1 Mar 1998 to 1 May 1998
DOGEE	0°–70°N, 100°W–15°E (North Atlantic)	0.2°	1 May 2007 to 1 Aug 2007

the wave number vector in the WDM. Due to the shorter spacing behavior (0.2 m) of the capacitance wave staffs, it was difficult to detect the propagation (i.e., phase difference) of longer waves. Therefore, the lowest and highest wave numbers analyzed were set at 1 (wave length: 6.3 m) and 10 rad/m

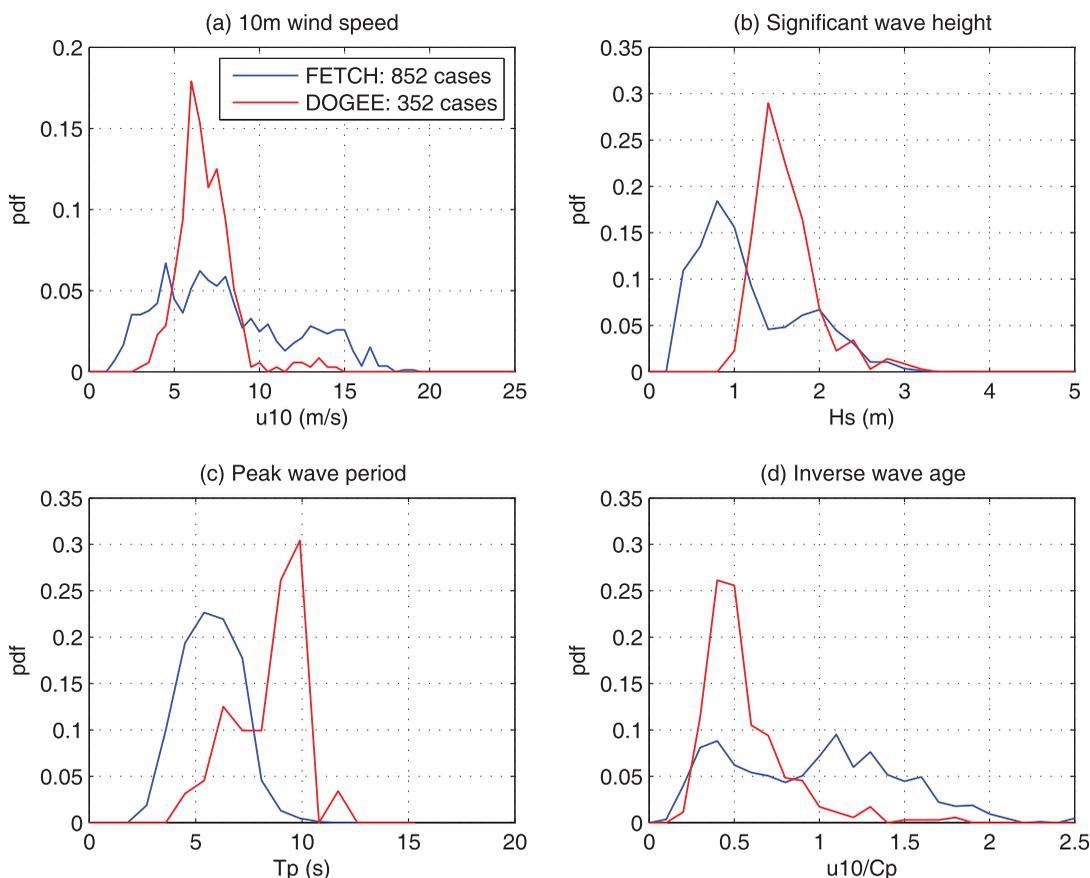
(wave length: 0.63 m), respectively. The observed wave number spectra were significantly contaminated by the overtopping of wave staffs during a severe storm from YD 180.7 to 182.6. After YD 183.2, two of the wave gauges were broken. We do not use any data following the storm event. A total of 352 wave number spectra were available for analysis.

### 3. Wave Hindcasts

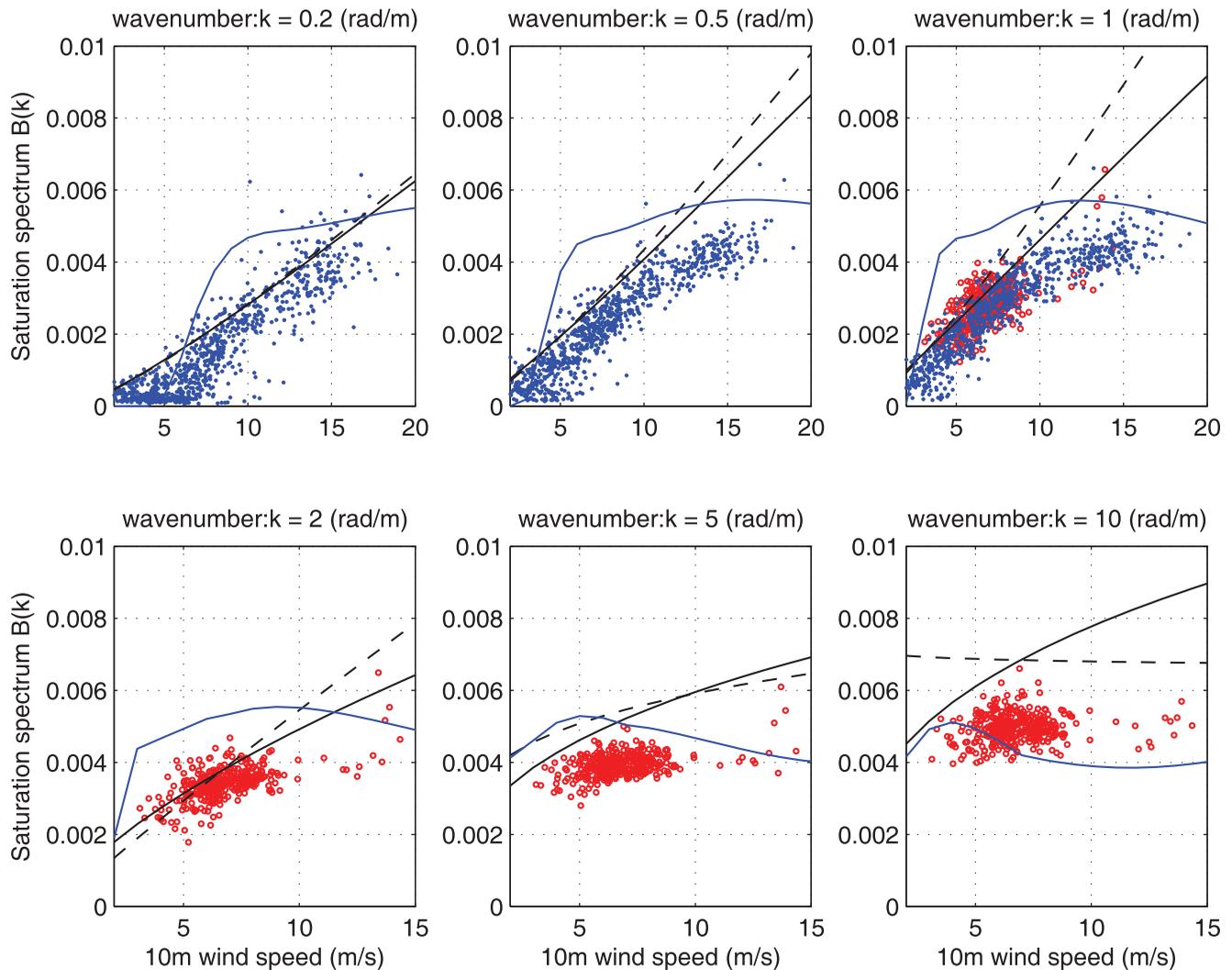
The spectral evolution of surface gravity waves can be represented by the following wave action equation:

$$D_t N = S_{in} + S_{nl} + S_{ds} \tag{1}$$

where  $D_t = \partial_t + c_g \cdot \nabla_x$  is the material derivative,  $c_g$  is the group velocity,  $\theta$  is wave direction,  $N = N(k, \theta)$  is the wave action ( $= F(k, \theta) / \sigma$ , where  $\sigma$  is the intrinsic frequency) with units of  $m^3 s$ ,  $S_{in}$  is wind input,  $S_{ds}$  is wave dissipation, and  $S_{nl}$  is the nonlinear transfer function. To discuss the spectral shape in the higher wave number domain, we also used the saturation spectrum,  $B(k)$ , defined as:



**Figure 1.** Probability density functions of observed wind and wave statistics: (a) surface wind at a height of 10 m,  $U_{10}$ ; (b) significant wave height,  $H_s$ ; (c) peak wave period,  $T_p$ ; and (d) inverse wave age,  $U_{10}/c_p$ , where  $c_p$  is the spectral peak phase speed. Blue and red lines indicate the data sets for FETCH (852 cases) and DOGEE (352 cases), respectively.

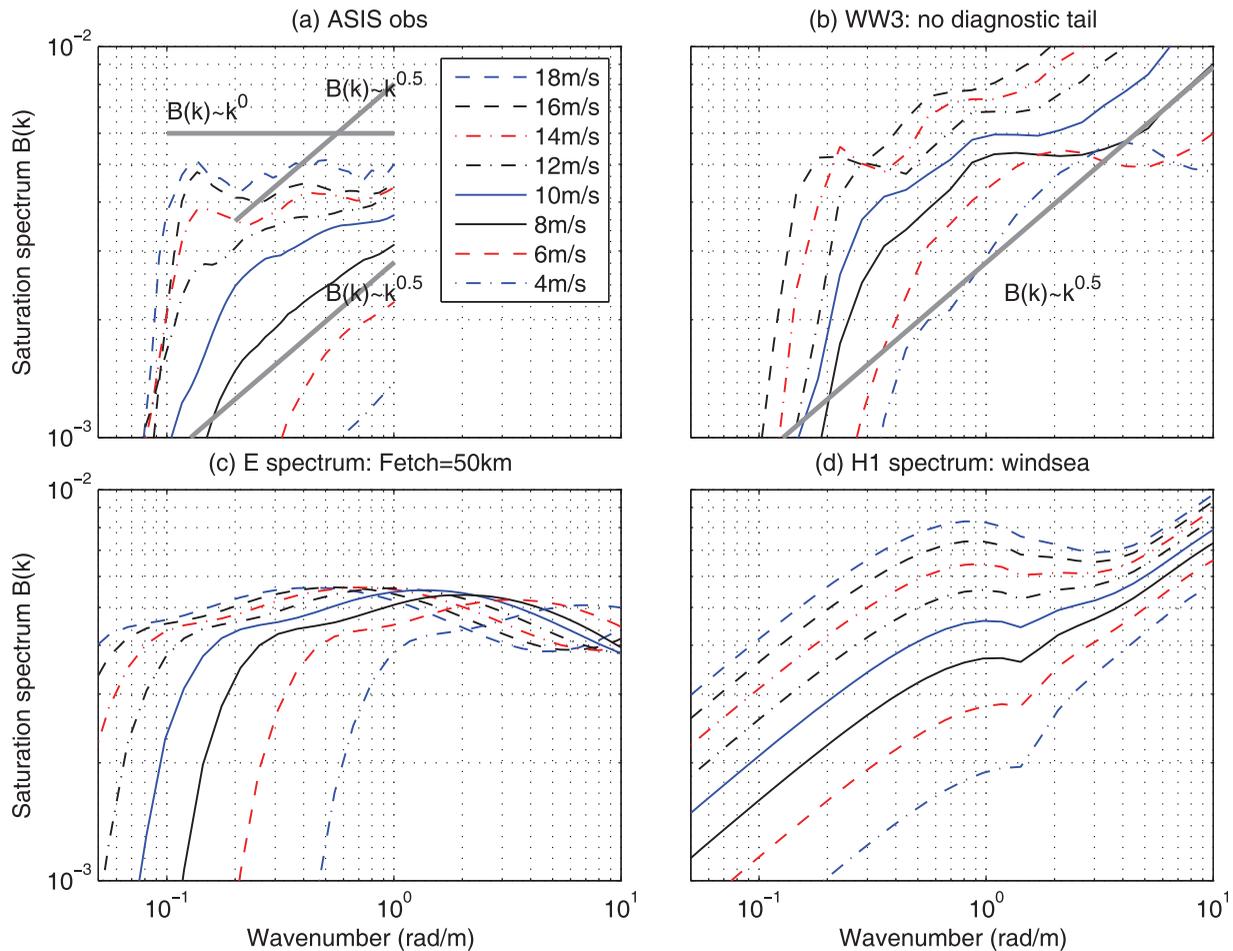


**Figure 2.** Saturation spectra,  $B(k)$ , plotted against wind speed for six different wave numbers (0.2, 0.5, 1.0, 2.0, 5.0, and 10.0 rad/m). Blue dots show the observed  $B(k)$  values for FETCH and red circles represent DOGEE. The  $B(k)$  models proposed by *Hwang* [2008] with swell effect (H4 spectrum: dashed lines) and without swell effect (H1 spectrum: solid lines) are also compared. Blue lines show the *Elfouhaily et al.* [1997] spectrum (E spectrum).

$$B(k) = k^3 F(k) = k^3 \int_{-\pi}^{\pi} \Psi(k, \theta) k d\theta \quad (2)$$

where  $\Psi(k, \theta)$  is the wave number spectrum in polar form with units of  $m^4$ . Wave hindcasts were conducted based on WAVEWATCH-III<sup>TM</sup> ver.3.14 (WW3) [Tolman, 2009]. The spectral space was discretized using 40 frequencies, ranging from 0.0412 to 3.995 Hz (relative frequency of 10%,  $f_{m+1} = 1.1f_m$ ), with 36 directions ( $\Delta\theta = 10^\circ$ ). The corresponding wave number range was  $6.8 \times 10^{-3} - 6.24 \times 10^1$  rad/m. Because one of our objectives was to investigate the spectral shape in the higher wave number region, we did not apply the diagnostic parameterization of the spectral tail. In addition, the minimum time step for source terms integration is set to be very small (1 s) to allow very small variation of wave spectra in higher wave number domain. Tolman and Chalikov's [1996, hereinafter TC96] source terms were used for  $S_{in}$  and  $S_{ds}$ . TC96 defined the dissipation terms of the lower and higher wave number domains separately. The wave energy dissipation in the higher wave number domain,  $S_{ds}^{high}$ , can be derived semiempirically from the source term balance between  $S_{ds}^{high}$  and  $S_{in}$ , and is expressed with the saturation spectrum,  $B(k)$ :

$$S_{ds}^{high} = -a'_0 (u_* / c)^2 k^{-3} B^{\beta+1}(k, \theta) \quad (3)$$



**Figure 3.** The mean saturation spectra for each wind speed range during the FETCH experiment.  $B(k)$  is averaged in the wind speed bins ( $\pm 1$  m/s) indicated in the legend. (a) ASIS observations ( $k$ : 0.05–1 rad/m) with reference power laws (gray lines) by Phillips [1958] ( $B(k) \sim k^0$ ) and Toba [1973] ( $B(k) \sim k^{0.5}$ ), (b) WW3 without the diagnostic tail, (c) Elfouhaily et al. [1997] for fetch=50 km (E spectrum), and (d) Hwang [2008] for windsea (H1 spectrum).

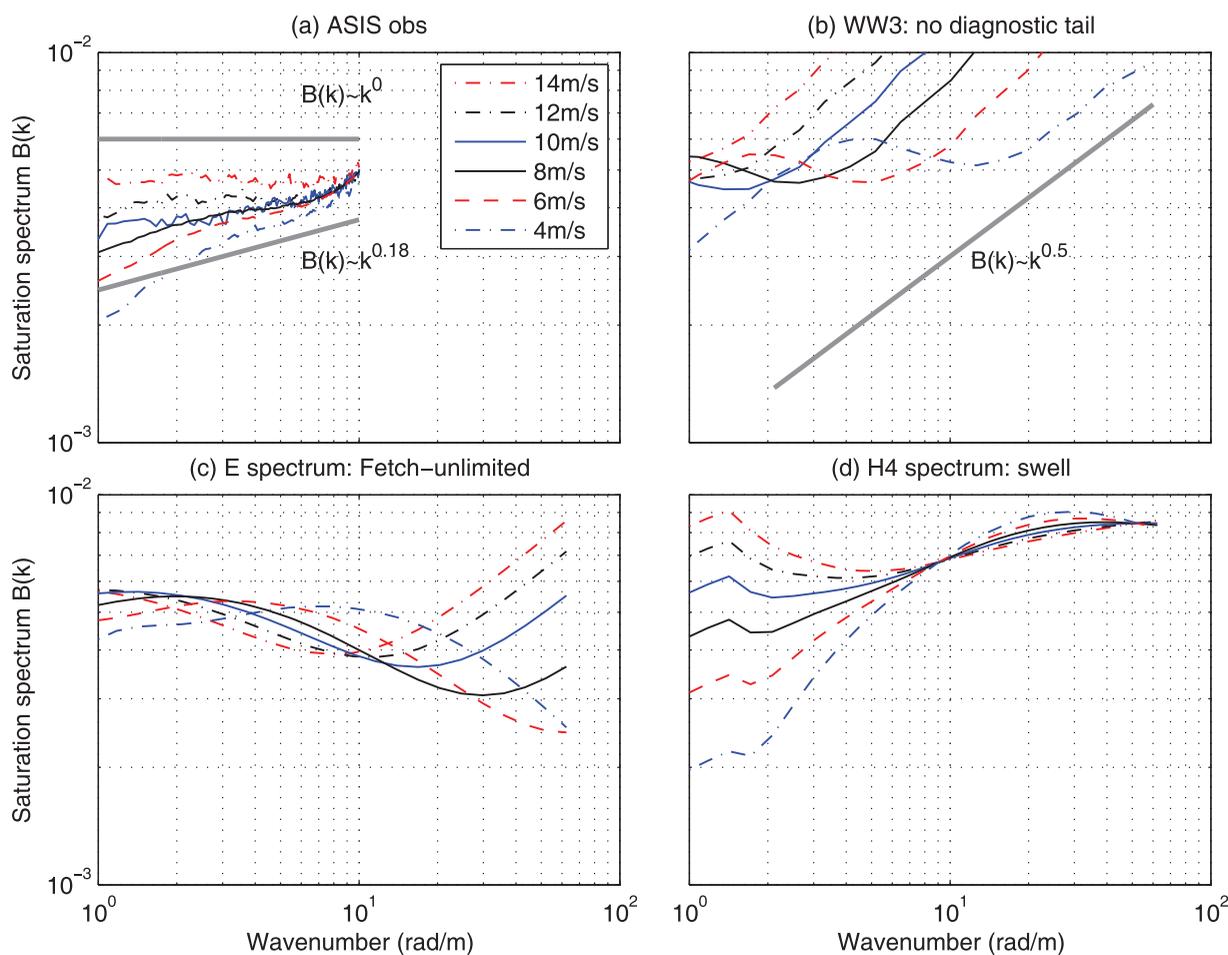
$$\beta = a'_1 (u_* / c)^{-a_2} \quad (4)$$

where  $a'_0$ ,  $a'_1$  and  $a_2$  are nondimensional tuning parameters,  $c$  is the wave phase speed, and  $\beta$  represents the nonlinearity of the dissipation term. Recent studies have indicated that the effect of nonlinearity on the dissipation term is important for accurate representation of observed wave number spectra in the high wave number domain [e.g., Donelan et al., 2012, Babanin et al., 2010]. This point is addressed in more detail in section 5 with modification of the  $S_{ds}^{high}$  term (equation (3)). We employed the discrete interaction approximation (DIA) method as the  $S_{nl}$  [Hasselmann et al., 1985]. Other source terms, such as linear input and bottom friction, were not used for the hindcast. The other computational parameters for the two experiments (FETCH and DOGEE) are summarized in Table 1. The wave model was driven by 6 hourly surface winds from the National Centers for Environmental Prediction/Climate Forecast System Reanalysis (NCEP/CFSR) product [Saha et al., 2010], in which the global surface data set has a resolution of  $\sim 38$  km (Gaussian Grid: T382). Two minute gridded elevations/bathymetry for world data (ETOPO2v2 Global Gridded Two-minute Database, National Oceanic and Atmospheric Administration, <http://www.ngdc.noaa.gov/mgg/global/etopo2.html>) was used to define the bottom topography and coastal lines. Hourly hindcast results were stored and statistical parameters and wave spectra at the observational location during the FETCH and DOGEE experiments were analyzed.

## 4. Results

### 4.1. Wind and Wave Conditions

The two data sets analyzed in this study show the characteristic features of surface wave fields, depending on the surface wind and geometrical fetch. Figure 1 shows probability density functions (pdf) for various

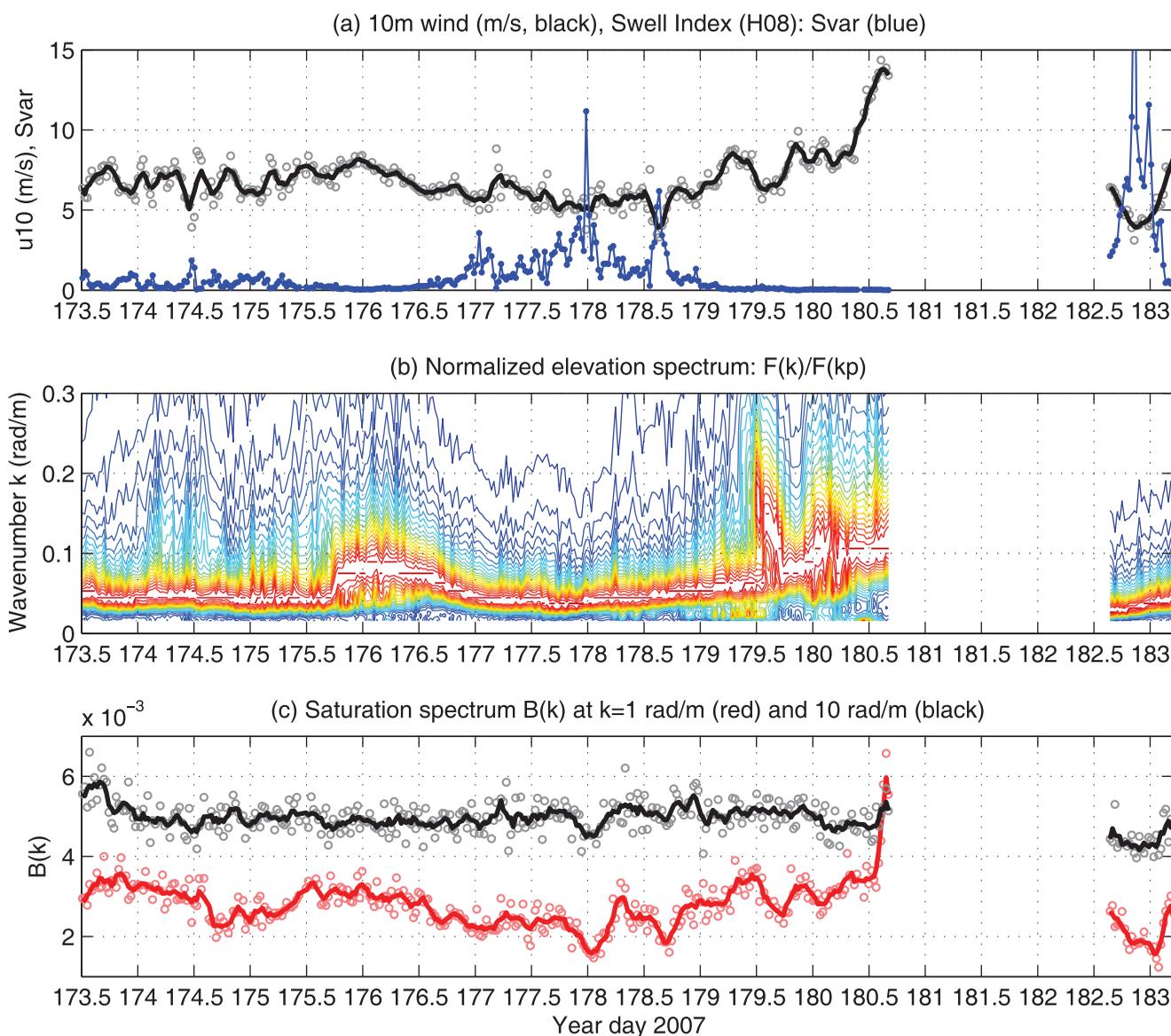


**Figure 4.** The mean saturation spectra for each wind speed range during the DOGEE. (a) ASIS observations ( $k$ : 1–10 rad/m) with reference power laws (gray lines) by *Banner et al.* [1989] ( $B(k) \sim k^{0.09 \pm 0.09}$ ), (b) WW3 without the diagnostic tail, (c) *Elfouhaily et al.* [1997] for the fetch unlimited case (E spectrum), and (d) *Hwang* [2008] with swell effect (H4 spectrum).

parameters. During the Mistral events, maximum wind speeds ( $U_{10}$ ) reached  $\sim 18$  m/s, and the pdf of the surface wind speed was widely distributed for the FETCH experiment (Figure 1a). Despite the strong wind, the maximum significant wave height ( $H_s$ ) during FETCH was limited to  $\sim 3$  m (Figure 1b), and the peak wave period  $T_p$  was distributed around 6 s due to the fetch-limited wave growth (Figure 1c). On the other hand, because of the prominent westerlies and the limited data available for DOGEE, the pdfs of  $U_{10}$  and  $H_s$  were narrower, but maximum  $U_{10}$  and  $H_s$  were around 14 m/s and 3 m, respectively. Accordingly, the wave data set for FETCH represents young windsea in the ocean, whereas the dominant sea state was a mature sea for DOGEE, which can also be confirmed in the pdf of inverse wave age (Figure 1d). The two data sets cover a wide range of sea states and represent the wave number spectra in open ocean conditions.

#### 4.2. The Spectral Form and Wind Dependence

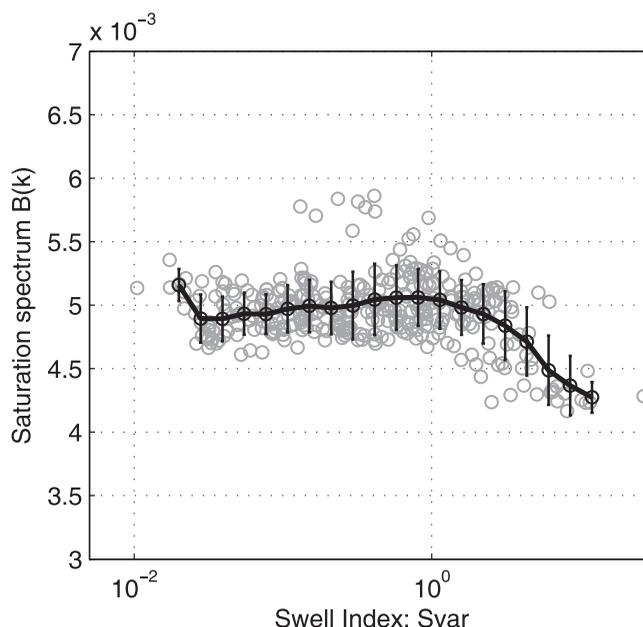
Figure 2 shows the saturation spectra,  $B(k)$ , plotted against wind speed for six different wave numbers. Observed  $B(k)$  values for FETCH (blue dots) were plotted for wave numbers ranging from 0.2 to 1 rad/m. For DOGEE (red dots), the wave numbers range from 1 to 10 rad/m, due to the difference in wave gauge arrangements (spatial resolution).  $B(k)$  measured in different sea states were quantitatively consistent at  $k = 1$  rad/m. Our evidence suggests that the estimated wave number spectra were robust, especially for short gravity wave regimes, and the WDM used for retrieving  $F(k)$  functioned reasonably well for the estimation. Both data sets showed that  $B(k)$  at  $k = 1$  rad/m linearly increased with  $U_{10}$  when wind speed was less than 10 m/s. For both data sets,  $B(k)$  at  $k = 1$  rad/m tended to be saturated under higher winds. On the other hand,  $B(k)$  in the intermediate range ( $5 < k < 10$ ) showed a weak dependence on  $U_{10}$  over the entire wind speed range.



**Figure 5.** Temporal variation in wind and swell parameters retrieved from the DOGEE data set; (a)  $U_{10}$  and Swell index,  $Svar$ , proposed by Hwang [2008], (b) normalized elevation spectra in the lower wave number domain ( $k$ : 0–0.3 rad/m), which is estimated from the global wavelet spectrum, and (c) saturation spectra  $B(k)$  at  $k = 1$  rad/m (red; circles: 30 min average, thick line: 2.5 h running average) and at  $k = 10$  rad/m (black).

We compared the  $B(k)$  proposed by Hwang [2008] (the so-called H1 and H4 spectra in Hwang and Plant [2010] and Elfouhaily et al. [1997, hereinafter E spectrum]) with the observations in Figure 2. Between the various models, there was a large discrepancy in representing the wave number spectrum, especially in the intermediate range of  $1 < k < 10$  rad/m [refer to Hwang and Plant, 2010, Figure 1]. For example, Hwang [2008] proposed different spectral forms for conditions with (H4 spectrum) and without (H1 spectrum) background swells. The spectral energy of the H1 spectrum weakly increases with increasing surface wind for windsea conditions, whereas it is suppressed against increasing wind for swell dominant conditions (H4 spectrum). The H1 and H4 spectra both capture the linear increase in  $B(k)$  for the lower wave number domain ( $k < 1$  rad/m), but only the H4 spectrum portrays the weak dependence of  $B(k)$  on wind speed at  $k = 10$  rad/m. The E spectrum shows similar behavior for the whole wave number range (0.2–10 rad/m); strong saturation patterns and local maxima of the spectral density for middle wind speeds ( $U_{10}$ : 5–10 m/s), which was not confirmed by observation.

In Figure 3, the mean saturation spectra for each wind speed range are shown during the FETCH experiment. The observed wave spectra (Figure 3a) clearly show the characteristic spectral shape and its wave



**Figure 6.** Scatterplot of the swell index,  $Svar$ , and the 2.5 h running average  $B(k)$  at  $k=10$  rad/m (gray circles). The black curve with bars shows the bin-averaged  $B(k)$ . The error bars correspond to one standard deviation.

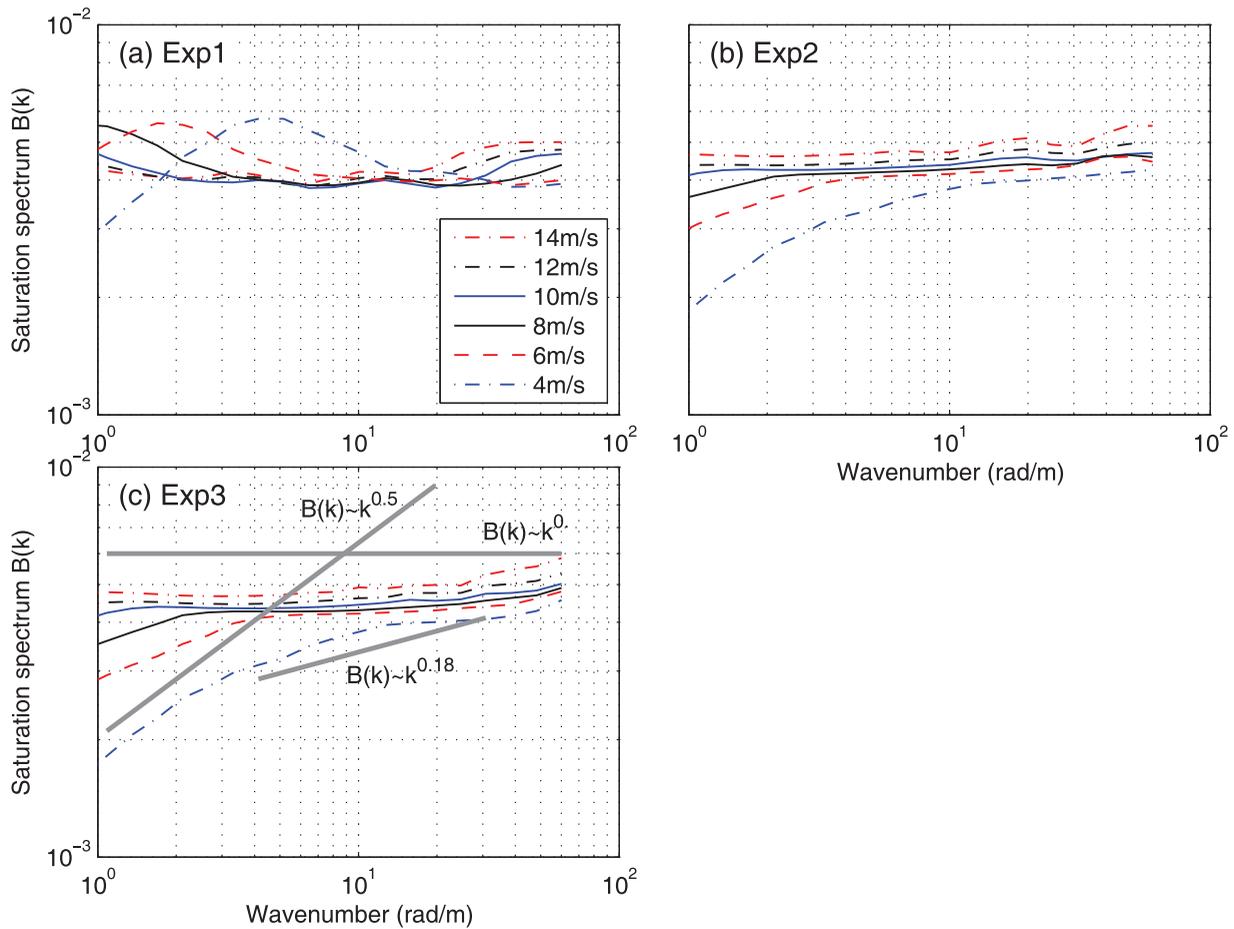
number power transition. When the wind speed is greater than 14 m/s, we see the following: an overshoot near the spectral peak ( $k$ : 0.1–0.2 rad/m) [e.g., Barnett and Wilkerson, 1967; Hasselmann et al., 1973] and a subsequent  $k^{-5/2}$  tail [e.g., Donelan et al., 1985]. The spectral shape transition from  $k^{-5/2}$  ( $B(k) \sim k^{1/2}$ ) to  $k^{-3}$  ( $B(k) \sim k^0$ ) [Forristall, 1981] is also shown in Figure 3a. When the wind speed is less than 10 m/s, the overshoot and  $k^{-3}$  tail are not easily discernible. Interestingly, the spectral bandwidth in which the  $k^{-2.5}$  power law appears depends on the wind speed; the bandwidth is broader for lower wind speeds ( $U_{10} < 10$  m/s) and narrower for higher wind speeds ( $U_{10} > 10$  m/s).

Third-generation wave models provide the most powerful tools for investigating the energy-containing region (the spectral peak) in the wave number domain, because they describe the temporal and spatial evolution of the wave spectrum without any prior assumptions about spectral shape. WW3 can reproduce the peak value of the overshoot and the spectral shape near the peak region ( $k$ : 0.1–0.3 rad/m), as shown in Figure 3b. On the other hand, modeled  $B(k)$  deviates substantially from observations in the higher wave number domain ( $k$ : 0.5–1 rad/m), where a diagnostic spectral tail is usually attached ( $B(k) \sim k^0$ , in the default settings of WW3). There was no clear overshoot and no wind speed dependence of the spectral shape in either the E spectrum with a 50 km fetch (Figure 3c) or the H1 spectrum (Figure 3d). This is because the spectral evolution near the peak region is strongly dependent on the temporal and spatial variation in the surface wind fields, such as the Mistral, which cannot be taken into account in the E and H1 spectra.

Figure 4 shows the mean saturation spectra for the DOGEE. Observed spectra (Figure 4a) were quantitatively very consistent with Banner et al. [1989]; that is, the wind speed dependence of  $B(k)$  was weak (see also Figure 2) and the power law of  $B(k)$  was mainly confined to between  $k^0$  and  $k^{0.18}$  in the wave number range of 2–10 rad/m. Moreover, the value of  $B(k)$  eventually became independent of the wind speed and showed the characteristic nodal point of saturation spectra at 10 rad/m, which can be seen in the H4 spectrum (Figure 4d). The E spectrum (Figure 4c) showed near-constant values of  $B(k)$  and its weak dependence on wind speed in the wave number range of 1–10 rad/m. However, the spectral shape is different from those of Banner et al. [1989] and our observations (Figure 4a). In addition, WW3 (Figure 4b) could not reproduce the observed spectra in this wave number domain, where  $B(k)$  is significantly overestimated and is proportional to the  $k^{-5/2}$  shape.

### 4.3. Swell Influence on Short Gravity Waves

The spectral evolution of short waves riding on long waves is complicated because, in addition to wind influences, the short waves are modulated by the longer waves [e.g., Longuet-Higgins and Stewart, 1960] and wave breaking is enhanced, as the short waves may break on the crest of the long waves. Following Smedman et al. [2003], Hwang [2008], and Sahlée et al. [2012], we separated swell energy from windsea components and investigated the influence of swells on short gravity waves. Hwang [2008] defined the swell index  $Svar$  as the ratio of the spectral energy in the frequency bands lower than the reference frequency,  $\sigma_r$ , to the spectral energy in the frequency bands higher than  $\sigma_r$ , where  $\sigma_r$  is the higher value between the measured spectral peak frequency,  $\sigma_p$ , and the empirical peak frequency,  $\sigma_0 = g/(1.2U_{10})$ . Figure 5 shows the temporal variation in wind and wave parameters retrieved from the DOGEE data set. The temporal variation in  $B(k)$  at



**Figure 7.** The mean saturation spectra  $B(k)$  with nonlinear dissipation terms. (a) Exp1:  $S_{in}^{TC96} + S_{ds}^{mTC96} + S_{nl}^{DIA}$ , (b) Exp2:  $S_{in}^{TC96} + S_{ds}^{D01} + S_{nl}^{DIA}$ , and (c) Exp3:  $S_{in}^{TC96} + S_{ds}^{D01} + S_{nl}^{WRT}$ .

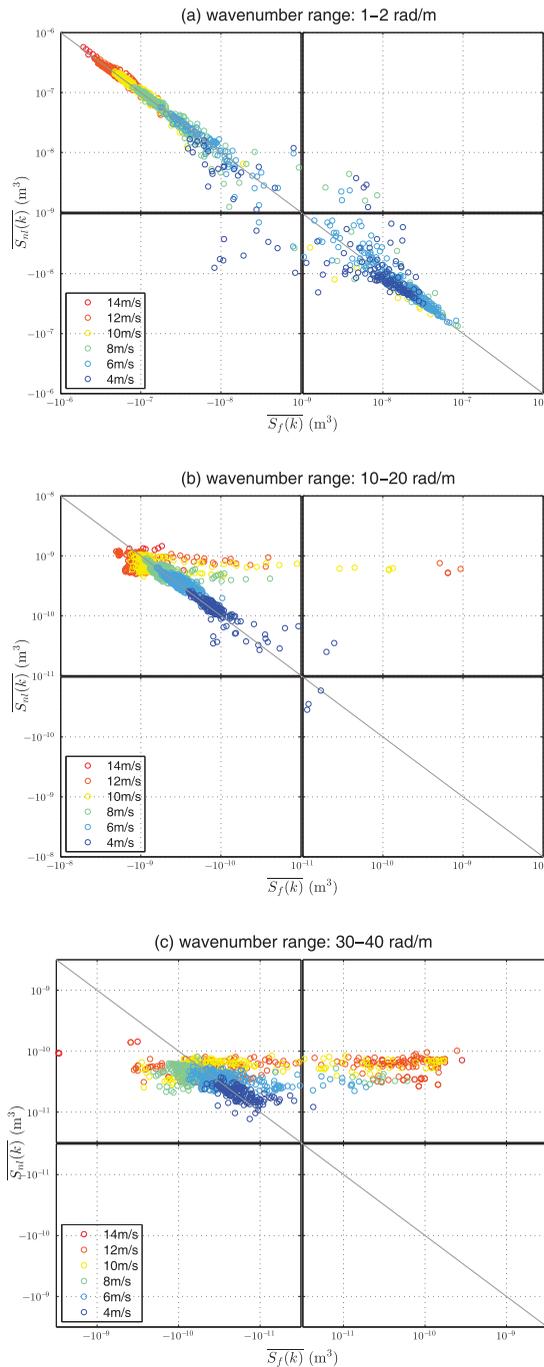
$k = 10$  rad/m was smaller than that at  $k = 1$  rad/m (Figure 5c) and its correspondence to the wind speed was very weak, as shown in Figure 2. Swell energy was significant during YD 177–179, and YD 182.6–183.2 (Figure 5a) and the corresponding spectral bandwidth was narrow (Figure 5b). Figure 6 shows the variation in the 2.5 h running average  $B(k = 10$  rad/m) plotted against  $Svar$ . Clearly,  $B(k)$  values decreased in significant swell conditions. The  $B(k = 10$  rad/m) values showed a similar pattern when plotted against an alternate parameter, defined as the swell energy at wave numbers less than 0.1 rad/m (not shown). *Hwang* [2008] proposed different spectral forms for when swells were present or absent; only when the swell effect was significant did the characteristic nodal point of the saturation spectra appear at wave number 10 (rad/m). On the other hand, our observations suggest that the spectral energy at  $\sim k = 10$  rad/m was almost constant, but short wave energy decreased when significant swells existed.

## 5. Discussion

### 5.1. Spectral Evolution and Nonlinear Dissipation Terms

Wave number spectra  $F(k)$  observed by ASIS buoys clearly showed different spectral forms in different wave number domains and the transition from  $k^{-2.5}$  to  $k^{-3}$ , which is consistent with *Forristall* [1981], *Hwang and Wang* [2001], *Resio et al.* [2004], and *Long and Resio* [2007]. This suggests that different physical properties exist in the different wave number domains. The estimated spectra by WW3 showed a  $k^{-2.5}$  form in the higher wave number domain (Figure 4b), which was different from our observations (Figure 4a). What causes this difference in spectral shape?

In the default settings of WW3, the diagnostic spectral tail of the  $k^{-3}$  form is attached beyond nine times the peak wave number. Therefore, the  $S_{ds}^{high}$  term (equation (3)) does not affect any spectral form in the



**Figure 8.** Scatterplot of the net external force,  $S_e$ , and the nonlinear transfer,  $S_{nl}$ , which were averaged in the wave number bins, (a)  $k$ : 1–2 rad/m, (b)  $k$ : 10–20 rad/m, and (c)  $k$ : 30–40 rad/m. The circles are color-coded according to the surface wind speed,  $U_{10}$ .

### 5.2. Wave Hindcasts With Nonlinear Dissipation Terms

We conducted three additional wave hindcasts for the DOGEE with nonlinear dissipation terms to validate the applicability of the nonlinear dissipation term. Wind input,  $S_{in}$ , and the wave energy dissipation in the lower wave number domain,  $S_{ds}^{low}$ , were the same as in TC96. In the first experiment,  $S_{ds}^{high}$ , as described by TC96 (equation (3)), was modified to reflect the strong nonlinear behavior; the nonlinear dependence of dissipation term on  $B(k)$  (i.e.,  $\beta + 1$  in equation (3)) was set at 3 (Exp1:  $S_{in}^{TC96} + S_{ds}^{mTC96} + S_{nl}^{DIA}$ ). In the second experiment, the nonlinear dissipation term proposed by D01 was applied (Exp2:  $S_{in}^{TC96} + S_{ds}^{D01} + S_{nl}^{DIA}$ ). Tunable parameters for

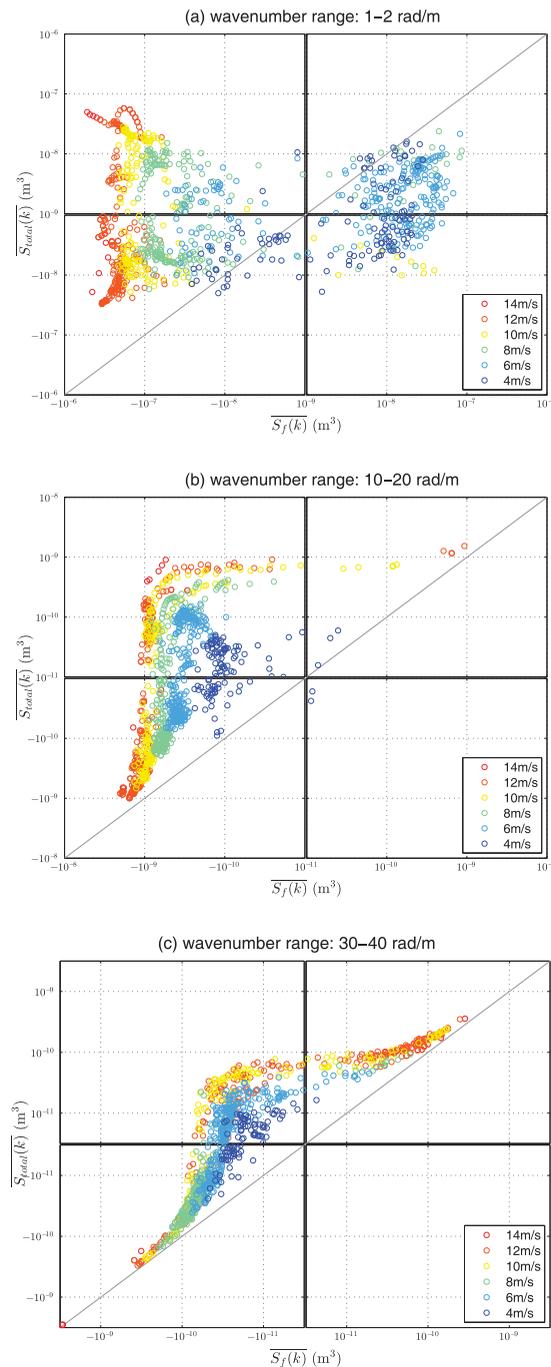
higher wave number domain. In addition, because WW3 focuses on modeling the spectral peak region, parameterization of  $\beta$  (equation (4)) in the wave number range of 1–10 rad/m is not taken into consideration. However, this range is the primary concern of this study, and if we calculate  $\beta$ , its parameters are  $10^{-2}$ – $10^{-1}$  for stronger winds ( $\sim 20$  m/s) and  $10^{-1}$ – $10^0$  for weaker winds ( $\sim 5$  m/s), suggesting a very weak dependence of nonlinearity, especially for moderate to high wind speeds.

Recent studies have proposed new source terms that include dissipation parameterizations [e.g., *Arduin et al., 2010; Rascle and Arduin, 2012; Thomson et al., 2013*]. It is interesting to speculate that nonlinear dissipation is crucial for accurate representation of the spectral form in the higher wave number domain. *Donelan* [2001, hereinafter D01] proposed the nonlinear dissipation function to balance wind input and dissipation due to wave breaking as follows:

$$S_{ds} = -A_2 [(1 + A_3 mss(k))^2 B(k)]^{2.5} k^{-3} B(k) \quad (5)$$

$$mss(k) = \int_0^k F(k) k^2 dk \quad (6)$$

where  $A_2$  and  $A_3$  are empirical coefficients [*Donelan et al., 2012*], and  $mss$  is the integrated mean square slope, which was introduced to represent the modulation of short waves and enhanced breaking (spilling) due to the orbital motion of long waves. This dissipation form is currently being applied to wave models to estimate higher wave number spectra and related wind stress [*Babanin et al., 2010; Donelan et al., 2012*]. In contrast to TC96, strong nonlinearity is included in the dissipation term, which effectively damps spectral growth in the higher wave number domain.



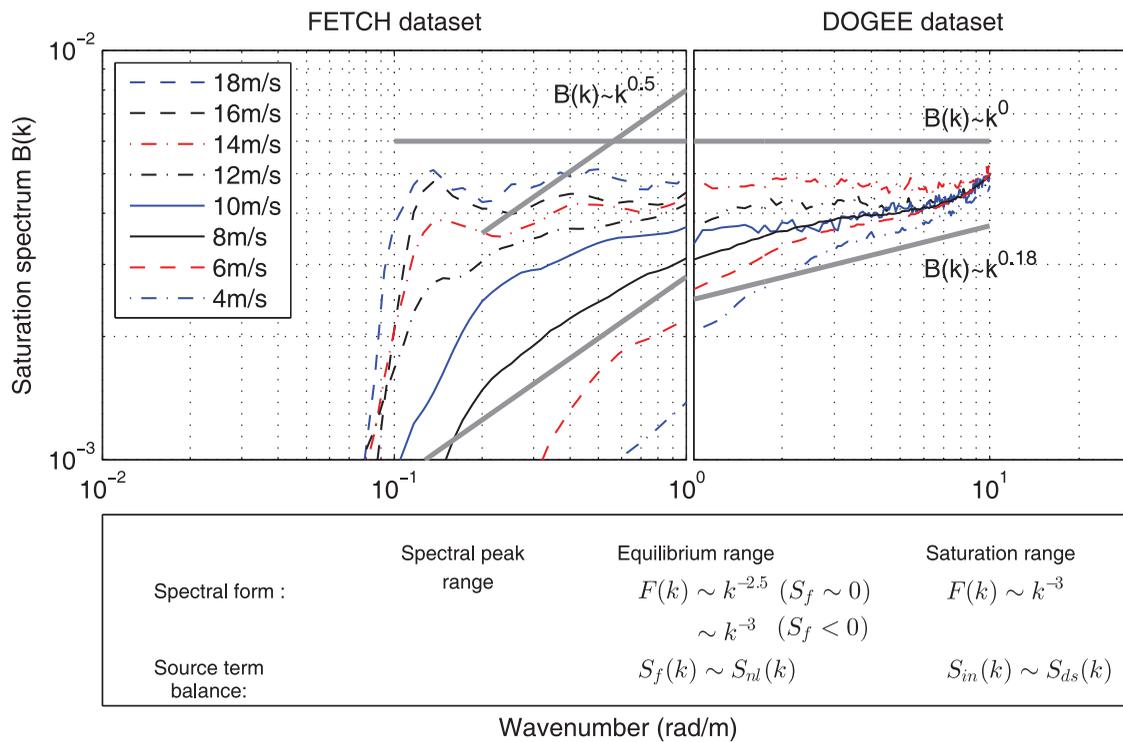
**Figure 9.** Scatterplot of the net external force,  $S_f$ , and the total source,  $S_{total}$ , which were averaged in the wave number bins, (a)  $k$ : 1–2 rad/m, (b)  $k$ : 10–20 rad/m, and (c)  $k$ : 30–40 rad/m. The circles are color-coded according to the surface wind speed,  $U_{10}$ .

suggesting that the role of  $mss$  is important for accurate representation of the observed spectra. Recently, the modulation of short waves by the underlying long waves has been widely studied, and from the modeling point of view, it is now seen as a cumulative dissipation [e.g., Babanin et al., 2010]. Our results also support the importance of the cumulative effect, in terms of enhanced breaking, for accurate representation of observed wave spectra in the higher wave number range.

The difference between Exp2 and Exp3 comes from the estimation of the  $S_{nl}$  term (i.e., DIA method versus WRT method). The resonant four-wave interactions estimated by DIA are not well established [e.g., Van

both hindcasts were as follows: (Exp1)  $a'_0 = 0.35$ ,  $\beta = 2.0$  in equation (3); (Exp2)  $A_2 = 5.8$ ,  $A_3 = 132.0$  in equation (5). In the third experiment, we employed an exact  $S_{nl}$  term estimated using the WRT method [e.g., Van Vledder, 2006] (Exp3:  $S_{in}^{TC96} + S_{ds}^{D01} + S_{nl}^{WRT}$ ). Due to the high computational cost for this experiment, we set a subdomain (40.8°N–44.8°N, 19°W–15°W) which is embedded in the model domain for DOGEE (0°–70°N, 100°W–15°E). The horizontal and spectral resolutions were same as for other experiments and the output spectra of Exp2 was used for the boundary condition of Exp3.

Figure 7 shows the saturation spectra for the three hindcast scenarios. All three models improved the spectral shape in the higher wave number domain; the spectral form tended to be saturated against the surface wind and exhibited  $B(k) \sim k^0$  instead of  $B(k) \sim k^{0.5}$  (Figure 4b). The spectral nodal point at  $k = 10$  rad/m was also represented in all three models. However, in Figure 7a (Exp1), the overshoot of  $B(k)$  was confirmed in the wave number range of 1–10 rad/m, whereas Exp2 (Figure 7b) and Exp3 (Figure 7c) performed significantly better and demonstrated the weak dependence of  $B(k)$  on wind speed. We also conducted hindcast experiments of Exp1 with larger values of  $\beta$  (2–4) in equation (3), but the spectral form showed a similar trend,



**Figure 10.** Schematic diagram of the spectral structure and source term balance in the equilibrium and saturation ranges.

Vledder et al., 2000; Tamura et al., 2008], and have not been validated in the higher wave number domain. Therefore, the applicability of DIA is unclear. However, the results (Figures 7b and 7c) showed similar behavior, implying that DIA was able to adequately reproduce the observation, but in the following discussions, we utilize the hindcast results from Exp3, which adequately represented the saturation spectrum in the higher wave number domain. Furthermore, we used an accurate method for evaluating  $S_{nl}$ , which should lead to reliable discussions of the source term balance.

### 5.3. Spectral Form and Source Term Balance

To maintain a stable spectral form in the higher wave number domain, the sum of the three source terms,  $S_{total}$  (i.e., the right side of equation (1)), approaches zero during wave growth. This is called the equilibrium condition. We compared the net external force,  $S_f (= S_{in} + S_{ds})$ , and the nonlinear transfer function,  $S_{nl}$ , for different wave number ranges to confirm the source term balance. Figures 8 and 9 show the scatterplots of  $S_{nl}$  and  $S_{total}$ , respectively, as a function of  $S_f$ , which were averaged for the wave number bins (a) 1–2 rad/m, (b) 10–20 rad/m, and (c) 30–40 rad/m. The results clearly demonstrate the different source term balance between  $S_f$  and  $S_{nl}$  for different wave number ranges; that is,  $S_{nl}$  was in balance with  $S_{nl}$  to cancel out the total source term for the wave number range of  $\sim 1$ –2 rad/m (Figures 8a and 9a), whereas they were unbalanced in the higher wave number domain of 30–40 rad/m (Figure 8c). We observe that  $S_f$  tends to approach  $S_{total}$  (Figure 9c).

Hwang and Wang [2001] revisited the concept of a saturation range in which the  $F(k) \sim k^{-3}$  form appeared. They treated the saturation range separately from the equilibrium range, and the saturation range started at the upper limit of the equilibrium range, according to the discussion of the *mss*. Here we also separated the equilibrium range and saturation range, according to the role of  $S_{nl}$  in source term balance (Figure 10). In the equilibrium range (Figure 8a),  $S_{nl}$  played a major role in the adjustment of the total source term balance during wave evolution, and the equilibrium condition was maintained by the  $S_{nl}$  [Tamura et al., 2010]. In addition, as described by Resio et al. [2004], any net gain or loss of wave energy within the equilibrium range tends to force the spectrum away from a  $k^{-2.5}$  shape. If the net force of  $S_f$  is negligible within the equilibrium range, the spectrum should have a  $k^{-2.5}$  form. This is again consistent with our hindcast results. The spectral form of  $k^{-3}$  appeared in the equilibrium range (Figure 7c:  $U_{10} > 8$  m/s) where the net external force  $S_f$  was negative

(Figure 8a). When  $U_{10}$  was less than 6 m/s, the spectral form approached the  $k^{-2.5}$  shape and the  $S_f$  values approached zero (Figure 8a). However,  $S_{nl}$  tended to be out of balance with  $S_f$  in the saturation range (Figure 8c).  $S_f$  approached the total source term when the  $S_{nl}$  term was not significant compared to  $S_f$  (Figure 9c), implying that the  $S_{ds}$  term was in balance with the  $S_{in}$  term in this wave number range.

## 6. Summary

We investigated the characteristics of the spectral form for short gravity waves. In addition, to understand the dynamic balance of three sources, we conducted wave hindcasts with different dissipation terms. The observed wave spectra clearly showed the characteristic spectral shape and its wave number power transition; that is, the overshoot near the spectral peak and the subsequent spectral form transition from  $k^{-2.5}$  to  $k^{-3}$  in the higher wave number domain, which is consistent with previous research. At the wave number  $k = 10$  rad/m, the saturation spectrum  $B(k)$  became constant over the entire wind speed range. The WAVEWATCH-III model reproduced the spectral form in the peak region, whereas it deviated substantially from observations in the higher wave number domain. The nonlinear dissipation term, with enhanced breaking due to longer waves (the cumulative effect), significantly improved the model results. In the equilibrium range, the nonlinear transfer function,  $S_{nl}$ , played a major role in maintaining the equilibrium conditions. The spectral form  $F(k)$  approached  $k^{-3}$  for negative values of  $S_f$ , whereas it approached  $k^{-2.5}$  for negligible  $S_f$ . On the other hand, in the saturation range, the spectral form approached  $k^{-3}$ , and  $S_{nl}$  did not balance with the other source terms. The  $S_{ds}$  term was in balance with  $S_{in}$  in this wave number range.

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