Wave Set-Up on Coral Reefs

ROBERT J. TAIT

Department of Oceanography, University of Hawaii, Honolulu, Hawaii 96822

Momentum flux considerations give reasonably good predictions on the amount of wave-induced set-up to be expected on coral reefs. It is possible that water levels on the outer edge of sea-level reefs may be raised as much as 20% of the incident wave height above the mean water level just seaward of the reef. Such wave set-up can significantly affect circulation and sediment behavior in the coral reef environment.

Mean water levels higher than the surrounding ocean were observed on the reef flat of Bikini Atoll by *Munk and Sargent* [1948]. These higher water levels on the reef flat were due solely to waves breaking on the reef and they created a significant circulation into the atoll lagoon [*von Arx*, 1948].

Circulation of a similar type was observed over a fringing reef on the island of Kauai in the Hawaiian chain by *Inman et al.* [1963]. Flow on the Kauai reef occurred over the reef flat, along the beach, and back out through a large inlet.

Theoretical investigations of changes in mean sea level near shore have been carried out by *Longuet-Higgins and Stewart* [1962, 1963, 1964], *Whitham* [1962], and *Lundgren* [1963] on the basis of conservation of momentum flux due to incident waves. Applied to beaches that extend above sea level, these models predicted a lowering of the water level, 'set-down,' as the waves approach the break point and a steady rise in sea level, 'set-up,' shoreward of the break point. *Bowen et al.* [1968] verified the main features of these models and introduced some slight modifications with a series of laboratory experiments.

In a slightly different context, *Longuet-Higgins* [1967] predicted a wave-induced change in sea level between the two sides of a submerged breakwater. However, this analysis was limited to unbroken waves.

With minor changes, the application of momentum flux can be used to predict 'set-up' over coral reefs due to waves. In addition, some qualitative suggestions can be made about related water circulation and sediment movement.

An Idealized Reef

The first step in the understanding of wave effects on reef environments will be the consideration of an idealized reef. The properties of such a reef are as follows:

1. The reef configuration does not vary with position along the shoreline.
2. In cross section the reef has three significant regions (see Figure 1), (1) a sloping reef front which rises to (2) a ridge or reef flat at sea level and (3) a wave absorbing shoreline which may or may not be separated from the reef flat by a lagoon.
3. There is no subsurface connection between the water on the reef flat or in the lagoon and the ocean. If the wave field is also constant along the shore, then considerations can be limited to a vertical section perpendicular to the shore line.

On this idealized reef the incident wave field will also be somewhat simplified. It will consist of a very narrow frequency band that does not change with time, i.e., the waves can be characterized by a period, T, and a mean height, H.

In a series of papers, *Longuet-Higgins and Stewart* [1960, 1962, 1964] showed that second-order considerations of momentum flux due to waves can be simplified by the introduction of a 2-dimensional tensor called the radiation stress tensor, **S**. This tensor is defined as the excess flux of momentum due to waves and is

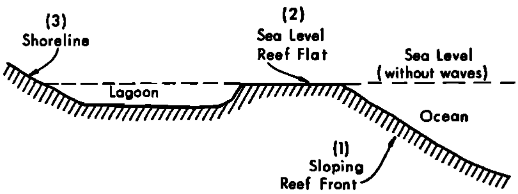

Fig. 1. Idealized coral reef in vertical cross section.

written as follows:

$$\mathsf{S} = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix}$$

where x is perpendicular to the shore and positive towards the shore and y is the coordinate along the shore. For waves approaching normal to the shore, the radiation stress tensor can be related to the wave energy per unit area E as follows:

$$\mathsf{S} = E\begin{bmatrix} \dfrac{2kh}{\sinh(2kh)} + \dfrac{1}{2} & 0 \\ 0 & \dfrac{kh}{\sinh(2kh)} \end{bmatrix} \tag{1}$$

In this expression, k is the wave number, and h is the water depth.

In the idealized reef situation, the only interest will be with the flow of x-directed momentum through a plane perpendicular to the x axis, S_{xx}. In shallow water the expression simplifies to

$$S_{xx} = \frac{3}{2}E = \frac{3}{16}\rho g H^2 \tag{2}$$

where ρ is the density of sea water and g is the acceleration of gravity.

If a steady state is assumed, then the shoreward flux of momentum must be independent of the distance from shore. With the beach slope small enough to neglect reflection of wave energy, and wave energy approximately conserved between wave rays, *Longuet-Higgins and Stewart* [1962] found the difference between the still-water level and the mean sea level in the presence of waves outside the break point to be

$$\langle\eta\rangle = -\frac{1}{8}\frac{H^2k}{\sinh(2kh)} \tag{3}$$

Where $\langle\eta\rangle$, the difference between the still-water level and mean sea level in the presence of waves, is always negative seaward of the break point of the waves.

Bowen et al. [1968] found that once the waves begin to break, the assumption that the wave height is proportional to the mean water depth is a good approximation. This is expressed in (4)

$$H = \gamma(\langle\eta\rangle + h) \tag{4}$$

At the seaward limit of breaking, the change in sea level due to waves will be small compared to the depth. Therefore, (3) can be rewritten for that specific location.

$$\langle\eta_b\rangle = -\frac{1}{8}\frac{\gamma^2 h_b^2 k}{\sinh(2kh_b)} \tag{5}$$

In (5) the subscript 'b' denotes the fact that this represents only that location where the waves first begin to break.

Using the assumption that S_{xx} remains $\frac{3}{2}E$ even after the waves begin to break [*Longuet-Higgins and Stewart*, 1963], and the fact that across the surf zone $\langle\eta\rangle$ may no longer be much smaller than the depth h, we find

$$S_{xx} = \frac{3}{16}\rho g\gamma^2(\langle\eta\rangle + h)^2 \tag{6}$$

and further [*Bowen et al.*, 1968]

$$\frac{d\langle\eta\rangle}{dx} = -\left[\frac{1}{1 + \dfrac{8}{3\gamma^2}}\right]\frac{dh}{dx} \tag{7}$$

If the beach slope $\tan\beta$ is a constant, then

$$\frac{d\langle\eta\rangle}{dx} = \left[\frac{1}{1 + \dfrac{8}{3\gamma^2}}\right]\tan\beta \tag{8}$$

This equation suggests that the mean water level begins to rise at a constant rate after the waves begin to break and this increase continues until the top of the sloping reef front is reached. From that point shoreward the water level stays constant unless further wave decay takes place.

The main interest here is in the actual level of the mean water surface on the reef flat and in any lagoon behind it. To find this value, the slope of the sea surface should be integrated from deep water to the top of the reef face. Equation 5 is the result of that integration to the break point, so only the integration of (8) from the break point to the reef flat is needed. The addition of that result and the value of (5) will give the difference between the still water level and the mean water level on the reef flat due to waves. Stated analytically

$$\langle \eta_{\mathrm{reef}} \rangle = -\frac{1}{8}\frac{\gamma^2 h_b^2 k}{\sinh(2kh_b)} + \int_{X_b}^{0} \left[\frac{1}{1+\left(\frac{8}{3\gamma^2}\right)}\right] \tan\beta \, dx \qquad (9)$$

where X_b is the x location of the break point. Carrying out the integration

$$\langle \eta_{\mathrm{reef}} \rangle = -\frac{1}{8}\frac{\gamma^2 h_b^2 k}{\sinh(2kh_b)} + \left[\frac{1}{1+\left(\frac{8}{3\gamma^2}\right)}\right](-X_b \tan\beta) \qquad (10)$$

and since $h_b = -X_b \tan\beta$

$$\langle \eta_{\mathrm{reef}} \rangle = -\frac{1}{8}\frac{\gamma^2 h_b^2 k}{\sinh(2kh_b)} + \left[\frac{1}{1+\left(\frac{8}{3\gamma^2}\right)}\right] h_b \qquad (11)$$

At first glance (11) does not seem to depend on beach slope at all. This appearance is somewhat misleading since γ is strongly dependent on beach slope. Data from *Putnam* [1945], *Iverson* [1953], *Bowen et al.* [1968], and *Inman et al.* [1971] produce a range of γ from 0.4 to 1.8 on beaches where $\tan\beta$ ranges from 0.02 to 0.12. On the steep reef fronts, γ could be higher than 1.8, but 2.0 will be the upper limit considered here.

In (11) the variables are h_b, k, and γ. Of these, k, has a very limited effect. In shallow water, by definition, kh is a small number as compared to 1. Therefore, $\sinh(2kh)$ is very close to $2kh$, and (11) simplifies to

$$\langle \eta_{\mathrm{reef}} \rangle = \left\{-\frac{1}{16}\gamma^2 + \left[\frac{1}{1+\frac{8}{3\gamma^2}}\right]\right\} h_b \qquad (12)$$

This leaves only h_b and γ as the variables and does not introduce more than ±5% error for all reasonable values of wave height versus wave number.

To apply this theory in practice, a potential source of error should be considered. *Bowen et al.* [1968] observed that very close to the break point, the actual measured set-down due to the unbroken waves deviated from the prediction of (3). Also, the data showed that the mean water level slope did not achieve the form of (8) until a short distance from the break point. These effects were due to the fact that the transition from unbroken waves to broken waves is not instantaneous, but takes place over a finite interval. The two variations of theory from reality are opposing and should at least partially cancel each other out.

A limited test of (12) can be made from the lab data of *Bowen et al.* [1968] in the following way: Although their experimental beach slope did not end at the still-water level, the amount of set-up over the still-water line on the beach should correspond to the value of (12). The five series in Figures 2 and 4 of *Bowen et al.* [1968] will be used to test (12). The slope used was $\tan\beta = 0.082$. In addition, one actual measurement of set-up over a reef can also be used as a rough test of (12) if a value for γ is assumed. This is the measurement by *Munk and Sargent* [1948] of set-up on Bikini Atoll. The sides of Bikini are very steep ($\tan\beta$ between 0.5 and 1.0) and so two values will be assumed for γ, 2.0 and 1.5. Since the only measured parameters on Bikini were the breaker height and the set-up, the depth of breaking h_b will vary between the two assumed values of γ. The results of lab and field data are seen in Table 1.

As Table 1 shows, the lab data seems to indicate that (12) gives too large a value for the set-up while the one field measurement suggests that the set-up should be somewhat larger than predicted. Obviously, more data is needed since the lab data is not taken from the exact circumstances needed and γ is not known for the field measurement. In addition, for the data on Bikini Atoll, the tide level is not known and could affect the amount of set-up.

To facilitate visualization of the effect of the variables in (12), Figure 2 shows the relationship between set-up on the reef flat, $\langle \eta_{\mathrm{reef}} \rangle$, and the still water depth at the break point h_b for several values of γ (the ratio of wave height to water depth in the surf zone). As a further aid, the dashed line illustrates how both the depth

TABLE 1. The Results of Laboratory and Field Data

h_b, cm	γ	$\langle \eta_{reef} \rangle$, cm*	$\langle \eta \rangle$, cm†
Lab Data: [*Bowen et al.*, *1968*]			
6.8	1.15	1.7	1.5
9.5	1.17	2.4	1.8
9.7	1.17	2.4	2.1
8.8	1.24	2.4	1.7
9.2	1.28	2.6	2.0
Field Data: [*Munk and Sargent*, *1948*]			
106	2.0	34	50–60
140	1.5	45	50–60

* Equation 12.
† Measured.

at the break point and the set-up on the reef flat vary with γ for a fixed breaker height H_b of 100 cm. It is interesting to note that for a fixed wave height at the break point, the highest set-up is achieved for γ equal to about 1.2. For other fixed breaker heights, the dashed line would have the identical form, but would be displaced to a different location.

Tide Level Effects

To this point it has been assumed that the still water level coincided exactly with the uppermost point on the sloping reef front, i.e., the level of the reef flat. If the tide or other effects should change the still water level from this point, the set-up on the reef flat can still be calculated.

Going back to (9), the only change will be an alteration of the upper limit of integration. The new upper limit will be $-h_T/\tan\beta$, where h_T is the height of the tide above the reef flat. The value of h_T will be positive if the tide is above the reef flat, and negative if the tide is lower than the reef flat. The effect of such a tide compensation on (12) will change it to

$$\langle \eta_{reef} \rangle = -\frac{1}{16}\gamma^2 h_b + \left[\frac{1}{1 + \frac{8}{3\gamma^2}}\right](h_b - h_T) \tag{13}$$

For (13) to remain valid, there are definite limits placed on h_T. The maximum positive value of h_T will be defined by the tide reaching a level such that waves no longer break on the reef, i.e., $h_T = h_b$. The maximum negative value of h_T is determined by the maximum height that the waves would run up on a sloping beach. There are at least two ways in which this value could be calculated. The slope indicated in (8) could be extended shoreward until it intersects the sloping beach. This approach would be consistent with the way in which the set-up is calculated over the reef flat, but has been shown by *Bowen et al.* [1968] not to be accurate. As the mean water depth in the presence of waves goes to zero, the slope of the mean water surface gets steeper, approaching tangency with the beach. The other way to get a number for this limit is to use the empirical run-up formula of *Hunt* [1959]. In this case

$$h_T \text{ (limiting)} = -(H_0 L_0)^{1/2} \tan\beta$$

where the subscript 0 denotes deep water values, and L_0 is the wave length. This limit is not as important as the high-tide limit since coral reefs are rarely much higher than the low-tide level.

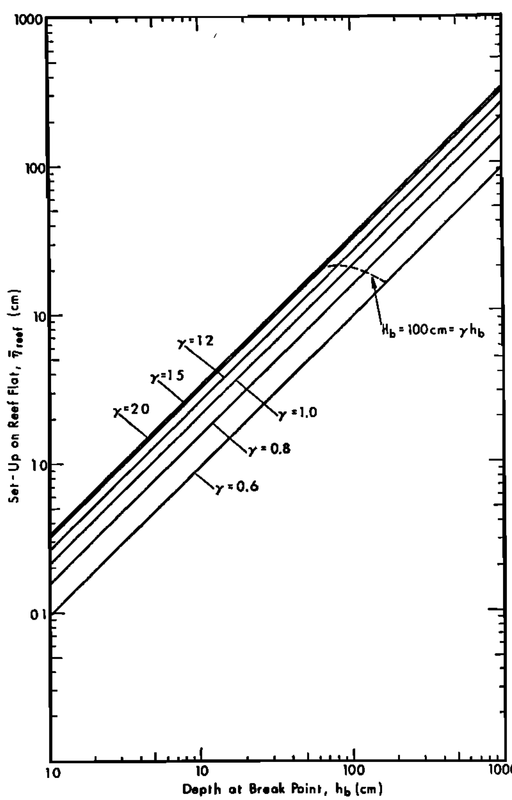

Fig. 2. Set-up on the reef flat versus water depth at the break point for various values of γ (the ratio of wave height to water depth in the surf zone). The dashed line illustrates the variation of parameters for a fixed height of the breaking wave.

In addition, the significant effects on circulation behind any reef will not occur when the tide level is much lower than the reef flat since this would diminish the net set-up above the reef flat.

Discussion

The calculations made here have been for the idealized reef pictured in Figure 1. Certain features of this model are rarely realized in the real world. In particular, the condition that there is no connection between the lagoon and the ocean except over the reef flat, is unrealistic. Often coral reefs have one or more relatively deep passages between the lagoon and ocean where waves do not break. In addition, there may be subsurface connections since coral reefs are rarely solid coral. The results of such connections can be that full set-up on the reef can not be achieved, or as on Bikini Atoll, the set-up can be at its peak value only out at the seaward edge of the reef flat [*Munk and Sargent*, 1948; *von Arx*, 1948].

If a set-up is achieved, currents will tend to flow in across the reef, as at Bikini, and back out to the ocean through either the subsurface or channel connections. For small waves the set-up induced circulation is not likely to be of great significance. Daily tidal currents will overwhelm it. In medium-sized waves (1–3 meters in height) it may no longer be true. The measurements by *von Arx* [1948] indicated that the set-up due to normal trade-wind-generated waves on Bikini Atoll (about 2.1 meters) created currents accounting for ⅓ of the lagoon volume in new water added each day. Further, large storm waves could produce a potentially large set-up on a reef and result in currents far larger than those of tidal origin. For example, 10 meter storm waves could, according to Figure 2, produce a set-up of up to 2 meters. This is considerably higher than the tidal range on most mid-ocean islands with fringing coral reefs. Sediment loss from behind reefs would probably occur at accelerated rates under such conditions. It is possible that this could aid in accounting for the channels through reefs which are usually attributed to cutting by streams at lower stands of sea level [*Moberly*, 1968]. Support is lent to this hypothesis by the observation of *Inman et al.* [1963] who suggested the channel through Kapaa Reef on Kauai, Hawaii was subject to scour from the rapid currents (several knots) heading out sea.

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