

## On the Response Characteristics of a Sonic Wave Gauge\*

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**Abstract:** This paper discusses the response characteristics of a remote sensing sonic wave gauge which measures wave height from a shipboom 7.5 meters high above the sea surface. It is operated by an echo-ranging method, using pulsed audible sound of 7 kHz at a sampling rate of 14 per second.

The measurement of long-crested waves generated in a water tank by the sonic wave gauge indicates that the apparent profile of a wave is distorted so that crests look flatter and troughs sharper and that the shorter the wave length is the smaller the measured wave height is. This distortion is due to the finiteness of the cross-section of the sound beam at the sea surface which is estimated approximately to be 1 meter in diameter. The results of observations of sea waves measured by the sonic wave gauge is found to agree well with those measured by a capacitance gauge in the frequency range lower than about 0.5 Hz.

### 1. Introduction

Wind waves play an important role in the process of momentum transfer from air to sea. In shipboard observations of wind waves, a remote sensing wave sensor may be required because it is not practical to have a suitable platform to fix a wave gauge near the sea surface.

This paper describes a sonic wave gauge which is operated by the echo-ranging method using pulsed, audible sound of 7 kHz at a sampling rate of 14 per second. Since the sound beam of the sonic wave gauge has a finite cross-section at the sea surface, the apparent profile of a wave is distorted. The response characteristics of the sonic wave gauge have been investigated by comparing the wave profiles of long-crested waves generated in a water tank and power spectra of the displacement of the sea surface measured by the sonic wave gauge and by the pole-type capacitance wave gauge.

Measurements of waves by the sonic wave gauge mounted at the shipboom is subject to errors due to ship motion. In practice, ship motion is measured by means of an accelerometer and a double-integrator to produce true wave height. This paper contains however the description of the sonic wave gauge, particularly its response characteristics, only.

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### 2. Description of the instrument

Our sonic wave gauge was designed primarily to be used aboard the R.V. Hakuho Maru of the Ocean Research Institute, the University of Tokyo. The ship has a boom which is projecting 10 m from the bow at height of 7.5 m over the sea surface and our gauge is mounted at the boom. The time  $\Delta t$  required for the echo to return to the gauge is given by  $\Delta t = 2H/c$  where  $H$  is the height of the sonic sensor from the sea surface and  $c$  is the sound velocity in the air. The range of distance detected by our gauge was designed to be  $\pm 2$  m. The longest sound path of 19 m ( $= 9.5 \text{ m} \times 2$ ) gives the maximum sampling rate of about 17 per second and we selected 14 per second.

The sound velocity in the air depends mainly on the air temperature. It may be regarded as a constant during one observational run which is usually several tens minutes. When this is the case, time  $\Delta t$  is proportional to the distance between the gauge and the sea surface. Its variation therefore gives the relative wave height measured by the sonic gauge.

It is apparent that the narrower the width of the sound beam is, the shorter the wave length of waves which can be detected by the sonic gauge is. The theoretical beamwidth at  $1/2$  power points at the operating frequency  $F$  is in proportion to  $\lambda/D'$ , where  $\lambda = c/F$  and  $D'$  is

the transmitter diameter. Therefore it is advantageous to use sound waves of high frequency to have a narrow beamwidth for a given transmitter. On the other hand the attenuation of sound waves in the air becomes larger as the frequency is increased. As a compromise, we have chosen 7 kHz audible sound. An audio tweeter for phonographs was used as the transmitter. A conical horn (26 cm high and 26 cm in diameter) was attached to make the sonic beamwidth narrow.

The block diagram of our sonic wave gauge is shown in Fig. 1. The oscillator unit generates 7 kHz waves. The same transducer converts electric waves to sonic waves in the transmitting state and sonic signals to electric signals in the receiving state. An electronic switch alternates the two states at a rate of 14 per second. The signal of the returned echo is amplified and fed into the echo-ranging unit.

The time sequence of the echo-ranging unit is shown in the lower schematic diagram of Fig. 1. A saw-tooth wave starts to grow linearly with time at the beginning of the receiving state and it is stopped by the returned echo. Consequently, the grownup voltage of the saw-tooth wave is in proportion to the time required for the returned echo to reach the transducer. The output of the echo-ranging circuit is fed into a

lowpass filter and then to the meter readout, and also to the output for recording.

The time sequence mentioned above is controlled by the oscillator unit. This is necessary because the error in time of one period (1/7000 sec.) gives the error in height of about 5 cm. The trigger to stop the saw-tooth wave must be made by the first return of the pulse. The output of the amplifier unit is also fed into the check terminal and we can observe it through an oscilloscope synchronized to the echo ranging unit. A sketch of the screen is shown in the upper portion of Fig. 2. The rectangular pulse composed of 7 kHz waves shows the echo returned from the sea surface. There is noise before and after the pulse. We have measured voltage of the echo ( $V_e$ ) and that of the noise ( $V_n$ ). The results are plotted in the lower portion of Fig. 2. The abscissa represents the measure of the gain control of amplifier. The values noted by open circles and dots were measured at the boom of the Hakuho Maru and at the marine observation tower at Hiratsuka respectively. In the figure, the higher values

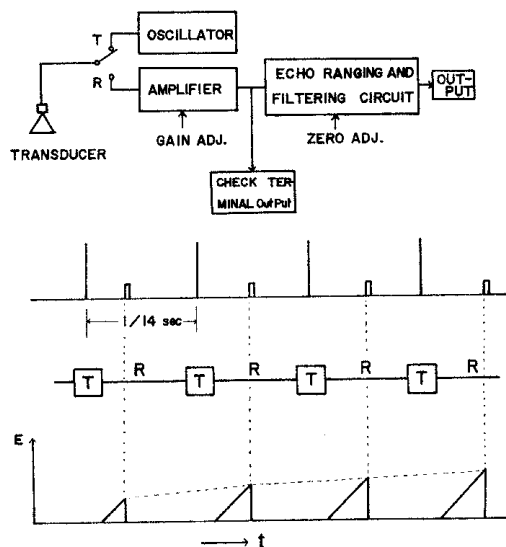
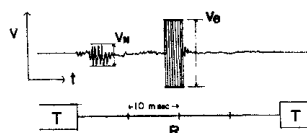


Fig. 1. Block diagram of sonic wave gauge and the time sequence of the echo-ranging.

#### SIGNALS AT THE CHECK TERMINAL



○ H.V. HAKUHO - MARU  
● HIRATSUKA OBS. TOWER

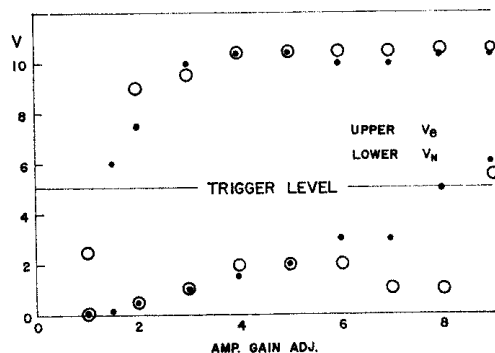


Fig. 2. Signals at the check terminal: (upper) oscilloscope, single sweep, (lower) voltage of the returned echo and voltage of noise.

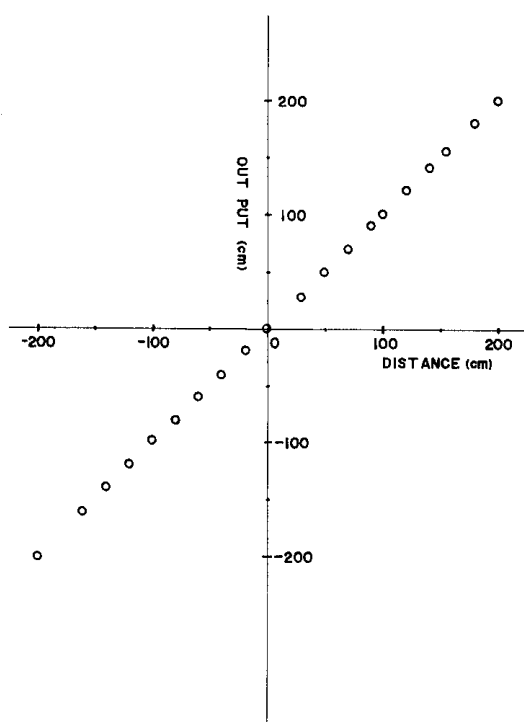


Fig. 3. Calibration curve of the sonic wave gauge.

show  $V_e$  and the lower  $V_n$  for the given gain. The trigger to stop the saw-tooth wave is made by the signal that exceeds the trigger level of 5 volts. With the sonic wave gauge, wave height can be measured if the echo voltage is higher than the trigger level and the noise voltage is lower than the trigger level. Fig. 2 indicates that the gain is 2-8 for the observation aboard the Hakuho Maru and 2-7 for that at Hiratsuka.

An examination of the linearity relation of the distance in the response characteristics of the gauge was made on the roof of our building. The wall of the building was the target surface. The result is plotted in Fig. 3. The abscissa represents the distance between the transducer and the wall adjusted to be zero at 7.5 m from the wall. It shows that the gauge has a good linearity of distance as well as sufficient accuracy of 1 cm.

### 3. Laboratory tests for long-crested waves

A sampling rate of 14 per second is considered to be sufficient to describe surface waves in the frequency range of our interest. In principle,

on the other hand, a sonic gauge is not capable of detecting those waves whose wave length is shorter than the diameter of the target area of the sound beam at the sea surface. In order to examine the response characteristics of the sonic gauge, we have made comparison between the profiles of long-crested wave measured by the sonic gauge and that by a pole-type capacitance wave gauge which measures the local water level within an area of 1 cm diameter. The test was made using a wave tank of 80 m square. Arrangement of the two wave sensors is shown in Fig. 4. The transducer unit of the sonic wave gauge was mounted at the top of a pole about 5 m high above the water surface. The sensing pole of the capacitance wave gauge was hung down from it.

The tank test was carried out in various conditions and the experimental conditions are tabulated in Table 1. Water wave length was varied from 2 m to 9 m. Waves were long-crested except in the last case (5.7). The results of the comparison are shown in Fig. 5. In Figs. 5-1~5-7, water level was read every 0.1 sec. and the time elapsed is taken on the abscissae. Solid curves in the figures show the waves measured by the capacitance wave gauge and the dots show those by the sonic wave gauge. The two records are arranged in the figure so that they are matched at the crest



Fig. 4. Arrangement of wave sensors in the tank test.

Table 1. Experimental conditions in the tank tests.

	Frequency	Wave length	Wave height 2a	Wave steepness	Gain of the sonic gauge	Wave height ratio
5.1	0.39 Hz	9 m	18 cm	2 %	G-4	1.0
5.2	0.43	7	21	3	G-4	0.90
5.3	0.51	6	19	3	G-3	0.98
5.4	0.51	6	20	5	G-6	
5.5	0.63	4	18	5	G-5	0.85
5.6	0.88	2	9	5	G-5	
5.7	0.88	2	18	9	G-6	0.58

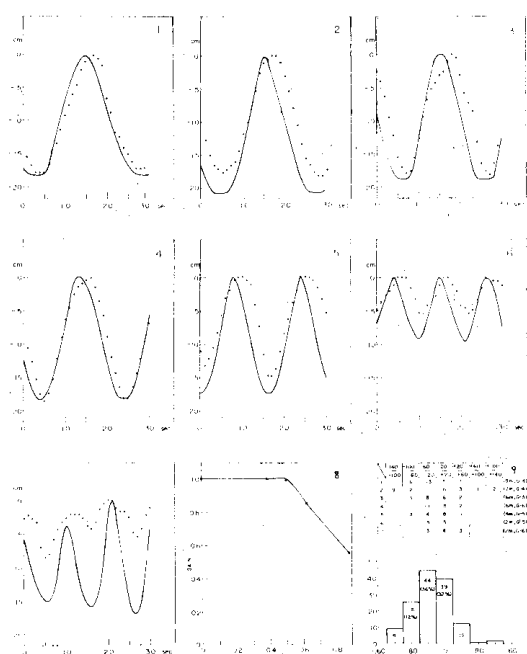


Fig. 5. Tests at a wave tank: (1-7) wave profiles of long-crested waves measured by the sonic wave gauge (dotted) and by the pole-type capacitance gauge (solid curves), (8) wave height ratio against wave frequency, (9) estimation of the sonic beam width.

level for the convenience of the following consideration.

The figures show that the wave height measured by the sonic gauge is smaller than that measured by the capacitance gauge. The ratio of the wave height measured by the sonic gauge to that by the capacitance gauge is plotted against the wave frequency in Fig. 5-8. The ratio is almost constant in the lower frequency range and it decreases rapidly with the frequency in

the higher frequency range. There is a 40 % loss in the highest frequency range in our experiment. The figures also show that the profile at a wave measured by the sonic gauge is distorted to be flatter near the crests and sharper at the troughs. We may then conclude that:

1. The sonic gauge measures the highest water level in the finite target area of the sound beam. This is because that the echo-ranging unit is operated by the first return from the water surface.

2. Therefore, the level of the crest is never missed and the level is maintained as far as the crest remains in the target area.

We can estimate the diameter of the target area from Figs. 5-1~5-7, by assuming that water waves propagate with their phase velocities while keeping their shapes unchanged. We first evaluate the time delay between two instances at which the sonic gauge and the capacitance gauge measure the same water level. The time delay multiplied by the phase velocity may correspond to the target radius of the sonic wave gauge. They are represented in Fig. 5-9 in the form of histogram. The figure indicates that all data are in the range of about 2 m and 80 % are in the range of one meter. We may therefore take 1 m as the diameter of the target\*. The center of the histogram is not located at

\* The distance is represented by a negative value when the level is recorded by the capacitance gauge earlier than by the sonic gauge. The range of the histogram, therefore, shows the target diameter. The histogram is rather dispersed. If we rearrange each pair of waves to have minimum discrepancy and reduce the rounding error, we could slow more precisely the target diameter (see the table in the upper portion of Fig. 5-9).

zero but at  $-20 \sim -40$  cm. It shows that the capacitance gauge was located closer to the wave maker.

The effect of the gain control of the sonic gauge on the response characteristics was examined by comparing the cases of 5.3 and 5.4. There is no significant difference in wave shapes or in diameters of the target areas (see the table in the upper portion of Fig. 5-9).

#### 4. Field test for ocean waves

We have observed ocean waves at the Marine Observation Tower off Hiratsuka with the sonic wave gauge and also with the capacitance wave gauge. The arrangement of the sensors is shown in Fig. 6. The transducer unit of the sonic gauge was mounted at the top of a pole, about 8 m high above the water surface. The sensing pole of the capacitance wave gauge was set at the base of the pole, about 1 m apart from the place right under the horn. Observation was made for various gain controls of the amplifier of the sonic gauge. Power spectra of wave height were calculated for each record. Coherence and gain (frequency response function of the sonic gauge compared with the capacitance gauge) were calculated using the two simultaneous records of the sonic gauge and of the capacitance gauge. The result is shown in Figs. 7, 8 and 9.

Fig. 7 shows the case where the gain control of the amplifier was set at 5. In the left figure, the dots and open circles show the power spectra measured by the capacitance gauge and by the sonic gauge respectively. The good agreement

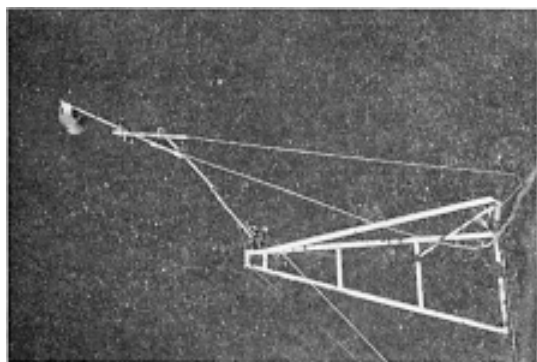


Fig. 6. Arrangement of wave sensors at the marine tower.

can be seen between two spectra in the low frequency range. In the frequency range higher than about 1 Hz, however, a significant difference can be seen between the two spectra. The degree of freedom in the spectra is 43 and the corresponding 95 % confidence limit is also shown in the figure. The gain of the record by the sonic gauge in comparison with that by the capacitance gauge is plotted in the upper right of the figure. It is almost constant in the lower frequency part. A remarkable decrease of the gain can be seen in the region higher than about 0.5 Hz. The coherence plotted in the lower right figure decreases rapidly in the range higher than 0.5 Hz, while the coherence exceeds 0.5 in the range lower than that frequency. Though the spectral density by the sonic gauge agrees with that by the capacitance gauge up to 1 Hz, the response function shows that the sonic gauge describes ocean waves with a good accuracy up to about 0.5 Hz.

The line  $\psi(f) = \beta g^2 (2\pi f)^{-5}$  is included in the left figure, with the dimensionless constant  $\beta$  set equal to  $1.17 \times 10^{-2}$  (suggested by PHILLIPS (1966)). There is a range of frequency near 1 Hz in which the measured spectral points by the capacitance gauge is well represented by this

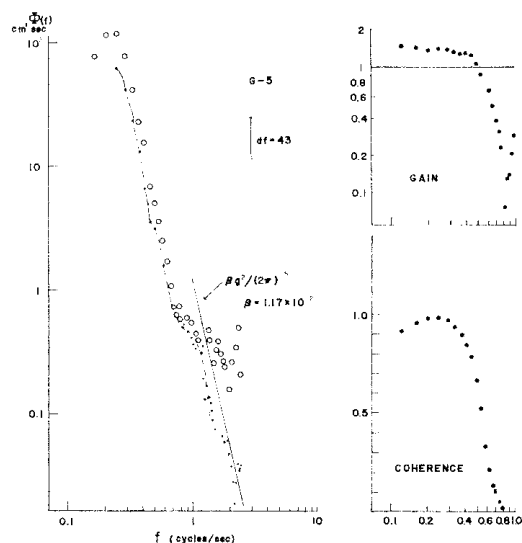


Fig. 7. Observation at the marine tower: gain control at 5, power spectra (left,  $\cdots$  by the sonic wave gauge,  $\cdots$  by the capacitance wave gauge) and frequency response function (right).

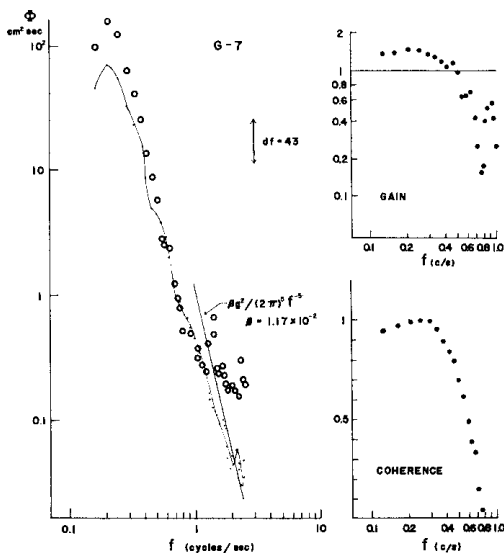


Fig. 8. Observation at the marine tower: gain control at 6, power spectra (left) and frequency response function (right).

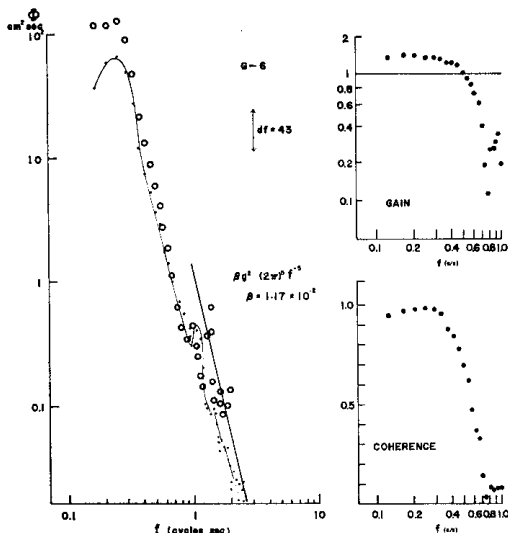


Fig. 9. Observation at the marine tower: gain control at 7, power spectra (left) and frequency response function (right).

line. The waves in this range is considered to be generated by the local wind. The main peak of the spectrum is located at about 0.25 Hz and this may correspond to swells.

Figs. 8 and 9 show the cases where the gain control of the amplifier of the sonic gauge was set at 6 and 7 respectively. There appears no

significant difference among Figs. 7, 8 and 9.

## 5. Discussion

Let us consider a simple model of the response characteristics of the sonic wave gauge. We first assume that sound waves of 7 kHz are reflected uniformly by the wavy surface. In other words, the reflection is assumed to be independent of the surface slope. The result of our tank experiment indicates that this is true up to the wave steepness of 9% (see Table 1). We also assume that the sonic gauge has a target area of  $D$  meters in diameter and that the gauge does not receive sound waves reflected from the outside of this target area. The gauge measures the distance from a certain point in the target area by means of the first return. The water waves are assumed to be sinusoidal long-crested waves and expressed as  $\eta = a \cos 2\pi x/\lambda$ , where  $a$  is amplitude and  $\lambda$  is wave length. If the gauge has a point target, the recorded waves must be unchanged. However the model gauge has a target of diameter  $D$ , and therefore the apparent profile of a wave must be distorted.

The solid line in Fig. 10 shows the original

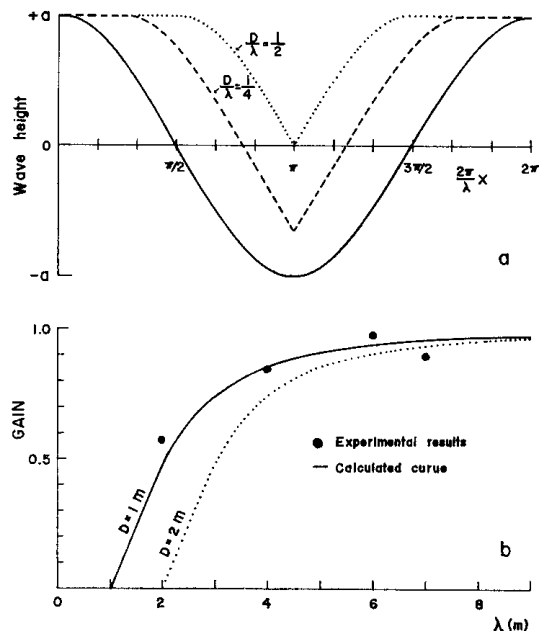


Fig. 10. Response characteristics of a model wave gauge: profiles of distorted waves (upper) and wave height ratio to the original wave (lower).

sinusoidal wave. The broken line and the dotted line show the profiles of a wave which are recorded by the model gauge with target of  $D/\lambda=1/4$  and  $D/\lambda=1/2$  respectively. The apparent waves are distorted to have the profile which is flatter near the crest and sharper at the trough than the original shape. The wave height becomes smaller as the ratio  $D/\lambda$  increases. A simple consideration shows that the ratio of the recorded wave height to the original is given by

$$\text{Gain} = \frac{\alpha\{1 - \cos(\pi \pm \pi D/\lambda)\}}{2a}, \quad D/\lambda \leq 1 \\ = 0, \quad D/\lambda > 1.$$

The value is plotted in the lower figure as a function of wave length. The solid and dotted lines show the cases of  $D=1$  m and  $D=2$  m respectively. Circles in the figure show the experimental results obtained in the tank test. The good agreement can be seen between the experimental and theoretical results. We may conclude then that our sonic gauge has response characteristics similar to this simple model and that our sonic gauge has a target area of 1 m in diameter. The latter conclusion is also based on the result shown in Fig. 5-9.

## 6. Summary

We have investigated the response characteristics of the sonic wave gauge to water waves. The sonic gauge is operated by the echo ranging method using pulsed audible sound of 7 kHz. By the first return of the pulse reflected from sea surface, the gauge measures the distance between the sea surface and the transducer unit. The measurement is made at a sampling rate of 14 per second.

Although the gauge has a good linearity of distance, the apparent shape of waves recorded by the sonic gauge have flatter crests and sharper troughs than the original waves. The height of the measured waves becomes smaller as the wave length is decreased. It is shown in this paper that the diameter of the target area is about 1 m and the distortion of the apparent profile is due to the finiteness of the target area of the sound beam. In order to have a better understanding of the response characteristics of

the wave gauge, a simple model is presented.

The comparison has been made between the power spectra of the displacement of the sea surface measured by the sonic gauge and by the capacitance wave gauge. The results indicate that two gauges give quite similar spectra in the frequency range lower than 0.5 Hz. It is concluded that the observation of the ocean waves with the sonic gauge is feasible with a good accuracy in the range of frequency lower than 0.5 Hz (corresponding to about 6 m in wave length in deep water). The fact suggests that the sonic wave gauge must have target area of a diameter smaller than 1/6 of the wave length which is required to be measured.

The same argument may be applied to other remote sensing wave sensors such as the gauges described by MARK (1962) and BARNETT (1967) and also to submarine ultra-sonic wave gauges set on the seafloor.

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## References

- BARNETT, T. P. and J. C. WILKERSON (1967): On the generation of ocean wind waves as inferred from airborne radar measurements of fetch-limited spectra. *Jour. Marine Res.*, **25** (3), 292-321.
- MARK, R. B. (1962): Shipboard ultrasonic wave height sensor. *Marine Sciences Instrumentation*, vol. 2. Proceedings of Symposium on Transducers for Oceanic Research at San Diego, California.
- PHILLIPS, O. M. (1966): The dynamics of the upper ocean. Cambridge University Press.

## 音波式波高計の応答特性について

平 啓 介      竹 田   厚

**要旨：** 海面上約 7.5 m の観測船白鳳丸の船首ブームに取り付けて風浪を測定するように設計した音波式波高計の応答特性について報告する。この波高計は約 7 kHz の可聴音波パルス海面に発射し、反射音が到着するのに要する時間を測定して水面の上下変動を測る方式で、1 秒間に 14 回の測定をくりかえす。

音波式波高計は音波ビームの幅のために、水面上で有限の面積のまともをもつ。そのために、この波高計で水槽に起こした long-crested の波を測ると、得られる記録は波長が短くなるに従い波高を小さめに評価し、また波形が峰では平坦に、谷では鋭くなるように変形する。水

面上での“まど”の直径が約 1 m であることが実験によって示された。簡単な模型化によって音波式波高計の応答特性をよく理解することができた。

観測塔における海洋波の観測で、音波式波高計の記録は同時に同じ場所得た、容量型波高計の記録と約 0.5 Hz より低い周波数領域で有意に一致することがわかった。

この論文で議論した音波式波高計の応答特性は、電波や超音波を用いて航空機あるいは海底から波高を測定する波高計にも適用できると思われる。