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Experimental Study of Wave Reflection by a Sloping Beach

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Abstract: Wave reflection by a sloping beach was studied with small-scale gravity waves in a small wave-tank 4 m long. The breaking of small-scale waves is much different from that observed on natural ocean beaches or in experimental tanks using large-scale waves. In a small tank, the waves lose their energy by generating capillary waves of several millimeters in length at wave fronts near the shoreline. However, the dependence of the reflection coefficient on beach slope and on wave steepness are similar to those for large-scale waves, and the critical beach slope for wave breaking at a given deep wave steepness can be predictable by Miche's theory. The difference occurs only in the slope range just larger than the critical beach slope where the small-scale waves lose a significant amount of their energy without breaking. The capillary waves which are generated near the shoreline and propagate offshore seem to have an important role in this energy loss.

The phase difference between the incident and reflected waves at the toe of the beach was also examined. The phase difference does not depend on wave steepness but depends on the ratio of wave length to beach length. The experimental results show that the phase difference is proportional to kL for a wide range of kL. For small values of kL, the experimental values agree with the calculated values based on a simple model.

1. Introduction

The phenomenon of wave reflection by a uniformly sloping beach may appear very simple, but many of the problems remain unsolved. Waves lose their energy mainly by breaking near the shore. The breaking is little understood theoretically because of its inherent strong nonlinearity. A singularity at the moving shoreline is also one of the difficulties in the theoretical works. The available theoretical works are mostly cases of the standing waves with no energy loss over the beach (LAMB, 1932; MICHE, 1944; LEWY, 1946; STOKER, 1947, 1957) and only one work by MICHE (1944) on the problem of the wave reflection with energy loss or the partial standing waves.

On setting a mathematical criterion on wave breaking, MICHE concluded that for a given beach slope there exists a critical value of incident wave steepness. The waves whose steepness is smaller than this critical steepness are reflected with no energy loss. He also gave the reflection coefficient as the function of incident wave steepness and of beach slope, assuming that when the incident wave steepness is larger than the critical wave steepness, the reflected waves have the steepness of critical value and the excess energy is lost in breaking.

Usually the perfect reflection cannot be observed in experiment even in a slope range where Miche's theory predicts no energy loss. MICHE (1951) introduced a roughness parameter to explain this discrepancy by attributing the energy loss to friction. The roughness parameter is determined empirically, and it is assumed to depend on the beach material and not on beach slope and wave steepness. The experimental reflection coefficient agrees with Miche's theoretical value within the experimental error on adopting this empirical parameter (*e.g.* GRESLOU and MAHE, 1954). Detailed interpretation, however, remains difficult because of large scattering in the data from tank experiments. One of the purposes of this paper is to investigate the energy loss without wave breaking.

In this paper wave reflection by a uniformly

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sloping beach was investigated experimentally by using a small-scale wave tank. Experimental wave height was only several millimeters, and wavelength was about a half meter. The breaking of gravity waves of such a small-scale is a type different from that seen on natural coasts. The waves lose their energy by generating capillary waves of several milimeters length at their wave fronts. The observed energy loss is compared with the results of Miche's theory and of Greslou and Mahe's experiments. Also, phase difference of the reflected waves to the incident waves at the toe of the sloping beach are investigated, and the parameter governing phase relation is found. The results are compared with the theory based on a simple model.

2. Apparatus and experimental Procedure

We used a small wave tank made of Lucite (Fig. 1). The length of the tank was 4 m, the height 30 cm, and the width 15 cm. A piston type wave-maker was located at one end of the tank. At the other end, an aluminum plate was used to simulate a uniformly sloping beach. A wave filter (a basket filled with film strips) was used both to regulate the generated waves and to minimize the influence of multi-reflection on the waves.

We used a resistance type wave gauge, the sensor of which consisted of two half-submerged parallel wires. The sensor was hung on a carriage as shown in Fig. 2. The carriage moved on a glass guide-rail and we could measure the wave height at any place within a 3 m range. A calibration was made by raising the sensor after each set of measurements.

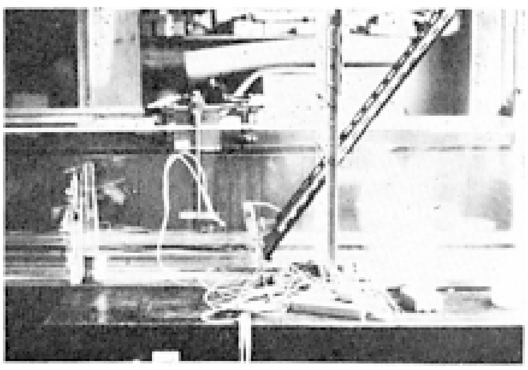

Fig. 1. Lucite wave tank, 4 m long, 15 cm wide, 30 cm deep.

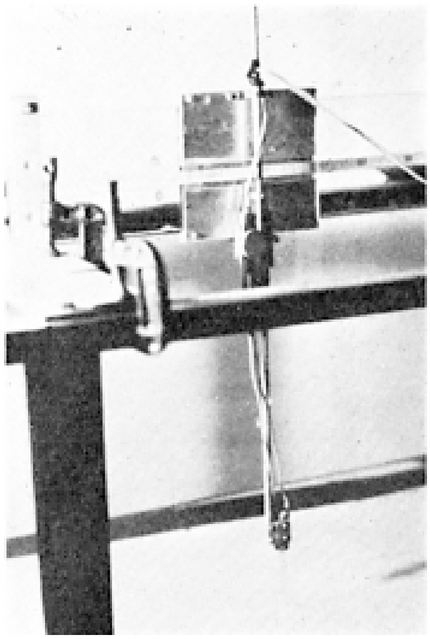

Fig. 2. Wave Gauge and Carriage.

The incident waves generated by the wave-maker lose some of their energy in breaking over the beach and the reflected waves have a smaller magnitude. Let x be a horizontal coordinate measured offshore from the toe of the sloping beach, and y be a vertical coordinate measured upward from the undisturbed surface. The undisturbed shoreline is located at $x=-L$ (see Fig. 3). Denoting the amplitudes of incident and reflected waves by a and b respectively, we can represent the surface elevation $\eta(x, t)$ by

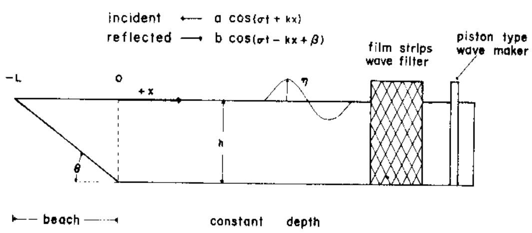

Fig. 3. The coordinate system. Position x is measured offshore from the toe of the beach. L denotes beach length, θ beach angle, h water depth, and η surface elevation.

$$\eta(x,t)=a\cos(\sigma t+kx)+b\cos(\sigma t-kx+\beta)$$
$$=E(x)\sin[\sigma t+\Theta(x)], \qquad (1)$$

where

$$\{E(x)\}^2=a^2+b^2+2ab\cos(2kx-\beta) \qquad (2)$$

and

$$\Theta(x)=\cot^{-1}\left\{\frac{-a\sin kx+b\sin kx\cos\beta-b\cos kx\sin\beta}{a\cos kx+b\cos kx\cos\beta+b\sin kx\sin\beta}\right\}.$$

Here σ and k are wave frequency and wave number respectively and are related by $\sigma^2=gk\tanh kh$. β is the phase difference between the incident and reflected waves at the toe of the sloping beach ($x=0$). $E(x)$ is the envelope of the partial standing wave. Then the reflection coefficient R can be calculated from the maximum and minimum of $E(x)$ by

$$R=\frac{b}{a}=\frac{\{E(x)\}_{max}-\{E(x)\}_{min}}{\{E(x)\}_{max}+\{E(x)\}_{min}}. \qquad (3)$$

The envelope $E(x)$ has extreme values when $2kx-\beta=n\pi$ (n is any integer). Let x_{max} and x_{min} be the positions where the envelope is a maximum and a minimum respectively, then β can be given by:

$$\beta=2kx_{max}-2n\pi$$

or

$$\beta=2kx_{min}-(2n+1)\pi, \qquad (4)$$

where n is an arbitrary integer.

The amplitude of the incident wave decreases shorewards and that of the reflected wave decreases offshore by dissipation due to friction. We measured the envelope at intervals of 1 cm and determined positions and amplitudes of several maxima and minima (see Fig. 4). Then we calculated apparent reflection coefficients for every successive pair of maximum and minimum by (3). From these, we determined the reflection coefficient R at the toe of the sloping beach by using the method of least squares and of

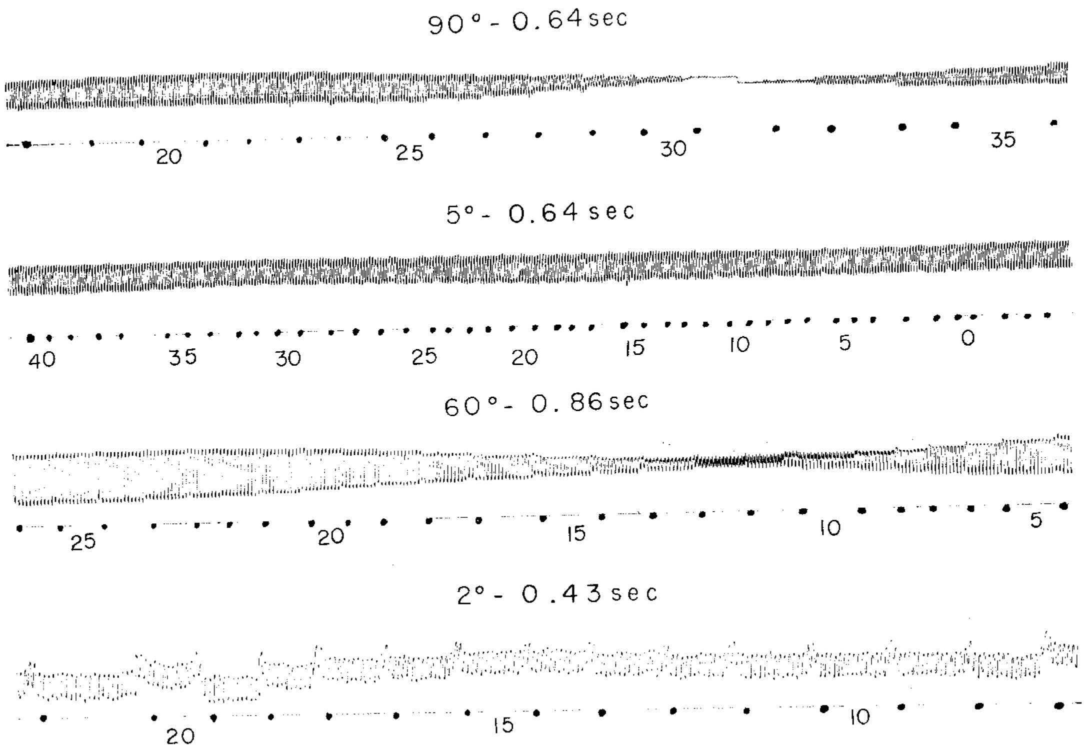

Fig. 4. Examples of wave records. Captions above each record (*e.g.* 90°-0.64 sec) indicate beach angle and wave period. Dots below each record indicate the time when the wave gauge was shifted. Numerals indicate positions from the toe of the beach in cm. The jump at position 31 in the first record is due to a zero adjustment.

linear extrapolation. The phase difference β between the incident and reflected waves at the toe of the beach was determined from observed positions of minima by (4).

3. Reflection coefficients

The dependence of reflection coefficient on slope angle is examined for waves of 0.43 sec, 0.64 sec, and 0.86 sec. Wave steepness and water depth were kept at 0.4% and 5.0 cm respectively throughout this experiment. The results are shown in Fig. 5. The reflection coefficient increases rapidly as the beach angle increases from 2° to 30°. The curves in Fig. 5 show some irregular variation for angles greater than 45°. In particular, the coefficients for 0.86 sec waves in the range between 60° and 90°, and the coefficient for 0.43 sec waves at 90° seem to be relatively low. In these cases, the secondary waves or the beats can be observed without exception in the wave records. Such examples are shown in Fig. 4; the secondary waves in the third record and the beats in the last record. As seen in the third record in Fig. 4, these irregularities in wave shape cause the large error in the estimation of the magnitude of $E(x)_{min}$ and so of the reflection coefficient. The secondary waves and beats seem to result from three sources, the irregular motion of wavemaker, the change of the efficiency of the wavemaker depending on the phase difference between the wavemaker and the raflected waves from both ends of the tank, and the free oscillations of the whole channel. Only in the case of 0.64 sec waves could we make closely sinusoidal waves* as shown in the first and second records in Fig. 4. In this section we use the results on 0.64 sec waves only.

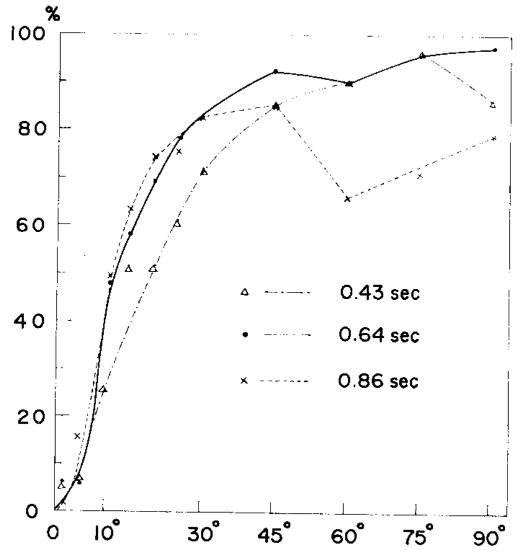

Fig. 5. Reflection coefficient R *vs.* the beach angle θ for waves of 0.43 sec, 0.64 sec, and 0.86 sec. Water depth is 5.0 cm and wave steepness is 0.4% in all cases. The coefficients for 0.86 sec waves for angles between 60° and 90° and that for 0.43 sec waves at 90° seem to be low. In these cases, the secondary waves or the beats are observed without exception in the wave records.

MICHE (1944) studied the theory of standing waves over a sloping beach of infinite length. From the surface condition, the slope of the water surface at the shoreline cannot exceed the slope of the beach. From this condition, the critical deep water wave steepness γ_{max} of the incident wave can be calculated for a given slope, and the waves having deep water wave steepness smaller than γ_{max} are reflected with no energy loss by the beach if friction is ignored. MICHE found that γ_{max} is a function of beach slope only, and is given by

$$\gamma_{max}=\sqrt{\frac{2\theta}{\pi}}\cdot\frac{\sin^2\theta}{\pi}, \tag{5}$$

where θ is the beach angle in radians. The incident wave whose steepness exceeds the critical value, γ_{max}, must break over the beach. MICHE assumed that in such a case the waves having the same steepness as γ_{max} are reflected by losing excess energy in breaking. The reflection coefficient R is then given by

$$R=\begin{cases}\gamma_{max}/\gamma_0 & \text{for} \quad \gamma_0\geqq\gamma_{max} \\ 1.0 & \text{for} \quad \gamma_0<\gamma_{max}\end{cases} \tag{6}$$

where γ_0 is the deep water wave steepness of the incident wave.

A wave steepness of 0.4% at the depth of 5.0 cm corresponds to that of 0.3% in deep water for 0.64 sec waves. The theoretical curve

* The records of 0.64 sec waves at a 60° angle show secondary waves of small magnitude. A small dip in the curve for 0.64 sec waves in Fig. 5 may be attributed to the existence of these secondary waves.

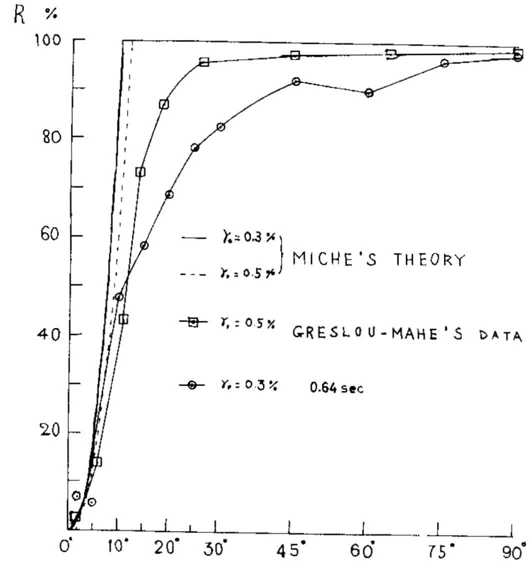

Fig. 6. Comparisons of the measured reflection coefficient to that given by Miche's theory. The experimental results of Greslou and Mahe are also shown.

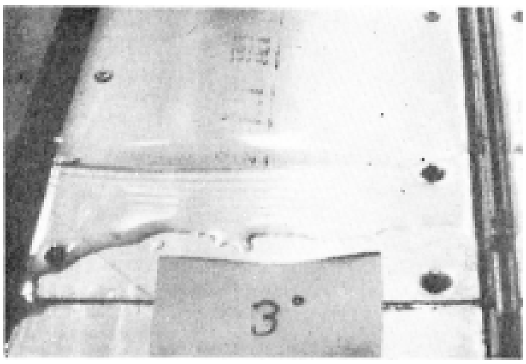

Fig. 7. Breaking of small-scale gravity waves. Wave height is 1.8 mm, wave steepness 0.3 % and beach angle 3°. The long crested capillary waves are generated at the wave front near the shoreline.

of the reflection coefficient for the deep water wave steepness of 0.3 % is shown by a solid line in Fig. 6 with the experimental curve for 0.64 sec waves. We also refer to the experimental values by GRESLOU and MAHE (1954) as representative of previous investigations. Their experimental waves have a deep water wave steepness of 0.5 %. Their results and the theoretical values corresponding to them are also plotted in the same figure.

Miche's theory predicts that, for an incident wave of 0.3 % steepness, breaking occurs over a beach with an angle of less than 9°. In our experiment, the reflection coefficients for angles less than 10° are considerably smaller than those for angles greater than 10°. The 0.64 sec waves have a wave height of only 1.8 mm, and the breaking of the wave is not the same as occurs on natural coasts, even in the case of beach slopes less than 10°. When the incident waves approach the shore, the slope of the wave front becomes steeper than the rear, and wave crests are sharpened. Then capillary waves of several millimeters in length are generated at wave fronts near the shoreline (Fig. 7). The generation of these capillary waves is not observed at beach angles greater than 15°. If we regard the generation of capillary waves at wave fronts as the breaking of small-scale gravity waves, then the critical angle of the breaking agrees fairly well with that predicted by Miche's theory.

Our experimental results agree with Miche's theory within experimental error, not only in critical angle but also in reflection coefficients for beach angles smaller than 10°. Under natural conditions and in the usual tank experiments, the breaking of the wave occurs not at the shoreline, but from a crest at some distance from the shore. According to Miche's criterion, however, the breaking of the wave must occur just at the shoreline. It is rather curious, therefore, that results of our experiment, Greslou and Mahe's experiment, and Miche's theory agree with one another. This shows that the small-scale waves can be used as a model of the large-scale waves if the energy relation is most significant in the systems considered.

For a beach steeper than the critical angle, Miche's theory predicts no energy loss and perfect reflection. On the contrary, both of the experimental results in Fig. 6 show some energy loss. The perfect reflection is rarely observed in tank experiments even in the case of reflection by a vertical wall. MICHE (1951) introduced a empirical roughness parameter to explain this discrepancy by attributing the energy loss to friction according to the following formula:

$$R' = \begin{cases} \rho \cdot \gamma_{max}/\gamma_0 & \text{for} \quad \gamma_0 \geqq \gamma_{max} \\ \rho & \text{for} \quad \gamma_0 < \gamma_{max} \end{cases}, \quad (7)$$

where R' is the predicted reflection coefficient and ρ is the roughness parameter which is assumed to depend only on beach material. The considerable energy loss for beach angles slightly greater than the critical angle, however, cannot be explained by this theory. Energy loss correlation can be improved by making ρ a function of θ, but the energy loss in our experiments seems to be too large to be explained by friction only.

In our experiment, for angles from 15° to 45°, we do not observe the breaking, that is, the generation of capillary waves at the wave fronts. We do observe the capillary waves propagating offshore which are generated near the shore in the instant of maximum run-up. These capillary waves are damped rapidly and disappear within a wave length of the incident waves. Since the generation of capillary waves at the wave fronts causes energy loss as the usual breaking, it is natural that the generation of these capillary waves propagating offshore should cause considerable energy loss. If the amplitude of these capillary waves depends mainly on the surface tension, and contact angle between water and beach material, but little on the magnitude of the incident wave height, the ratio of the energy loss to the incident wave energy is in inverse proportion to the magnitude of the incident wave energy. Since the wave height of Greslou and Mahe's experimental waves is about five times greater than that of ours, it is reasonable that our reflection coefficients for angles from 15° to 45° are much smaller than those of GRESLOU and MAHE.

It is well known that for the large-scale waves the reflection coefficients depend mainly on deep water wave steepness for a given beach angle just as Miche's theory predicts. It is worth-while to examine whether this is true for our small-scale waves. The experiments were done with various wave steepness for an angle of 6°. The results are shown in Fig. 8. For comparison, Greslou and Mahe's experimental results for an angle of 1/10 (approximately 6°) and Miche's theoretical values are plotted in the same figure. We made additional experiments in the 10 m wave tank at the Earthquake Research Institute, University of Tokyo, with waves of the same order of magni-

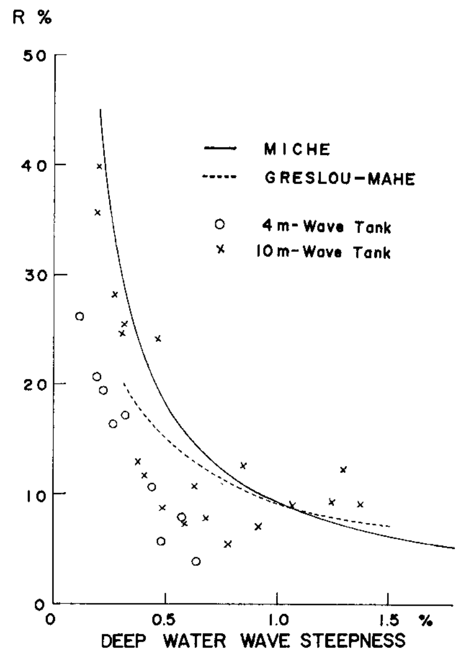

Fig. 8. Reflection coefficient R *vs.* deep water wave steepness at a beach angle of 6°. The solid line indicates the reflection coefficient given by Miche's theory for a beach slope of 6° and the dotted line is that referred from the results of Greslou and Mahe's experiment.

tude as in Greslou and Mahe's experiment. The result is also shown in Fig. 8. It is clear from this figure that the reflection coefficient for small-scale waves also depends on wave steepness and it decreases with increase of steepness in the same manner as for large-scale waves.

For the smaller wave steepness, the reflection coefficient for small-scale waves in Fig. 8 appears to be a little smaller than for large-scale waves. This may be explained by the difference in breaking of the small-scale waves. In the case of the smallest steepness (0.12 %)*, we do not observe clear breaking (*i.e.* the generation of capillary waves at the wave front), although 74 % of the energy is lost. This energy loss must be attributed to other mechanisms such as the generation of the offshore capillary waves. In the next four experiments, the long crested capillary waves as seen in Fig. 7 were generated

* The value agrees roughly with Miche's critical steepness of 0.09 % for an angle of 6°.

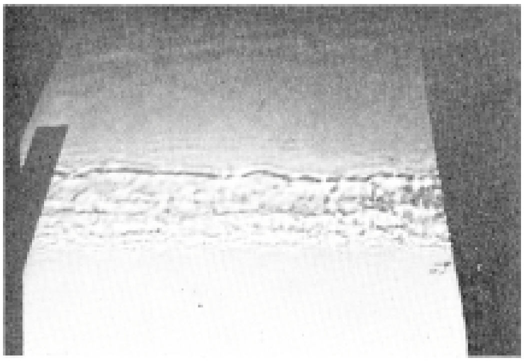

Fig. 9. Breaking of small-scale gravity waves. The short crested capillary waves are generated at the wave front just near the shoreline. This type of breaking occurs in waves a little larger than those shown in Fig. 7.

at the wave front, and the reflection coefficient for small-scale waves is somewhat smaller than that for large-scale waves. On the other hand in the experiment with the steepness greater than 0.45 %, the capillary waves generated at the wave front are short crested and have very irregular shapes as seen in Fig. 9, and no significant difference in reflection coefficient can be seen between small-scale and large-scale waves.

4. Phase difference between the incident and reflected waves

Some examples of the observed envelopes are shown by small circles in Fig. 10 for various wave steepnesses. The wave length, the beach slope and the water depth were kept contsant throughout this experiment. The solid curves are drawn from harmonic analysis of each envelope on the assumption that these nine envelopes have the same phase. It is clear from this figure that the phase difference does not depend on the wave steepness, at least within an experimental error, while the reflection coefficient changes greatly from 3.8 % to 26.2 %. The phase differences for the large-scale waves were also examined in the 10 m wave tank and shown in Fig. 11 for various conditions. The phase difference shows no clear dependency on the wave steepness, and so it must be independent of energy loss or reflection coefficient at least in the first approximation.

The change of the phase difference β against the beach angle θ was examined for the waves of 0.43 sec, 0.64 sec, and 0.86 sec. The results are shown in Fig. 12. In this case, we so selected the arbitrary integer n in (4) that the phase difference for the reflection by a vertical wall was approximately zero, and that the phase difference changes smoothly with the beach angle θ. Secondary waves and beats do not influence the curves of the phase difference in Fig. 12, as they do not affect the estimation of x_{min} as that of $\{E(x)\}_{min}$. The absolute value of the phase difference β decreases with increase of wave period or with increase of wave length for a given beach angle. Also, $|\beta|$ decreases with increase of beach angle or with decrease of beach length.

We calculated the effective beach length l, which is defined as $l=-\beta/2k$. If we replace the sloping beach by an imaginary wave channel of uniform depth having a vertical wall at its end, $-l$ corresponds to the position of this wall. In Fig. 13, the effective beach length, l, is plotted against the beach angle instead of β. The value of l is independent of wave period for the constant water depth. For comparison, the beach length L is also shown in the same figure by a solid curve. It is interesting to note that the effective beach length l is roughly equal to the real beach length L. If waves are mainly reflected at the shore line, l must be larger than L because wave length decreases with decrease of water depth over the beach. If we can neglect the effect of decrease of wave length over the beach, and, if the reflection occurs in the middle of a sloping beach or partly at the toe, l must be smaller than L. In reality, these effects must cancel each other.

Since the phase difference does not depend on wave steepness and therefore is not affected by wave breaking, and the effective beach length corresponds roughly to the real beach length, the phase difference appears to be mainly governed by the ratio of wave length to beach length. It is therefore better to consider the phase difference β, as a function of a nondimensional parameter kL. This result is shown in Figs. 14 and 15.

If we assume that the wave length is very

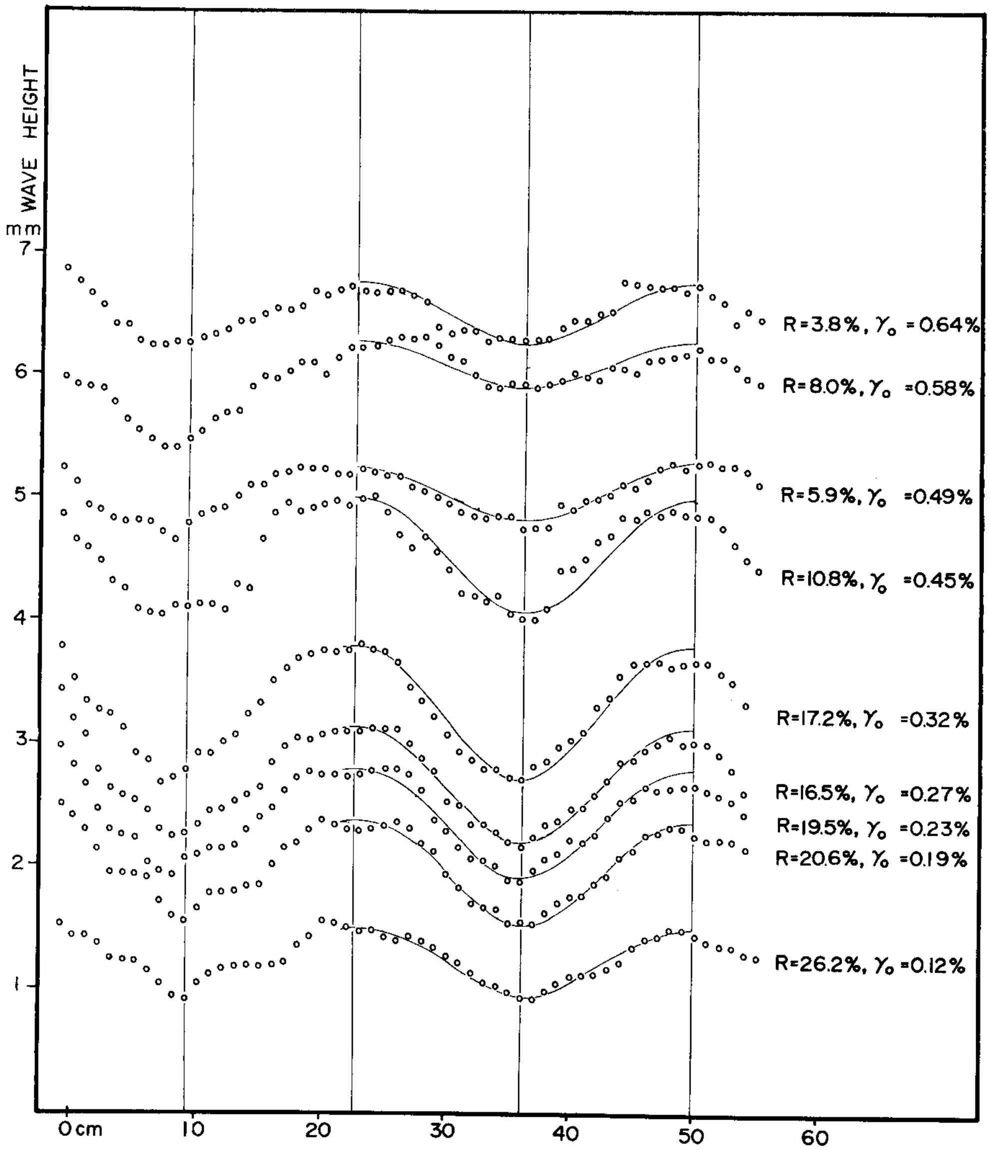

Fig. 10. Envelopes of partial standing waves for various wave steepness. Beach angle, water depth and wave period are kept constant at 6°, 5 cm and 0.8 sec respectively. The solid lines are drawn from harmonic analysis on the assumption that these nine envelopes have the same phase. It is clear that the phase difference does not depend on the wave steepness.

large and the wave amplitude is very small compared with the water depth, and that the waves are reflected perfectly by the beach, we can estimate theoretically the value of β as a function of kL. In the region of sloping beach the surface elevation, η_B, is described by:

$$\eta_B(x, t) = AJ_0(2\sqrt{q(x+L)}) \cos(\sigma t+\varepsilon), \quad (8)$$

where $q=\sigma^2 \cot\theta/g$ (LAMB, 1932). In the region of constant depth the surface elevation, η_c, is given by

$$\eta_c(x, t) = a\{\cos(kx+\sigma t)+\cos(kx-\sigma t+\beta)\}. \quad (9)$$

The surface elevation and the volume flux must be continuous at the toe of the sloping beach.

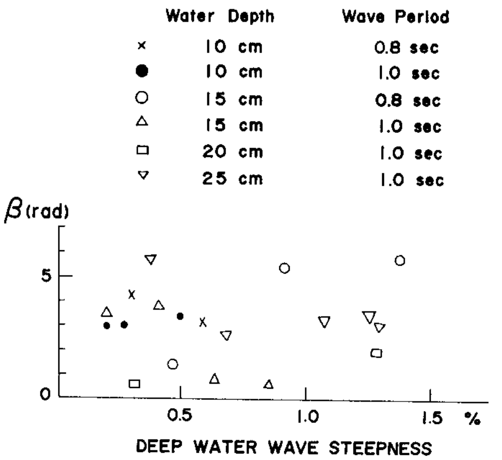

Fig. 11. Phase difference between incident and reflected waves at the toe of the beach measured for large-scale waves in 10 m wave tank for various conditions.

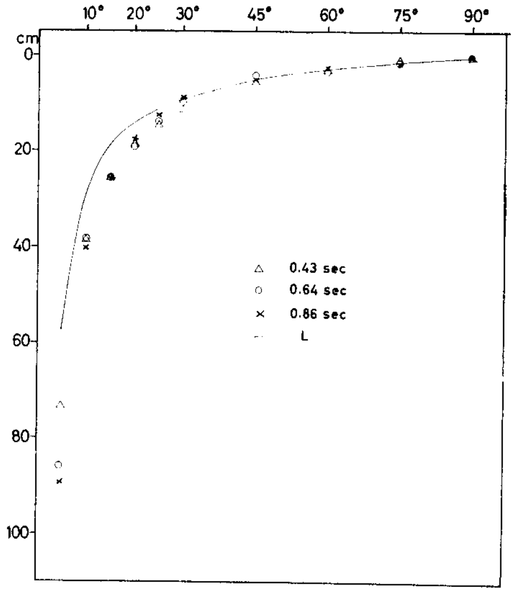

Fig. 13. Effective beach length $l=-\beta/2k$ *vs.* beach angle θ. The real beach length L is shown by the solid line.

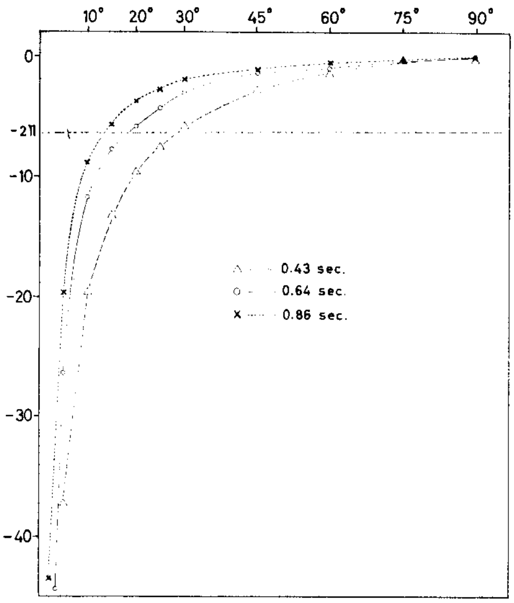

Fig. 12. Phase difference between incident and reflected waves at the toe of the beach *vs.* beach angle for 0.43 sec, 0.64 sec, and 0.86 sec waves.

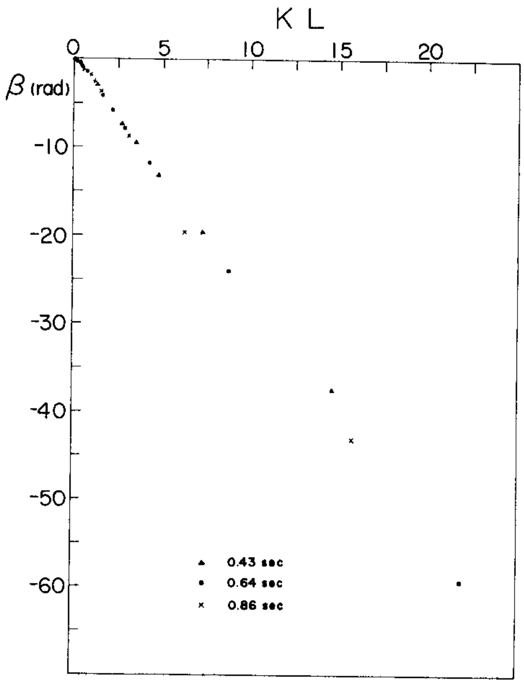

Fig. 14. Phase difference β between the incident and reflected waves at the toe of the beach *vs.* kL.

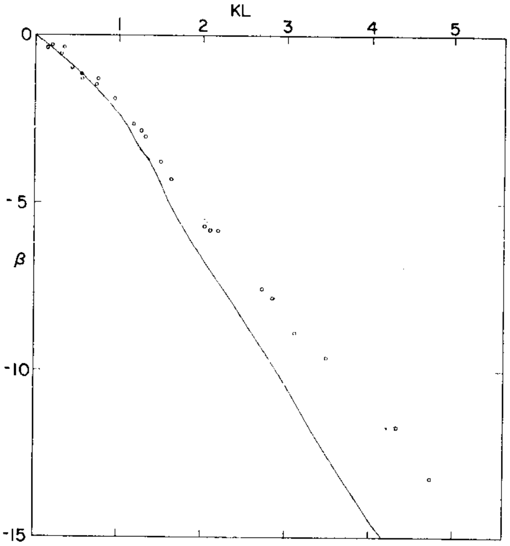

Fig. 15. Calculated phase difference from a simple model *vs.* kL (solid line). For small values of kL, the theoretical curve agrees with the experimental values.

Then β must have the following form;

$$\beta=\tan^{-1}\left\{-\frac{2\dfrac{J_1(2\sqrt{qL})}{J_0(2\sqrt{qL})}}{1-\dfrac{J_1(2\sqrt{qL})}{J_0(2\sqrt{qL})}}\right\}=\tan^{-1}\left\{-\frac{2\dfrac{J_1(2kL)}{J_0(2kL)}}{1-\dfrac{J_1(2kL)}{J_0(2kL)}}\right\} \tag{10}$$

The phase difference calculated from this equation is shown by a solid line in Fig. 15. It agrees with the observed phase difference for small values of kL. The theory gives higher values of $|\beta|$ for large values of kL. The long wave assumption is not satisfied in the experiment; the wave length in the experiment is shorter than the assumed long waves for given water depth as follows: 22% for 0.43 sec waves, 9% for 0.64 sec waves and 5% for 0.86 sec waves. This is one explanation of the discrepancy between theoretical and experimental values of $|\beta|$ for larger values of kL. The large energy loss due to the breaking may cause secondary effects on the phase difference for extremely large values of kL. It is very interesting to note that the experimental values of $|\beta|$ are approximately proportional to kL for a very wide range of kL, though the results are not as reliable for large values of $|\beta|$ because of the arbitrary selection of n in (4).

5. Discussion

We investigated the wave reflection by a uniformly sloping beach in a small wave tank. Experimental wave height was small, and the breaking of the waves at the shore is different from that on the natural coast or in the experiments using a large tank. In a small tank, the waves lose their energy by generating capillary waves of several millimeters in length at their wave fronts near the shoreline. However, the change of the reflection coefficient to the incident wave steepness and to the beach slope is almost the same as that in large-scale experiments. We also cannot find the significant difference of the critical wave steepness for wave breaking between small-scale and large-scale experiments. The phase difference between incident and reflected waves at the toe of the sloping beach is mainly governed by the ratio of wave length to beach length and is not affected by wave breaking.

These results suggest that model experiments can be conducted in a small tank using small-scale waves, if the phenomena under investigation is mainly related to energy loss in breaking and phase shift at the beach. As the experimental conditions are controlled easily and economically in small wave tanks, further investigations of the possibilities and limitations of small-scale experiments would be worth-while.

High frequency motion of water surfaces such as capillary waves cannot be measured by resistance type wave gauges. Therefore, only qualitative discussions of the energy losses by the generation of capillary waves are possible in this paper. The quantitative studies remain for future investigation.

Acknowledgements

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斜面による波の反射の実験的研究

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要旨 長さ 4 m の小型水槽を用いて斜面による波の反射を研究した．波長数 10 cm，波高数 mm の小さな波の砕波は，海岸で観測される砕波やスケールの大きい波を用いる水槽での砕波と異なる．小型水槽では汀線近くで波の前面に波長数 mm の表面張力波が発生して波のエネルギーが消滅する．しかし波の反射率の斜面傾斜，波形勾配に対する依存は Miche の理論と一致する．砕波が生じなくなる臨界の斜面傾斜より大きい傾斜の範囲でも，小さなスケールの波は，砕波は生じないが多量のエネルギーを失う．このエネルギーロスは，汀線附近で発生して沖側に発散する表面張力波によってもたらされているように思われる．

斜面の toe における入射波と反射波との位相の差も調べた．位相差は波形勾配にはよらず，波長と斜面の長さとの比によってきまる．波数 k，斜面の長さ L とすると，実験結果は kL の広い範囲で，位相差が kL に比例することを示す．簡単なモデルについて計算した位相差は，kL の小さい範囲で実験値と一致した．