A Measurement of Visco-Elastic Constants of Sea Ice*

By

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Introduction

Sea ice consists of minor pure ice crystals that surround brine-filled cells. By the reason of this, marked differences exist in mechanical properties between fresh-water ice and sea ice. It is well known that the fresh-water ice of 3–4 cm thickness is strong enough to skate upon whilst it is barely permissible to walk upon sea ice of 10 cm thickness.

A number of measurements of visco-elastic constants of sea ice have been carried out at several places on the Okhotsk sea coast of Hokkaido; *viz.*, at Abashiri and Lake Notoro (brackish water) in 1948, at Lake Tofutsu (brackish) in 1950 and at Monbetsu in 1955. The principle of measurement was quite similar to the bending method which is frequently applied to obtain Young's modulus of solid specimen. An ice bar was deflected under static loading, elastic constant was obtained from observing the amount of deflection and viscosity was derived from the strain-time curve.

Method of Measurement

Since the brine may bleed out from the ice samples which are raised into the air from the surface of the sea, if was necessary to make the measurements in the natural state of ice cover. The scheme of measurement is shown in Fig. 1–a and b.

A rectangular ice bar was sawed out as shown in Fig. 1–a, b; the bar is fixed at one or both the two ends. In the case of Fig. 1–a, the free end *A* was loaded and its

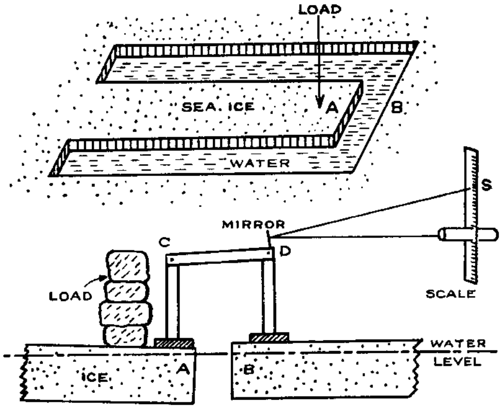

Fig. 1-a. Schematic diagram of the method of measurement.

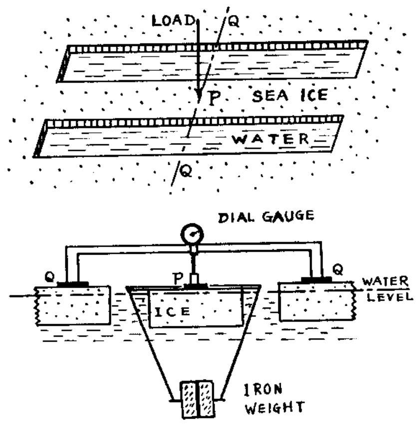

Fig. 1-b. Schematic diagram of the method of measurement.

sinking was read in relation to *B*. The depression of the ice bar was measured by the method of optical lever and scaled-telescope. In the case of Fig. 1-b, the weight was loaded at the centre *P*, and the amount of bending was read by dial gauge in relation to *Q*. The method of optical lever was used before 1955. Vibration of bar caused by swell or wind ripples of the

* Contribution No. 282 from the Institute of Low Temperature Science.

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surrounding water prevented the obtainment of precise readings of deflection.

Results

A. Figures 2-a and 2-b show an example of strain-time curve of ice bar. The ordinate denotes the depression of bar and abscissa the elapsed time. The results of measurements which were made by the method of Fig. 1-a are shown in Fig. 2-a. A certain weight W_1 for which an ice block was substituted was loaded and the depression at A was measured in the range of from 1 to 2 minutes. Successive loading (W_2, W_3, ……) was continued accompanied by the measurements of depression of bar. When unloading, reverse operations were repeated. The dimensions of ice bar were: thickness 15-40 cm, length 1.5-3 m and the width 10-80 cm.

It may easily be seen from Fig. 2-a that the elastic deformation took place immediately after loading or unloading and followed the creep of the bar. In this

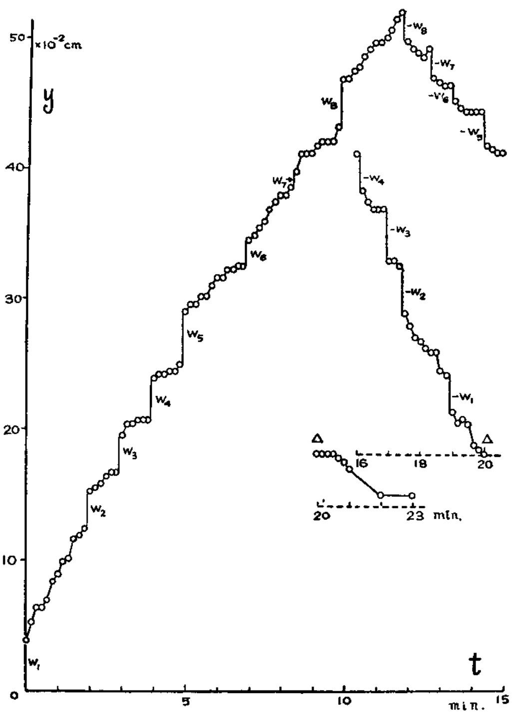

Fig. 2-a. Strain-time curve of Sea-ice (Successive loading)

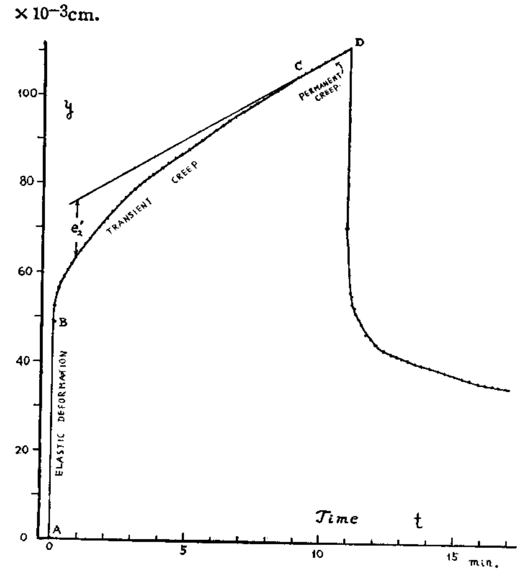

Fig. 2-b. Strain-time curve of Sea-ice (Single weight loaded)

measurement, after the settlement of creep the next weight was loaded or unloaded. Accordingly each deformation of bar caused by the successive loading or unloading may be treated as an independent one. If it be assumed that the elastic constant of ice remains unaffected in the course of a series of loading and unloading, the same elastic deformation may be observed. Thus the same elastic deformation could be observed under loading and unloading of the same weight. Averaging those two deformations, the elastic constant E is given by

$$E=\frac{4pl^3}{yd^3b}, \qquad (1)$$

where p is the weight of load, l the length of ice bar, d the thickness, b the width and y the elastic deformation (depression). However, it is necessary to make a correction of bending moment due to the increase of buoyancy caused by the sinking of the ice bar into the water.

It is easily be seem from Fig. 2-a that the transient creep is clearly observable in comparison with the permanent creep. Within the limit of accuracy of the present measurements, it may be said without doubt that the creep increased linearly with the lapse of time. Therefore, the viscosity η may be expressed by

$$\eta=\frac{4l^3}{y'd^3b}pt \qquad (2)$$

where t is the duration of loading and y' is the amount of creep in t.

B. Fig. 2-b represents an example of results obtained in 1955 by the method shown in Fig. 1-b. Instead of using successive loadings, a strain-time curve was obtained by loading of a single weight, because an objectional point of the method shown in Fig. 1-a had already been indicaten by the present writers (Fukutomi, Kusunoki and Tabata 1954).

Inspecting the strain-time curve shown in Fig. 2-b, AB and CD may be taken as straight lines. e'_2 may be expressed by logarithmic curve with respect to time t. In applying the dynamical model of the theory of visco-elasticity, the observed strain-time corve of ice bar may have the same nature as that constructed by a series connection of Maxwell unit and Voigt unit (Fig. 3).

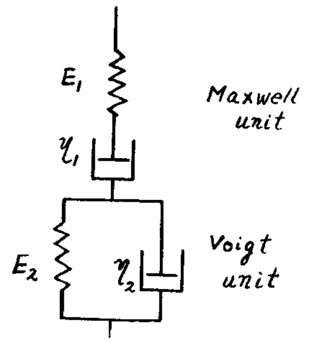

Fig. 3. Model of visco-elasticity of sea ice.

As is shown in Fig. 3, elastic constants of springs and viscosity of dashpots of the model are taken as E_1, E_2 and η_1, η_2 respectively, and the following expessions are taken:

e_1=strain exerted upon the spring of the Maxwell unit,

$e_2(t)$=strain exerted upon the dashpot of the Maxwell unit,

$e'_2(t)$=strain exerted upon the Voigt unit.

Then the following equations are obtained.

$$\begin{cases} E_1e_1=p' \\ \eta_1\dfrac{de_2}{dt}=p' \\ E_2e_2'+\eta_2\dfrac{de_2'}{dt}=p' \end{cases} \qquad (3)$$

where $p'=pl^3/4d^3b$, p is the weight of load, l the length of ice bar, b the width and d the thickness. Assuming p=constant, and taking the next initial conditions

$$t=0\,;\ e_2=e_2'=0 \qquad (4)$$

the solutions are expressed by solving the the equations (3) under condition (4).

$$\begin{cases} e_1=\dfrac{p'}{E_1} \\ e_2=\dfrac{p'}{\eta_1}t \\ e_2'=\dfrac{p'}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right) \end{cases}$$

As stated above, the ice bar gains buoyancy caused by the sinking into the water. Thus a decrease of bending moment takes place due to the loading and, strictly speaking, the moment decreases with the depression of bar. In the present measurement the amount of depression was very small; the required correction will be assumed to be constant within the range of the error of measurement. Then the total strain of whole system shown in Fig. 3 is expressed by

$$e=e_1+e_2+e_2'$$

Accordingly,

$$e=\frac{p'}{E_1}+\frac{p'}{E_1}\frac{t}{\tau_1}+\frac{p'}{E_2}\left(1-e^{-\frac{t}{\tau_2}}\right)$$

where $\tau_1=\eta_1/E_1$, $\tau_2=\eta_2/E_2$; τ_1 and τ_2 are relaxation time and retardation time respectively.

C. The results are made into tabular form of Table 1 and Table 2. The figures in Table 1 were obtained in the winters of 1948 and 1950, while those in Table 2 in the winter of 1955. Successive loading was made in case of Table 1, so the several values of eleastic constant are computed from each single loading. But only the maximum and the minimum values are shown. In Table 2, visco-eleastic constants of ice as well as relaxation and retardation

Table 1. Elastic constant and visosity of sea ice
(Successive loading)

No.	Elastic constant $E(\times 10^{10}$ dyne/cm$^2)$	Viscosity $\eta(\times 10^{10}$ dyne · min/cm$^2)$	Density of ice	Chlorinity of ice (‰)
1	5.6-1.0		0.91	0.97
2	2.0-0.4		0.89	2.51
2′	2.2-0.4			
3	2.6-1.0	1.1-4.2	0.90	0.54
4	2.2-1.0	0.5-1.6		
5	2.1-1.1	1.2-4.2		
6	2.7-2.3	1.2-1.5	0.92	
7	3.5-1.9	1.6-2.6		
8	2.9-1.4	1.4-3.2	0.92	0.38
9	1.5-1.0	0.6-2.6		
10	5.8-1.9	1.0-3.2	0.89	0.26
11	2.6-2.4	0.3-1.5		
12	4.0-1.9	1.9-4.0		

Table 2. Elastic constant and viscosity of sea ice
(Single weight)

No.	E_1 (dyne/cm^2)	η_1 (dyne min/cm^2)	E_2 (dyne/cm^2)	η_2 (dyne min/cm^2)	τ_1 (min.)	τ_2 (min.)
1	4.8×10^{10}	9.1×10^{11}	8.9×10^{10}	4.3×10^{11}	19	3.6
2	$9.0\times$ 〃	$12.2\times$ 〃	10.3×10^{11}	$3.5\times$ 〃	14	3.4
3	$2.4\times$ 〃	$9.2\times$ 〃	11.1×10^{10}	$1.0\times$ 〃	39	0.9
4	$2.5\times$ 〃	$4.8\times$ 〃	$7.5\times$ 〃	$1.8\times$ 〃	19	2.4
5	$3.2\times$ 〃	$12.0\times$ 〃	$18.2\times$ 〃	$1.2\times$ 〃	37	0.7

time are compulted from the basic idea that the ice is represented by a model of a serial connection of Maxwell unit and Voigt unit.

Elastic constants shown in Table 1 are those corresponding to E_1 of Table 2. The values of viscosity in Table 1 have equivalent meanings to η_2 of Table 2, for *CD* of Fig. 2-b was assumed as a straight line.

It will be recognized that E_1 and E_2 of sea ice have the same order of magnitude. It will easily be expected that the elastic constant of sea ice will depend upon the temperature, salinity, thermal history and the conditions when the ice was formed. In the present study, the temperature changed within a narrow range and there not being a sufficient number of measurements, the dependencies on those factors are not clearly known. When the ice temperature was low and one felt resistance in sawing the ice, a large value of the elastic constant was obtained as No. 2 of Table 2. The η_1 is larger than η_2 in its order of magnitude. The former is larger in its one order. As the same order of magnitude of visco-elastic constants was obtained from ice which differed in its period of formation and in its location, it may safely be said that E_1 (or E) and η_2 (or η) of sea ice are 10^{10} dyne/cm^2 and $10^{10}\sim10^{11}$ dyne-min/cm^2 respectively.

For reference, it may be added that Weinberg (Zubov 1945) summarized the elastic constant of fresh-water ice as $5.9\times 10^9\sim1.8\times10^{11}$ dyne/cm^2 and the values obtained by vibration method or explosion method are $4.9\sim9.4\times10^{10}$ dyne/cm^2. For deposited snow, Kojima (1954) obtained 10^7 dyne/cm^2 for elastic constant and 10^7 dyne-min/cm^2 for viscosity.

Discussion

In the present study, the applied weight was under the static condition, thus the strain would not be uniformely distributed in the ice bar, but especially stressed at the fixed end. Accordingly, the elongated part and contracted part would take place inner part of ice bar. In order to overcome these imperfection, an improved method which will yield uniform distribution of strain in the ice is desirable, and moreover, dynamical method would be preferable.

Standard fixed point was taken, as shown by B and Q in Fig. 1-a, -b, on the same ice cover. Vertical displacement at a fixed point will be examined in the future.

At present, the measurement of visco-elastic constants of sea ice *in situ* is described with some other critical analyses. However, the order of magnitude of visco-elastic constants will be obtained from the present paper.

Acknowledgment: This study was partly supported by the Scientific Research Fund of the Ministry of Education.

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