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EMPIRICAL AND THEORETICAL RELATIONS BETWEEN WIND, SEA, AND SWELL

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Prior to the War there existed a wide gap between the theoretical knowledge of surface waves and the empirical knowledge of sea and swell, sea being defined as waves raised by the direct action of the wind, swell as waves that have emerged from a storm area and travel through regions of calm or weak winds. A few theoretical conclusions have been verified by experience. Thus, it had been established that the wind waves have, in general, the character of deep water waves for which speed, period, or length are interrelated. The speed is proportional to the period and proportional to the square root of the length:

$$C = L/T = (g/2\pi)T = \sqrt{(g/2\pi)L}$$

where C is wave speed, L wave length, T period, and g acceleration of gravity.

On the basis of these results it is safe to assume that a number of the other characteristics required by the theory apply, for instance, that the average energy per wave length is proportional to the square of the wave height (vertical distance from crest to trough), that only the wave form advances, and that the water particles move nearly in circles, the radii of which decrease rapidly with depth.

The knowledge of the generation of waves and of the advance of swell was, however, very inadequate, particularly for the purpose of forecasting sea and swell on the basis of weather maps from ocean areas. For this purpose it is necessary to know the relations between wind and sea, and the manner in which swell advances and decays.

One might believe that after a century of observations, empirical laws had been established which would answer these practical purposes, but such is not the case. Anyone who has examined

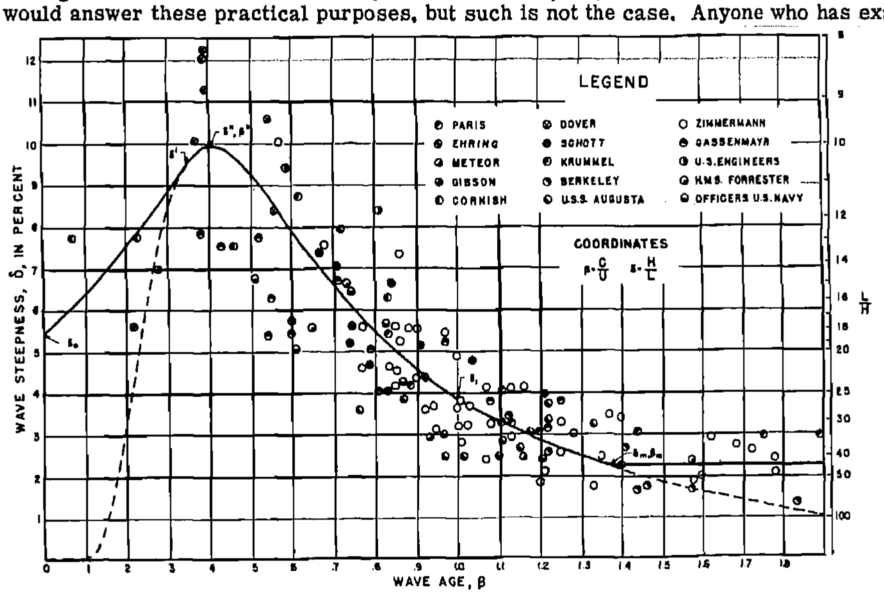

Fig. 1--Wave steepness plotted against the ratio of wave velocity to wind velocity

the results of statistical compilations knows that these are incomplete and often contradictory. Thus, some authors claim that the speed of the waves never exceeds the speed of the wind that generates them; whereas others present evidence to show that the waves can travel with a speed which exceeds the wind speed up to 50 per cent. Similarly, some authors find that the wave height is proportional to the wind speed, others that it increases with the square of the wind speed, and some authors find that the steepness of the waves, as expressed by the ratio between wave height and wave length, is independent of the wind speed, others find that steepness decreases with increasing wind speed, and others again that it increases.

There are two reasons for these discrepancies. In the first place, the sea does not depend upon the wind speed only, but it also depends upon the dimensions of the water surface over which the wind is blowing, the fetch of the wind, or upon the length of time the wind has been blowing, the duration of the wind. In the second place, the wind never raises a uniform sea, that is, it does not generate a single train of well-defined waves which all have similar heights and periods. To the contrary, the wind generates a spectrum of waves ranging from ripples to large billowing seas. Furthermore, these waves travel in different directions at small angles with the prevailing wind, and the consequent crisscrossing leads to a checkerboard pattern of crests and troughs which is constantly changing. The sea is therefore always confused and it is extremely difficult to describe it by means of two single parameters, such as wave height and wave period.

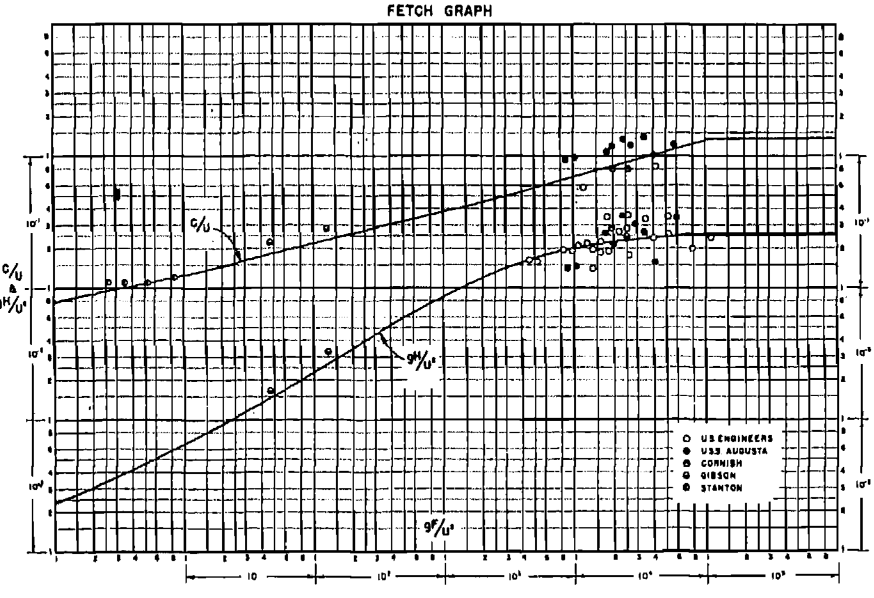

Fig. 2--Parameters C/U and gH/U^2 plotted against gF/U^2

Actually, the sea should be described by means of adequate statistical terms, but this has not been done in the past, and the true meaning of numerous observations is therefore obscure. In order to obtain consistent observations of the sea, it is now proposed to introduce the concept "significant waves," and to define these as waves having the average height and period of the one-third higher waves. This definition is not exact, because it depends upon the extent to which small waves have been recorded. It is suggested, therefore, that ripples and small waves of heights less than one ft should be disregarded.

Exact heights and periods of the significant waves can be obtained by laborious evaluations of continuous records, but for practical purposes it is necessary to make short-cuts. Experience so far indicates that an observer who attempts to determine the characteristics of the higher waves present tends to record values which approximate those of the significant waves as defined above. One of the goals of future work should be to establish methods of observation by which comparable results could be obtained by different observers in different localities.

When dealing with earlier observations, we have to assume that these apply to the higher waves present and can be considered applicable to the significant waves. However, a considerable scatter of the values may be expected, because the observations have not been made by following a standard procedure. The empirical data which are available for the establishment of rational relations between wind and sea are therefore of poor quality, but it will be shown that in spite of this they can be fitted into a consistent framework.

A consistent framework can be established by considering the manner in which waves grow by energy being transferred to the waves from the wind. This transfer takes place by two processes; by the effect of <u>normal pressures</u>, and by the effect of the <u>tangential stress</u> which the wind exerts on the sea surface. The effect of normal pressures was first examined by Jeffreys. Jeffreys points out that when the wind blows over waves that travel at a speed less than the wind speed, there will be exerted a surplus pressure against the upwind slope of the crest, whereas a deficit in pressure develops on the downwind slope. Under these conditions energy is transferred from the wind to the wave; but if the wave travels faster than the wind, the wave meets air resistance and loses energy. Jeffreys believed that energy transfer takes place by this process only, and reasoned therefore that the waves could not travel faster than the wind. However, energy is also transferred by the tangential stress (the drag) which the wind exerts on the sea surface. The effect of the drag is to speed up the motion of the particles at the wave crest and to slow down the motion of the particles at the trough, but the speed-up is greater than the slow-down, so that a net increase in energy results even if the wave moves with greater speed than the wind.

In this connection it should be remembered that only the wave form advances with a great speed; the water particles move nearly in circles and advance very slowly in the direction of wave travel.

When applying the above considerations to the growth of waves, two empirical results must be considered: (1) If the wind blows over a limited fetch, a steady state will be reached, that is, when the wind has blown for a sufficient length of time the significant waves will no longer increase in height or period anywhere in the fetch. (2) When the wind blows over an unlimited fetch, the waves will grow in height and period at the same rate in all localities.

DURATION GRAPH

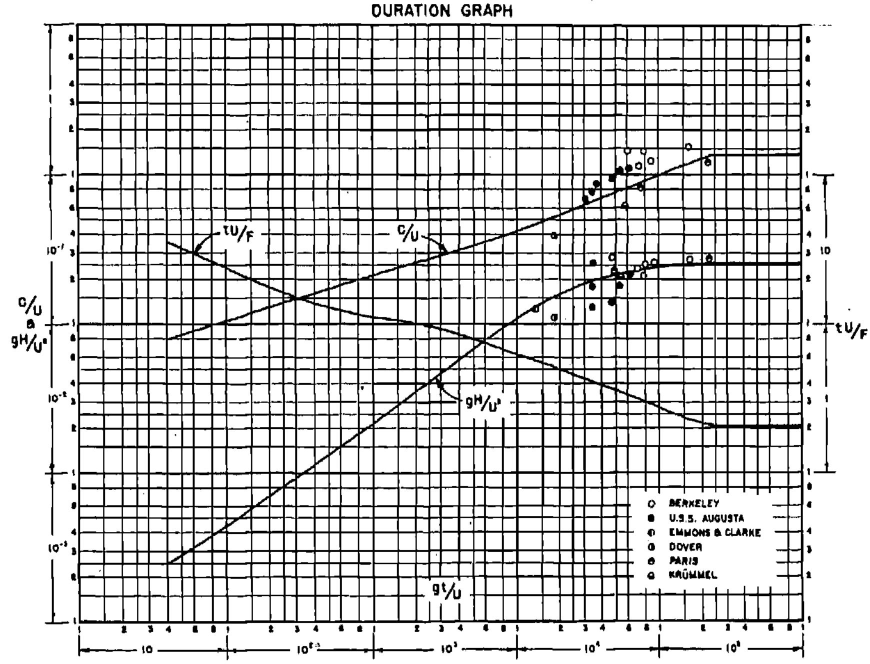

Fig. 3--Wave height and speed as functions of duration, t, and wind speed, assuming unlimited fetch

These considerations, together with knowledge of the manner in which energy advances with the wave form, lead to the establishment of two differential equations which apply to the two cases stated above, provided that the wind blows with a steady speed, U.

Steady state:

$$(C/2)(dE/dx) + (E/2)(dC/dx) = R_T \pm R_N \left\} \begin{matrix} + & C < U \\ - & C > U \end{matrix} \right. \quad \ldots\ldots\ldots\ldots\ (1)$$

Transient state:

$$dE/dt + (E/C)(dC/dt) = R_T \pm R_N \left\} \begin{matrix} + & C < U \\ - & C > U \end{matrix} \right. \quad \ldots\ldots\ldots\ldots\ (2)$$

where E is the average energy per wave length, which is proportional to H^2 (H is wave height), and R_T and R_N are the rates at which energy is transferred by the effects of tangential stress and normal pressure. Each of these equations contains two unknown variables, energy and wave speed, but energy depends upon wave height only, and the unknowns can therefore be considered as wave height and wave speed. In order to solve them, a third equation must be established, and for this purpose one has to turn to the empirical data. Certain considerations suggest that a relationship exists between wave steepness, as defined by the ratio H/L, and wave age, as defined by the ratio C/U. These two quantities are plotted against each other in Figure 1, which contains all the observations available to the authors (128 sets). The observations show a considerable scatter which at least in part can be attributed to the inconsistent manner in which they have been made, but they also show an unmistakable relationship which can be well represented by the solid curve in the graph. This curve has not been drawn simply to fit the observations, but is derived theoretically after having selected certain numerical constants to obtain a fit.

The relationship between wave steepness and wave age, which has been established in this manner, can be used in solving equations (1) and (2). The solutions can all be presented as relations between non-dimensional ratios and can be compared with observed values.

Figure 2 shows the parameters C/U and gH/U^2 plotted against gF/U^2 (F is fetch), that is, it shows wave height and wave speed as functions of fetch and wind speed, assuming unlimited duration. The range is so large that 4 × 5 cycle logarithmic paper is used. The curves represent the theoretical results, and the dots represent observations. The general agreement is satisfactory over a wide range, particularly since the observed values are of poor quality. Figure 3 shows in a similar manner the wave height and speed as functions of duration, t, and wind speed, assuming unlimited fetch, and again the agreement with observations is adequate.

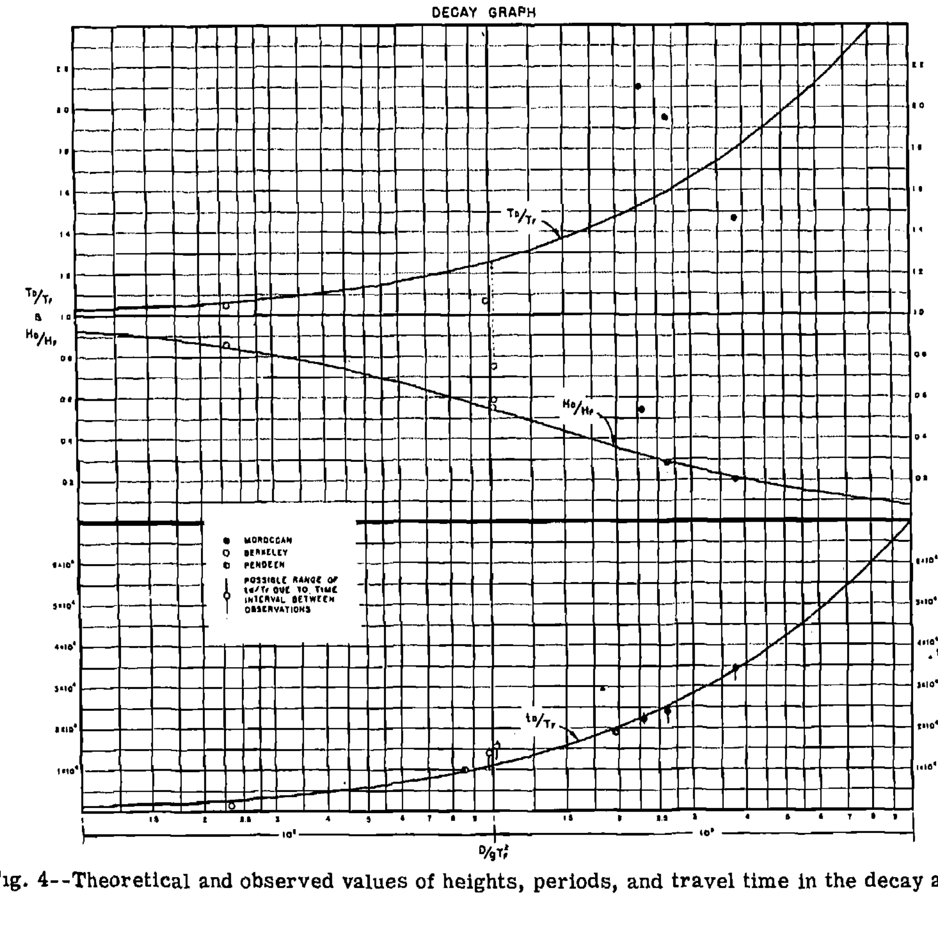

Fig. 4--Theoretical and observed values of heights, periods, and travel time in the decay area

Figure 3 contains a third curve from which one can obtain the duration needed for establishing a steady state over a given fetch. If the observed value of the parameter tU/F falls below the curve marked tU/F, a steady state has not been reached and corresponding values of H and C must be obtained from the duration graph, but if the observed value falls above the curve, a steady state has been established and values of H and C must be taken from the fetch graph.

Turning to the swell, it can first be stated that the swell is also irregular and must be described by the "significant waves." When swell advances through regions of calm it loses energy because it meets an air resistance, but no effect of tangential stress need be considered. The behavior of swell can be examined without introducing any new assumptions or constants. One arrives at the conclusion that the period increases and the wave height decreases with increasing distance from the end of the fetch. This distance is called the distance of decay, D. The ratio between period at the end of the distance of decay and at the end of the fetch, T_D/T_F, and the ratio between the corresponding wave heights, H_D/H_F, as well as the ratio between travel time and period at the end of the fetch, t_D/T_F, can all be represented as functions of the same parameter, D/gT_F^2.

Figure 4 shows the theoretical curves and the few observed values which are available for comparison. The observed heights and travel times of swell agree well with the theory, but the changes in wave period show a wide scatter. It should be noted, however, that the ratio between the periods is a ratio between two quantities which are difficult to establish, and, furthermore, that the assumed period increase enters in computing the travel time, for which reason the good agreement between observed and computed travel time confirms the conclusions as to increase in period. It should be added that actually only the values of D, T_D, H_D, and t_D are observed, and that the values of H_F and T_F have been derived from weather maps by means of Figures 2 or 3.

The relations which have been discussed here are not presented in a manner which is convenient for practical purposes, but they can all be combined in a nomogram or an alignment diagram by means of which the significant waves in the generating area and the character of the swell arriving at a distant coast can be obtained in a few steps, if wind velocity, fetch, and duration have been derived from weather maps.

The relationships shown here have been used with considerable success during the War for forecasting sea, swell, and surf, but the establishment of these relationships represents only a first step. It is hoped that attempts will be made to obtain systematic observations of sea and swell, that the reason for the scatter of the observed values will be studied, and that the forecasting technique will be improved. It is also hoped that future theoretical work will clarify several steps in the theory leading to the relationships shown in the figures, and will bring an understanding of the physical contents of assumptions which now appear as mathematical manipulations [see "Reference" below, SVERDRUP and MUNK, 1946].

Reference

SVERDRUP, H. U., and MUNK, W. H., Wind, sea, and swell; theory of relations for forecasting, U. S. Hydrographic Office, Technical Bulletin in Oceanography, no. 1, in press.

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