# Surface waves affect frontogenesis

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release (LA-UR-15-29048)



This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1002/2015JC011563

© 2015 American Geophysical Union Received: Dec 15, 2015; Revised: Apr 22, 2016; Accepted: May 02, 2016

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Key Points.

- Overturning circulation of a submesoscale, down-Stokes front is energized by Stokes shear force.
- Overturning circulation extracts momentum and kinetic energy from surrounding submesoscale eddies.
- Stokes shear force appears alongside buoyancy and modifies the hydrostatic balance.

Abstract. This paper provides a detailed analysis of momentum, angular momentum, vorticity, and energy budgets of a submesoscale front undergoing frontogenesis driven by an upper-ocean, submesoscale eddy field in a Large Eddy Simulation (LES). The LES solves the wave-averaged, or Craik-Leibovich, equations in order to account for the Stokes forces that result from interactions between nonbreaking surface waves and currents, and resolves both submesoscale eddies and boundary layer turbulence down to  $4.9\,\mathrm{m}$  imes $4.9\,\mathrm{m} \times 1.25\,\mathrm{m}$  grid scales. It is found that submesoscale frontogenesis differs from traditional frontogenesis theory due to four effects: Stokes forces, momentum and kinetic energy transfer from submesoscale eddies to frontal secondary circulations, resolved turbulent stresses, and unbalanced torque. In the energy, momentum, angular momentum, and vorticity budgets for the frontal overturning circulation, the Stokes shear force is a leading-order contributor, typically either the second or third largest source of frontal overturning. These effects violate hydrostatic and thermal wind balances during submesoscale frontogenesis. The effect of the Stokes shear force becomes stronger with increasing alignment of the front and Stokes shear and with

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a nondimensional scaling. The Stokes shear force and momentum transfer from submesoscale eddies significantly energize the frontal secondary circulation along with the buoyancy.

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# 1. Introduction

Fronts with ageostrophic secondary circulations, or *frontal overturning circulations*, and wave-driven Langmuir turbulence both have significant horizontal divergence and are expected to impact transport and dispersion of chemical substances in the upper ocean, such as oil, pollutants, biological tracers, and flotsam. This paper quantifies a commonality between fronts and Langmuir turbulence: they are both significantly energized by surface gravity waves. Fronts are ubiquitous across the global ocean [Ferrari and Rudnick, 2000; Capet et al., 2008a] and stem from frontogenesis mechanisms similar to those proposed by Hoskins and Bretherton [1972]. Their solutions for the frontal overturning circulation are found by solving the equation proposed by Sawyer [1956] and Eliassen [1962]. The dynamics of oceanic fronts [Capet et al., 2008a] affect and are affected by air-sea interactions [Boutin et al., 2008], winds [Thomas and Lee, 2005], submesoscale instabilities [Spall, 1997; Boccaletti et al., 2007; Taylor and Ferrari, 2009, 2010; Nencioli et al., 2013; Haney et al., 2015], boundary layer or convective turbulence [Parker and Thorpe, 1995; Nagai et al., 2006, 2009; McWilliams et al., 2009; D'Asaro et al., 2011], phytoplankton productivity [Mahadevan and Archer, 2000; Lima et al., 2002], and surface gravity waves [McWilliams and Fox-Kemper, 2013]. In the upper ocean, fronts vary in width from 100 m to 100 km [Pollard and Regier, 1992; Hosegood et al., 2006; Nencioli et al., 2013; Shcherbina et al., 2014, and they typically originate from straining of density gradients by mesoscale or submesoscale eddies. The primary emphasis of this paper is on studying what energizes and torques submesoscale frontal overturning circulation in the presence of surface gravity waves. We will address these questions by analyzing a Large Eddy Sim-

ulation (LES) dataset [*Hamlington et al.*, 2014] of upper ocean processes which resolves straining of fronts by submesoscale eddies.

Surface gravity waves are also ubiquitous in the ocean. Their short wavelengths (0.1 m <  $\lambda < 100$  m) and fast periods (1-10 s) require casting their interactions with larger, slower phenomena – such as large-scale boundary layer turbulence or longshore currents – in a multi-scale framework: namely, the wave-averaged equations [*Craik and Leibovich*, 1976; *Holm*, 1996; *McWilliams et al.*, 1997; *McWilliams et al.*, 2004]. The LES analyzed in this paper solves these equations. For simplicity, the wave-averaged equations neglect intermittent breaking wave effects (although similar models have considered these effects [*Sullivan et al.*, 2007]) and account for the effects of nonbreaking gravity waves through a Stokes drift velocity that decays rapidly from the surface. The wave interaction forces acting on larger, slower flows in the wave-averaged equations can be organized into three categories: 1) the advection of momentum and tracers by the Stokes drift, 2) the Stokes Coriolis force, and 3) the Stokes shear force [*Suzuki and Fox-Kemper*, 2015]. These forces are referred to as Stokes forces throughout.

Stokes forces in the wave-averaged equations are equivalent to the radiation stresses in other forms of the equations [Longuet-Higgins and Stewart, 1964; Lane et al., 2007], but in the context of this study Stokes forces are easier to understand dynamically. Stokes forces exist within only a shallow surface region of the mixed layer or turbulent boundary layer because Stokes drift rapidly decays with depth, with an e-folding depth of  $\lambda/4\pi$ , which is usually less than 10 m. However, momentum and energy injected by Stokes forces are transported outside of this shallow region via inertia, pressure perturbations, and stresses. Hence, Stokes forces can lead to circulation effects that are much larger

and deeper than the shallow surface region where Stokes drift itself is large [Polton and Belcher, 2007; McWilliams and Fox-Kemper, 2013]. Stokes effects also have a particularly sensitive dependence on the direction of flow structures as compared to the wave direction [Van Roekel et al., 2012; Suzuki and Fox-Kemper, 2015].

Langmuir turbulence present in the ocean boundary layer is partly driven by the Stokes shear force [Suzuki and Fox-Kemper, 2015] and involves Langmuir circulations. Typically, Langmuir circulations are 1 m-50 m deep and wide and 100 m to 1 km long, and their axial directions are aligned with the wind and waves [Craik and Leibovich, 1976; Thorpe, 2004; Teixeira and Belcher, 2010]. Ocean mixing by Langmuir turbulence has important climatic effects [Kukulka et al., 2009; Fan and Griffies, 2014; Li et al., 2015]. LES of the wave-averaged equations allow for systematic study of Langmuir turbulence and its dependence on parameters defining the upper ocean state, such as wind strength and direction. [Skyllingstad and Denbo, 1995; McWilliams et al., 1997; Harcourt and D'Asaro, 2008; Van Roekel et al., 2012]. An important feature of the LES analyzed in this paper, as discussed in Hamlington et al. [2014], is that nearly all small-scale motions due to Langmuir turbulence are explicitly resolved by the simulations, rather than parameterized using a subgrid-scale model. Therefore, this LES simulates natural development of both Langmuir turbulence and submesoscale eddies.

The LES in this study neglects intermittent breaking wave effects. It is theoretically unclear, however, to what extent the wave-averaged equations must be modified in order to account for breaking waves. Moreover, the extent of such modifications may depend on the types of breaking (e.g., microscale versus whitecap breaking waves) as well as the frequency of wave breaking. As observational data of breaking waves in the ocean is not well

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constrained [*Kleiss and Melville*, 2010, 2011], speculating on the importance of breaking waves relative to nonbreaking waves is difficult. However, as Langmuir circulations and, hence, Stokes forces are common features present for modest winds similar to the winds considered in this study, we expect that the wave-averaged equations and the effects of Stokes forces highlighted here are largely valid.

The Stokes Coriolis force and additional related advection of momentum and tracers by Stokes drift may have profound effects on coastal circulation, upper ocean fronts, internal waves, and instabilities [e.g., *Olbers and Herterich*, 1979; *McWilliams and Restrepo*, 1999; *Monismith et al.*, 2007; *McWilliams et al.*, 2012; *Lentz and Fewings*, 2012; *Breivik et al.*, 2015; *Haney et al.*, 2015]. These effects are present in the simulations and theory presented here, and on occasion they are significant in the energetics and angular momentum balance of the frontal overturning circulation.

McWilliams and Fox-Kemper [2013] explore the effect of Stokes forcing on features much larger than Langmuir cells: namely, fronts, filaments, and the associated geostrophic currents. Of particular relevance here is the effect of the Stokes shear force on those currents [Suzuki and Fox-Kemper, 2015]. In McWilliams and Fox-Kemper [2013], inviscid, adiabatic dynamics were assumed for ease of analysis. By contrast, the dynamics in this paper simultaneously include turbulent stresses, diabatic mixing, and Stokes forces.

The most intense upper ocean geostrophic currents – fronts and filaments – result from buoyancy anomalies strained by a confluent flow. Buoyancy anomalies are related to shear in the currents through the thermal wind relation. The effect of waves – or, more precisely, the Stokes shear force – in a front can be compared to the buoyancy anomaly; that is, the Stokes shear force acting on a thermal wind current scales as the buoyancy anomaly

multiplied by the dimensionless parameter [McWilliams and Fox-Kemper, 2013; Suzuki and Fox-Kemper, 2015]

$$\epsilon = \frac{V^S}{f\ell} \frac{H}{H^S} = \frac{V^S}{\underbrace{fH^S}_{\text{wave}}} \frac{H}{\underbrace{\ell}}.$$
(1)

Here  $V^S$  is a characteristic velocity of the Stokes drift, f is the Coriolis parameter, H and  $\ell$  are the depth and width of the front, and  $H^S$  is the Stokes drift decay depth (which is  $\lambda/4\pi$  for monochromatic waves, or 1-10 m for typical conditions). Thus, despite the shallow  $H^S$ , the effects of the Stokes shear force are substantial since the local magnitude of the Stokes shear force relative to the buoyancy anomaly increases inversely with  $H^S$ . In other words, the ratio of the depth-integrated Stokes shear force and depth-integrated buoyancy anomaly forms the dimensionless parameter  $V^S/(f\ell)$ . The Stokes-front interaction parameter ( $\epsilon$ ) can also be expressed as the aspect ratio of the front multiplied by another dimensionless parameter grouping the wave parameters. The latter dimensionless parameter is typically O(10 - 100). The aspect ratio may be estimated by the cross-front isopycnal slope, which is O(1/10 - 1/100) for submesoscale fronts and O(1/100 - 1/1000) for mesoscale fronts. Thus, for given wave parameters, the Stokes shear force effects are strongest on steep submesoscale fronts. The front analyzed in detail here has  $\epsilon = O(20)$ .

As already mentioned, the simulation [Hamlington et al., 2014] analyzed in this paper explicitly resolves both submesoscale eddies and most scales of Langmuir and boundary layer turbulence. In Fig. 1, fronts in two simulations examined in Hamlington et al. [2014] are shown using snapshots of vertical vorticity. These two simulations differ only by their Stokes forcing: one (left panel) has Stokes forcing and the other one (right panel) does not. The former Stokes-forced case is the same simulation analyzed here. Fronts are indicated by adjacent stripes of positive and negative vorticity on the flanks of each front. In the

left panel, fronts that are parallel to the wave (Stokes drift) direction are much stronger than fronts in other directions. In contrast, no clear directionality is preferred in the right panel depicting a simulation with wind forcing only. Thus, some aspect of Stokes forces enhances the fronts in one direction. Preliminary examination of observational data by *Shcherbina et al.* [2013] was inconclusive as to whether similar sharpening occurs in the real ocean.

Frontal enhancement in the Stokes-forced case may represent an indirect coupling between Langmuir turbulence and fronts, or it may result from a direct connection between fronts and Stokes forces. Hamlington et al. [2014] have examined the interactions between Langmuir turbulence, mixed layer eddies [Boccaletti et al., 2007], and fronts in these simulations. However, in addition to the indirect effects of Stokes forces on fronts through the intermediary of Langmuir turbulence, there is a direct effect of Stokes forcing on ocean fronts [McWilliams and Fox-Kemper, 2013]. This direct effect is the dominant reason for the selection of the preferred frontal direction. The interaction between boundary layer turbulence – whether forced by waves, winds, or convection – and fronts is indeed sometimes important and will be quantified here, and further analysis will be carried out to assess the distinction between indirect and direct effects.

The fronts examined here (see again Fig. 1) have sharpened through frontogenesis acting on the strain field of submesoscale mixed layer eddies. These mixed layer eddies have an O(1) Rossby number and evolve faster and hold less energy than mesoscale eddies [Boccaletti et al., 2007; Fox-Kemper et al., 2008]. As will be shown, they transfer an appreciable fraction of their energy to the frontal overturning circulation which alters the submesoscale strain field. To study these changes, it is important to use a simulation

with an active eddy field that can respond as the front extracts energy and evolves, rather than an imposed background eddy field that does not evolve [e.g., *Hoskins and Bretherton*, 1972].

#### 2. Simulation

The simulation in this study is described in detail in *Hamlington et al.* [2014] and Smith et al. [2016]. As a result, here we only briefly summarize the simulation details. The simulation models the spindown of two horizontal buoyancy gradients subject to the effects of winds, waves, and modest cooling. Both gradients are initially O(1 km)-wide. The spindown takes place in the presence of modest initial cooling  $(-5 \text{ W m}^{-2} \text{ for } 10 \text{ days},$ none afterward), modest winds arriving at a  $30^{\circ}$  angle to the initial fronts (10-meter height wind speed  $U_{10} = 5.46 \,\mathrm{m \, s^{-1}}$  and water-side friction velocity  $u^* = 5.46 \times 10^{-3} \,\mathrm{m \, s^{-1}}$ ), and fully-developed waves aligned and consistent with the imposed winds (surface Stokes drift  $V^s = 6.3 \times 10^{-2} \,\mathrm{m \, s^{-1}}$ , turbulent Langmuir number  $\sqrt{u^*/V^s} = 0.29$ , phase speed at the peak wave frequency  $C_p = 6.68 \,\mathrm{m \, s^{-1}}$ , and wave age  $C_p/U_{10} = 1.2$  places the majority of the Stokes drift shear in the upper 4 meters). The prescribed Stokes drift is horizontally uniform and constant in time. Hence, it is assumed that the surface wave field has a large energy reservoir and a ready supply of energy from the winds, so interactions with currents negligibly affect the Stokes drift. Because of the interactions between the front and the Ekman flow, only the initial front that is downwind is unstable to mixed layer eddies [Thomas, 2005].

The LES domain is  $20 \text{ km} \times 20 \text{ km} \times 160 \text{ m}$ , and the simulation explicitly resolves both submesoscale and boundary layer turbulence down to the  $4.9 \text{ m} \times 4.9 \text{ m} \times 1.25 \text{ m}$  computational grid. This domain is much larger than the submesoscale eddies and fronts in the

LES and, thereby, allows natural development of the submesoscale features in a realistic larger-scale flow field. The fine grid explicitly resolves nearly all small-scale motions due to Langmuir turbulence, minimizing the role of the subgrid-scale parameterization. Therefore, the LES simulates realistic multi-scale flow interactions between the submesoscale flow features and both larger-scale and smaller-scale flow features.

#### 3. Selected Front and its Flow Environment

This section characterizes the front analyzed in detail in the following sections and the flow environment in which this front is embedded. After evolving submesoscale mixed layer eddies for just over 12 days (Fig. 1), one submesoscale front oriented in the down-Stokes direction is singled out for detailed analysis; it is shown in the center of the yellow box in the left panel of Fig. 1. To select this front, a number of fronts in different locations and orientations were preliminarily examined. Subsequently, it was found that all of the fronts having a strong frontal overturning circulation are oriented in the down-Stokes direction. There is no up-Stokes front having a clear overturning circulation. The importance of the Stokes shear force relative to the buoyancy depends on the degree of alignment between the front and the Stokes drift shear as well as the strength of the surrounding submesoscale eddies and the buoyancy gradient. For some down-Stokes fronts, the energy input by the Stokes shear force to the overturning circulation is as large as the energy input by buoyancy. For the selected front, however, the relative importance of the Stokes shear force is moderate; hence, it is a typical down-Stokes front. It also has a long stretch of straight front, which makes the along-front averaging used in the following analysis robust.

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The boxed region in Fig. 1 is illustrated in plan and section views in Figs. 2 and 3. These figures also indicate the "front region" (solid box) and the "surrounding region" (dashed box) used for the along-front averaging and flow decomposition detailed in Section 4.3. Figure 2 shows the orientation of the coordinate system used in this study as well as flow anomalies (indicated with a prime) from the horizontal averages. Looking from above, the submesoscale front is at the center of four pressure anomalies p' (Fig. 2a) in a checkerboard pattern; the low pressures are in the positive-x-positive-y and negative-xnegative-y quadrants, and the high pressures are in the positive-x-negative-y and negativex-positive-y quadrants. This arrangement of pressure fields leads to a strong confluence  $(\partial_x u - \partial_y v \gg 0)$  near the center of the front region (solid box). This front is in near alignment with the direction of the Stokes drift, which is 30° above the horizontal axis in these figures (as indicated in Fig. 1). The front has a sharp gradient of buoyancy anomaly b' (Fig. 2b) and strong along-front jets apparent in velocity anomaly in the x-direction, u' (Fig. 2c). The front features a strong overturning circulation, which is visible as the downwelling jet along the front (Fig. 2d) even among the Langmuir downwelling jets that exist throughout the domain. The overturning circulation is also visible in the cross-front velocity anomaly v' that is positive near the surface (filling the solid box in Fig. 2e) and negative near the base of the mixed layer (filling the solid box in the same location in Fig. 2f).

Section views of the front and surrounding regions are shown in Fig. 3. This section lies on the local coordinate y-z plane cutting through the center of the solid and dashed boxes shown in Fig. 2. The variables shown here are smoothed using 10-minute time averaging and also along-front (i.e., x-direction) averaging across the solid box in Fig. 2

(see Section 4.1 for details of the averaging procedure). The buoyancy anomaly of the front is apparent in temperature [Fig. 3a, compare to observations such as *Pollard and Regier*, 1992]. This buoyancy anomaly is associated with several frontal circulation modes (Fig. 3b-e) which are diagnostically defined in the next section.

The along-front velocity anomaly,  $u^H$ , from the surrounding flow shows a 3 cm s<sup>-1</sup> baroclinic shear over the mixed layer depth (Fig. 3b), sensibly co-located with the horizontal temperature gradient (Fig. 3a). Although the front has modest velocity, it has large Rossby number, or equivalently significant vertical relative vorticity normalized by f (Fig. 1). The strong vertical vorticity is largely due to the horizontal shear of the along-front jets:  $\partial_u u^H \sim \pm 3 \times 10^{-4} \text{s}^{-1}$ .

The frontal overturning circulation,  $\psi$  (Fig. 3c), reveals the clockwise overturning pattern expected from the vertical shear of cross-front velocity anomaly shown in Fig. 2e-f and the frontal downwelling shown in Fig. 2d. Vertical velocity, although overlapped with small-scale Langmuir signatures, shows downwelling jets spanning the whole of the mixed layer on the cold side of the front and a broader upwelling on the warm side of the front (Fig. 3d). The downwelling jet at the "nose" of the front, where the front reaches the surface, is enhanced by the Stokes shear force incited by the positive  $u^H$  peaking at the nose (Fig. 3b) and causes the temperature field to form a vertical wall there (Fig. 3a). The Stokes shear force is explained in more detail in Section 4.2.

The cross-front flow  $v^H$  associated with the submesoscale eddies pinches the front by a fairly uniform confluence  $-\partial_y v^H \sim \partial_x u^H \sim 3 \times 10^{-5} \text{s}^{-1}$  (Fig. 3e). Compared to the confluence, the magnitude of  $\partial_x v^H$  is much smaller, although it increases to  $2 \times 10^{-5} \text{s}^{-1}$  at

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the front in the lower mixed layer. The largest component of the Q vector [Hoskins, 1982] resulting from this submesoscale eddy field is  $-\partial_y u^H \partial_x b - \partial_y v^H \partial_y b \sim -5 \times 10^{-12} \text{s}^{-3}$ .

In addition to these frontal circulation modes (Fig.3b-e), the region shown in figure 3 has another flow mode  $(u^B, v^B, 0)$  that is horizontally uniform (Fig. 4a). It consists of horizontal flows (namely, an Ekman spiral, an inertial oscillation, and a geostrophic current) that have much larger scales than the front region; hence these phenomena are approximately horizontally uniform within the front region. The cross-front component  $v^B$  has a vertical shear (mainly, due to the Ekman spiral) that is roughly  $-1.5 \,\mathrm{cm}\,\mathrm{s}^{-1}$  over 50 m. This shear is opposite to the vertical shear of the overturning circulation, which is roughly  $+3 \,\mathrm{cm}\,\mathrm{s}^{-1}$  over 50 m (Fig. 2e, 2f, 3c). In summary, the frontal overturning circulation is superimposed with 1) the horizontally-uniform, down-front Stokes drift confined near the surface  $(u^S, v^S \approx 0)$ , 2) the narrow, along-front baroclinic jets and the confluent submesoscale eddies  $(u^H, v^H)$ , and 3) the horizontally-uniform flow  $(u^B, v^B)$ , whose cross-front component  $v^B$  opposes the vertical shear due to the overturning circulation.

Finally, note that the orange line in Fig. 3c is the bounding streamline of the overturning circulation, where  $\psi$  is positive. Inside of this orange line is the integration domain for the energy, vorticity, and angular momentum budgets shown in the following analysis.

#### 4. Theory

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The following analyses involve many terms and symbols. Hence, for the reader's convenience, a glossary of terms is provided at the end of this paper after the appendices.

#### 4.1. Momentum and Buoyancy Equations

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The LES model [*McWilliams et al.*, 1997; *Sullivan and Patton*, 2011] solves the waveaveraged equations with an LES subgrid-scale model suitable for boundary layer turbulence [*Sullivan et al.*, 1994]. Our primary interest in this section is to present the versions of these equations averaged in time and along one particular front. These averaging operations are necessary for the following analyses to filter out the strong signal of small-scale turbulence and isolate the dynamics pertaining to the larger-scale flows.

The LES resolution used is sufficient to explicitly resolve Langmuir turbulence, which has a much smaller characteristic scale than the frontal features studied here. As mentioned already, Fig. 2d shows a snapshot of vertical velocities due to Langmuir turbulence and those due to the longer front. In magnitude and width of downward jet, these features are quite similar, but they differ substantially in length and timescale of evolution. Langmuir turbulence is three-dimensional, exists inside and outside of the front, and acts as a stress to the other larger-scale flow modes. Without averaging, Langmuir turbulence appears at leading order in the flow dynamics and obscures the dynamics of the largerscale flow modes. Hence, the signal of Langmuir turbulence dynamics may be considered as "noise" in the desired frontal analysis.

To reduce such signal, all flow variables are averaged by a simple moving average in time as well as in the x-direction (i.e., the along-front direction). Experimentation with the particular front examined here allowed the determination of the largest region consistent with a straight front that also avoids introducing uncertainty from other anomalies in the nearby flow field, which can be seen in Fig. 2. The time averaging window is 10 minutes, and during this window the front position moves less than one horizontal grid cell, thus the sharp front is minimally blurred. The window of the along-front averaging corresponds

to the x dimension of the boxes shown in Fig. 2. Again, the sharp front is minimally blurred by the along-front averaging because submesoscale frontal features line up well in the x-direction. Although these averaging operations do not perfectly remove the signal of small-scale turbulence, they certainly reduce it (Fig. 3).

The equations of averaged motion describe the dynamics of the flow modes larger than small-scale turbulence. With a horizontally-uniform Stokes drift used in the LES, these equations are:

$$\left(\partial_t + u_j^L \partial_j\right) u = -\partial_x p' + f v^L - \partial_j L_{1j},\tag{2}$$

$$\left(\partial_t + u_j^L \partial_j\right) v = -\partial_y p' - f u^L - \partial_j L_{2j},\tag{3}$$

$$\left(\partial_t + u_j^L \partial_j\right) w = -\partial_z p' + \partial_z \langle \tau_{33}^L \rangle - \partial_j L_{3j} + b' - u_j' \partial_z u_j^S,\tag{4}$$

$$\left(\partial_t + u_j^L \partial_j\right)\theta = -\partial_j L_{\theta j},\tag{5}$$

$$\partial_j u_j = 0, \tag{6}$$

where j = 1, 2, 3 and Einstein summation is implied on repeated indices. All variables are taken as averages along the front and in time, but notation of the averaging operators is omitted except in Appendix A where these equations are derived systematically. As already shown in Figs. 2 and 3, the coordinates are oriented such that x, y, z, or equivalently  $x_1, x_2, x_3$ , are in the along-front, cross-front, and vertical directions, respectively. Velocity components u, v, w, or equivalently  $u_1, u_2, u_3$ , are in the x, y, z directions, respectively. The mean Eulerian velocity is (u, v, w), the Stokes drift is  $(u^S, v^S, w^S)$  where  $w^S = 0$ , and the mean Lagrangian velocity is  $(u^L, v^L, w^L) = (u + u^S, v + v^S, w + w^S)$ . The Stokes drift used in the LES is horizontally uniform and constant in time. The potential temperature is denoted by  $\theta$ .

The presence of small-scale turbulence results in the stress tensors in these equations. The stress tensor  $\tau_{ij}^L$   $(i = 1, 2, 3, \theta)$  is the sum of the Lagrangian Reynolds stress tensor and the LES subgrid-scale (SGS) stress tensor. Its full definition is given by (A4)-(A5) in Appendix A1. This tensor represents the averaged flux of momentum or temperature due to the full (i.e., not averaged) turbulent flow. In (2)-(5), the advection term (i.e.,  $-\partial_j u_i u_j^L$  or  $-\partial_j \theta u_j^L$ ) is separated from  $-\partial_j \tau_{ij}^L$ . This results in the Leonard stress tensor  $L_{ij} = \tau_{ij}^L - u_i u_j^L$ . Here,  $u_i u_j^L$  can be interpreted as the flux due to the averaged flow. The full definition of the Leonard stress tensor is given by Eqs. (A25)-(A26) in Appendix A4.

Angle brackets  $\langle \rangle$  in (4) indicate the horizontal mean taken over the entire simulation domain, and the prime notations in (2)-(4) indicate deviations from this horizontal mean (i.e.,  $\phi' = \phi - \langle \phi \rangle$ ). Correspondingly, p' is the pressure deviation divided by the constant background density, and b' is the buoyancy deviation where the buoyancy is proportional to temperature as defined in the notation chart at the end of this paper. The presented form of (4) does not contain the dynamically-unimportant background hydrostatic balance:  $-\partial_z \langle p \rangle - \partial_z \langle \tau_{33}^L \rangle + \langle b \rangle - \langle u_j^L \rangle \partial_z u_j^S = 0$  (see Appendix A3). The term  $\langle \tau_{33}^L \rangle$  appears in (4) because this background hydrostatic balance has been removed in (4). That is,  $\partial_z (L_{33} - \langle \tau_{33}^L \rangle)$  – rather than  $\partial_z L_{33}$  alone – appears in (4), in the same way as  $b' = b - \langle b \rangle$ rather than b appears in (4). This is also true for  $\partial_h (L_{3h} - \langle \tau_{3h}^L \rangle)$  with h = 1, 2. However,  $\langle \tau_{31}^L \rangle$  and  $\langle \tau_{32}^L \rangle$  do not appear in (4) because  $\partial_x \langle \tau_{31}^L \rangle = 0$  and  $\partial_y \langle \tau_{32}^L \rangle = 0$ . The last term of the RHS of equation (4) is the Stokes shear force deviation (hereafter, SSF) [Suzuki and Fox-Kemper, 2015] from the horizontal mean,  $-\langle u_j^L \rangle \partial_z u_j^S$ . Again, just as the buoyancy, only the deviating part of the Stokes shear force is dynamically important.

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The temperature budget (5) for the front is not equilibrated. The front overall is undergoing cooling, in part because the large-scale flow is delivering cold water via  $v^L \partial_y \theta$ [*Thomas and Lee*, 2005], and in part because there is upwelling of cold water by  $w \partial_z \theta$ near the base of the mixed layer by turbulent entrainment. Near the surface, turbulent buoyancy fluxes are significant and mix the cold water advected to the front.

#### 4.2. Stokes Shear Force

This section describes how Stokes forces—especially, Stokes shear force—drive the frontal overturning circulation. Understanding this mechanism is critical in making sense of the quantitative analyses presented in the following sections.

In Eqs. (2)-(5), advection of all velocities and temperature is by the Lagrangian velocity; that is, the advective effect of the Stokes drift is distinguished from other wave effects and represented by these terms. In general, advection of buoyancy and momentum may have an important effect on fronts (e.g., change of stratification due to buoyancy advection). However, the effect of the Stokes advection is negligible for the front analyzed here because the front is nearly two-dimensional and aligned with the Stokes drift velocity.

Waves also affect the horizontal momentum equations through the Stokes Coriolis force  $(fv^S, -fu^S)$ , which is part of the Lagrangian Coriolis force in Eqs. (2) and (3). The Stokes drift and, thereby, the Stokes Coriolis force in this study have a large horizontal scale (namely, horizontally uniform). The results of the LES show that this Stokes Coriolis force contributes to the formation of a large-scale Ekman spiral and is typically canceled by the turbulent stress gradients (Section 4.4). Because it is balanced, the Stokes Coriolis force cannot directly force the frontal overturning unless the turbulent stresses in the front region deviate from the large-scale stresses. Hence, it is the frontal perturbation to the

turbulent stresses or equivalently, the resulting disruption of the Ekman balance (rather than the Stokes Coriolis force itself) that affects the frontal overturning dynamics. This point will be apparent in (15) where the Stokes Coriolis force does not directly appear in the momentum equations for the overturning circulation.

Stokes shear force, in contrast, forces and energizes the frontal overturning circulation directly. With a horizontally-uniform Stokes drift (see Appendices A2 and A3), the Stokes shear force (SSF) appears only in the vertical momentum equation (4), and it is useful to consider how it pairs with the buoyancy force that also appears only in that equation. If the anomalous Lagrangian flow  $\mathbf{u}'$  has a component in the direction of the Stokes drift shear  $\partial_z \mathbf{u}^S$  (namely,  $\mathbf{u}' \cdot \partial_z \mathbf{u}^S > 0$ ), then the Stokes shear force pushes the water down like a negative buoyancy anomaly (b'). On the other hand, if the anomalous Lagrangian flow and Stokes drift shear are oppositely oriented  $(\mathbf{u}' \cdot \partial_z \mathbf{u}^S < 0)$ , then the SSF pushes the water up like a positive buoyancy anomaly. When there is a down-Stokes  $(\mathbf{u}' \cdot \partial_z \mathbf{u}^S > 0)$ flow anomaly at small scales, the SSF triggered is downward, drives a downward jet, and transfers wave energy to the jet; that is, the work done by the SSF,  $w(-\mathbf{u}' \cdot \partial_z \mathbf{u}^S)$ , is positive. This is why Langmuir circulations have strong downward jets and gain energy from surface waves [Suzuki and Fox-Kemper, 2015]. The same forcing mechanism of SSF applies to larger-scale flow anomalies such as frontal jets near the surface. When the along-front flow is down-Stokes, both the SSF and buoyancy are downward, and they work in concert. In contrast, when the along-front flow is up-Stokes, then the SSF is upward and the buoyancy is downward; hence they work against each other.

Since the Stokes-front interaction parameter  $-\epsilon$  defined in (1) that determines the relative size of the SSF versus buoyancy-is large for this front, the SSF near the surface is larger

than the buoyancy. Thus, this front is non-hydrostatic and is embedded within a field of non-hydrostatic Langmuir turbulence. Although non-hydrostatic effects are often small for submesoscale fronts when the Stokes shear force is neglected [*Mahadevan and Tandon*, 2006], they are not small here and observations suggest that the Stokes shear force is not negligible for submesoscale fronts worldwide [*McWilliams and Fox-Kemper*, 2013].

Some authors combine the SSF and advection terms and regroup them into a Stokes vortex force and a Stokes Bernoulli effect [*McWilliams et al.*, 1997; *Holm*, 1996; *McWilliams* and Fox-Kemper, 2013; Suzuki and Fox-Kemper, 2015]. While that form is mathematically identical, the form used here simplifies the analysis of the frontal circulation, angular momentum and potential vorticity dynamics, and energetics. Lagrangian advection appears in every prognostic equation, which means that derived quantities, such as energy and potential vorticity, will be advected by the Lagrangian velocity as well. Furthermore,  $O(\epsilon)$  terms are collected into one centralized SSF term for easy examination. Also, while neither Stokes advection nor Lagrangian advection transfers energy between the waves and currents, the SSF does; in fact, the SSF and the Stokes Coriolis forces are the only ways in this system to transfer energy between the waves and currents.

#### 4.3. Flow Decomposition

As already shown in Fig. 3 and Fig. 4a, the averaged motion (u, v, w) defined in Section 4.1 consists of not only the frontal overturning circulation (Fig. 3c) but also the other frontal circulation modes – namely, the along-front baroclinic jets (Fig. 3b) and the submesoscale confluent eddies (Fig. 3e and the nearly barotropic flow in Fig. 3b) – as well as non-frontal circulation modes (Fig. 4a) which encompass the front and have much larger horizontal scales than the cross-frontal scale of the front. Non-frontal circulation modes

are horizontally uniform in the frontal region and are in the background of the frontal features. They may be large-scale types of inertial oscillations, wavy Ekman spirals, and geostrophic currents. Thus, Eqs. (2) and (3) contain the dynamics of many flow modes and do not single out the dynamics of the frontal circulations. It is, therefore, useful to distinguish the frontal circulation modes from the background flow mode and decompose the frontal circulation modes structurally to examine underlying balances.

In particular, the flows are decomposed into: 1. small-scale turbulence (already removed by averaging), 2. the background mode, which is horizontally uniform and has no vertical velocity (denoted by a superscript B and  $w^B \equiv 0$ ), and 3. frontal circulation (i.e., frontal deviation from the background; denoted by a superscript C). The frontal circulation is further divided into: 3a. circulations of submesoscale eddies and the along-front jet, both of which are mostly horizontal motions (denoted by a superscript H), and 3b. an overturning circulation that lies on a y-z plane (denoted by a superscript  $\psi$  and  $u^{\psi} \equiv 0$ ). Therefore, the decomposition is performed as

$$u = u^B + u^C = u^B + u^H \qquad , \tag{7}$$

$$v = v^{B} + v^{C} = v^{B} + v^{H} + v^{\psi},$$
(8)

$$w = w^C = w^H + w^{\psi}, \tag{9}$$

where  $\partial_h u_i^B = 0$  for h = 1, 2 (as this mode is horizontally uniform), and  $\partial_y v^{\psi} + \partial_z w^{\psi} = 0$  (as this mode is a circulation on a y-z plane).

In general, the flow in the front region (solid box in Fig. 3) contains large anomalies from the "background" flow that are present in both the front region and the surrounding regions (dashed boxes in Fig. 3). To diagnose the background mode without being biased by the frontal anomalies, the background mode is defined as the horizontal average of

*u* and *w* in the surrounding regions, thereby excluding the anomalies in the front region (see Appendix B). We will denote this horizontal averaging over the surrounding regions by  $\overline{\phantom{a}}^B$ . Thus,  $u_h^B \equiv \overline{u_h}^B$  and  $u_h^C \equiv u_h - u_h^B$  for the horizontal components: h =1,2. Note that, unlike  $(u^B, v^B)$ , the vertical component  $w^B$  is not defined using the  $\overline{\phantom{a}}^B$  operation. Instead,  $w^B$  is 0 by definition – hence, it is not equivalent to  $\overline{w}^B$  – because  $(u^B, v^B)$  has no horizontal divergence and the rigid lid condition does not permit a vertical motion associated with this mode. In contrast, a horizontally-uniform flow  $\overline{w}^B$  may become non-zero when the horizontal divergence of the submesoscale eddies  $(u^H, v^H)$  is non-zero in the surrounding regions where the  $\overline{\phantom{a}}^B$  operation is taken. Hence,  $\overline{w}^B$  is associated to the submesoscale eddy motion (H-mode or  $w^H$ ) rather than to the larger-scale motions (B-mode). Experimentation with this particular front determined the largest surrounding regions that include only nearby flow features relevant to this front (Fig. 2), and uncertainties in budgets are estimated by varying the width of the front region and surrounding regions up to 100 m.

The decomposition of the circulation modes (i.e., the decomposition of C-mode into  $\psi$ and H-mode), in practice, requires care and is detailed in Appendix C. Here we note that, although a significant part of small-scale turbulence has been removed, some remains in the averaged motion. A part of this residual turbulence forms circulations in x-z planes and contaminates  $u^H$  and  $w^H$  with small-scale fluctuations. Our results show that the average of  $(w^H)^2$  in the front region is less than 6% of the average of  $w^2$ , the average of  $(\partial_t w^H)^2$  is less than 8% of the average of  $(\partial_t w)^2$ , and the predominant structure of  $w^H$ is small-scale turbulence fluctuations. Therefore, to leading order, and especially when small-scale fluctuations are removed,  $w^H$  is negligible; that is,  $w \approx w^{\psi}$ .

In summary, Fig. 4a shows  $u^B$  and  $v^B$ , Fig. 3b shows  $u^C = u^H$ , Fig. 3e shows  $v^H$ , and Fig. 3c shows the overturning circulation  $\psi$  (hence  $v^{\psi} = -\partial_z \psi$  and  $w^{\psi} = \partial_y \psi$ ; also  $w^{\psi}$  is nearly identical to w shown in Fig. 3d).

### 4.4. Momentum Budget for the Background Mode

To further characterize the background mode, the force balances involved in the background mode are analyzed in this section. This analysis shows what types of flows constitute the background mode.

An application of the -B operation to Eqs. (2) and (3) shows that the background mode  $(u^B, v^B)$  obeys

$$\partial_t u^B + \overline{u_j^L \partial_j u}^B - f(v^S + v^B) = -\overline{\partial_x p'}^B - \overline{\partial_j L_{1j}}^B, \tag{10}$$

$$\partial_t v^B + \overline{u_j^L \partial_j v}^B + f(u^S + u^B) = -\overline{\partial_y p'}^B - \overline{\partial_j L_{2j}}^B, \tag{11}$$

where j = 1, 2, 3. These balances for the present front are shown in Fig. 4b-c.

The dashed blue lines in Fig. 4b-c show the turbulent wavy Ekman balance described in *McWilliams et al.* [2012] and *Haney et al.* [2015]; that is,  $(-f(v^S + v^E), f(u^S + u^E)) =$  $-(\overline{\partial_j L_{1j}}^B, \overline{\partial_j L_{2j}}^B)$  where  $(u^E, v^E)$  is the horizontally-uniform Eulerian Ekman spiral. Note that  $v^S \approx 0$  for this front as it is nearly aligned with the Stokes drift (and wind) direction. By comparing the solid yellow lines to the solid blue lines, it is clear that  $(u^E, v^E)$  is a main component of  $(u^B, v^B)$ . The red line in Fig. 4a or the solid blue line in Fig. 4b shows that there is an Ekman transport in the cross frontal direction for this down-front wind. A further analysis (not shown here) shows that the frontal anomaly in the along-front stress gradient from its background value (i.e.,  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$ ) is not correlated with  $fv^{\psi}$  or  $fv^H$ . This indicates that the along-front stress deviation does not

establish a local Ekman balance at this horizontal scale, and hence neither  $v^{\psi}$  nor  $v^{H}$  has an Ekman spiral component.

In addition to the horizontally-uniform Ekman spiral, the background mode  $(u^B, v^B)$ has a weaker geostrophic component (Fig. 4b-c red) and an inertial oscillation component (Fig. 4b-c black). Note that, according to (10) and (11), the acceleration  $\partial_t(u^B, v^B) =$  $(\partial_t + u_h^L \partial_h)(u^B, v^B)$  (where h = 1, 2) is influenced not only by the Coriolis force pertaining to inertial oscillations but also by nonlinear interactions with the circulation modes, namely,  $\overline{-u_j^L \partial_j u_i^B} = \overline{-u_j^L \partial_j u_i^{CB}} + \overline{-w^C \partial_z u^B}^B$ . Thus, the acceleration minus the nonlinear interactions (Fig. 4b-c black) pertains to the inertial oscillations. Notice also that the near-surface increase of  $\overline{-\partial_x p'}^B$  (Fig. 4b red) is balanced by the acceleration (Fig. 4b black) instead of by the Coriolis force  $fv^B$  (Fig. 4b yellow). Hence, these increases are not related to geostrophic or inertial oscillation modes. Both the geostrophic and inertial oscillation modes are almost vertically uniform. This near-surface pressure gradient and the resultant acceleration occur because the SSF triggered by u' (Fig. 2c) sets up the pressure via (4) in the same way that buoyancy sets up pressure [*Suzuki and Fox-Kemper*, 2015]. As a result, the existing large-scale gradient of u' yields this pressure gradient.

#### 4.5. Overturning Circulation Dynamics

#### 4.5.1. Momentum Budget for the Overturning Circulation

This section shows the equations of motion governing the frontal overturning circulation. These equations reveal the forces involved in the frontal overturning and lay the foundation for the torque and energy analyses presented in the following sections.

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Subtraction of Eqs. (10) and (11) from (2) and (3), respectively, shows that the frontal circulation mode obeys

$$(\partial_{t} + u_{j}^{L}\partial_{j})u^{C} - \overline{u_{j}^{L}\partial_{j}u^{C}}^{B} = -\left(\partial_{x}p' - \overline{\partial_{x}p'}^{B}\right) - \left(\partial_{j}L_{1j} - \overline{\partial_{j}L_{1j}}^{B}\right) + fv^{C} - \left(w\partial_{z}u^{B} - \overline{w\partial_{z}u^{B}}^{B}\right)$$

$$(12)$$

$$(\partial_{t} + u_{j}^{L}\partial_{j})v^{C} - \overline{u_{j}^{L}\partial_{j}v^{C}}^{B} = -\left(\partial_{y}p' - \overline{\partial_{y}p'}^{B}\right) - \left(\partial_{j}L_{2j} - \overline{\partial_{j}L_{2j}}^{B}\right) - fu^{C} - \left(w\partial_{z}v^{B} - \overline{w\partial_{z}v^{B}}^{B}\right)$$

$$(13)$$

$$(\partial_{t} + u_{j}^{L}\partial_{j})w^{C} = -\partial_{z}p' - \left(\partial_{j}L_{3j} - \partial_{z}\langle\tau_{33}^{L}\rangle\right) + b' - u'_{j}\partial_{z}u_{j}^{S},$$

$$(14)$$

where  $w = w^C$  [(9)] and j = 1, 2, 3. Equations (13) and (14) can then be rewritten to highlight the dynamics of the frontal overturning  $(v^{\psi}, w^{\psi})$ . First, using  $(u^C, v^C) = (u^H, v^H + v^{\psi})$  in (13) yields the momentum budget for  $v^{\psi}$  as

$$\left(\partial_t + u_j^L \partial_j\right) v^{\psi} = -\left(\partial_y p' - \overline{\partial_y p'}^B\right) - \left(\partial_j L_{2j} - \overline{\partial_j L_{2j}}^B\right) - f u^H - F^h - F^v \tag{15}$$

where

$$F^{h} = \partial_{t}v^{H} + u_{j}^{L}\partial_{j}v^{H} - \overline{u_{j}^{L}\partial_{j}v^{C}}^{B}, \qquad (16)$$

$$F^{v} = w\partial_{z}v^{B} - \overline{w\partial_{z}v^{B}}^{B}.$$
(17)

Next, because  $w^C \approx w^{\psi}$ , as quantified in Section 4.3, (14) yields the momentum budget for  $w^{\psi}$  as

$$\left(\partial_t + u_j^L \partial_j\right) w^{\psi} \approx -\partial_z p' - \left(\partial_j L_{3j} - \partial_z \langle \tau_{33}^L \rangle\right) + b' - u_j' \partial_z u_j^S.$$
(18)

As  $v^S \approx 0$  for this down-Stokes front and also  $u' \approx u^H$ , the Stokes shear force here is essentially  $-u'_j \partial_z u^S_j \approx -u^H \partial_z u^S$ . The mechanism of how the Stokes shear force drives the overturning circulation is schematically shown in Fig. 5.

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Note that Eqs. (11), (15), and (16) are related by the forces  $\overline{u_i^L \partial_j v^C}^B$ ,  $F^h$ , and  $F^v$  as background mode  $v^B$ :  $(\partial_t + u_j^L \partial_j) v^B = -\overline{u_j^L \partial_j v^C}^B + F^v + \cdots,$ horizontal circulation mode  $v^H$ :  $(\partial_t + u_j^L \partial_j)v^H = +\overline{u_j^L \partial_j v^C}^B + F^h$ overturning circulation mode  $v^{\psi}$ :  $(\partial_t + u_i^L \partial_j) v^{\psi} =$  $-F^h-F^v+\cdots,$ where  $\partial_h v^B = 0$  for h = 1, 2 is used to rearrange (11). Note that – in theory, or aside from the remaining turbulence signals – the frontal overturning circulation  $(v^{\psi}, w^{\psi})$  is negligible in the surrounding regions (i.e., outside the front) where the -B averaging is taken; that is,  $\overline{u_j^L \partial_j v^C}^B \approx \overline{(u_h^S + u_h^B + u_h^H) \partial_h v^H}^B$ . This term thus represents nonlinear interactions between the horizontal circulation (submesoscale eddy) mode and the background mode. Likewise,  $F^{v}$  in theory reduces to  $w^{\psi} \partial_z v^B$ . Hence, hereafter,  $F^{v}$  is addressed as the nonlinear interaction between the background mode and the overturning circulation. In a similar way as the other two forces,  $F^h$  appears with alternate signs between two modes. This term is a forcing on the horizontal circulation and enters as  $-F^h$  in the dynamics of the overturning circulation. This relation allows momentum partitioning or "interaction" between the horizontal circulation and the overturning circulation. Indeed, the largest contribution to  $F^h$  comes from the frontal deviation in a nonlinear interaction  $v^L \partial_y v^H \overline{v^L \partial_y v^H}^B$ , which is mostly  $v^{\psi} \partial_y v^H$  in the frontal region. Hereafter,  $-F^h$  is addressed as the interaction between the horizontal circulation and the overturning circulation. The typical appearance of  $-F^h$  is schematically shown in Fig. 5.

Quantitative exploration of the overturning dynamics [Eqs. (15) and (18)] will be carried out by analyzing integrated angular momentum, vorticity, and energy budgets for the overturning  $\psi$  variables in the next section. Here, however, the leading-order momentum

balances revealed by examining the LES dataset are given without illustration. First, the leading order balance in (18) is a quasi-hydrostatic, or "wavy-hydrostatic", balance in which the leading-order pressure perturbation is determined by the sum of buoyancy and SSF, namely

$$\partial_z p' \approx b' - u'_j \partial_z u^S_j. \tag{19}$$

Although a small imbalance of this relationship and the associated acceleration of  $w^{\psi}$  will be shown to be important for this front in the following sections, the wavy-hydrostatic balance is, by far, the leading-order vertical balance and determines the leading-order pressure perturbation. Next, the cross-frontal flow associated with the submesoscale eddy  $(v^H)$  satisfies a geostrophic balance plus a small near-surface correction for Stokes advection:

$$fv^{H} \approx \partial_{x} p' - \overline{\partial_{x} p'}^{B} + \left( u^{S} \partial_{x} u^{H} - \overline{u^{S} \partial_{x} u^{H}}^{B} \right).$$

$$(20)$$

Removing this balance and also neglecting terms that are expected to be generally small in (12) yields an approximate momentum balance for the along-front component of the frontal circulation ( $u^{C} = u^{H}$ ) as

$$\left(\partial_t + u\partial_x + v^L\partial_y + w\partial_z\right)u^H - \overline{\left(u\partial_x + v^L\partial_y\right)u^H}^B \approx -\left(\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B\right) + fv^\psi - w\partial_z u^B.$$
(21)

In particular, for this nearly 2D ( $\partial_x \ll \partial_y$ ), downwind ( $u^B < v^B$ ), and down-Stokes ( $v^S \ll u^S$ ) front, the largest terms are  $(\partial_t + v\partial_y)u^H$ ,  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$ , and  $fv^{\psi}$ . McWilliams et al. [2015] find a similar equation in turbulent filaments, referred to there as the "turbulent thermal wind." However, here the large Rossby number keeps the time derivative and advection term in the dominant balance. Unlike the turbulent thermal wind mechanism, the Coriolis term is balanced by the material derivative term instead of the stress gradient term. The Coriolis term and the stress gradient term are rather independent from each

other. Hence, this front is not a front driven by the turbulent thermal wind mechanism (see section 5.4 for more detail).

The force balance of the cross-frontal overturning momentum in (15) is more complicated. Every term takes part in the leading order balance at some location of the front, and a simple relationship such as a geostrophic balance for  $u^H$  does not hold well. However, the complexity posed by the spatial variations can be ameliorated by studying integrated budgets of energy, angular momentum, and vorticity. Sources and sinks of these budgets highlight the average dynamics of overturning circulation and thereby that of the frontal strength  $u^H$  [via the Coriolis turning of  $v^{\psi}$  in (21)]. The rest of section 4 reviews the theory of such integrated budgets, then in section 5 the simulation results are used to quantify the sources and sinks in the budgets.

# 4.5.2. Integrated Angular Momentum Budget for the Overturning Circulation

Vorticity and the related local balances such as the thermal wind balance are key concepts in traditional theories of fronts. However, in the presence of strong small-scale turbulence, local vorticity is dominated by the signal due to the small-scale turbulence rather than that of the flows and forces pertaining to the frontal overturning (even after the time and along-front averaging). Therefore, understanding the frontal overturning dynamics requires use of a quantity that is less sensitive to small-scale rotating features and that can capture the circulation at the frontal-scale. For this reason, our primary analyses are based on the angular momentum and energy, both of which are quantities that can be integrated over the entire overturning cell to measure the dynamics and energetics of the entire overturning circulation. This is in contrast to the integration of vorticity, which

results in the circulation only along a line (Stokes's theorem). The angular momentum budget is explained in this section, and the energy budget is explained in Section 4.5.3.

In the front region, a water parcel carries the angular momentum associated to the overturning motion  $(v^{\psi}, w^{\psi})$  about the center of mass of the overturning cell. This angular momentum is defined as

$$\varepsilon_{1ij}(r_i - r_{0i})u_j^{\psi} = (y - y_0)w^{\psi} - (z - z_0)v^{\psi}$$
(22)

where i, j = 1, 2, 3,  $\varepsilon_{1ij}$  is the Levi-Civita symbol,  $(r_2, r_3) = (y, z)$ , and  $(r_{02}, r_{03}) = (y_0, z_0)$ is the center of mass of the overturning cell (see Fig. 3c). Following the water parcel, the angular momentum changes as

$$(\partial_t + u_k^L \partial_k) \varepsilon_{1ij} (r_i - r_{0i}) u_j^{\psi} = \varepsilon_{1ij} (r_i - r_{0i}) (\partial_t + u_k^L \partial_k) u_j^{\psi} + \varepsilon_{1ij} u_i^L u_j^{\psi}$$

$$= (y - y_0) (\partial_t + u_k^L \partial_k) w^{\psi} - (z - z_0) (\partial_t + u_k^L \partial_k) v^{\psi} + v^L w^{\psi} - w v^{\psi}$$

$$\approx (y - y_0) (\partial_t + u_k^L \partial_k) w^{\psi} - (z - z_0) (\partial_t + u_k^L \partial_k) v^{\psi} + (v^L - v^{\psi}) w^{\psi}$$

$$(23)$$

where k = 1, 2, 3, and the last approximation is due to  $w \approx w^{\psi}$ . The first two terms on the right-hand side are the torques by the vertical [(18)] and horizontal [(15)] forces driving the overturning circulation. The last term on the right-hand side comes from commuting the moment arm with the material derivative, and it does not vanish when the water parcel moves with the total velocity  $v^L$ , which is different from the overturning velocity  $v^{\psi}$ . This term represents the change of angular momentum due to dislocation of the vertical overturning motion  $(w^{\psi})$  by the fluid velocity other than the overturning velocity itself  $(v^L - v^{\psi} = v^S + v^B + v^H)$ . Hence, this term will be referred to in the following as the "dislocation cost."

Equation (23) shows that the forces in Eqs. (15) and (18) contribute toward torquing the front and driving the overturning circulation. The overall contribution from each of these forces can be revealed by integrating (23) over the overturning cell (Fig. 3c). These contributions are presented in Table 1 and will be discussed in detail later in Section 5.1.

## 4.5.3. Integrated Overturning Energy Budget

Multiplying Eqs. (15) and (18) by  $v^{\psi}$  and  $w^{\psi}$ , respectively, and combining them allows computation of the overturning kinetic energy (KE) budget. The equation governing this budget is

$$(\partial_{t} + u_{j}^{L}\partial_{j})\frac{v^{\psi}v^{\psi} + w^{\psi}w^{\psi}}{2} = \underbrace{w^{\psi}b' - v^{\psi}F^{h} - w^{\psi}u'_{j}\partial_{z}u_{j}^{S}}_{\text{major sources}} - fv^{\psi}u^{H} + v^{\psi}\left(\overline{\partial_{y}p'}^{B} + \overline{\partial_{j}L_{2j}}^{B}\right) + L_{kj}\partial_{j}u_{k}^{\psi} \qquad (24)$$

$$\underbrace{-\partial_{k}(u_{k}^{\psi}P) - \partial_{j}(u_{k}^{\psi}L_{kj}) - v^{\psi}F^{v}}_{\text{negligible}}$$

where  $P \equiv p' - \langle \tau_{33}^L \rangle$ , j = 1, 2, 3, and k = 2, 3. The kinetic energy budget for the whole overturning circulation is diagnosed by integrating (24) over the overturning cell. The result is presented in Table 2 and will be discussed in Section 5.2. Note that the pressure transport  $\int_A -\partial_k (u_k^{\psi} P) dA$  vanishes because the overturning motion  $(v^{\psi}, w^{\psi})$  is parallel to the boundary of the overturning cell area A.

Here we note interpretations of the terms in (24). The three major sources of energy for the frontal overturning circulation are the buoyancy production, the work done by  $-F^h$ , and the work done by the SSF. In an up-Stokes front, the SSF work would constitute a sink of energy rather than a source. The work done by  $-F^h$  is the rate of change in  $v^{\psi}v^{\psi}/2$  due to the interaction between the horizontal circulation (confluent eddies) and the overturning circulation (Section 4.5.1).

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The three major sinks of energy are the Coriolis conversion of  $v^{\psi}$  into the along-front jet  $u^{H}$ , the work done against the Coriolis force due to the background wavy-Ekman and geostrophic modes, and the generation of small-scale shear turbulence by the shear in the overturning circulation. The interpretation of the second sink term is due to the definitions that the background wavy Ekman mode  $u^{S} + u^{E}$  satisfies  $\overline{\partial_{j}L_{2j}}^{B} = -f(u^{S} + u^{E})$  and the background geostrophic mode  $u^{G}$  satisfies  $\overline{\partial_{y}p'}^{B} = -fu^{G}$  (Section 4.4). Hence, the second sink term is equal to  $-fv^{\psi}(u^{S} + u^{E} + u^{G})$ . The appearance of  $fv^{\psi}$  – i.e., the Coriolis turning of  $v^{\psi}$  – in (21) but not in (10) shows that  $v^{\psi}$  turns and becomes the along-front jet  $u^{H}$ , which is a deviation from the background, but does not affect the background  $u^{B}$ . This results in an increase of energy in the jet  $u^{H}u^{H}/2$  by  $u^{H}fv^{\psi}$  [according to  $u^{H}$  times (21)], corresponding to the first sink term of (24).

The second sink term, on the other hand, does not change the along-front jet energy  $u_i^H u_i^H/2$  nor the background flow energy  $u_i^B u_i^B/2$ . Note that the energy of the full flow  $u_i u_i/2$  consists of not only  $u_i^B u_i^B/2$ ,  $u_i^H u_i^H/2$ , and  $u_i^{\psi} u_i^{\psi}/2$  but also cross-mode terms  $u_i^B u_i^B$ ,  $u_i^B u_i^{\psi}$ , and  $u_i^H u_i^{\psi}$ . Conservation of energy applies to the full flow energy. The second sink term  $-fv^{\psi}(u^S + u^E + u^G)$  represents the work done by the Stokes Coriolis force  $(-fu^S v^{\psi})$  and the energy transfer from  $v^{\psi}v^{\psi}/2$  to a cross-mode energy component involving the jet and the background mode,  $(u^G + u^E)u^H$  [according to Eqs. (10) and (12)]. Because  $v^{\psi}$  turns and increases  $u^H$  to  $u^H + fv^{\psi}\Delta t$  in a short time  $\Delta t$  without changing  $u^G + u^E$ , the cross-mode energy component  $(u^G + u^E)u^H$  changes to  $(u^G + u^E)(u^H + fv^{\psi}\Delta t)$ . This rate of increase in the cross-mode energy corresponds to the energy transfer in the second sink term  $-fv^{\psi}(u^G + u^E)$ .

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If one wishes to facilitate comparison between the energy budget and the angular momentum budget, the torques involving pressure must be added together. That is,  $\int_{A} (z_0 - z)(-\partial_y p' + \overline{\partial_y p'}^B) + (y - y_0)(-\partial_z p') dA.$ This can be readily compared to the work done by the same forces  $\int_{A} v^{\psi}(-\partial_y p' + \overline{\partial_y p'}^B) + w^{\psi}(-\partial_z p') dA = \int_{A} v^{\psi} \overline{\partial_y p'}^B dA.$ 

# 4.5.4. Integrated Overturning Vorticity Budget

The vorticity budget and the angular momentum budget over a region are closely related because the conservation of vorticity and the first and second moments of vorticity imply conservation of angular momentum [*Batchelor*, 1973, Section 7.3]. However, pressure terms appear in the angular momentum budget, whereas they do not in the vorticity budget. In this turbulent flow, where the Langmuir cells have axial vorticity of O(30f), local vorticity balance is very noisy. This turbulence signal can be reduced by integrating the vorticity budget over an area. Then, the integrated budget shows the dynamics of the circulation due to  $(v^{\psi}, w^{\psi})$  along the contour bounding the area (Stokes's theorem). Although this budget does not describe the overturning dynamics as a whole, it may be still useful in studying the circulation at a specific location (e.g., inner core, outer boundary, etc.). Hence, even though the vorticity, or circulation, budget is not our primary analysis, it is included here for completeness.

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The x-component of the vorticity dynamics associated with the overturning circulation  
is obtained by subtracting the z-derivative of (15) from the y-derivative of (18), giving  

$$\partial_t (-\partial_z v^{\psi} + \partial_y w^{\psi}) = \underbrace{\partial_y b' + \partial_y (-u'_j \partial_z u^S_j) + \partial_z F^h}_{\text{major sources}} + \underbrace{\left(\partial_z (\partial_j L_{2j} - \overline{\partial_j L_{2j}}^B) - \partial_y \partial_j L_{3j}\right) + f \partial_z u^H + \left(\partial_z (u^L_j \partial_j v^{\psi}) - \partial_y (u^L_j \partial_j w^{\psi})\right)}_{\text{major sinks}}$$
  
 $\underbrace{-\partial_z \overline{\partial_y p'}^B + \partial_z F^v}_{\text{negligible}}.$ 
(25)

Integration of this equation over the overturning cell, for example, yields the budget for the overturning circulation along the overturning cell boundary. The result is shown in Table 3 and will be discussed in Section 5.3.

Note that the curl of the Lagrangian advection of the overturning circulation  $\nabla \times (\mathbf{u}^L \cdot \nabla) \mathbf{u}^{\psi}$  is equal to  $(\mathbf{u}^L \cdot \nabla) \omega^{\psi} - (\omega^{\psi} \cdot \nabla) \mathbf{u}^L - \nabla \times (u_j^{\psi} \nabla (u_j^S + u_j^B + u_j^H))$  where  $\omega^{\psi} = \nabla \times \mathbf{u}^{\psi}$  is the overturning vorticity. The first term is the Lagrangian advection of the overturning vorticity, the second term is the Lagrangian tilting and stretching of the overturning vorticity, and the last term is overturning vorticity generation by nonlinear mode interactions. In the last term, there is a term that resembles the curl of the Stokes shear force  $\nabla \times (-u_j' \nabla u_j^S)$ , but they are not the same (e.g.,  $u' \approx u^H$ , but  $u^{\psi} = 0$ ). For this down-Stokes  $(v^S \approx 0)$  front with  $w^H \approx 0$  and  $u^{\psi} = 0$ , the last term reduces to  $-\nabla \times (v^{\psi} \nabla (v^B + v^H))$ .

## 5. Results and Discussion

#### 5.1. Forces and Torques

First, we visualize how the forces in Eqs. (15) and (18) torque the front by showing their integrals at each moment arm. For example, according to (18), the total buoyancy

torque is  $\int_A (y - y_0) b' dA$  where A is the area of the overturning cell (Fig. 3c). This can be rewritten as  $\int_y (y - y_0) \left( \int_{z(y)} b' dz \right) dy$ . Thus, the vertical integral of buoyancy  $\int_{z(y)} b' dz$  at a given y is the net buoyancy that torques the front at the corresponding moment arm  $(y - y_0)$ .

Figure 6a shows (18) integrated vertically within the overturning cell at each y. These terms multiplied by the horizontal moment arm  $(y - y_0)$  are the torques exerted by vertical forces. The location of  $y_0$  is indicated with the vertical dashed black line. The wavy-hydrostatic balance [(19)] is apparent in the dominant balance between the pressure gradient (red), buoyancy (yellow), and Stokes shear force (black dashed-dot). However, the combination of these three terms (green) is not exactly balanced and causes acceleration (black solid). Although the visible signals in the green and black solid lines are small-scale fluctuations, these lines also contain a variation whose spatial scale is of the front width. This variation can be seen by filtering out the small-scale fluctuations (not plotted because it is of order of  $10^{-6}$ ). As a result, the sum of the vertical forces turns the fluid in the same sense of rotation as the overturning circulation (clockwise). This point will be quantified below in Table 1.

Figure 6b shows (15) integrated horizontally within the overturning cell at each z. These terms multiplied by the vertical moment arm  $z - z_0$  are the torques exerted by the horizontal forces. The location of  $z_0$  is indicated by the horizontal dashed black line. Many forces are important, and it is clear that the imbalance of these forces produces acceleration (black) both near the surface and at depth. This acceleration, or net force, torques the front in the same sense of rotation as the overturning circulation (clockwise). The along-front component of the frontal circulation  $u^H$  (yellow) is not fully in a geostrophic

balance with the frontal pressure gradient (red). At depth, the pressure gradient and the Coriolis force due to  $u^H$  are the largest terms. Near the surface, the importance of  $u^H$  decreases and that of other terms increases. The frontal imbalance in the wavy Ekman relation  $-\partial_j L_{2j} + \overline{\partial_j L_{2j}}^B$  (dark solid blue) is the imbalance between the local turbulence  $-\partial_j L_{2j}$  (blue dashed) and the Coriolis force on the background wavy Ekman mode  $-f(u^S + u^E) = \overline{\partial_j L_{2j}}^B$  (blue dashed-dot). The imbalance in  $-\partial_j L_{2j} - f(u^S + u^E)$ cancels part of the frontal pressure gradient  $-\partial_y p' + \overline{\partial_y p'}^B$  (red) and thereby reduces the clockwise torque by the pressure term. The interaction with the submesoscale confluent flow (cyan;  $-F^h$ ) also torques the front clockwise. The nonlinear interaction with the background mode  $v^B$  (magenta;  $-F^v$ ) is overall negligible.

The frontal imbalance in the wavy Ekman relation here occurs because the cross-front stress term  $-\partial_j L_{2j}$  is affected by the frontal anomaly in the vertical shear of the total cross-front velocity:  $\partial_z v = \partial_z v^B + \partial_z v^H + \partial_z v^{\psi}$ . As described in Section 3,  $\partial_z v^B$  is negative,  $\partial_z v^H$  is negligible compared to  $\partial_z v^B$  or  $\partial_z v^{\psi}$ , and  $\partial_z v^{\psi}$  is positive inside the front and its magnitude is about twice as large as the magnitude of  $\partial_z v^B$ . Hence,  $\partial_z v^{\psi}$  reverses the sign of  $\partial_z v$  inside the front. Especially near the surface,  $\partial_z v^{\psi}$  is significant, even more so than the along-front baroclinic shear  $\partial_z u^H$  (Fig. 5). As a result,  $\partial_j L_{2j}$  significantly deviates from  $\overline{\partial_j L_{2j}}^B$ , or equivalently  $-f(u^S + u^E)$ , near the surface inside the front. Finally, it should be clearly noted that the stress anomaly in Figure 6b is the cross-front one  $(-\partial_j L_{2j} + \overline{\partial_j L_{2j}}^B)$  rather than the along-front one  $(-\partial_j L_{1j} + \overline{\partial_j L_{1j}}^B)$ . The anomaly in the cross-front stress acts on the overturning circulation  $v^{\psi}$  and opposes the overturning. In contrast, the along-front stress does not directly act on  $v^{\psi}$ . However, it may indirectly induce  $v^{\psi}$  by vertically diffusing the baroclinic jets  $u^H$  and thereby reducing the yellow

line in Figure 6b. Such diffusion of  $u^H$  weakens the torque preventing the front from overturning. Hence, it has a tendency of inducing or enhancing overturning.

The net torques exerted on the overturning cell by these forces are computed by integrating (23) over the overturning cell and are quantified in Table 1. The sum of all the torques is not balanced and at this rate of angular acceleration the current overturning circulation could have been created in only  $8\pm1$  hours without the displacement cost and  $11\pm1$  hours with the displacement cost. Because of the wavy-hydrostatic relation (19), the torques by the buoyancy and Stokes shear force are largely canceled by the torque due to the vertical pressure gradient. However, the small imbalance in the wavy-hydrostatic relation still produces a net torque that is as large as the torque by the net horizontal force. Thus, nonhydrostatic effects are important in determining the overturning angular momentum.

Among horizontal forces, the largest source of overturning angular momentum is the torque due to the frontal deviation in the cross-frontal pressure gradient from its background value, where the background value is set by the background geostrophic mode  $(\overline{\partial_y p'}^B = -f u^G)$ . Only 68% of this torque is balanced by the Coriolis torque due to the along-front jet  $u^H$ ; thus, the geostrophic balance is only partial. Another 27% is balanced by the frontal imbalance in the wavy Ekman relation. However, this loss is recovered by another significant source: the torque generated by the interaction between the submesoscale confluent flow  $v^H$  and the overturning circulation. This torque is as large as 26% of the buoyancy torque. Therefore, the buoyancy, the interaction between the submesoscale eddies and overturning circulation, and the SSF are the major sources
of overturning torque. The pressure plays the important role of redirecting the vertical forcing of buoyancy and SSF to a force driving the cross-front flow.

### 5.2. Energy Sources and Sinks

Table 2 quantifies the integrated budget for the KE of the overturning circulation. The KE budget has essentially the same sources and sinks as the angular momentum budget, but it is not so heavily influenced by the wavy-hydrostatic balance [because the vertical forces are multiplied by large moment arms  $|y - y_0| \gg |z - z_0|$  in the torque formula and small velocities  $|w^{\psi}| \ll |v^{\psi}|$  in the work formula]. Following the overturning cell, there is hardly any net incoming or outgoing turbulent transport of the overturning circulation KE through the cell boundary. The overturning circulation KE is vigorously increasing. In particular, the energy uptake by the overturning circulation is as large as nearly half of the largest energy source; namely, buoyancy production. The buoyancy production is joined by energy inputs due to the SSF and the interaction with the confluent flow (leading to extraction of momentum and energy from the submesoscale eddies). The two uncommon sources – surface waves and KE of submesoscale eddies – together produce energy at a rate nearly as large as the more common buoyancy production.

Horizontal velocities of the overturning circulation  $v^{\psi}$  turn due to the Coriolis force and generate along-front jets. This effect represents the largest energy loss from the overturning circulation, but only 69% of the buoyancy production or 38% of the net production is used for this. The work done against  $-f(u^S + u^E + u^G)$  is the next largest sink. As mentioned in Section 4.5.3, the energy lost by this work does not change the momentum nor energy of  $u^E + u^G$ .

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Interestingly, the loss of the overturning circulation KE to small-scale, shear turbulence is relatively small: only 16%. This may be due to the fact that  $L_{23}$  is not as large as one may expect from the significant  $\partial_z v^{\psi}$ . However,  $L_{23}$  depends on  $\partial_z v = \partial_z v^B + \partial_z v^H + \partial_z v^{\psi}$ rather than  $\partial_z v^{\psi}$  alone. As mentioned in Section 5.1,  $\partial_z v^{\psi}$  and  $\partial_z v^B$  have opposite signs, resulting in a smaller magnitude of  $\partial_z v$ . Hence,  $L_{23}\partial_z v^{\psi}$  becomes smaller in the presence of  $\partial_z v^B$ .

### 5.3. Vorticity Sources and Sinks

Table 3 shows the integrated budget for the vorticity of the overturning circulation  $(v^{\psi}, w^{\psi})$ . This is equivalent to the tendency of the overturning circulation along the overturning cell boundary (Stokes's theorem). The table reinforces the results of the angular momentum and energy budgets: relative to the buoyancy (largest source term), the SSF as well as the interaction between the confluent flow and overturning circulation are significant.

As already shown in Section 4.5.4, the curl of the Lagrangian advection term  $\nabla \times (-\mathbf{u}^L \cdot \nabla)\mathbf{u}^{\psi}$  consists of the Lagrangian advection, stretching, and tilting  $-(\mathbf{u}^L \cdot \nabla)\omega^{\psi} + (\omega^{\psi} \cdot \nabla)\mathbf{u}^L$ and a vorticity generation by mode interactions  $\nabla \times (u_j^{\psi} \nabla (u_j^S + u_j^B + u_j^H))$ . The mode interactions for this front are approximately equal to  $\nabla \times (v^{\psi} \nabla (v^B + v^H))$  and do not correlate with the curl of the SSF.

Unlike the angular momentum and energy budgets, the negative contribution from the frontal imbalance in the local and background turbulence (mostly, wavy Ekman relation) is larger than the Coriolis contribution due to the along-front jet. This is because the circulation budget reflects contributions along the cell boundary, and the wavy Ekman imbalance near the cell top is very large (c.f. the blue solid and yellow lines in Fig. 6b).

In contrast, the angular momentum and energy budgets reflect the whole interior of the overturning circulation.

### 5.4. Comparison to Traditional Mesoscale Frontogenesis

The budgets clearly show that this submesoscale front is not in a balanced state, and the unbalanced net torque directly drives the overturning. This is in contrast with the conventional mesoscale frontogenesis models, in which the front is in a nearly balanced state. In these models, the net overturning torque is zero and does not drive the overturning. Instead, an overturning circulation exists because there is a force (e.g., from mesoscale straining) that tends to reduce the integrated torque due to  $-fu^H$  (i.e., the mesoscale counterpart to yellow line in Fig.6b). The overturning circulation, once generated, can continue because it continually generates vertically-sheared  $u^H$  and thereby cancels the torque reduction. In this way, the net torque stays balanced and the overall overturning circulation stays unforced.

Some examples of such processes are depicted in Fig.7. The conventional quasigeostrophic model considers the torque reduction due to advection by a confluent flow  $v^H$  (Fig.7a). As the advection narrows and reduces the area occupied by  $u^H$  without changing the magnitude of it,  $-fu^H$  integrated between  $y_1$  and  $y_2$  at each vertical moment arm weakens and reduces the overall torque due to  $-fu^H$  (note that the buoyancy and, hence, the hydrostatic pressure at  $y_1$  or  $y_2$  does not change due to the advection). This weakening in  $\int_{y_1}^{y_2} -fu^H dy$  can be compensated if an overturning circulation generates more vertical shear in  $u^H$ . Note that the description here is given from the perspective of a fixed control volume, whereas the more common description of this frontogenesis is given from the Lagrangian perspective. This is because the control volume perspective is less

sensitive to small-scale fluctuations than the common description based on a point-wise local balance (Section 4.5.2). From the Lagrangian perspective, the horizontal pressure gradient acting on a water parcel in a jet  $u^H$  increases as  $v^H$  moves the isopycnals closer to each other. At the same time, the jet remains unchanged. Hence, the pressure gradient force on the parcel starts to overwhelm  $-fu^H$  and overturns the front. This local imbalance is reduced if the overturning circulation enhances the jet (hence,  $-fu^H$ ) as the pressure gradient increases. Similarly to Fig. 7a, Fig. 7b uses the control volume perspective to understand the conventional semi-geostrophic model, which considers advection by the overturning circulation in addition to the advection shown in Fig. 7a. The advection in Fig. 7b again weakens  $\int_{y_1}^{y_2} -fu^H dy$  and results in the same effect.

In the submesoscale front, there are similar processes that tend to reduce the torque by  $-fu^{H}$ . To visualize the rate of change of this torque, (12) is multiplied by -f and integrated in the y-direction at each height within the overturning cell, and the result is shown in Fig. 8 (note  $u^{C} = u^{H}$ ). The frontal advection of  $u^{H}$  by the horizontal circulation (black dashed-dot) and that by the overturning circulation (black solid) tend to reduce the torque by  $-fu^{H}$ , in the same way as in Fig. 7a,b. This reduction is opposed by the generation of along-front jet due to the Coriolis turning of the overturning circulation (magenta) and by the frontal anomaly in the advection of the submesoscale eddy  $u^{H}$  due to the background flows (black dashed). The frontal anomaly in the stress gradient (dark blue) has little systematic relation to the overturning circulation (magenta), indicating that this frontogenesis is not related to the turbulent thermal wind mechanism studied by McWilliams et al. [2015]. The net change (cyan) in the torque by  $-fu^{H}$  is small. As a result, this submesoscale frontogenesis cannot develop a torque by along-front jets

strong enough to balance the source torques (i.e., buoyancy, SSF, interaction between confluent flow and overturning) and stays unbalanced. Hence, the unbalanced net overturning torque keeps accelerating the overturning at a high rate of angular momentum and overturning energy productions (time-scale of hours).

The advection shown by the black dashed line in Fig. 8 simplifies as  $-(u_h^S + u_h^B)\partial_h u^H + \overline{(u_h^S + u_h^B)\partial_h u^H}^B \approx -v^B (\partial_y u^H - \overline{\partial_y u^H}^B)$  for the current front, and it is mainly due to the cross-front Ekman flow advecting the along-front component of the horizontal circulation  $u^H$  associated to the barotropic component of the submesoscale eddies (seen in Fig. 3b) as depicted in Fig. 7c. This term can contribute to the vertical shear of  $\int_{y_1}^{y_2} \partial_t u^H dy$ , typically when there is a vertical shear in  $v^B$  (e.g., background Ekman spiral) and a sudden jump at the front in the barotropic part of  $\partial_y u^H$  (as in Fig. 7c). Both of these conditions are likely in a natural confluent eddy field.

In the previous theories of frontogenesis involving stress [e.g., Garrett and Loder, 1981; Thomas and Lee, 2005; McWilliams et al., 2015], it is essential that 1) a frontal anomaly in the along-front component of the stress gradients (namely,  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$  with j = 1, 2, 3) is produced by the along-front baroclinic jets, and 2) this stress gradient acting on the jets form a balance with the Coriolis force due to the overturning circulation. However, the LES result shows that  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$  is not significantly produced by the baroclinic jets and is also independent from  $fv^{\psi}$  (or  $fv^H$ ). Determination of the reason for this result is beyond the scope of this study, but the following causes are likely. The small-scale motions in this front is heavily affected by the vertical stratification and the Stokes shear force. The vertical shear of the baroclinic jets is less than half of the Stokes drift shear. Hence, the vertical shear of the jets does not have a dominant effect on the turbulence

production, resulting on the little relation between the jets and  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$ . Another important factor is the narrowness and rapid evolution of this front. The observed stress fluctuation occurs at scales below 200 m and evolves significantly over a day, making it difficult for the Coriolis effect to establish a local balance. Therefore, for a wider, slower front with a larger horizontal buoyancy gradient, larger baroclinic shear, and less wave influence, the turbulence may be more dominated by shear turbulence and  $\partial_j L_{1j} - \overline{\partial_j L_{1j}}^B$ may establish a local balance with the Coriolis effect.

Another distinction between this front, formed in the confluence between four submesoscale eddies, and a similar front formed by a strain field of mesoscale eddies is transfer of energy from the eddies to the front. In traditional theory, the strain field is held fixed while the front evolves [e.g., Hoskins and Bretherton, 1972], implying that the source of energy from the strain field is of such magnitude that the front only negligibly affects it. However, consistent with arguments about the important role of fronts in the forward cascade of energy through the submesoscale [Capet et al., 2008a, b], the extraction of energy by the front in this case is not negligible. If the rate of conversion of kinetic energy from the strain field  $(v^H)$  to the frontal flow (Table 2) is compared to the eddy kinetic energy in the four eddies surrounding the examined front (approximately  $0.22 \,\mathrm{J\,m^{-3}}$  over a 4 km by 4 km region nearby), only 9 days would be required to drain all of the eddy kinetic energy (assuming the frontal extraction rate remained constant). If both kinetic and potential energy conversion are considered, then it would require only 5 days for the conversion of all of the eddy kinetic energy and available potential energy (approximately  $0.18 \,\mathrm{J\,m^{-3}}$  over the same region) to be transferred to the front. This rapid timescale reflects both the small scale of submesoscale eddies, but also the intensity and efficiency of

the chosen front. Furthermore, under such rapidly evolving circumstances the diagnosed time tendency and imbalance in the energy, angular momentum, and vorticity budgets are not surprising. Note that, although in general time tendency may oscillate (centering around a thermal wind balance) due to inertial oscillation of a background flow or that of a frontal flow, the time tendency observed here is not oscillatory. The background inertial oscillation in this study has little vertical shear (Section 4.4); hence it cannot tilt and oscillate the frontal isopycnal slope. Also, as shown in Fig. 8, much of the energy transferred to the along-front jet does not increase the torque to the extent that could overshoot, reverse the overturning, and cause oscillation.

As mentioned in the introduction and in *McWilliams and Fox-Kemper* [2013], the parameter  $\epsilon$  is larger for submesoscale fronts than for mesoscale fronts. This parameter, referred to here as the Stokes-front interaction parameter, quantifies the relative importance of Stokes shear force to buoyancy in the vertical momentum budget. If the wave parameters are the same for a mesoscale and a submesoscale front, it is the isopycnal slope (aspect ratio) that governs the magnitude of  $\epsilon$ , and submesoscale fronts. Thus, the Stokes-front interaction ( $\epsilon$ ) is large for this front, and Stokes shear force causes a significant deviation from traditional hydrostatic balance (Table 1), although it might still be the case that a wavy-hydrostatic balance holds, where the combined effect of buoyancy and Stokes shear force dominate the vertical momentum balance. However, the stratification in this front is also small, so both aspect ratio times Froude number are large. Non-hydrostatic effects scale as the square of aspect ratio times Froude number squared [e.g., *McWilliams*, 1985]. Table 1 indicates that indeed the combination of buoyancy force and Stokes shear force do

largely balance the vertical pressure gradient, indicating a leading order wavy-hydrostatic balance. However, the table also shows that the large aspect ratio and Froude number are sufficient to make true deviations from hydrostasy – wavy-hydrostatic imbalances – as important as horizontal sources of angular momentum.

Finally, it is worth emphasizing that the particular values of the budget terms presented in this case study may change in other fronts in other flow environments, although similar balances hold for most of the fronts in this simulation. We have already mentioned that the alignment between the Stokes drift shear and the front, the Stokes drift magnitude, and the isopycnal slope are important factors. Another potentially important factor is the vertical shear of  $v^B$ . A different  $\partial_z v^B$  may change, for example, the contribution of the nonlinear interaction term  $v^{\psi}(-F^{v})$  in the energy budget through the vertical advection of  $v^B$  by  $w^{\psi}$  (which is analogous to the shear production of boundary layer turbulence). The process shown in Fig. 7c also depends on this shear. Other than the aforementioned factors, the horizontal density gradient is also an important factor. For a larger  $\partial_y b$ , the vertical shear of the baroclinic jets would be larger. This may cause a larger frontal anomaly in the stress diffusing the jets ( $L_{13}$ ). Thereby, the turbulent thermal wind mechanism [ $McWilliams \ et \ al.$ , 2015] shown by the dark blue line in Fig. 8 may become more significant.

### 6. Conclusions

In the energy, momentum, angular momentum, and vorticity budgets for the frontal overturning circulation, the Stokes shear force is a leading-order contributor, typically either the second or third largest source of frontal overturning. Because the front examined here is oriented in the down-Stokes direction, the Stokes shear force pushes down the

along-front flow at the cold side of the front and produces a downward jet there, in much the same way as it drives Langmuir circulations where jets along the Stokes shear direction develop into convergence zones (windrows). As a result, the Stokes shear force accelerates the overturning circulation in concert with the buoyancy, leading to sharpening of the front and, under the effect of the Coriolis force, enhancement of the along-front jet. Hence, the fronts that are down-Stokes are stronger than those that are up-Stokes or cross-Stokes (Fig. 1). The Stokes-front interaction parameter ( $\epsilon$ ) in (1) measures the strength of the Stokes shear force versus buoyancy for submesoscale and mesoscale flows. Given that realistic values of  $\epsilon$  are often as large as in this study [Fig. 1 of *McWilliams and Fox-Kemper*, 2013], and given that the fraction of the mixed layer occupied by the Stokes shear in this study is in a typical range, the relative importance of the Stokes shear force found here is a sign that it should often be important in the real-world ocean.

The stronger overturning circulation interacts with the surrounding submesoscale eddies more and extracts not only more potential energy but also more momentum and kinetic energy from submesoscale eddies. This leading order effect potentially plays a significant role in the cascade of upper ocean energy toward smaller scales [e.g., *Capet et al.*, 2008b; *Molemaker and McWilliams*, 2010; *Thomas and Taylor*, 2010]. A greater proportion of the energy produced went into strengthening the frontal overturning in this case rather than enhancing dissipation, but it is not clear how typical this transfer is beyond the context of this simulation.

The results here indicate that the Stokes shear force should be implemented alongside the buoyancy whenever the Stokes-front interaction parameter ( $\epsilon$ ) is expected to be large. While the wavy-hydrostatic balance was closely held here and could be implemented eas-

ily in hydrostatic models by adding the Stokes shear force wherever the buoyancy occurs [Suzuki and Fox-Kemper, 2015], deviations from this balance were as important in the angular momentum budget as the horizontal accelerations. It is difficult to gauge if this fully non-hydrostatic effect is only a result of the front in question being a small, submesoscale example. The Stokes Coriolis force and Stokes advection play a significant role here and elsewhere [Lentz and Fewings, 2012; McWilliams et al., 2012; Haney et al., 2015; Breivik et al., 2015]. Thus, direct substantive impacts of Stokes forces on submesoscale and mesoscale phenomena in the real world and front-permitting simulations are expected, even when boundary layer and Langmuir turbulence are parameterized. Direct observation of these effects would be ideal validation but may be difficult because, even in the model diagnosis here, quantification of the effects required closing budgets to a level of accuracy rarely achieved at sea. However, study of the orientation of frontal strength versus wave direction is a tractable starting point.

### **Appendix A: Derivation of equations** (2)-(6)

In Appendix A, equations (2)-(6) are systematically derived from the original LES equations of motion. In and only in Appendix A, the symbols of the time averaging  $\sim$  and the along-front averaging {} are explicitly shown. That is, any variable  $\phi$  in Appendix A is a variable without being averaged. In contrast,  $\phi$  in all the other sections refers to the quantity already averaged in time and along-front direction. Hence,  $\{\tilde{\phi}\}$  in Appendix A is equal to  $\phi$  in all the other sections.

### A1. Equations of Resolved-Scale Motion

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Resolved-scale flow in the LES satisfies the incompressible wave-averaged, or Craik-Leibovich, equations with Boussinesq approximation, a temperature conservation equation, and the incompressibility equation. Namely,

$$\partial_t u_i = -\partial_i p - \varepsilon_{i3j} f u_j^L + \delta_{i3} b - \partial_j (u_i u_j^L + \tau_{ij}^{SGS}) - u_j^L \partial_i u_j^S, \tag{A1}$$

$$\partial_t \theta = -\partial_j (\theta u_j^L + \tau_{\theta j}^{SGS}), \tag{A2}$$

$$\partial_j u_j^L = \partial_j u_j = 0. \tag{A3}$$

In these equations,  $\mathbf{u} = (u_1, u_2, u_3) = (u, v, w)$  is the resolved Eulerian velocity;  $\mathbf{u}^S = (u_1^S, u_2^S, u_3^S) = (u^S, v^S, w^S)$  is the prescribed Stokes drift;  $\mathbf{u}^L = \mathbf{u} + \mathbf{u}^S$  is the resolved Lagrangian velocity;  $\theta$  is the resolved virtual potential temperature;  $\tau_{ij}^{SGS}$  are components of the subgrid-scale stresses;  $\tau_{\theta j}^{SGS}$  are components of the subgrid-scale fluxes of  $\theta$  from Sullivan et al. [1994]; p is the pressure divided by  $\rho_o = 1000 \text{ kgm}^{-3}$ ;  $b = -g(1 + \beta_T(\theta_o - \theta))$  is the buoyancy where  $g = 9.81 \text{ms}^{-2}$ ,  $\beta_T = 2 \times 10^{-4} \text{K}^{-1}$ , and  $\theta_o = 290.16 \text{K}$ ;  $\varepsilon_{ijk}$  is the Levi Civita symbol; and  $\delta_{ij}$  is the Kronecker delta. Using

$$\tau_{ij}^L \equiv u_i u_j^L + \tau_{ij}^{SGS},\tag{A4}$$

$$\tau^L_{\theta j} \equiv \theta u^L_j + \tau^{SGS}_{\theta j},\tag{A5}$$

we can write equations (A1)-(A2) as

$$\partial_t u_i = -\partial_i p - \varepsilon_{i3j} f u_j^L + \delta_{i3} b - \partial_j \tau_{ij}^L - u_j^L \partial_i u_j^S, \tag{A6}$$

$$\partial_t \theta = -\partial_j \tau^L_{\theta j}.\tag{A7}$$

### A2. Horizontally Uniform Stokes Drift

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For a horizontally uniform Stokes drift – as consistently used here – with  $w^S = 0$ , Equation (A6) becomes

$$\partial_t u_i = -\partial_i p - \varepsilon_{i3j} f u_j^L - \partial_j \tau_{ij}^L + \delta_{i3} (b - u_j^L \partial_z u_j^S).$$
(A8)

### A3. Removal of Dynamically Unimportant Balance

To elucidate the frontal dynamics, the dynamically-unimportant balance between the horizontal averages of vertical forces is removed from (A8). Let  $\langle \phi \rangle$  be the horizontal average of a variable  $\phi$  over the entire LES domain, and let  $\phi'$  be the deviation from this horizontal average; thus,  $\phi(x, y, z, t) = \langle \phi \rangle(z, t) + \phi'(x, y, z, t)$  for any  $\phi$ . Note that  $\langle w \rangle = 0$  for this LES. The horizontal averaging operator commutes with the temporal and spatial differential operators as the boundary conditions in this LES are horizontally periodic. First, write the vertical component of (A8):

$$\partial_t w = -\partial_z p - \partial_j \tau^L_{3j} + b - u^L_j \partial_z u^S_j.$$
(A9)

Next, rewrite (A9) as:

$$\partial_t (\langle w \rangle + w') = -\partial_z (\langle p \rangle + p') - \partial_j \left( \langle \tau_{3j}^L \rangle + (\tau_{3j}^L)' \right) + \langle b \rangle + b' - \langle u_j^L \rangle \partial_z u_j^S - u_j' \partial_z u_j^S.$$
(A10)

Because  $\partial_h \langle \phi \rangle = 0$  for h = 1, 2 and also  $\langle w \rangle = 0$  and w = w', Equation (A10) becomes

$$\partial_t w = -\partial_z p' - \partial_j (\tau_{3j}^L)' + b' - u'_j \partial_z u_j^S + (-\partial_z \langle p \rangle - \partial_z \langle \tau_{33}^L \rangle + \langle b \rangle - \langle u_j^L \rangle \partial_z u_j^S).$$
(A11)

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The last term of the RHS of (A11) is zero for horizontally periodic boundary conditions because (A9) implies

$$0 = \partial_t \langle w \rangle$$

$$= \langle \partial_t w \rangle$$

$$= \langle -\partial_z p - \partial_j \tau_{3j}^L + b - u_j^L \partial_z u_j^S \rangle$$

$$= -\partial_z \langle p \rangle - \partial_z \langle \tau_{33}^L \rangle + \langle b \rangle - \langle u_j^L \rangle \partial_z u_j^S.$$
(A12)

The last equality is valid with horizontally periodic conditions. Using this and the fact that  $\partial_x p = \partial_x P$  and  $\partial_y p = \partial_y P$  where  $P \equiv p' - \langle \tau_{33}^L \rangle$ , we can rewrite (A8) as

$$\partial_t u_i = -\partial_i p' - \varepsilon_{i3j} f u_j^L - \partial_j (\tau_{ij}^L - \delta_{i3} \langle \tau_{3j}^L \rangle) + \delta_{i3} (b' - u'_j \partial_z u_j^S)$$

$$= -\partial_i P - \varepsilon_{i3j} f u_j^L - \partial_j \tau_{ij}^L + \delta_{i3} (b' - u'_j \partial_z u_j^S)$$
(A13)

or equivalently,

$$\partial_t u = -\partial_x P + f v^L - \partial_j \tau^L_{1j}, \tag{A14}$$

$$\partial_t v = -\partial_y P - f u^L - \partial_j \tau^L_{2j},\tag{A15}$$

$$\partial_t w = -\partial_z P - \partial_j \tau_{3j}^L + b' - u'_j \partial_z u_j^S.$$
(A16)

### A4. Equations of Averaged Motion

To eliminate small-scale and fast turbulent fluctuations and elucidate the dynamics of submesoscale flows, smoothing filters are applied to the equations of motion. The filters used are a simple moving average in time (denoted by  $\sim$ ) and a simple moving average in the along-front (i.e., x-) direction (denoted by { }). These filtering operators commute with the differential operators. Then, equations (A14)-(A16) and (A7) imply:

$$\partial_t \{ \widetilde{u} \} = -\partial_x \{ \widetilde{P} \} + f\{ \widetilde{v^L} \} - \partial_j \{ \widetilde{\tau^L_{1j}} \}, \tag{A17}$$

$$\partial_t \{ \widetilde{v} \} = -\partial_y \{ \widetilde{P} \} - f\{ \widetilde{u^L} \} - \partial_j \{ \widetilde{\tau^L_{2j}} \}, \tag{A18}$$

$$\partial_t \{\widetilde{w}\} = -\partial_z \{\widetilde{P}\} - \partial_j \{\widetilde{\tau_{3j}^L}\} + \left\{\widetilde{b'}\right\} - \{\widetilde{u'_j}\}\partial_z u_j^S,\tag{A19}$$

$$\partial_t \{ \widetilde{\theta} \} = -\partial_j \{ \widetilde{\tau_{\theta_j}^L} \}.$$
 (A20)

Combining these equations with  $\{\widetilde{u_j^L}\}\partial_j\{\widetilde{\phi}\} = \partial_j(\{\widetilde{\phi}\}\{\widetilde{u_j^L}\})$ , we can express the equations for the averaged flow as

$$\partial_t \{\widetilde{u}\} + \{\widetilde{u_j^L}\} \partial_j \{\widetilde{u}\} = -\partial_x \{\widetilde{P}\} + f\{\widetilde{v^L}\} - \partial_j L_{1j}$$
(A21)

$$\partial_t \{\widetilde{v}\} + \{\widetilde{u_j^L}\} \partial_j \{\widetilde{v}\} = -\partial_y \{\widetilde{P}\} - f\{\widetilde{u^L}\} - \partial_j L_{2j}$$
(A22)

$$\partial_t \{\widetilde{w}\} + \{\widetilde{u_j^L}\} \partial_j \{\widetilde{w}\} = -\partial_z \{\widetilde{P}\} - \partial_j L_{3j} + \{\widetilde{b'}\} - \{\widetilde{u'_j}\} \partial_z u_j^S \tag{A23}$$

$$\partial_t \{ \widetilde{\theta} \} + \{ \widetilde{u_j^L} \} \partial_j \{ \widetilde{\theta} \} = -\partial_j L_{\theta j}$$
(A24)

where the Leonard stresses for momentum and temperature are

$$L_{ij} \equiv \{\widetilde{\tau_{ij}}\} - \{\widetilde{u}_i\}\{\widetilde{u_j}\}$$

$$= \{u_i u_j^L\} + \{\widetilde{\tau_{ij}^{SGS}}\} - \{\widetilde{u}_i\}\{\widetilde{u_j}\}$$

$$= \{u_i \widetilde{u_j} + u_i u_j^S\} + \{\widetilde{\tau_{ij}^{SGS}}\} - \{\widetilde{u}_i\}\{\widetilde{u_j} + u_j^S\}$$

$$= \{u_i \widetilde{u_j}\} + \{\widetilde{u}_i\}u_j^S + \{\widetilde{\tau_{ij}^{SGS}}\} - \{\widetilde{u}_i\}\{\widetilde{u_j}\} - \{\widetilde{u}_i\}\{\widetilde{u_j}\}$$

$$= \{\widetilde{u}_i \widetilde{u_j}\} + \{\widetilde{\tau_{ij}^{SGS}}\} - \{\widetilde{u}_i\}\{\widetilde{u_j}\},$$

$$L_{\theta j} \equiv \{\widetilde{\tau_{\theta j}}\} - \{\widetilde{\theta}\}\{\widetilde{u_j}\}$$

$$= \{\widetilde{\theta}\widetilde{u_j}\} + \{\widetilde{\tau_{\theta j}^{SGS}}\} - \{\widetilde{\theta}\}\{\widetilde{u_j}\}.$$
(A25)
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### Appendix B: Defining the Background Mode

Let  $\overline{\phi}^B$  be the horizontal average of a variable  $\phi$  over the surrounding regions shown in Figs. 2-3. That is,

$$\overline{\phi}^B(z,t) \equiv \frac{\phi_1(z,t) + \phi_2(z,t)}{2},\tag{B1}$$

where  $\phi_1$  is the horizontal average of  $\phi$  over one of the surrounding regions, and  $\phi_2$  is the horizontal average over the other one. Note that the areas of the surrounding regions are equal to each other, and the positions of the surrounding regions do not change with time [within the duration of O(10 min) used in the budget analysis of this study]. This averaging is idempotent  $(\overline{\phi}^B{}^B = \overline{\phi}^B)$  and commutes with  $\partial_t$  and  $\partial_z$ . In addition,  $\partial_x \overline{\phi}^B = \partial_y \overline{\phi}^B = 0$ whereas  $\overline{\partial_x \phi}^B$  or  $\overline{\partial_y \phi}^B$  may not be zero.

Using this averaging, the background (horizontally-uniform) flow is defined as

$$u^B \equiv \overline{u}^B, \quad v^B \equiv \overline{v}^B, \quad w^B \equiv 0.$$
 (B2)

Then, deviations from the background flow are

$$(u^{C}, v^{C}, w^{C}) \equiv (u, v, w) - (u^{B}, v^{B}, w^{B}).$$
 (B3)

Deviations are denoted by a superscript C because they are frontal circulations. Note  $w^B = 0$  by definition. Hence,  $w^C = w$ . Because both (u, v, w) and  $(u^B, v^B, w^B)$  are divergence free,  $(u^C, v^C, w^C)$  is also divergence free.

### Appendix C: Defining the Frontal Overturning Circulation

Frontal circulations are composed of overturning circulations ( $\psi$ ) that lie on y-z planes and circulations of submesoscale eddies and along-front jets, both of which have vertical velocities negligibly small compared to vertical velocities of overturning circulations (hence, they virtually lie on x-y planes within a frontal region). For the analysis of frontal overturning dynamics, it is essential to distinguish these modes without introducing undue noise or uncertainty. However, such identification is not trivial because even after the time and along-front averaging, there are some remaining turbulence signals (e.g., see Fig. 3d) which make accurate diagnosis of  $\psi$  difficult. However, after experimentation,

it was found that the confluent submesoscale eddy flow  $v^H$  is governed in a tractable way; that is, turbulence fluctuations in (20) are small enough compared to the signals of submesoscale flows, and it allows accurate diagnosis of  $v^H$ . Therefore, our diagnosis starts with finding  $v^H$  and, then, finds  $\psi$  implied by the  $v^H$ . Namely, the components of  $v = v^B + v^H + v^{\psi}$  are found in the following way:

- 1. Obtain  $v^B$  by  $v^B = \overline{v}^B$ .
- 2. Obtain  $v^{C} = v^{H} + v^{\psi}$  by  $v^{C} = v v^{B}$ .

3. Let  $\langle \phi \rangle_{DA}$  be the depth-average mode of a variable  $\phi$ ; namely,  $\langle \phi \rangle_{DA} = \frac{1}{z_t - z_b} \int_{z_b}^{z_t} \phi \, dz$ where  $z_t$  and  $z_b$  are the top and bottom of the frontal region, respectively. Let  $\langle \phi \rangle_{BC}$  be the baroclinic mode  $\langle \phi \rangle_{BC} = \phi - \langle \phi \rangle_{DA}$ . We find  $\langle v^H \rangle_{DA}$  and  $\langle v^H \rangle_{BC}$  separately. First, obtain  $\langle v^H \rangle_{DA}$  using  $\langle v^H \rangle_{DA} = \langle v^C \rangle_{DA}$ , which is true because  $\langle v^{\psi} \rangle_{DA} = 0$ . The last equality is true because the overturning satisfies  $v^{\psi} = -\partial_z \psi$  and the stream function is  $\psi = 0$  at the top and bottom of the frontal region (Fig. 3c).

4. Obtain  $\langle v^H \rangle_{BC}$  by an empirical force balance. First, the data shows that  $fv^H \approx \partial_x p' + u^S \partial_x u - \overline{(\partial_x p' + u^S \partial_x u)}^B$  is a good approximation when the terms on the RHS are additionally filtered by a Gaussian filter in the cross-front direction to remove its small-scale turbulent fluctuations. In addition, the LES data shows that  $\langle \partial_x p' + u^S \partial_x u - \overline{(\partial_x p' + u^S \partial_x u)}^B \rangle_{BC} \approx \langle \partial_x p'_b - \overline{\partial_x p'_b}^B \rangle_{BC}$  is also a very good approximation when turbulent fluctuations are removed and where  $p'_b$  is defined by  $\partial_z p'_b = b'$ . Therefore, obtain  $\langle v^H \rangle_{BC}$  by  $f \langle v^H \rangle_{BC} \approx \langle \partial_x p'_b - \overline{\partial_x p'_b}^B \rangle_{BC}$  where small-scale fluctuations in the RHS are removed. This procedure allows us to find not only  $v^H$  but also  $\partial_t v^H$  by replacing  $p'_b$  with  $\partial_t p'_b$ . We can obtain  $\partial_t p'_b$  from  $\partial_t b'$ .

5. Obtain  $v^{\psi}$  by  $v^{\psi} = v^C - v^H$ .

6. Finally, obtain  $w^{\psi}$  using  $\partial_y v^{\psi} + \partial_z w^{\psi} = 0$  with  $w^{\psi} = 0$  at the surface.

A good Gaussian filter width is found by trial-and-error seeking the following conditions. 1) The frontal circulations analyzed have two dominant components: circulations on yz planes and those on x-y planes. Any circulation on a x-z plane is due to small-scale turbulence. Thus, divergence  $\partial_y v^H$  should cancel the submesoscale pattern seen in  $\partial_x u$ and minimize the residual  $\partial_x u + \partial_y v^H$ , which is essentially small-scale fluctuations. 2) For the same reason, we expect that  $\partial_y v^{\psi} + \partial_z w \approx 0$ ; that is, the vertical velocity is dominantly due to the overturning. Thus, maximize  $w \approx w^{\psi}$ . 3)  $v^{\psi}$  is mainly ageostrophic. 4) The extent of the overturning is reasonably contained near the frontal region.

### Notation

horizontal average over the entire domain. deviation from the domain-wise horizontal average: such as  $\phi' = \phi - \langle \phi \rangle$ . simple moving average in time. Notation omitted unless otherwise noted. { } simple moving average in the alongfront direction. Notation omitted unless otherwise noted. t time coordinate. along-front, cross-front, and vertical x, y, zcoordinate, respectively. u, v, w along-front, cross-front, and vertical velocity, respectively.  $\rho_o$  constant background density: 1000 kgm<sup>-3</sup>. potential temperature.  $\theta$ constant background potential tem- $\theta_{o}$ perature: 290.16 K.  $\beta_T$  thermal expansion coefficient: 2 ×  $10^{-4} \mathrm{K}^{-1}$ .

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$$\begin{array}{lll} f & \text{Coriolis parameter: } 0.7 \times 10^{-4} \mathrm{s}^{-1}.\\ p & \text{pressure divided by } \rho_o.\\ P & p' = \langle \tau_{33}^L \rangle.\\ b & \text{buoyancy: } b \equiv -g \big( 1 + \beta_T (\theta_o - \theta) \big)\\ \text{where } g = 9.81 \mathrm{ms}^{-2}.\\ \tau_{ij}^L & \text{Lagrangian stresses defined by Eqs. (A4)-}\\ & (A5).\\ L_{ij} & \text{Leonard stresses defined by Eqs. (A25)-}\\ & (A26).\\ u_i^S & \text{Stokes-drift velocity.}\\ u_i^L & \text{Lagrangian velocity: } u_i + u_i^S.\\ \hline & \text{-}^B & \text{operation used to find the back-}\\ & \text{ground mode: namely, horizontal}\\ & \text{average taken over the two box re-}\\ & \text{gions surrounding the front region.}\\ \end{array}$$

Acknowledgments. The authors would like to acknowledge fruitful conversations with Eric D'Asaro, Sean Haney, and James McWilliams. This work would not be possible without the use of the NCAR LES code written by Peter Sullivan and collaborators. N.S. is supported by Brown University and a grant from the Rhode Island Science & Technology Advisory Council. B.F.-K.'s contribution was made possible in part by a grant from The Gulf of Mexico Research Initiative and in part by NSF 1258907. P.E.H was supported

in part by NSF 1258995. The data analyzed in this paper is available from the authors upon request (nobuhiro\_suzuki@brown.edu).

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Table 1. Integrated Budget for the Angular Momentum of the Overturning Circulation<sup>a</sup>a) Torques by Vertical Forces

	responsible force : torque term	relative value
$\mathbf{C}$	net vertical force : $(y - y_0)(\partial_t + u_j^L \partial_j)w^{\psi}$	$26\pm7\%$
Sources		
	buoyancy anomaly : $(y - y_0)b'$	$13371 \pm 1209\%$
	Stokes shear force anomaly : $(y - y_0)(-u'_i\partial_z u^S_i)$	$3517\pm229\%$
frontal a	anomaly in turbulent vert. stress : $(y - y_0) \left( -\partial_j L_{3j} + \partial_z \langle \tau_{33}^L \rangle \right)$	$0\pm2\%$
Sinks	vertical pressure grad. : $(y - y_0)(-\partial_z p')$	$-16866 \pm 1431\%$
b) Torqu	ies by Horizontal Forces	
	responsible force : torque term	relative value
	net horizontal force : $(z_0 - z)(\partial_t + u_j^L \partial_j)v^{\psi}$	$38\pm8\%$
Sources		
frontal	anomaly in horiz. pressure grad. : $(z_0 - z) \left( -\partial_y p' + \overline{\partial_y p'}^B \right)$	100%
	interaction with $v^H$ : $(z_0 - z)(-F^h)$	$32\pm6\%$
	nonlinear interaction with $v^B$ : $(z_0 - z)(-F^v)$	$2\pm1\%$
Sinks	Coriolis on along-front current : $(z_0 - z)(-fu^H)$	$-68 \pm 5\%$
frontal in	belong in ways Elemen relation : $(z_1 - z)(-\partial I - \frac{\partial I}{\partial I}^B)$	$37 \pm 3\%$
	Ibarance in wavy Exman relation . $(z_0 - z)(-\partial_j L_{2j} + \partial_j L_{2j})$	$-27 \pm 370$
c) Disloc	cation Cost	nolotivo voluo
	$\frac{1}{1} \frac{1}{1} \frac{1}$	$\frac{10 \pm 107}{10}$
	dislocation cost : $(v - v^{\dagger})w^{\dagger}$	$-19 \pm 170$
<sup>a</sup> All term	as are integrated over the overturning cell and normalized versu	s horizontal pres
adient tor	que, $\int (z - z_0) (\partial_y p' - \overline{\partial_y p'}^B) dy dz = -0.31 \pm 0.01 \text{m}^4 \text{s}^{-2}$ . Post	tive values speed
nd negative	e values slow down the overturning circulation. Uncertainty is d	ue to variation in

values of decomposed modes. This variation results from variation in the widths of front region and surrounding regions.

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Table 2.Integrated Budget for the Kinetic Energy of the Overturning CirculationaRate of Change of Overturning Circulation KE

name : term	relative value
total : $(\partial_t + u_j^L \partial_j) \frac{v^{\psi} v^{\psi} + w^{\psi} w^{\psi}}{2}$	$45\pm6\%$
Sources	
buoyancy production : $w^{\psi}b'$	100%
energy increase due to interaction with $v^H$ : $v^{\psi}(-F^h)$	$49\pm5\%$
Stokes shear force work : $w^{\psi}(-u'_i\partial_z u^S_i)$	$24\pm1\%$
energy increase due to nonlinear interaction with $v^B$ : $v^{\psi}(-F^{v})$	$7\pm1\%$
Sinks	
generation of along-front jet by Coriolis turning of $v^{\psi}$ : $-fv^{\psi}u^{H}$	$-69\pm3\%$
work done against Coriolis of background flows : $v^{\psi} \left( \overline{\partial_y p'}^B + \overline{\partial_j L_{2j}}^B \right)$	$-45\pm3\%$
generation of shear turbulence : $L_{kj}\partial_j u_k^{\psi}$	$-16\pm1\%$
turbulent transport through the cell boundary : $-\partial_j(u_k^{\psi}L_{kj})$	$-2\pm0.4\%$
<sup>a</sup> All terms are integrated over the overturning cell and normalized versus buoya	ancy production,
$\int w^{\psi} b' dy dz = (1.23 \pm 0.05) \times 10^{-4} \text{m}^4 \text{s}^{-3}$ . Uncertainty is due to variation	in the values of
decomposed modes. This variation results from variation in the widths of f	ront region and
surrounding regions.	

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### Table 3.Integrated Budget for Overturning VorticityaRelative Tendency of Overturning Circulation along the Cell Boundary

	J
responsible force : vorticity term	relative value
net tendency : $\partial_t (-\partial_z v^{\psi} + \partial_y w^{\psi})$	$11\pm8\%$
Sources	
buoyancy anomaly : $\partial_y b'$	100%
Stokes shear force anomaly : $\partial_y(-u'_j\partial_z u^S_j)$	$44 \pm 4\%$
interaction with $v^H : -\partial_z (-F^h)^{"}$	$44\pm8\%$
frontal anomaly in pressure gradient : $-\partial_z \left( -\frac{\partial_y p'}{\partial_y p'} + \overline{\partial_y p'}^B \right) + \partial_y \left( -\partial_z p' \right)$	)
$= -\partial_z \partial_y p'^D$	$6\pm9\%$
nonlinear interaction with $v^B : -\partial_z(-F^v)$	$2 \pm 1\%$
Sinks	
frontal turbulence anomaly : $-\partial_{\tau}(-\partial_{\lambda}L_{2\lambda}+\overline{\partial_{\lambda}L_{2\lambda}}^{B})$	
(mostly, imbalance in wavy Ekman relation) $+\partial_{\alpha}(-\partial_{\beta}L_{2\beta} + \partial_{\beta}(\tau_{2\alpha}^{L}))$	$-82 \pm 11\%$
Coriolis on along-front jet : $-\partial_r(-fu^H)$	$-66 \pm 2\%$
Lagrangian advection of $(v^{\psi}, w^{\psi}) : -\partial_z (-u^L_L \partial_i v^{\psi}) + \partial_w (-u^L_L \partial_i w^{\psi})$	$-36 \pm 7\%$
	00 _ 0,0
<sup>a</sup> All terms are integrated over the overturning cell and normalized versus the	e buoyancy term,
$\int \partial_y b' dy dz = (-1.14 \pm 0.01) \times 10^{-3} \text{m}^2 \text{s}^{-2}$ . The integration of vorticity tend	dency yields the
tendency of the circulation around the overturning cell. Positive values speed	up and negative
- less the simulation. Il containte is due to multiplice in the order	f. ]
values slow down the circulation. Uncertainty is due to variation in the value	s of decomposed
modes. This variation results from variation in the widths of front region and sur	counding regions.

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Figure 1. A snapshot of near-surface vertical vorticity scaled by the Coriolis parameter from each of the two runs analyzed in *Hamlington et al.* [2014]: [left] wind and wave (Stokes) forcing and [right] only wind forcing. The strongest frontal features are parallel to the direction of wave forcing in the wave-forced (left) panel, while fronts in other directions are much weaker. In the wind-only simulation, frontal strength is more isotropic. The yellow box indicates the location of Fig. 2.

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Figure 2. Instantaneous horizontal plane views of the region encompassing the front analyzed in detail. The "front region" (solid box), the "surrounding region" (dashed boxes), and the orientation of the coordinate system (x and y) are indicated in each panel. Each primed variable shown represents the difference from the horizontal mean. The depths (-11 m and -45 m) shown are representative of other depths near the top and bottom of the mixed layer, respectively.

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Figure 3. Sections in y-z plane of time-averaged and along-front averaged variables. The cross-front coordinate y, the front region (solid box), and the surrounding region (dashed boxes) corresponding to those in Fig. 2 are shown in every panel. a) Temperature reveals a sharp gradient, and sloping isotherms throughout the mixed layer. b) Along-front current  $u^H$  shows a vertical shear, typical of thermal wind-like pattern. c) Overturning stream function shows the cross-frontal and vertical velocity of the front. The overturning cell where  $\psi \geq 0$  is indicated by the orange line. The cross indicates  $(y_0, z_0)$ , which is the center of mass of the overturning cell. d) Vertical velocity shows a noisy pattern, but note the downward frontal jets on the cold side of the front and a broader upwelling on the warm side of the front. e) Large-scale cross-frontal flow shows the confluence contributing to frontogenesis. Details of averaging given in the text. The print 21, 2016, 3:01pm

## Figure 5

Figure 5. Schematic diagram showing how the overturning circulation is forced by some forces in equations (15) and (18). The black contours are  $\psi$ , and the overturning circulation is clockwise. Darker gray indicates denser water. The buoyancy anomaly b' turns the front clockwise. The Coriolis force due to the along-front jets  $u^H$  turns the front counter-clockwise. 1) The Stokes shear force  $-u'\partial_z u^S \approx -u^H \partial_z u^S$  triggered by the near surface jet pushes the overturning circulation clockwise. 2) The overturning circulation advects the momentum of the confluent eddy field  $v^H$  by  $-v^{\psi}\partial_y v^H$ , and this interaction converts  $v^H$  to  $v^{\psi}$ . As a result, this term forces the overturning circulation clockwise. 3)  $\partial_z v^{\psi}$  is especially high in this near-surface region and disturbs the cross-front stress gradient  $\partial_z L_{23}$ , leading to an imbalance in the wavy Ekman relation which forces the front counter-clockwise.

Figure 7. Schematic diagram showing the effect of advection by a) the confluent flow, b) the overturning circulation, and c) the background flow. The vertical solid lines are isopycnals. The dashed lines indicate the area of integration between  $y_1$  and  $y_2$ . In panel c), the  $u^H$  shown is that of the larger-scale barotropic flow. Notice that the buoyancy hence the hydrostatic pressure at  $y_1$  or  $y_2$  does not change due to the advection shown; thus, the integrated torque due to the horizontal pressure gradient does not change. On the other hand, the integrated torque due to the along-front current  $-fu^H$  changes due to the advection of  $u^H$ .

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Figure 8. The horizontally-integrated rate of change of a major torque component,  $-fu^H$ . The equation of motion for  $u^H(=u^C)$  [(12)] is multiplied by -f and integrated in the y-direction inside the overturning cell. The integration symbol is omitted in the legend for ease of notation. The terms not shown are all small. Before the integration, the depth average of each term is subtracted to highlight its vertical shear.

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Along Initial Front Direction (km)








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a) confluent flow  $(v^H)$  effect on  $u^H$ 



b) overturning flow  $(v^{\psi})$  effect on  $u^H$ 



c) background flow  $(v^B)$  effect on  $u^H$ 



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$$-f\partial_{t}u^{H} - f^{2}v^{\psi} -f\left(-\partial_{j}L_{1j} + \overline{\partial_{j}L_{1j}}^{B}\right) - -f\left(-(u_{h}^{S} + u_{h}^{B})\partial_{h}u^{H} + \overline{(u_{h}^{S} + u_{h}^{B})}\partial_{h}u^{H}^{B}\right) - -f\left(-u_{h}^{H}\partial_{h}u^{H} + \overline{u_{h}^{H}}\partial_{h}u^{H}^{B}\right) - -f\left(-v^{\psi}\partial_{y}u^{H} - w^{\psi}\partial_{z}u^{H} + \overline{v^{\psi}}\partial_{y}u^{H} + w^{\psi}\partial_{z}u^{H}^{B}\right)$$

