CHARACTERISTICS OF WAVE REFLECTION SPECTRA

By James Sutherland[1] and Tom O'Donoghue[2]

ABSTRACT: Experimental results of irregular wave reflection from impermeable walls and rubble mound breakwaters suggest that there is a characteristic form of wave reflection spectrum. The results indicate that the reflection coefficient at a particular frequency within a random wave spectrum is determined by an Iribarren number based on that frequency. An empirical relationship between reflection coefficient and local, frequency-dependent Iribarren number is provided for both structure types, although the range of parameters used in the rubble mound tests is limited. Therefore, the reflection coefficient spectrum can be determined from the structure slope, frequency, incident significant wave height, and two fitted coefficients that vary with structure type. Specification of the reflection coefficient spectrum becomes more critical as the width of the incident wave spectrum increases and as the Iribarren number decreases. Otherwise, random wave reflection can be adequately determined using the bulk reflection coefficient across the frequency range of the incident sea.

INTRODUCTION

The wave kinematics in front of a coastal structure are highly dependent on the wave reflection characteristics defined by the reflection coefficient spectrum and the phase-shift spectrum. The reflection coefficient spectrum relates the magnitude of the reflected wave spectrum to that of the incident wave spectrum at each frequency and is inherently linked to the spectral variation in energy dissipation. The phase-shift spectrum relates the phase of the reflected wave to that of the incident wave at the toe of the structure for each frequency. Details of a recent treatment of the phase-shift spectrum can be found in Sutherland and O'Donoghue (1998), which includes empirical and theoretical expressions. It is common in engineering simply to use the bulk reflection coefficient to characterize the magnitude of the wave reflection. The bulk reflection coefficient relates the reflected significant wave height to the incident significant wave height and is a single value for each sea state. This averaging process means that potentially important information is missing from the treatment of the waves so that, for example, the wave spectrum at a position in front of the structure (including incident and reflected waves) may be incorrectly calculated. It is only by using the reflection coefficient spectrum that the details of the wave field in front of a reflecting structure may be determined. This paper examines possible ways of characterizing the reflection coefficient spectrum for impermeable and rubble mound structures.

Prediction of Bulk Reflection Coefficients

There have been many attempts to find a formula that will accurately predict the bulk reflection coefficient given a particular incident sea state and coastal structure. Many of them fit measured bulk reflection coefficients to various nondimensional parameters. The most common parameter used for reflections from coastal structures is the Iribarren number or the surf-similarity parameter ξ, which relates the wall slope to the wave steepness and is defined as

$$\xi = \frac{\tan\alpha}{\sqrt{\frac{H_s}{L_o}}} = \frac{\tan\alpha}{f_p}\sqrt{\frac{g}{2\pi H_s}} \tag{1}$$

where α = wall slope; H_s = incident significant wave height; $L_o = g/(2\pi f_p^2)$ = linear theory deepwater wavelength calculated at the peak frequency, f_p, of the spectrum. Examples of this work include Seelig and Ahrens (1981), Allsop and Hettiarachchi (1988), and Davidson et al. (1996a,b), all of which include reviews of other research. Seelig and Ahrens (1981) expressed the bulk reflection coefficient as

$$C_{rb} = \frac{a\xi^p}{b + \xi^p} \tag{2}$$

where a, b, and p = coefficients that vary with the type of structure. Seelig and Ahrens (1981) set $p = 2$ and recommend $a = 1$ and $b = 5.5$ for smooth, impermeable walls and $a = 0.6$ and $b = 6.6$ for rubble mound breakwaters. The limiting value of the reflection coefficient for high Iribarren numbers is given by a, and so this is set to 1 for idealized smooth, impermeable structures.

Observations of Reflection Coefficient Spectra

There has not been many papers that have looked specifically at the characteristics of reflection coefficient spectra. Several papers have presented measured reflection coefficient spectra and have commented on their form. Papers dealing with field measurements of waves in front of coastal structures include Takezawa et al. (1993), Davidson et al. (1994), and Dickson et al. (1995), which all show measured reflection coefficient spectra. Generally, the reflection coefficients have decreased as frequency increased and wall slope decreased (Davidson et al. 1994) without depending strongly on incident wave height (Dickson et al. 1995).

Some reflection coefficient spectra have been presented for sea swell on natural beaches, for example, by Elgar et al. (1994), and Elgar et al. (1997) who found that the reflection coefficient decreased as frequency or incident significant wave height increased and as the beach slope decreased. Thus similar trends to those for coastal structures are observed for beaches that have much lower frequencies and slopes.

Examples of reflection coefficient spectra have been included in papers concerned mainly with the methods of determining reflection coefficients by, among others, Mansard and Funke (1980), Seelig and Ahrens (1981), and Teisson and Benoit (1994), all of whom showed reflection coefficients decreasing with frequency.

Laboratory tests on breakwaters have sometimes reported measurements of reflection coefficient spectra. Kobayashi et al. (1990) measured irregular waves at the toe of a 1:3 im-

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permeable, rough slope. Three seas were used with Iribarren numbers in the range 1.9–5.0. The measured reflection coefficient spectra and surface elevation time series were compared to the results from their numerical model for wave behavior at and on coastal structures. The agreement was generally good, and both measured and computed reflection coefficients decreased fairly smoothly as frequency increased for those frequencies for which there was appreciable energy. No attempt was made to parameterize the reflection coefficients.

Isaacson et al. (1996) compared the reflection coefficient spectrum from a random sea with a peak frequency of 0.625 Hz and significant wave height of 0.12 m to the bulk reflection coefficients from a series of tests using regular waves with frequencies from 0.4 to 1.0 Hz and a height of 0.1 m. A rubble mound breakwater was used. They found that the regular wave bulk reflection coefficients gave roughly the same variation with frequency as the reflection coefficient spectrum from the random sea. This suggests that random wave reflection can possibly be addressed as a largely linear process in that similar results come from a random sea as from a succession of separate, regular waves.

Parameterization of Reflection Spectra

The study of Hughes and Fowler (1995) is the first that the writers are aware of that attempts to characterize the reflection coefficient spectrum and the phase-shift spectrum for irregular waves. They measured irregular waves in front of smooth, impermeable walls (with three front slopes) and a rubble mound breakwater and used a colocated velocities method to determine reflection coefficient spectra. Three peak frequencies and three toe depths were used with a slightly different significant wave height for each toe depth. The reflection coefficients were plotted against a nondimensional parameter, here called χ, defined as

$$\chi = \frac{f_n}{\tan\alpha}\sqrt{\frac{d_t}{g}} = \frac{x_m}{L_s} \tag{3}$$

where f_n = frequency of nth component; d_t = water depth at the toe of the structure; $x_m = d_t/\tan\alpha$ = cross-shore length of structure from toe to still water level; and $L_s = \sqrt{gd_t}/f_n$ = linear theory shallow water wavelength. Empirical relationships for the reflection coefficient and phase spectra were found for the impermeable walls and the rubble mound breakwater.

The parameter χ is one that Hughes and Fowler (1995) and, subsequently, Sutherland and O'Donoghue (1998) used to determine the spectrum of wave phase shift on reflection. This works well because the phase shift is, for practical purposes, independent of the wave height. However, the reflection coefficient is expected to have a dependence on wave height so that it is unlikely that χ can characterize the reflection coefficient spectrum over a wide range of incident wave heights.

The present paper uses small-scale laboratory test data in an attempt to characterize reflection coefficient spectra, using a broader range of parameters than in Hughes and Fowler (1995), thereby, giving greater credibility to the resulting empirical relationship. It will also show that χ is not an appropriate parameter for characterizing the reflection coefficient spectrum and that better results are obtained by using a frequency-dependent Iribarren number.

There is no universally agreed upon formula for predicting bulk reflection coefficients [although variations of Eq. 2 from Seelig and Ahrens (1981) appear to be the most commonly used], and so it would be unrealistic to expect a universally applicable relationship for the reflection coefficient spectrum to come out of the present study. This study does suggest, however, that there is a characteristic form for the wave reflection spectrum and indicates the form that it might take.

PROBLEM OUTLINE

Two-dimensional linear wave theory is used so that the incident random sea is treated as the linear superposition of a number of component waves, each with its own amplitude a_n, circular frequency $2\pi f_n$ and randomly allocated phase at the origin ε_n. The coordinate system is shown in Fig. 1 where x and z are the cross-shore and vertical directions respectively. The frequency of the nth component is $f_n = nf_s/N$, where N is the number of samples and f_s is the sampling rate. Each component is reflected separately with the nth reflected component related to the incident component by its reflection coefficient C_{rn} and phase shift γ_n taken from their respective spectra.

The surface elevation $\eta(x, t)$, at a position x in front of the structure and at a time t is given by

$$\eta(x, t) = \sum_{n=0}^{\infty} a_n\,[\cos(k_n x + \varepsilon_n - 2\pi f_n t)$$

$$+ C_{rn}\cos(k_n x + \gamma_n - \varepsilon_n + 2\pi f_n t)] \tag{4}$$

where k_n = wave number of the nth component determined from the linear theory dispersion relationship, $(2\pi f_n)^2 = gk_n \tanh k_n d$ where d is the water depth.

Wave gauges are used to measure time series of surface elevation in front of the structure where the incident and reflected waves interact. The nth component of surface elevation energy density spectrum $S_{\eta\eta}(x, f_n)$, at a point x, is given by

$$S_{\eta\eta}(x, f_n) = S_{ii}(f_n)[1 + C_{rn}^2 + 2C_{rn}\cos(2k_n x + \gamma_n)] \tag{5}$$

where $S_{ii}(f_n)$ = energy density spectrum of the incident waves (m^2/s). The mean-square value of the incident wave surface elevation is given by the zeroth-order spectral moment of the incident spectrum m_{0i}

$$m_{0i} = \int_0^{\infty} S_{ii}(f_n)\,df \tag{6}$$

The incident significant wave height H_s is then $H_s = 4\sqrt{m_{0i}}$.

The least-squares analysis method of Mansard and Funke (1980) is used to separate the incident and reflected wave spectra from measured time series from three wave gauges. The nth component of the reflected energy density spectrum is denoted $S_{rr}(f_n)$ and the zeroth-order spectral moment of the reflected spectrum m_{0r} is

$$m_{0r} = \int_0^{\infty} S_{rr}(f_n)\,df \tag{7}$$

The reflected significant wave height is $H_{sr} = 4\sqrt{m_{0r}}$ and the bulk reflection coefficient is defined by

$$C_{rb} = \sqrt{\frac{m_{0r}}{m_{0i}}} = \frac{H_{sr}}{H_s} \tag{8}$$

The nth component of the reflection coefficient spectrum is

$$C_{rn} = \sqrt{\frac{S_{rr}(f_n)}{S_{ii}(f_n)}} \tag{9}$$

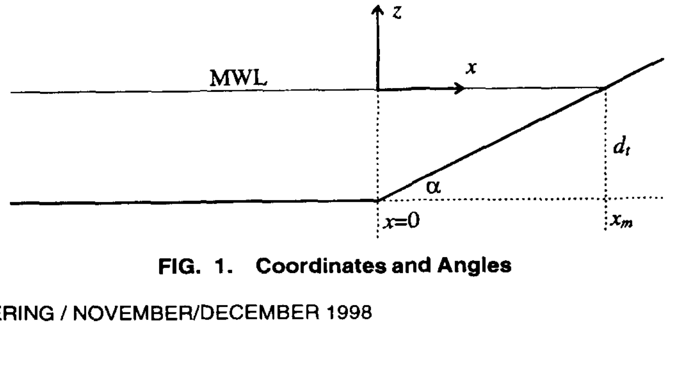

FIG. 1. Coordinates and Angles

Linear theory may be applied easily to obtain expressions for the velocity spectra, given the incident wave, reflection coefficient, and the phase-shift spectra as in Hughes and Fowler (1995), O'Donoghue and Goldsworthy (1995) and Sutherland and O'Donoghue (1997).

DESCRIPTION OF TESTS

Flume Tests at Aberdeen University

The flume tests were carried out in the 20-m-long wave flume at Aberdeen University. The flume is 0.45 m wide and has a still water depth at the wave paddle of 0.70 m. Waves are generated by a bottom-hinged, absorbing paddle. For the tests conducted for this study, the flume contained a perspex beach consisting of a 1:20 sloping section that brings the water depth to 0.145 m, followed by a 4.1-m-long horizontal section as shown in Fig. 2.

Tests were conducted using a smooth, impermeable wall and a rubble mound structure. The impermeable wall was tested using two toe depths, 0.145 m (Test 1) and 0.37 m (Test 2) as illustrated in Fig. 2(a). The rubble mound breakwater tests (Test 4) were conducted with the structure on the 1:20 slope with a toe depth of 0.23 m as illustrated in Fig. 2(b). An absorbing foam beach was placed on the flat section behind the structure to absorb any transmitted waves. The rubble mound breakwater had a front slope of 1:2, a crown height of 0.35 m, a crest width of 0.16 m, a rear slope of 2:3, and no core. The rocks had a characteristic diameter D = 45.6 mm found from the median mass of the stones and their density.

The waves in front of the structure were measured by three wave gauges in an array between 0.2 and 0.3 m long. The sample rate was f_s = 8 Hz and the number of samples was N = 4,096/gauge. The wave gauge spacing was based on the criteria set out by Mansard and Funke (1980), but the same spacing was used for all seas. Some repeat tests were done with a wider spacing (appropriate for the lowest frequency sea), but the reflection coefficient spectrum results were insignificantly different from those using the narrower spacing. Sensitivity tests were carried out to gauge the effect of the sloping beach on the results of the reflection analysis. There were no significant differences between reflection coefficient spectra obtained using the depth as the inshore wave gauge, the depth at the offshore wave gauge, and the average depth over the three wave gauges. At frequencies away from the peak of the spectrum the coherence between gauges fell away, and the reliability of the reflection coefficients dropped. Experience showed that results with significant noise and low coherence were filtered out by including only frequency components about the peak, where the incident spectral density was >10% of the maximum incident spectral density.

The ratio of the two toe depths for the impermeable wall experiments was chosen to be quite large (here 2.55) to ensure that any dependence on toe depth in intermediate to shallow water was brought out in the experiments. The parameter χ includes the square root of the toe depth so that using the same input wave spectra with the same slope of structure at the two depths tests whether χ can be used to characterize the wave reflection spectrum. Four seas of the same spectral shape, but different peak frequencies, were generated at two significant wave heights—one low, the other high. The four target peak frequencies were chosen to cover the working frequency range of the paddle while being close enough together so that the incident wave spectra overlapped. The incident wave spectra were adjusted so that all four low wave spectra had approximately the same measured incident significant wave height as did all four high wave spectra. The high seas were as high as possible without having more than very occasional wave breaking between the structure toe and the wave gauges. The low seas were as low as possible to give a large ratio between the high and low significant wave heights while still giving breaking on the structure for low slopes or high peak frequencies.

Table 1 summarizes all of the test conditions, including those tests carried out in the U.K. Coastal Research Facility (CRF).

Basin Tests at CRF

Reflection coefficients were measured for an impermeable sloping wall and a rubble mound breakwater in the CRF. The CRF is a 36 by 27 m wave basin with, for the experiments reported here, a water depth at the paddles of 0.5 m, as described in Sutherland and O'Donoghue (1998). A 1:20 rough

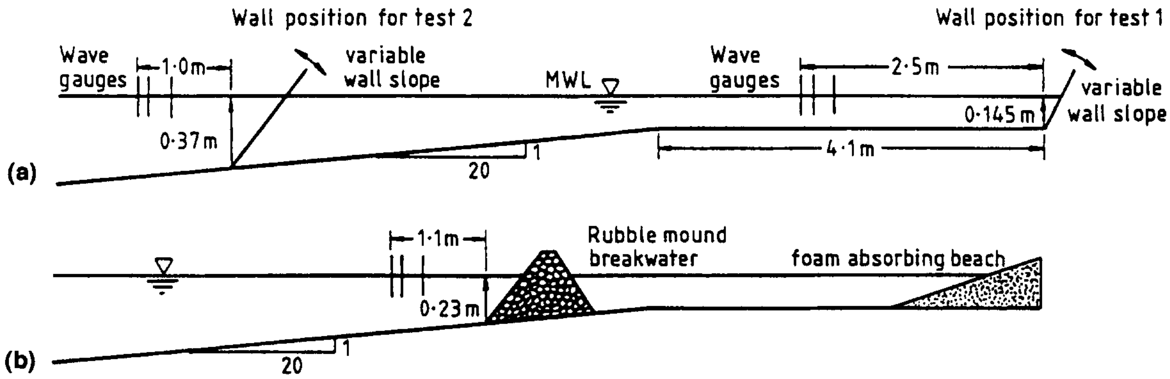

FIG. 2. Setup for Impermeable Wall and Rubble Mound Breakwater Experiments in the Aberdeen University Wave Flume: (a) Tests 1 and 2; (b) Test 4

TABLE 1. Summary of Experimental Conditions

Test number (1)	Location (2)	Structure type (3)	d_t (mm) (4)	H_s (mm) (5)	f_p (Hz) (6)	Wall slope (V:H) (7)
1	Aberdeen	Impermeable	145	15, 43	0.62, 0.87, 1.12, 1.31	1:1, 1:2, 1:3, 1:4
2	Aberdeen	Impermeable	370	15, 43	0.62, 0.87, 1.12, 1.31	1:1, 1:2, 1:4
3	CRF	Impermeable	230	53, 90	0.5, 0.7, 1.0	1:2
4	Aberdeen	RMB	230	44, 78	0.68, 1.0, 1.25	1:2
5	CRF	RMB	215	49, 89	0.38, 0.50, 0.7, 1.0	1:2

beach extended from the 0.5 m depth to beyond the mean water level (MWL). The breakwaters were built on the beach with a toe depth of 0.23 m in the case of the impermeable, sloping wall (Test 3) and 0.215 m in the case of the rubble mound breakwater (Test 5). Each of the structures had a front slope of 1:2 and was placed parallel to the depth contours. The lengths of the structures were 7.2 m for the sloping wall and 8 m (plus rounded ends) for the rubble mound breakwater.

The wave field in front of the structures was measured by four wave gauges that measured 4,096 samples at 10 Hz. The cross section and stone size of the rubble mound breakwater were the same as in the Aberdeen University tests. The waves were analyzed using three of the gauges. Some seas were also analyzed using a second set of three gauges and using all four gauges. These results differed typically by ±2%. The analysis was also found to be insensitive to the changes in depth along the array of wave gauges.

RESULTS OF IMPERMEABLE WALL TESTS

Bulk Reflection Coefficients

Bulk reflection coefficients and Iribarren numbers were calculated for the Aberdeen University and CRF impermeable wall tests. The bulk reflection coefficients C_{rb} are plotted against bulk Iribarren numbers ξ in Fig. 3, which identifies each set of tests by the toe depth and incident significant wave height used. Fig. 3 also shows the Seelig and Ahrens (1981) equation for impermeable walls (equation 2 with $p = 2$, $a = 1$, and $b = 5.5$), which generally agrees well with the measured values. However, the measured coefficients appear to level off for Iribarren numbers above 10, rather than increasing toward the theoretical maximum value of 1.0. Some dependence of reflection coefficient on depth is also apparent at the high Iribarren number, with reflection being slightly lower in shallower water. Reflection coefficients at the very high Iribarren number can be expected to be slightly <1.0 because of some energy dissipation at the flume side walls, at the bed, at the structure face and at the edges of the structure, which did not perfectly meet the sides of the flume. However, the waves in front of the structure are nonlinear, and reflection coefficients for conditions of high reflection of nonlinear waves are underestimated by linear reflection analysis methods such as the one used here; as the depth decreases the nonlinearity and the underestimate increase. Moreover, some of the results may have been affected by the gauges being less than one wavelength from the toe of the structure for some of the low frequency tests. For these reasons it is likely that the measured reflection coefficients at the high Iribarren number in Fig. 3 underestimate the actual reflection levels.

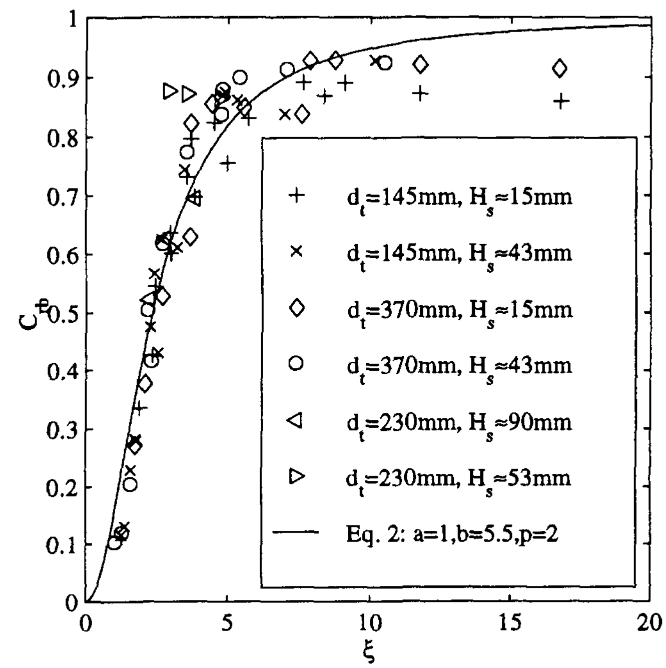

FIG. 3. Bulk Reflection Coefficients against Iribarren Number ξ for Impermeable Wall Experiments

Incident Wave and Reflection Coefficient Spectra

The incident wave spectra of all four Aberdeen University flume tests with the same wall type, slope, incident significant wave height, and toe depth, but different peak frequencies, were plotted on the same graph alongside a graph of their corresponding reflection coefficient spectra. The results for the impermeable wall tests at a toe depth of 145 mm and $H_s \approx$ 15 mm are shown in Fig. 4. The left-hand column of Fig. 4

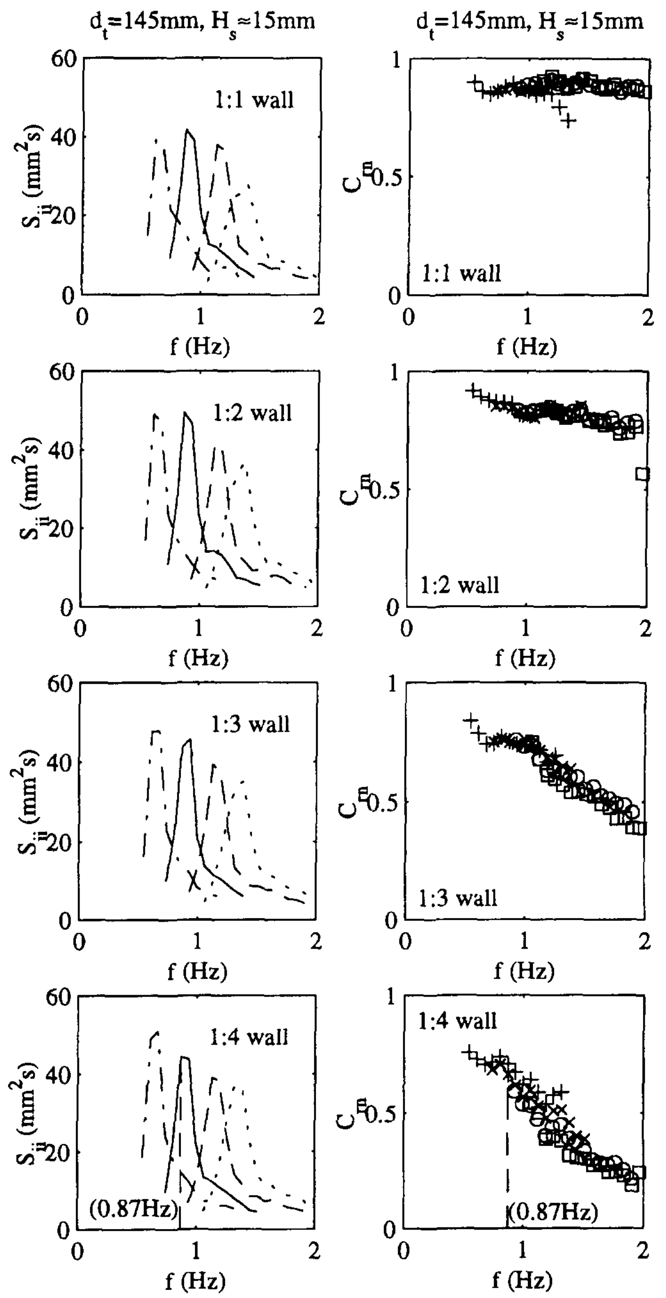

FIG. 4. Incident Wave Spectra and Reflection Coefficient Spectra for Aberdeen University Impermeable Wall Experiments with d_t = 145 mm and $H_s \approx$ 15 mm

presents the incident wave spectra; the reflection coefficient spectra are presented in the right-hand column, and each row corresponds to a different wall slope. The different incident wave spectra are identified by different line types, and the corresponding reflection coefficient spectra are identified by different symbols (plus, cross, circle, and square for incident wave f_p = 0.62, 0.87, 1.12, and 1.31 Hz). The spectra for the impermeable wall tests at a toe depth of 145 mm and $H_s \approx$ 43 mm are shown similarly in Fig. 5.

Figs. 4 and 5 show the reflection coefficients decreasing as frequency increases except where there is no real breaking on the structure and the reflection coefficients remain very high across the frequency range. Moreover, the reflection coefficients decrease as the wall slope decreases. These trends are as expected and are the same as those noted by several authors from field tests on structures and beaches and from laboratory tests on structures, as referenced earlier.

It is important to note that two seas with different peak frequencies, but the same significant wave height, give similar reflection coefficients at frequencies where the spectra overlap. This is despite the fact that the incident spectral densities of the two seas may be very different at the overlapping frequencies. This can clearly be seen in both Figs. 4 and 5. For example, in Fig. 4 on the bottom row (the 1:4 wall) the spectral density at 0.87 Hz is 15 mm^2/s for the first spectrum and 44 mm^2/s for the second, while the corresponding reflection coefficients are 0.71 and 0.66. This pattern is repeated for the other impermeable wall tests, which have not been shown.

This suggests that there is a general reflection coefficient spectrum for waves of a given significant wave height (and possibly spectral shape, although this is not shown here) reflecting off each impermeable structure with a particular toe depth and wall slope. Seas with different peak frequencies will occupy different parts of the spectrum but will have the same reflection coefficient at frequencies where they overlap. This new observation leads the writers to suggest that there will be a more general form of reflection coefficient spectrum linking the spectra from different wave heights, wall slopes, and toe depths.

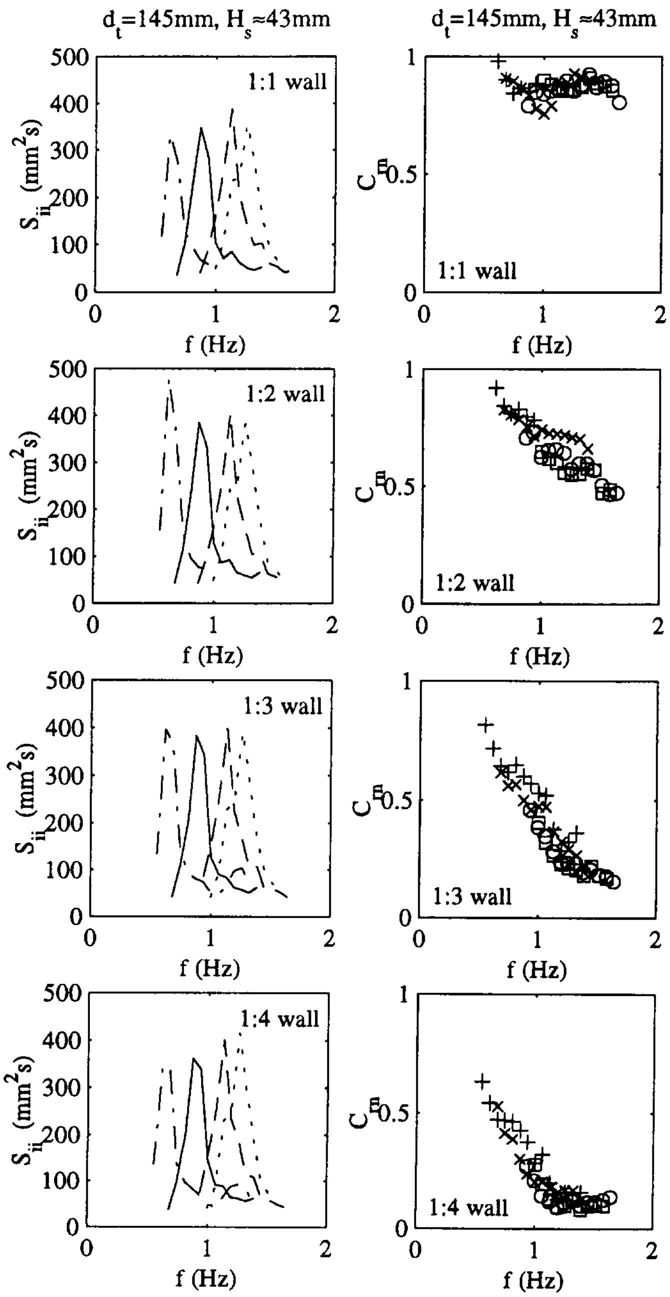

FIG. 5. Incident Wave Spectra and Reflection Coefficient Spectra for Aberdeen University Impermeable Wall Experiments d_t = 145 mm and $H_s \approx$ 43 mm

Reflection Coefficient versus χ

The reflection coefficient spectra from the Aberdeen University impermeable wall tests are plotted against χ in Fig. 6 with each subplot containing results from a different toe depth, significant wave height combination. The results from each wall slope are plotted using a different symbol, which is the same for the results from all four peak frequency seas. There is a reasonably smooth variation in reflection coefficient with χ for each wave height, toe depth combination but there is a different variation for each one.

If χ represented the variation of reflection coefficient with depth correctly, then there would be no difference in the results from Figs. 6(a and b) or in the results from Figs. 6(c and d), which use the same wall slopes and sea states but have different toe depths. This is not the case as can be seen most clearly in comparing Figs. 6(c and d). Similarly, if the reflection coefficient did not vary with incident significant wave height, then the results from Figs. 6(a and c) would be the same as would the results from Figs. 6(b and d). Again, this is not the case. This shows that χ is not a suitable parameter for characterizing reflection coefficient spectra.

Hughes and Fowler (1995) plotted reflection coefficient against χ and fitted a curve to their data. The results contained some scatter even though the range of parameters for the tests was quite limited. The present tests use a broader range of parameters and show that χ does not adequately characterize the reflection coefficient spectrum. This explains why O'Donoghue (1996) was unable to get good estimates of wave kinematics in front of reflecting structures using the equations presented in Hughes and Fowler (1995) to determine reflection coefficient and phase-shift spectra even though the equation for the phase shift was found by Sutherland and O'Donoghue (1998) to be quite accurate.

Reflection Coefficient versus ξ_f

A frequency-dependent Iribarren number ξ_f for each wave component can be defined as follows:

$$\xi_f = \frac{\tan \alpha}{f_n} \sqrt{\frac{g}{2\pi H_s}} \quad (10)$$

Note that ξ_f is the same as the bulk Iribarren number [(1)] except that the peak frequency f_p in (1) is replaced by the local frequency within the spectrum. The value of H_s used to calculate ξ_f is the incident significant wave height (for the whole spectrum), which is the same at each frequency.

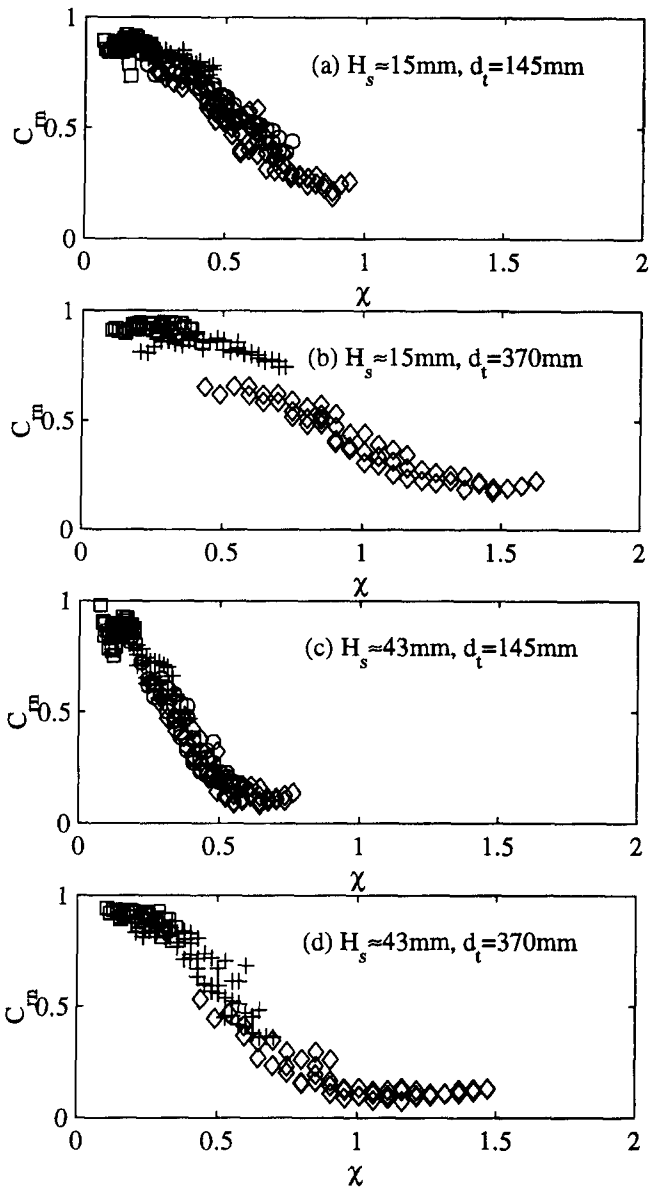

FIG. 6. Reflection Coefficient against χ for Impermeable Wall Experiments

The impermeable wall reflection coefficients are plotted against the frequency-dependent Iribarren number in Fig. 7, with each set of tests identified by its toe depth and incident significant wave height and containing spectra from different peak frequencies and/or wall slopes. The measured results show a consistent trend in behavior for all of the wave height, toe depth combinations, and, as for the bulk reflection coefficients, the results tend to level off for high Iribarren numbers at a value less than the theoretical limit of 1.

The best-fit line with the same form as (2) but using ξ_f rather than ξ and setting $a = 1$ is

$$C_m = \frac{\xi_f^{2.58}}{7.64 + \xi_f^{2.58}} \qquad (11)$$

and is also shown in Fig. 7. This equation produces a good fit to the data in the area of greatest curvature and at lowest values of ξ_f. Setting $a = 1$ forces the curve to a maximum value of 1, which is the theoretical limit. Fitting the data to a hyperbolic tangent, and allowing the results to tend to a maximum value <1 at the high Iribarren number, results in a root-mean-square error marginally lower than that obtained using (11). However, for reasons given earlier, data at the high Iribarren number may not be reliable and (11) is therefore preferred.

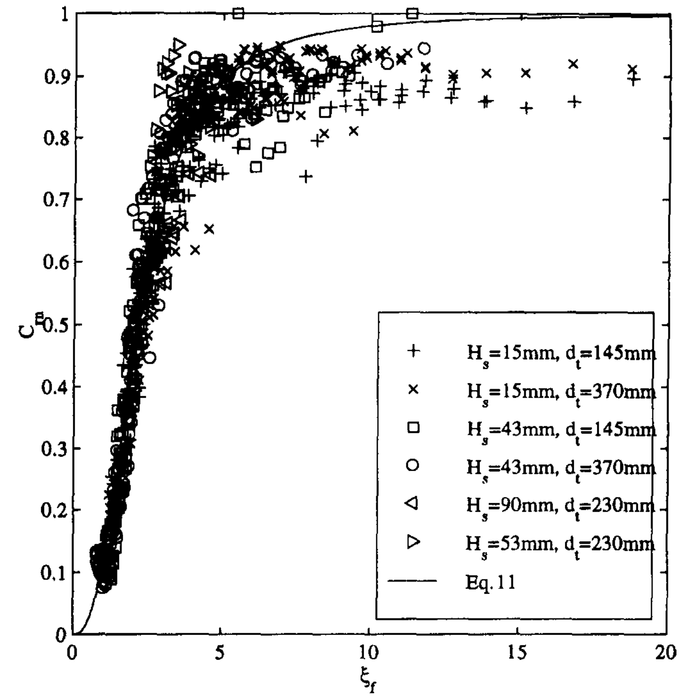

FIG. 7. Reflection Coefficient against Frequency-Dependent Iribarren Number ξ_f for Impermeable Wall Experiments

Eq. (11) predicts the reflection coefficient spectrum and, hence, the reflected wave spectrum for a given random sea incident on an impermeable wall. Of course the bulk reflection coefficient can then be determined [(8)] and, if (11) is valid, the bulk reflection coefficient should be in line with the bulk reflection coefficient calculated using (2), with $p = 2$, $a = 1$, and $b = 5.5$. To check this, bulk reflection coefficients were calculated in this way for eight Jonswap seas with $H_s = 0.5$ and 1.5 m, and $f_p = 0.10$, 0.14, 0.18, and 0.22 Hz incident on 1:1, 1:2, and 1:4 wall slopes. Fig. 8 shows that the 24 calculated bulk reflection coefficients are in line with (2) indicating that good estimates of the bulk reflection coefficient are recovered through (11) for reflection coefficient spectra.

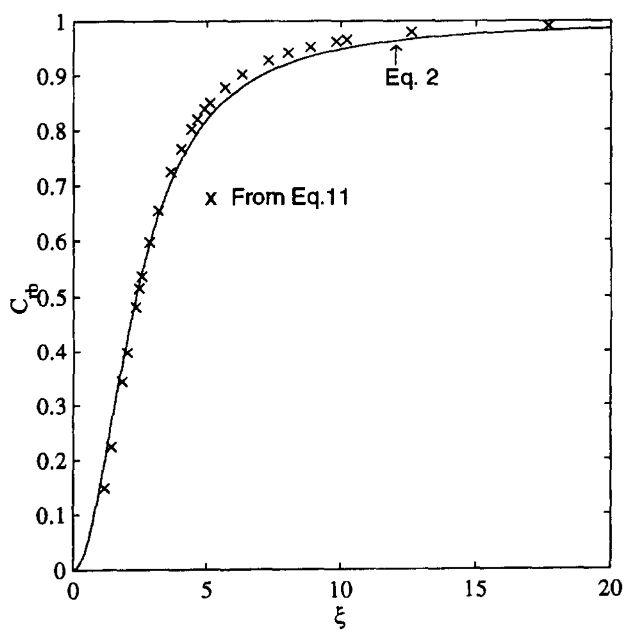

FIG. 8. Predicted Bulk Reflection Coefficients Based on Eq. (11) Compared with Eq. (2) for Impermeable Walls

RESULTS OF RUBBLE MOUND BREAKWATER TESTS

A less extensive set of tests was carried out involving similar rubble mound breakwaters in the Aberdeen University flume and in the CRF. The same cross section was formed using the same stones, with a nominal diameter of 45.6 mm.

Bulk Reflection Coefficients

The bulk reflection coefficients are plotted against the Iribarren number in Fig. 9 with results from different peak frequency seas but the same toe depth and incident significant wave height shown by the same symbol. Also shown is (2) with $p = 2$, $a = 0.6$, and $b = 6.6$ for rubble mound structures from Seelig and Ahrens (1981). The curve does not tend to a maximum value of 1 as dissipation and transmission will always occur with rubble structures, even for steep slopes and mild waves. These tests involved relatively low Iribarren numbers and so the variation of reflection coefficient with high Iribarren numbers is not determined.

Reflection Coefficient Spectra

The incident wave spectra and reflection coefficient spectra for the CRF rubble mound breakwater tests are shown in Fig. 10. The top row of Fig. 10 presents results from the tests with $H_s \approx 49$ mm, while the bottom row presents results from the tests with $H_s \approx 89$ mm. In both cases the reflection coefficients vary smoothly with frequency with different seas having similar reflection coefficients at frequencies where the spectra overlap, even when the incident spectral densities are different at these frequencies. The results for the rubble mound breakwater show the same trends as those from the impermeable walls and again indicate that there is likely to be a characteristic form of the reflection coefficient spectrum.

Reflection Coefficient versus ξ_f

The rubble mound breakwater reflection coefficient spectra are plotted against ξ_f in Fig. 11 along with (12) which represents the best-fit curve to all the rubble mound data with the same form as (2) with $p = 2$

$$C_m = \frac{0.82\xi_f^2}{22.85 + \xi_f^2} \tag{12}$$

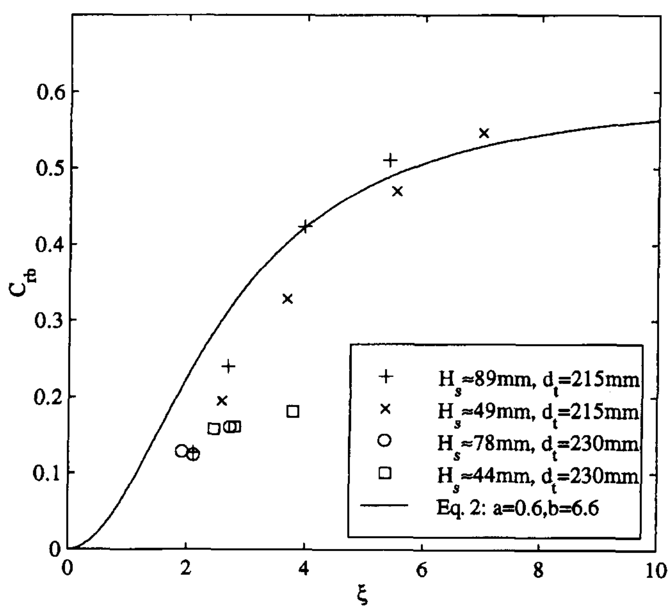

FIG. 9. Bulk Reflection Coefficients against Iribarren Number ξ for Rubble Mound Breakwater Experiments

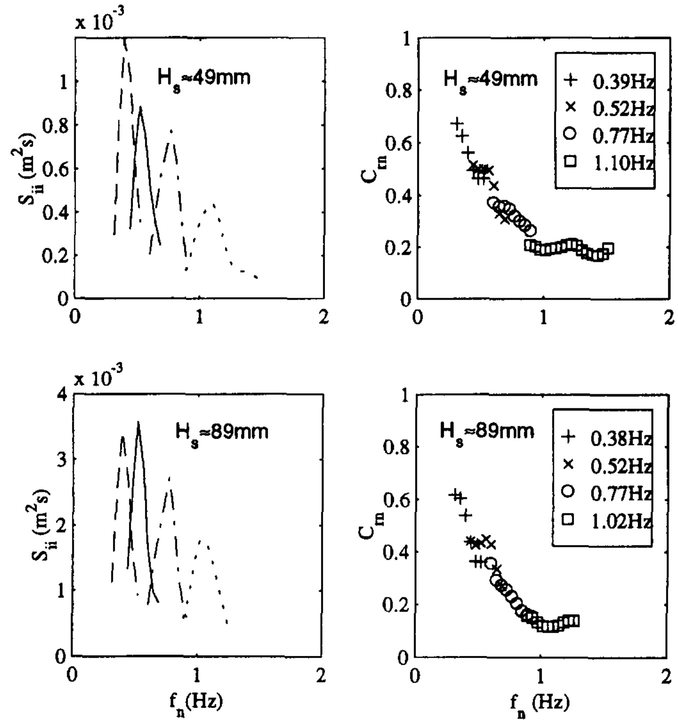

FIG. 10. Incident Wave Spectra and Reflection Coefficient Spectra for Rubble Mound Breakwater Experiments (Frequency Values Shown Are Peak Spectral Frequencies)

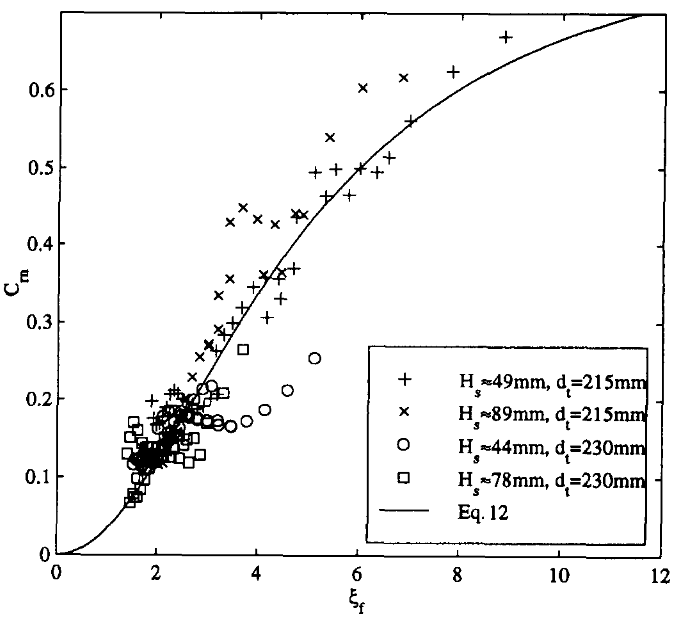

FIG. 11. Reflection Coefficient against Frequency-Dependent Iribarren Number ξ_f for Rubble Mound Breakwater Experiments

The range of rubble mound breakwater tests is more limited than the range of impermeable wall tests and so (12) is very tentative. It remains to be proven whether porosity and possibly other structure characteristics can be fully accommodated by appropriate values of the coefficients in (12). The important point is that the results indicate that a general form of reflection coefficient spectrum, characterized by the local Iribarren number, exists for rubble structures as well as for impermeable structures.

RELATIVE IMPORTANCE OF REFLECTION COEFFICIENT SPECTRA

Equations like (11) and (12) can be used to calculate the reflection coefficient spectrum for a given incident wave spectrum and structure characteristics. The variation in the magnitude of the reflection coefficient across the frequency range of the incident wave spectrum depends on the Iribarren number and on the width of the incident wave spectrum. This is illustrated in Fig. 12.

Figs. 12(a and b) show an incident Jonswap spectrum with $f_p = 0.1$ Hz and $H_s = 2.44$ m and the corresponding reflection coefficient spectrum for the waves incident on a smooth, impermeable wall of slope 1:4 [Fig. 12(a)] and slope 1:2 [Fig. 12(b)]. The spectral density has been normalized by dividing by the peak spectral density, giving S_{ii}^*. Figs. 12(c and d) are similar to Figs. 12(a and b), but the incident spectrum is a Pierson-Moskowitz spectrum and is, therefore, much broader than the Jonswap spectrum. The reflection coefficient spectra C_{rn} in Fig. 12 were calculated using (11). Fig. 12 also shows the bulk reflection coefficient C_{rb} for each case where C_{rb} is calculated using (2).

It is clear from the examples shown in Fig. 12 that reflections at frequencies below the peak frequency of the wave spectrum will be underestimated and reflections at frequencies above the peak frequency will be overestimated if the bulk reflection coefficient is used to calculate the reflected wave spectrum. However, the difference between C_{rb} and C_{rn} is small in the vicinity of the peak of the wave spectrum, that is, in the area of highest incident wave energy. This means that for narrow incident wave spectra, such as in Fig. 12(b), there will be little difference between reflected wave spectra calculated using the bulk reflection coefficient at all frequencies and calculated using (11) for smooth, impermeable walls or an equation of similar form for other structure types. For such cases, therefore, specification of a reflection coefficient spectrum is not critical. However, the difference between using the reflection coefficient spectrum and the bulk reflection coefficient becomes significant as the width of the incident wave spectrum increases [compare Figs. 12(a and b) with Figs. 12(c and d)]. Moreover, it also becomes more significant as the Iribarren number decreases [compare Fig. 12(a) with Fig. 12(b) and Fig. 12(c) with Fig. 12(d)] because of the greater sensitivity of C_{rn} to changes in ξ_f at lower values of ξ_f as seen in Fig. 7.

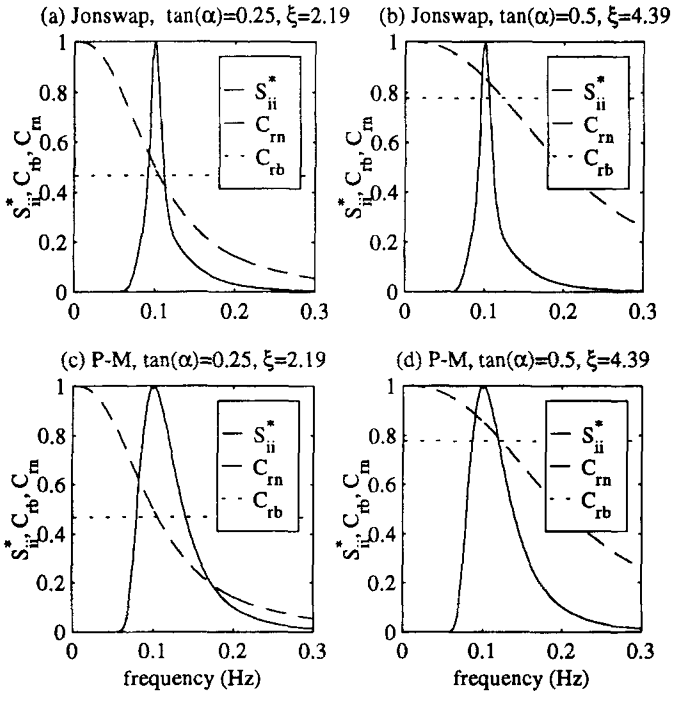

FIG. 12. Nondimensionalized Incident Wave Spectra, Reflection Coefficient Spectra from Eq. (11) and Bulk Reflection Coefficients from Eq. (2)

CONCLUSIONS

A number of wave reflection experiments have been carried out in an attempt to characterize the form of the reflection coefficient spectrum for random wave reflection from coastal structures. Tests were performed on smooth, impermeable walls and ruble mound breakwaters. The parameter proposed by Hughes and Fowler (1995) for characterizing the reflection coefficient spectrum χ [given by (3)] does not adequately account for the effects of toe depth and significant wave height. However, seas of different peak frequencies, but the same incident significant wave height (and approximately the same spectral shape) reflecting off a given structure, have similar reflection coefficients at frequencies where the spectra overlap and where there is a significant amount of energy. Moreover, a local, frequency-dependent Iribarren number ξ_f [given by (10)] can be used to characterize the reflection coefficient spectra for seas of different peak frequency and significant wave height reflecting off a structure.

Equations similar to those given by Seelig and Ahrens (1981) for bulk reflection coefficient can be used to calculate the reflection coefficient spectrum as a function of ξ_f with the fitted coefficients depending on the type of structure. Therefore, the reflection coefficient can be determined from wall slope, frequency, incident significant wave height, and two fitted coefficients. Eq. (11) is recommended for the prediction of reflection coefficient spectra for smooth, impermeable walls. The rubble mound breakwater tests cover such a limited range of parameters that no equation is recommended at this stage, but it is expected to have the same form as (11) with the fitted coefficients varying with structure porosity and, possibly, other structure characteristics.

Specification of the reflection coefficient spectrum becomes more critical as the width of the incident wave spectrum increases and as the Iribarren number decreases. Otherwise, random wave reflection can be adequately determined using the bulk reflection coefficient across the frequency range.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

a, b, p = fitted coefficients;
a_n = amplitude of nth component;
C_{rb} = bulk reflection coefficient;
C_{rn} = frequency-dependent reflection coefficient;
d_t = water depth at toe of structure;
f_n = frequency of nth component;
f_p = peak frequency of incident wave spectrum;
f_s = sampling frequency;
g = gravitational acceleration;
H_s, H_{sr} = incident and reflected significant wave height, respectively;
k_n = linear theory wave number;
L_o, L_s = linear theory wavelength in deep and shallow water, respectively;
m_{0i}, m_{0r} = incident and reflected mean-square surface elevation, respectively;
N = number of samples in time series;
S_{ii}, S_{rr} = incident and reflected surface elevation spectral energy density, respectively;
$S_{\eta\eta}$ = surface elevation spectral energy density;
t = time;
x = cross-shore coordinate;
x_m = width of breakwater from toe to still water line;
α = wall slope;
γ_n = phase shift on reflection of nth component;
ε_n = random phase of nth component at origin;
η = surface elevation;
ξ = Iribarren number;
ξ_f = frequency-dependent Iribarren number; and
χ = parameter used to characterize wave reflection.