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WAVE REFLECTION AND TRANSMISSION AT PERMEABLE BREAKWATERS OF ARBITRARY CROSS-SECTION

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ABSTRACT

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A theory is formulated to predict wave reflection and transmission at an infinite rubble-mound breakwater. The breakwater may be a multilayered structure with arbitrary cross-section. It is assumed that the incident wave is normal to the structure and the wave may be described by linear theory. A hybrid method has been applied to solve the boundary value problem.

Comparison between experimental and theoretical results shows reasonable agreement.

1. INTRODUCTION

Various types of permeable structure, including the well known rubble-mound breakwater, have been constructed for the purpose of protecting coastal and inland basins from ocean waves. Permeable structures have several advantages compared with impervious ones. The permeable structure is much more effective in decreasing the wave run-up, in reducing reflected wave height, and eventually in reducing pressures acting on it. The construction of a rubble-mound breakwater is also recommended when the protected water area is deep and the bottom consists of weak soil.

Several laboratory studies have been conducted to investigate the reflected and transmitted waves for specific types of permeable structures (Loewy, 1967; Iwasaki and Numata, 1970; Shore Protection Manual, 1973; Dattatri et al., 1978; Mani, 1981). Analytical studies have been done for practical purposes — to predict the wave height of reflected and transmitted waves (Le Méhauté, 1957; Kondo, 1970; Sollitt and Cross, 1972a,b; Ijima et al., 1974; Madsen, 1974; Massel and Mei, 1977; Madsen et al., 1978; Massel and Butowski, 1980). The existing solutions of this problem are valid for structures of rectangular cross-section and linear wave theory, or for the trapezoidal breakwaters and linear theory of long waves (Madsen et al., 1978). In the second case, additional experimental information is necessary to

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determine the energy dissipated on the seaward slope of the breakwater. It is also possible to develop an approximate procedure for the prediction of wave reflection and transmission at trapezoidal breakwaters by considering an equivalent breakwater of rectangular cross-section (Sollitt and Cross, 1972a,b; Kondo and Toma, 1974; Madsen and White, 1976; Massel and Butowski, 1981).

In this paper, a theory of wave transmission and reflection at an infinite porous rubble-mound breakwater is presented. The breakwater may be a multilayered structure with arbitrary cross-section.

The theoretical approach used in this study is based on the unsteady Forchheimer equation of motion in the pores of a coarse, granular medium. The equation is linearized using Lorentz's hypothesis of equivalent work. The linear wave theory is applied and the excitation is provided by a monochromatic incident wave that is normal to the structure. The boundary value problem is solved by using a hybrid method which employs the boundary element methods in the breakwater body and in the vicinity of the breakwater with a boundary solution procedure in the exterior regions extending to infinity.

The numerical results are compared with experimental data.

2. THEORETICAL FORMULATION

2.1. The equation of motion in porous media

Fluid motion in a body of this structure is described in terms of seepage velocity and pressure. These are conceptual quantities which are averaged over finite and continuously distributed pore volumes. The equation of motion then reduces to the form:

$$\frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{\rho}\nabla(p + \gamma z) + \text{resistance forces} \tag{2.1}$$

where $\boldsymbol{v}$ is the seepage velocity vector at any point, p is the corresponding pressure, t is time, ∇ is the gradient operator, ρ is the fluid mass density, γ is the fluid weight density, z is the vertical coordinate.

In order to represent the resistance forces, the relation proposed by Forchheimer is used (Bear, 1972). In the present application we add to the motion equation an additional term which evaluates the added resistance caused by the virtual mass of discrete grains within the medium (Sollitt and Cross, 1972b; Hannoura and McCorqoudale, 1978). Thus, eq. (2.1) may be rewritten in the form:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\nabla(p + \gamma z) - \frac{\nu\epsilon}{K}\,\boldsymbol{v} - \frac{C_f\epsilon^2}{K^{1/2}}\,|\boldsymbol{v}|\,\boldsymbol{v} - \frac{1-\epsilon}{\epsilon}\,C_M\,\frac{\partial \boldsymbol{v}}{\partial t} \tag{2.2}$$

where ν is the kinematic viscosity, ϵ is the porosity of the medium, K is the intrinsic permeability, C_f is the dimensionless turbulent resistance coefficient, C_M is the virtual mass coefficient of medium grains which is a known quantity for isolated simple shapes, but generally is unknown for random, densely packed materials.

In order to complete the set of equations in the porous media, we add the equation of continuity for an incompressible fluid. Thus:

$$S\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\nabla(p+\gamma z) - \frac{\nu\epsilon}{K} - \frac{C_f\epsilon^2}{K^{1/2}}\;|\boldsymbol{v}|\boldsymbol{v} \tag{2.3}$$

$$\nabla\bullet\boldsymbol{v} = 0 \tag{2.4}$$

where S is an inertial coefficient, and:

$$S = 1+\frac{1-\epsilon}{\epsilon}\,C_M \tag{2.5}$$

Linearization of eq. (2.3) is done using a technique that approximates the nonlinear damping condition inside the porous media. The dissipative nonlinear stress term in eq. (2.3) is replaced by an equivalent stress term, linear in $\boldsymbol{v}$, i.e.:

$$\frac{\nu\epsilon}{K}\boldsymbol{v} + \frac{C_f\epsilon^2}{K^{1/2}}\;|\boldsymbol{v}|\boldsymbol{v} \rightarrow f\omega\boldsymbol{v}$$

where ω is the angular frequency of the periodic motion and f is a dimensionless friction (damping) coefficient.

To evaluate f in terms of the known damping law, it is required that both the linear and nonlinear friction laws account for the same amount of energy dissipation during one wave period — Lorentz's hypothesis (Lean, 1967). From Lorentz's condition of equivalent work we obtain:

$$f = \frac{1}{\omega}\;\frac{\int_{\bar{R}} \mathrm{d}\bar{R} \int_t^{t+T_1} \epsilon^2\left(\frac{\nu v^2}{K} + \frac{C_f\epsilon}{K^{1/2}}\;|\boldsymbol{v}|^3\right)\mathrm{d}t}{\int_{\bar{R}} \mathrm{d}\bar{R} \int_t^{t+T_1} \epsilon v^2\,\mathrm{d}t} \tag{2.6}$$

where T_1 is a wave period, $\bar{R}$ is the porous domain and f is considered to be constant within $\bar{R}$.

Thus the equation of motion reduces to the following form:

$$S\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\nabla(p+\gamma z) - f\omega\boldsymbol{v} \tag{2.7}$$

with

$$\nabla \cdot \boldsymbol{v} = 0 \tag{2.8}$$

2.2. Statement of the problem

The situation considered for analysis is shown schematically in Fig. 1. Additionally, we assume that:

(1) In the infinite rubble-mound breakwater L homogeneous porous layers (l = 3, 4, ..., L + 2, Fig. 1) can be distinguished with known physical (ϵ_l) and hydraulic (K_l, C_{f_l}) properties. The breakwater has the same cross-section over the entire length.

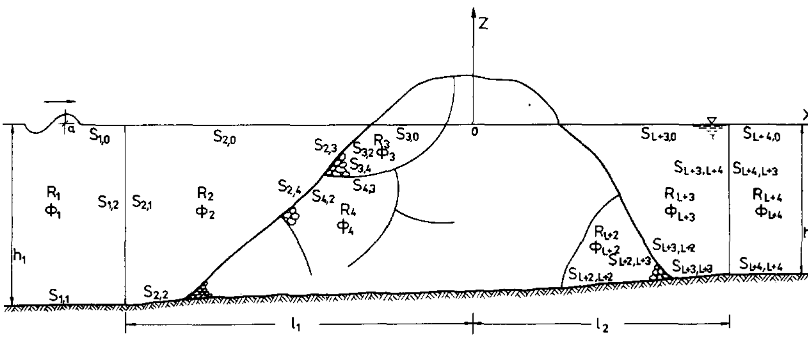


Fig. 1. Definition sketch and coordinate system.

(2) The semi-infinite domains R_1 and R_{L+4} have constant depths, h_1 and h_2, respectively.

(3) The sea bottom is impervious.

(4) A train of simple harmonic waves (ω) of small amplitude (a) is approaching the breakwater such that the wave crests are parallel to the longitudinal axis of the breakwater.

(5) The amplitude of the wave and the largest dimensions of the elements which form the porous layers are small, compared to the water depths h_1 and h_2 and to the wavelength.

(6) The fluid is inviscid and incompressible.

(7) The velocity in each flow domain has a potential.

(8) The only forces acting on fluid in the domains R_1,R_2,R_{L+3},R_{L+4} are gravity forces. Additionally, in porous layers the damping force proportional to the fluid velocity is acting on a fluid element.

According to these assumptions the wave field can be specified by a velocity potential of the form:

$$\Phi_l(x,z,t) = \mathrm{Re}[\phi_l(x,z)\mathrm{e}^{-\mathrm{i}\omega t}] \qquad l = 1,2,\ldots,L+4 \tag{2.9}$$

where Re denotes the real part and $\mathrm{i} = \sqrt{-1}$.

The wave field is completely specified if $\phi_l(x,z)$ is known. The boundary value problem for $\phi_l(x,z)$ may be written as follows:

$$\nabla^2\phi_l = 0 \qquad l = 1,2,\ldots,L+4 \tag{2.10}$$

on $S_{l,0}$:

$$\frac{\partial\phi_l}{\partial z} - \frac{\omega^2}{g}(S_l + \mathrm{i}\, f_l)\phi_l = 0, z = 0 \quad \text{— free surface combined condition} \tag{2.11}$$

$f_l = 0,\ S_l = 1 \quad$ for $l = 1,2,L+3,L+4$

On $S_{l,l}$:

$$\frac{\partial\phi_l}{\partial n_{l,l}} = 0 \qquad \text{— no normal velocity on the bed} \tag{2.12}$$

On $S_{l,m}$ and $S_{m,l}$:

$$(S_l + \mathrm{i}\, f_l)\phi_l = (S_m + \mathrm{i}\, f_m)\phi_m \qquad \text{— continuity of pressure} \tag{2.13}$$

$$\epsilon_l \frac{\partial\phi_l}{\partial n_{l,m}} = -\epsilon_m \frac{\partial\phi_m}{\partial n_{m,l}} \qquad \text{— continuity of normal velocity} \tag{2.14}$$

$f_{l(m)} = 0$ and $S_{l(m)}, \epsilon_{l(m)} = 1$ for $l(m) = 1,2,L+3,L+4$:

$$x \to -\infty: \quad \frac{\partial\hat{\phi}_1}{\partial x} + \mathrm{i}\, k_1\hat{\phi}_1 = 0 \qquad \text{— radiation condition at } -\infty \tag{2.15}$$

$$x \to +\infty: \quad \frac{\partial\phi_{L+4}}{\partial x} - \mathrm{i}\, k_2\phi_{L+4} = 0 \quad \text{— radiation condition at } +\infty \tag{2.16}$$

where
$S_{l,0}$ ($S_{l,l}$) is part of the R_l boundary domain-free water surface (sea bottom),
$S_{l,m}$ and $S_{m,l}$ are common boundaries of R_l and R_m domains,
$\partial\phi_l/\partial n_{l,m}$ is the outward normal derivative of ϕ_l at $S_{l,m}$,
S_l is the inertial coefficient for R_l domain (see 2.5),
k_1, k_2 are the wave numbers corresponding to the fluid depths in domains R_1 and R_{L+4},

$$\phi_1 = \phi_{\text{inc}} + \hat{\phi}_1,$$

where ϕ_{inc} is the velocity potential due the incident wave.

3. METHOD OF SOLUTION

3.1. Analytical solution

In domain R_1, the function $\phi_1(x,z)$ should be ($x \leqslant -l_1$):

$$\phi_1(x,z) = \frac{-iga}{\omega}\, e^{ik_1(x+l_1)} \frac{\cosh k_1(z+h_1)}{\cosh k_1 h_1} + \frac{-igR}{\omega}\, e^{-ik_1(x+l_1)} \frac{\cosh k_1(z+h_1)}{\cosh k_1 h_1}$$

$$+ \sum_{\alpha_n} \frac{-igR_{\alpha_n}}{\omega}\, e^{\alpha_n(x+l_1)} \frac{\cos \alpha_n(z+h_1)}{\cos \alpha_n h_1} \tag{3.1}$$

where:

$$-\frac{iga}{\omega}\, e^{ik_1(x+l_1)} \frac{\cosh k_1(z+h_1)}{\cosh k_1 h_1} = \phi_{\mathrm{inc}} \tag{3.2}$$

a is the amplitude of the incident wave,
R is the unknown amplitude of the reflected wave,
R_{α_n} is the unknown amplitude of the local standing wave (with exponentially decaying amplitude),
g is the acceleration due to gravity.

In domain R_{L+4} the function $\phi_{L+4}(x,z)$ should be ($x \geqslant l_2$):

$$\phi_{L+4}(x,z) = \frac{-igT}{\omega}\, e^{ik_2(x-l_2)} \frac{\cosh k_2(z+h_2)}{\cosh k_2 h_2}$$

$$+ \sum_{\beta_n} \frac{-igT_{\beta_n}}{\omega}\, e^{\beta_n(l_2-x)} \frac{\cos \beta_n(z+h_2)}{\cos \beta_n h_2} \tag{3.3}$$

where T is the unknown amplitude of the transmitted wave and T_{β_n} is the unknown amplitude of the local standing wave (with exponentially decaying amplitude).

The eigenvalues $k_1, k_2, \alpha_n, \beta_n$ are roots of the equations:

$$\frac{\omega^2}{g} = k_1 \operatorname{tgh}(k_1 h_1) = k_2 \operatorname{tgh}(k_2 h_2) \tag{3.4}$$

and

$$\frac{\omega^2}{g} = -\alpha_n \operatorname{tg}(\alpha_n h_1) = -\beta_n \operatorname{tg}(\beta_n h_2) \tag{3.5}$$

3.2. *Numerical solution*

A boundary-element method is used to write the equations for velocity potential in the domains $R_2, R_3, \ldots, R_{L+3}$ (Fig. 1).

The mathematical basis stems from Green's identities. Assuming that (Sternberg, 1926; Jaswon and Symm, 1977):

(1) the domain $\hat{R}$ is bounded by the surface $\hat{S}$ which has a continuous normal, except for finite numbers of corners;
(2) ϕ is continuous in $\hat{R}+\hat{S}$;

(3) ϕ is differentiable to at least the second order in $\hat{R}$;
(4) ϕ satisfies Laplace's equation in $\hat{R}$; and
(5) $V = \ln r$, where r is the distance from an arbitrary point, P, to a point, Q, on the boundary $\hat{S}$;

we obtain from Green's second identity:

$$\int_{\hat{S}} \left(\phi \frac{\partial v}{\partial n} - V \frac{\partial \phi}{\partial n} \right) \mathrm{d}\hat{S} = 0 \tag{3.6}$$

The outward normal derivative $(\partial\phi/\partial n)$ should be piecewise continuous on $\hat{S}$. Note that a discontinuous normal derivative boundary condition does not contradict our original assumption of the continuity of ϕ (Bai and Yeung, 1974).

The first step in the numerical solution of the boundary value problem formulated in terms of integral equations, is the subdivision of the boundary $\hat{S}$ into suitably small N_l straight-line segments. Then eq. (3.6) yields:

$$\sum_j \int_{\hat{S}_j} \left[\phi \frac{\partial}{\partial n} (\ln r) - \ln r \frac{\partial \phi}{\partial n} \right] \mathrm{d}\hat{S}_j = 0 , \qquad j = 1,2,\ldots,N_l \tag{3.7}$$

If we assume now that:
(a) the point P_i is placed at the beginning of the i-th segment;
(b) the beginning of a local (ξ,η) coordinate system (Sulisz, 1982) is located at point P_i (Fig. 2); and
(c) between a pair of node points Q_j, Q_{j+1} (Fig. 2), the potential and its normal derivative are written as:

$$\phi = \frac{\phi_{j+1} - \phi_j}{\xi_{j+1} - \xi_j} \xi + \frac{\xi_{j+1}\phi_j - \xi_j\phi_{j+1}}{\xi_{j+1} - \xi_j} \qquad \xi_j \leqslant \xi \leqslant \xi_{j+1} \tag{3.8}$$

$$\frac{\partial \phi}{\partial n} = \frac{\left(\frac{\partial \phi}{\partial n}\right)_{j+1} - \left(\frac{\partial \phi}{\partial n}\right)_j}{\xi_{j+1} - \xi_j} \xi + \frac{\xi_{j+1}\left(\frac{\partial \phi}{\partial n}\right)_j - \xi_j\left(\frac{\partial \phi}{\partial n}\right)_{j+1}}{\xi_{j+1} - \xi_j} \qquad \xi_j \leqslant \xi \leqslant \xi_{j+1} \tag{3.9}$$

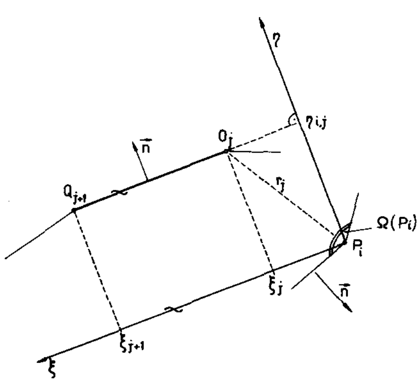


Fig. 2. ξ-η coordinate system.

then substitution of eqs. (3.8) and (3.9) in eq. (3.7) yields (for P_i) an algebraic equation of the following form (Sulisz, 1982):

$$\sum_j \int_{\xi_j}^{\xi_{j+1}} \left[\phi \frac{\partial}{\partial n} (\ln r) - \frac{\partial \phi}{\partial n} \ln r \right] d\xi = \sum_j (I_{i,\mathrm{I}} + I_{i,\mathrm{II}}) = 0, \quad j = 1,\ldots,N_l \quad (3.10)$$

In a well-posed problem, either ϕ or $\partial\phi/\partial n$ or the linear relation:

$$\delta_i \phi + \delta_2 \cdot \frac{\partial \phi}{\partial n} = \delta_3 \quad (3.11)$$

between ϕ and $\partial\phi/\partial n$ is known at each point of $\hat{S}$ (Jaswon and Symm, 1977).

Applying eq. (3.10) at each point P_i (i=1,2,...,N_l) we obtain N_l equations in either $\phi_i(\partial\phi/\partial n)_i$, or some combination of ϕ_i and $(\partial\phi/\partial n)_i$:

$$\sum_j (I_{i,\mathrm{I}} + I_{i,\mathrm{II}}) = 0 \qquad i,j = 1,2,\ldots,N_l \quad (3.12)$$

The solution of the above equations provides the boundary data that can be used in eq. (3.6) to find the solution at any interior point.

3.3. The solution of the boundary value problem

The boundary value problem formulated in item 2.2 is solved in the following way:

(1) We assume the friction coefficient f_l (l=3,...,L+2). Usually as a starting approximation we apply $f_l = 1 \div 2$.

(2) We apply the numerical solutions at the domains $R_2, R_3, \ldots, R_{L+3}$. Using the analytical solutions (eqs. 3.1, 3.3, 3.4, 3.5) and boundary conditions (eqs. 2.11, 2.12, 2.13, 2.14) we obtain N equations for N unknowns (complex), where $N = \sum_{l=2}^{L+3} N_l$. After solving such a system of equations, the quantities $R, R_{\alpha_n}, T, T_{\beta_n}$ and the value of potential function and its normal derivative at each point of the boundary (of domains R_l) will be known.

(3) Next, we calculate the new values of f_l, $l = 3,\ldots,L$+2 (eq. 2.6) and compare them with the assumed values f_l and iterate if necessary (return to 2). The iteration scheme typically closes after two to four cycles.

As a test case, the hybrid method discussed in this paper was first applied to calculate wave reflection and transmission at a permeable breakwater of rectangular cross-section. Figure 3 shows a comparison of the reflection and transmission coefficients obtained by Sollitt and Cross (1972a) and by the method presented. The reflection (RC) and transmission (TC) coefficients are defined by:

$$RC = |R|/\mathrm{a} \quad (3.13)$$

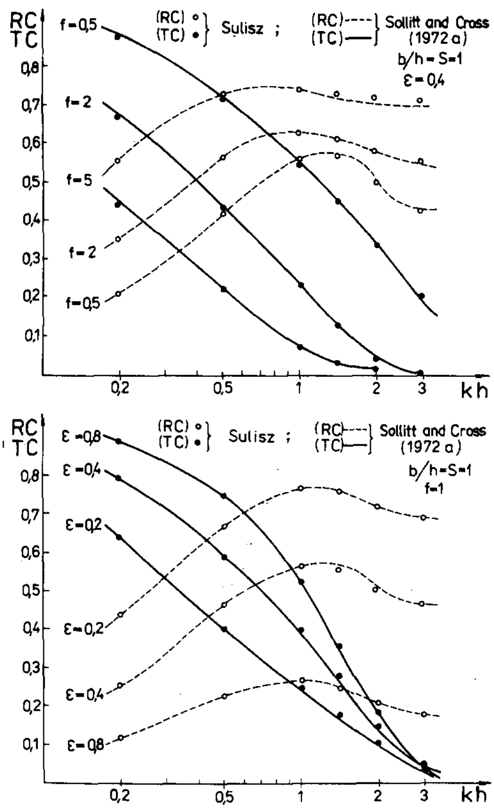


Fig. 3. Comparison of the reflection (RC) and transmission (TC) coefficients obtained by Sollitt and Cross (1972a) and the present method.

$$TC = |T|/a \tag{3.14}$$

Additional comparisons of the test results (wave forces on large pipelines, diffraction problems), obtained by the present method with the results obtained by other authors have already been published by Sulisz (1982, 1983b).

4. EXPERIMENTAL RESULTS

The experiment was done in two steps (Sulisz, 1983a). First, the physical (ϵ) and hydraulic (K, C_f) properties of two kinds of crushed rock were determined.

The porosity was obtained by weighing a gravel sample dry and submerged, subtracting the one from the other to yield the weight of water occupying the pores, and dividing the pore water weight by the weight of water occupying the same gross volume as that of the sample.

The hydraulic properties (permeability, K and turbulent damping coefficient, C_f) of the medium were determined using a large permeameter. These quantities are evaluated from steady state tests by measuring the pressure gradient through a sample of medium as a function of an imposed discharge velocity. Then K and C_f were determined from eq. (2.3) ($S\,\partial v/\partial t = 0$) in terms of the measured quantities. A summary of the physical and the hydraulic properties are listed in Table 1.

TABLE 1

Physical and hydraulic properties of the medium

Size, d m	Equivalent mean sphere diameter, d_b m	Porosity, ϵ dimensionless	Permeability, K m^2	Turbulent damping coefficient, C_f dimensionless
0.023–0.037	0.025	0.442	1.923×10^{-7}	0.270
0.01–0.02	0.012	0.468	0.919×10^{-7}	0.387

In the second step of the experiment, a rubble-mound breakwater of crushed rock (of known ϵ, K, C_f) was constructed in the channel and the wave reflection from and transmission through the structure were determined. The amplitudes of the incident, reflected and transmitted waves were obtained using Fourier's analysis of the surface elevation records in the domains up- and downstream of the structure (Jolas, 1962; Bendykowska, 1966).

5. COMPARISON OF THEORY AND EXPERIMENT

Theoretical reflection and transmission coefficients are found using the iteration procedure discussed previously. The virtual mass coefficient, C_M is

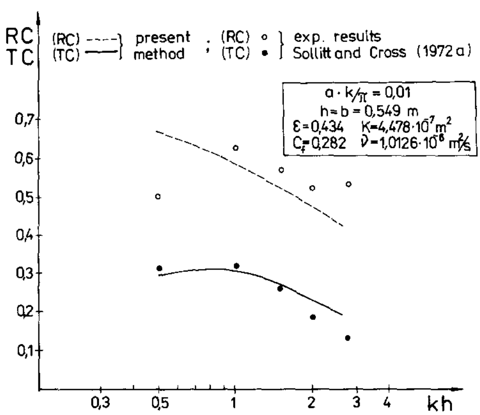


Fig. 4. Reflection and transmission coefficients for rectangular homogeneous breakwater dependence on wave number.

unknown, and it is taken as equal to zero by default. The results are presented as continuous (TC) and dashed (RC) lines on the experimental plots. Figure 4 presents the reflection and transmission coefficients for a rectangular breakwater as a function of dimensionless wave number. The structure is a homogeneous, vertical-walled breakwater composed of gravel contained in a wire screen crib (Sollitt and Cross, 1972a). The breadth (b) of this structure is equal to the depth of the water in the channel (h).

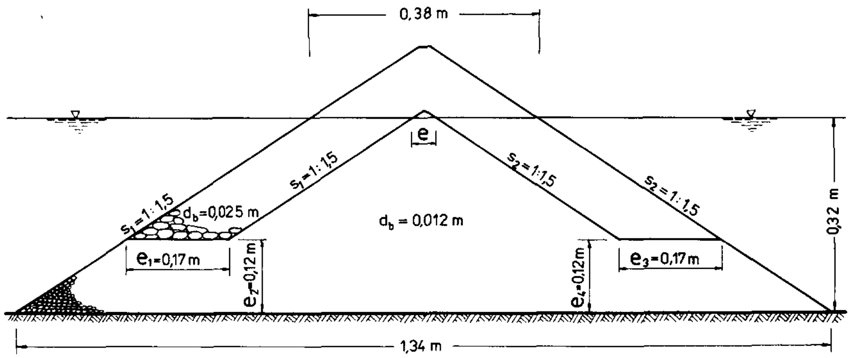


Fig. 5. Trapezoidal layered breakwater (TW-2).

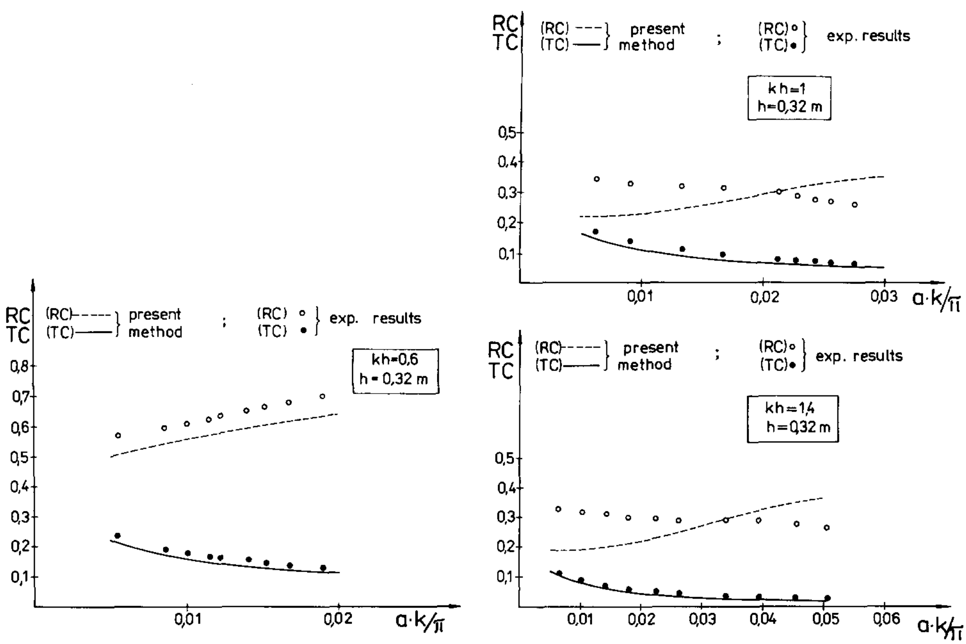


Fig. 6. Reflection and transmission coefficients for trapezoidal layered breakwater (TW-2) dependence on wave steepness.

Fig. 7. Reflection and transmission coefficients for trapezoidal layered breakwater (TW-2) dependence on wave steepness.

The second breakwater is a two-layered trapezoidal-shaped structure dimensioned as in Fig. 5. The medium properties are given in Table 1 ($g = 9.81$ m/s^2, $\nu = 1.17 \times 10^{-6}$ m^2/s). The reflection and transmission coefficients are presented as functions of wave steepness in Figs. 6, 7 and 8.

The multilayered breakwaters are built from core, secondary armour and primary armour. The seaward and leeward faces of the primary (secondary)

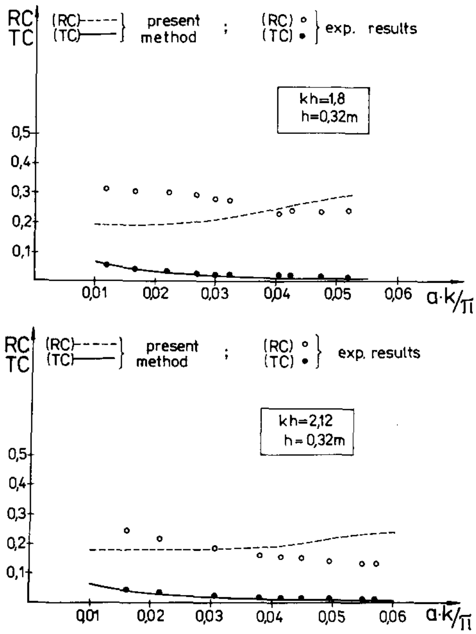
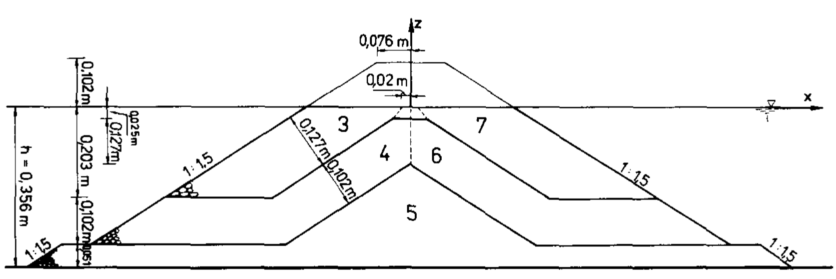


Fig. 8. Reflection and transmission coefficients for trapezoidal layered breakwater (TW-2) dependence on wave steepness.



layer	ε	K	C_f
	1	m^2	1
3,7	0,434	$4{,}478\cdot10^{-7}$	0,282
4,6	0,439	$1{,}057\cdot10^{-7}$	0,295
5	0,430	$0{,}3484\cdot10^{-7}$	0,4058

$g = 9{,}815$ m/s^2; $\nu = 1{,}0125\cdot10^{-6}$ m^2/s

Fig. 9. Trapezoidal layered breakwater (TW-3).

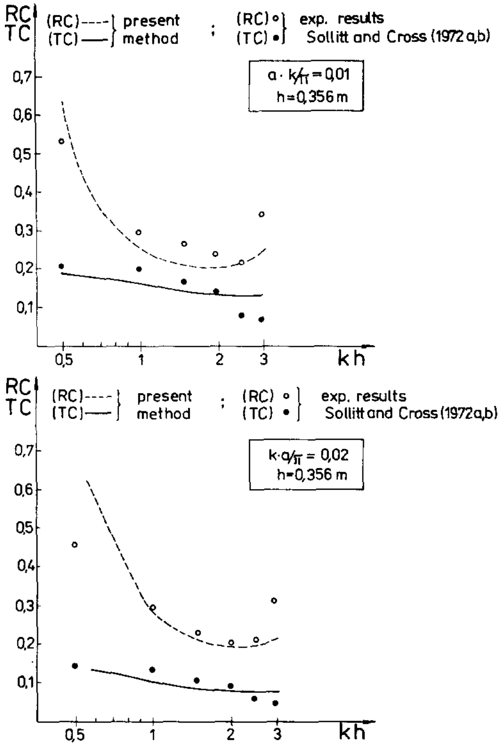


Fig. 10. Reflection and transmission coefficients for trapezoidal layered breakwater (TW-3) dependence on wave number.

armours are usually constructed from different media (Shore Protection Manual, 1973; Hydro Delft, 1983). Therefore, a computational program which calculates the reflection and transmission coefficients for a five-layered breakwater (core, seaward and leeward face of second armour, seaward and leeward face of first armour) has been developed. Using this program, the wave reflection from and transmission through a three-layered trapezoidal structure (continuous line) dimensioned as in Fig. 9 (Sollitt and Cross, 1972a) were calculated. The media properties are the same as tabulated in the figure. The dashed line in Fig. 9 denotes the division of the second armour and the small change carried into the physical model (extension of secondary armour to free surface) in order to use a computer program. The reflection and transmission coefficients are presented as a function of wave number for a constant wave steepness in Fig. 10.

6. FURTHER DISCUSSION AND CONCLUSIONS

The experimental and theoretical transmission coefficients correlate rather well. For the reflection coefficient the correlation is satisfactory only for small k. Note that for small wave numbers (k) the differences between the theoretical reflection coefficient and experimental results obtained by Sollitt and Cross (1972a, b) (presented in Fig. 4 and Fig. 10), are due to the experimental procedure (determination of the reflection coefficient by measuring the heights of loop and node), which underestimates the value of the reflection coefficient for small k (Bendykowska, 1966; Madsen and White, 1976). For the remaining k values considered the theory generally underestimates the experimental results at low steepnesses and overestimates the results at high steepnesses.

In general, the observed discrepancies may be due to a number of factors including:

(I) Assumption that $S = 1$.

(II) Loss of energy in the domain R_2, particularly for high wave steepnesses.

(III) Unsteady modification of the steady-state damping law.

The theory underestimates the experimental results (RC) at low steepnesses probably due to factor I or II. The correlation is substantially improved by taking nonzero values for the virtual mass coefficient (Fig. 11); $S = 2$ as proposed Le Méhauté (1957). However, it is not possible to predict the magnitude of this coefficient a priori as the virtual mass of densely packed fractured stone is not known. The experimental results show very

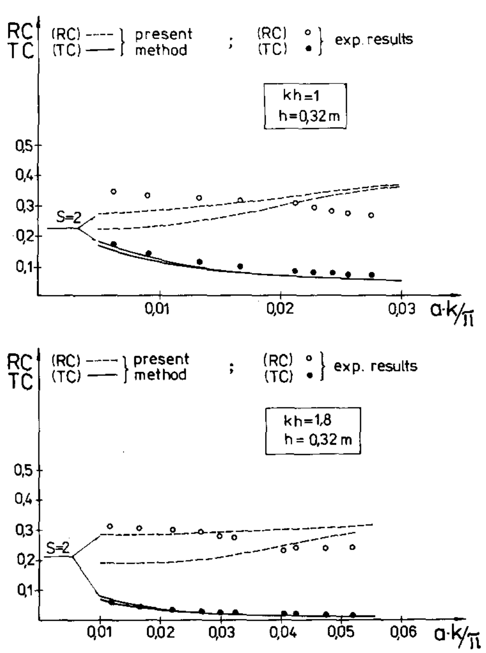


Fig. 11. Reflection and transmission coefficients for trapezoidal layered breakwater (TW-2) dependence on wave number and on inertial coefficient ($S = 1, S = 2$).

large scatter (Hannoura and McCorquodale, 1978). Thus, the results obtained by using $S > 1$ can be treated only as a possible explanation of the discrepancies between the theoretical and experimental reflection coefficients at low wave steepnesses.

The theory overestimates the experimental results (RC) at high wave steepnesses, probably due to factor II or III. In the theoretical model we have assumed that there is no loss of energy in domain R_2. This assumption was involved with the difficulty of the theoretical estimation of the energy loss in this domain.

The discrepancies between the experimental and theoretical reflection coefficient may also be due to linearization of the motion equation and boundary condition. However, one must not forget, that in the domain before the breakwater, the resultant wave amplitude is the sum of the amplitudes of the incident and reflected waves. Thus, in spite of the discrepancies between the experimental and theoretical reflection coefficient, the theory estimates the resultant wave amplitude before the structure (very important design quantity) within an error of only several percent.

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