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SHORTER CONTRIBUTION

Dispersion of the directional spectrum of short gravity waves in the Kuroshio Current

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Abstract—The dispersion of the directional spectrum of short gravity waves was obtained from air-photographs of the Kuroshio using the hologram method. Low-frequency components in the direction of the averaged stream of the Kuroshio may be clearly seen. On the contrary, the directional spectrum of waves outside the Kuroshio Current is closely related to the direction of local winds with a wavelength shorter than 1 m.

The present analysis suggests that the directional dispersion of the power spectrum is caused by the total reflection and trapping of wave-trains in the shear flows of the Kuroshio and by wave-current interaction especially for waves with short wavelengths.

INTRODUCTION

THE DIRECTIONAL two-dimensional spectrum of wind-generated waves has been measured using various methods from an aircraft in the North Atlantic Ocean (COTE *et al.*, 1960) with a buoy oscillating vertically with surface waves (LONGUET-HIGGINS, CARTWRIGHT and SMITH, 1963), with water-pressure gauges located on the sea bottom (MUNK, MILLER, SNODGRASS and BARBER, 1963) and with P-type electromagnetic current meters (NAGATA, 1964). BARBER (1954) first used an optical method, recently called the hologram method. This has been further developed and placed on a firm theoretical basis (STILWELL, 1969). It has now been applied to field observations (SUGIMORI, 1972).

The present article reports an investigation of the characteristics of the interaction between a shear flow of large magnitude and gravity waves in the Kuroshio region with the directional spectrum obtained from the optical analysis of aerial photographs.

FIELD OBSERVATION

The real distribution of surface temperature was measured to determine the meander of the Kuroshio using an infra-red radiation thermometer. Aerial photographs of the reflected image of the sea surface were also taken from an aircraft of the Maritime Safety Agency of Japan on the 17th of August in 1971 (Fig. 1).

The speed and direction of the wind (Fig. 2) at a height of 450 m was determined from the drift of the aircraft with LORAN-C. The wind speed at the sea surface may possibly be less than at higher levels but it may be assumed that the wind direction is not very different at various levels. Nearly four hundred air-photographs were taken with an almost vertical angle. In some the sea surface was more or less covered by clouds and the reflected images of sunlight were not sufficiently clear. Thirty-two were free of clouds and were thus chosen for the present analysis (Fig. 3).

ANALYSIS OF OBSERVATION RESULTS

The photographs were optically analyzed with a hologram device constructed by the author (SUGIMORI, 1972). Fraunhofer images were obtained from the aerial photographs of Fig. 3 (Fig. 4). Symmetry of the images is merely due to the optical treatment. The center of the image corresponds to an infinite wavelength. The wavelength becomes smaller with increasing radial distance from the center. By scanning these images with a microphotodensitometer, the contours of the directional spectrum may be drawn (Fig. 5).

A model experiment using a test pattern demonstrated that the instrumental error of the power spectrum due to the superposition of higher order Fraunhofer images and to the instability of the

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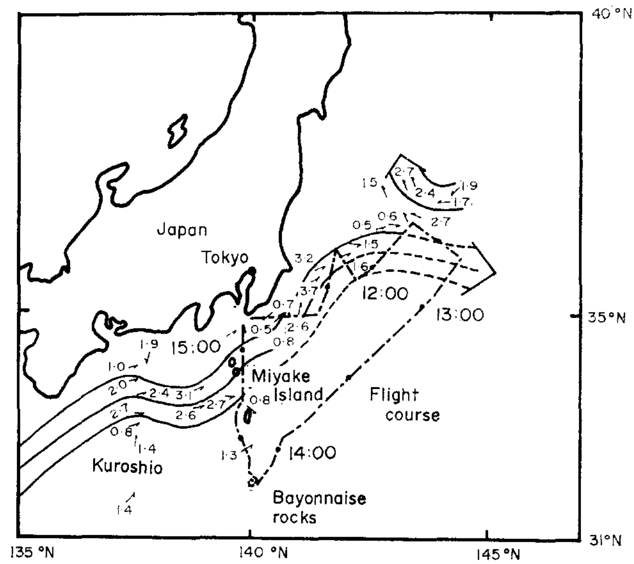


Fig. 1. Flight course over the Kuroshio Current, 17 August, 1971. Solid lines indicate the boundaries of the meandering Kuroshio which were clearly defined by surface temperature observations and current data. The dashed lines are the extrapolated Kuroshio boundaries where data are unavailable. Arrows and numbers show the direction and velocities of the local current—(in knots)* as measured by an infra-red radiation thermometer and G.E.K. Broken lines show the flight course and the attached numbers the flight time.

*1 knot = 0·51 m/sec.

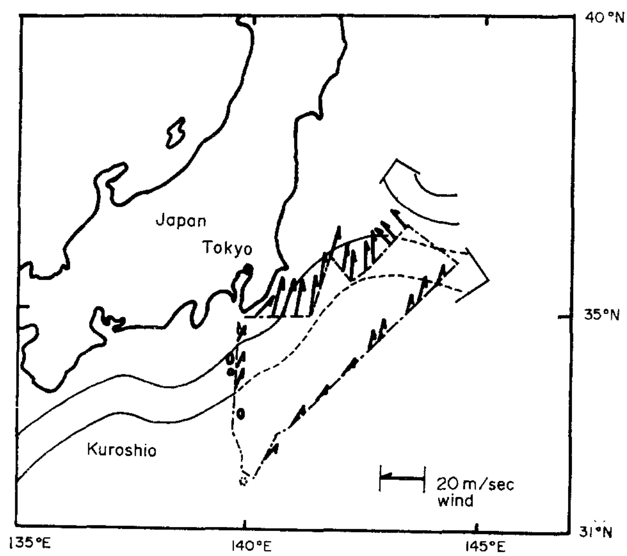
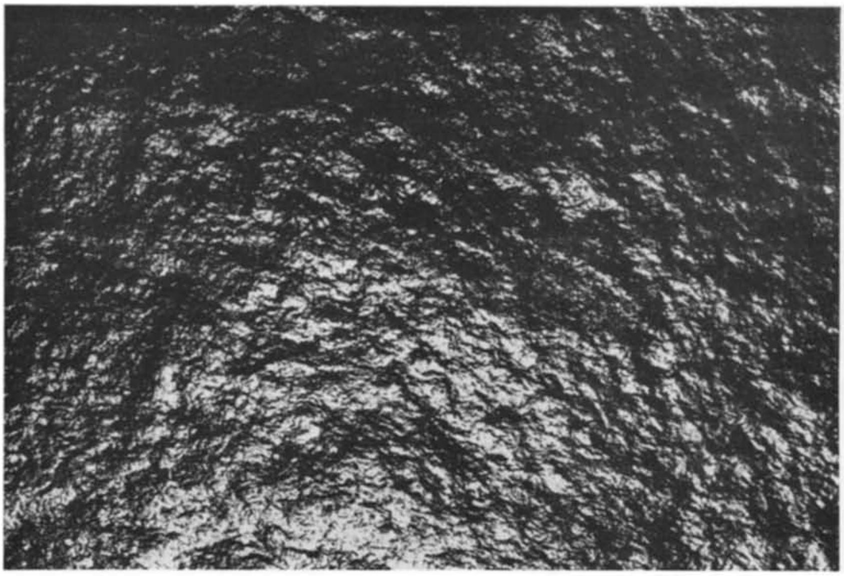
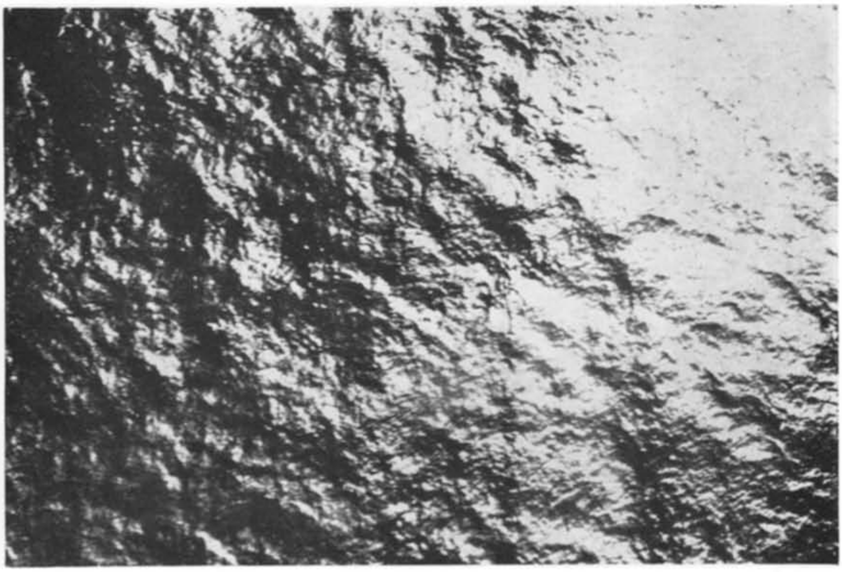


Fig. 2. Wind field over the Kuroshio Current at a height of 450 m.

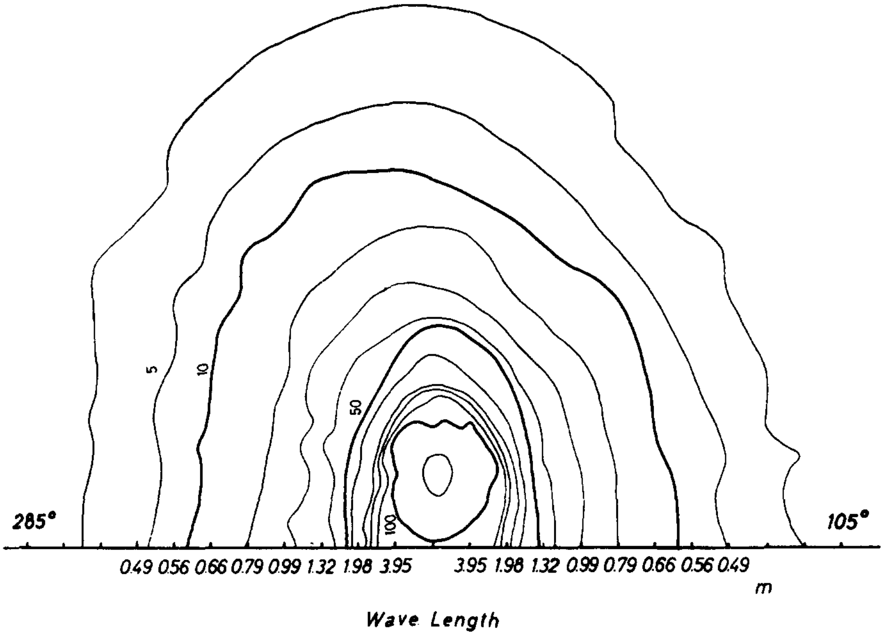


(a)

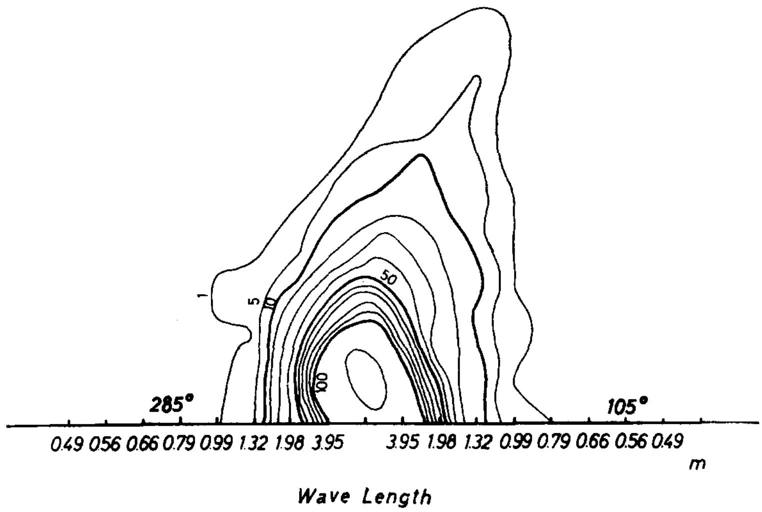


(b)

Fig. 3. Air-photograph of the sea surface (295 $\times$ 205 m) taken on a course from Bayonnaise Rocks to Tokyo Bay. (a) Inside the Kuroshio Current (14 hr 55 min, 17 August, 1971). (b) Outside the Kuroshio Current (15 hr 09 min, 17 August, 1971).



(a)

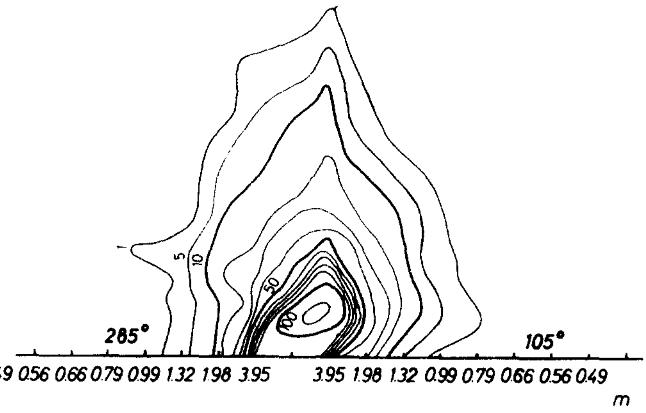


(b)

Fig. 5. Contours of directional power spectra. (a) Inside the Kuroshio Current (14 hr 55 min, 17 August, 1971). (b) Outside the Kuroshio Current (15 hr 09 min, 17 August, 1971).

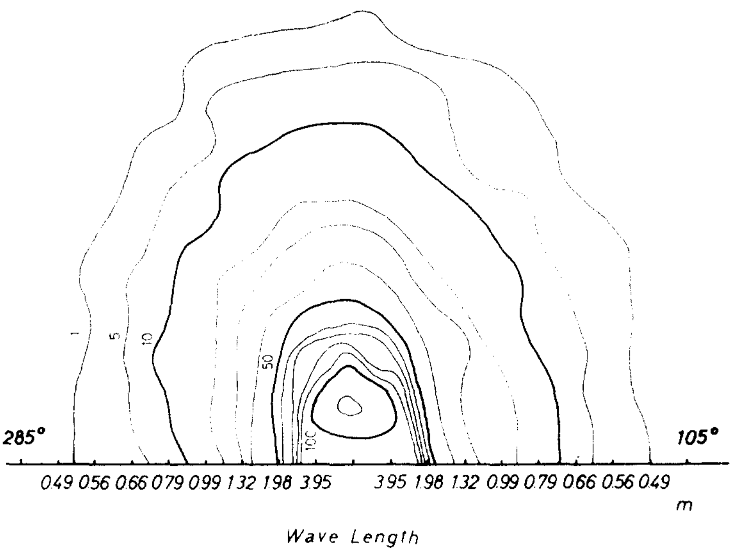
laser power does not exceed 13 % of the spectrum power (SUGIMORI, 1972). The resolving power of photographs at an altitude of 450 m is limited to about 30 cm in wavelength because of the pitch and roll of the moving aircraft.

It is clear (Fig. 5) that the directional spectrum of surface waves with a wavelength shorter than 1 m outside the Kuroshio was predominantly about parallel to the direction of the local wind, while it was uniformly dispersed inside the Kuroshio. In other words, surface waves with wavelengths shorter than 1 m occurred in all directions inside the Kuroshio, but only larger wavelengths had a directional distribution (Fig. 6). All power spectra except that shown in Fig. 6(c) inside the Kuroshio had a similar



(6a)

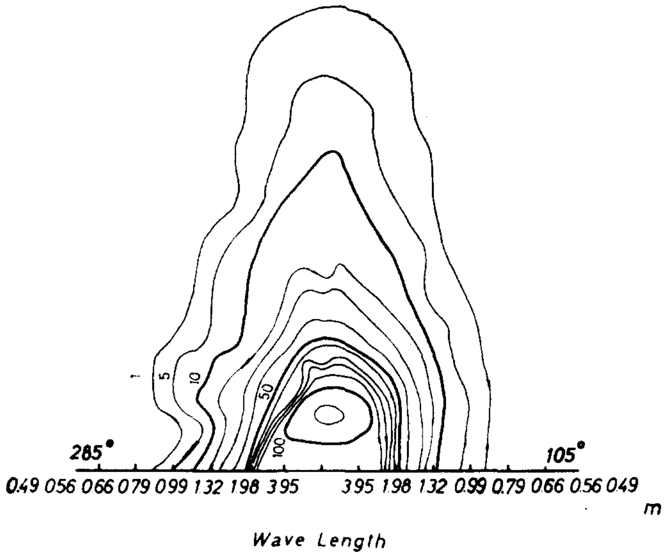
Fig. 6. Contours of directional power spectra of the short gravity waves without those for 14 hr 55 min and 15 hr 09 min on the course from Bayonnaise Rocks to Tokyo Bay; (a) is a contour of the directional power spectrum outside the Kuroshio Current, (b)–(e) are those inside the Kuroshio Current, (c) was obtained near Miyake Island.



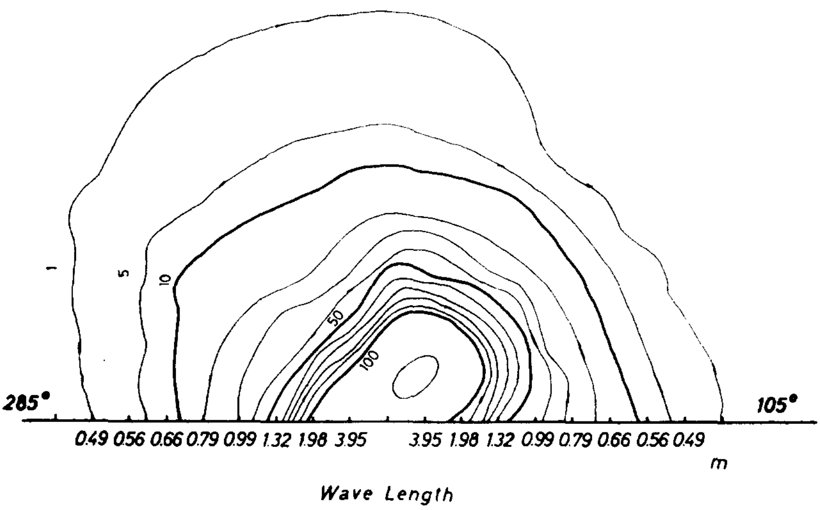
(6b)

directional dispersion for the shorter wavelength but the direction of surface waves with wavelengths longer than 1 m coincided with that of the averaged stream of the Kuroshio. Most spectra outside the Kuroshio were predominantly in the direction of the local winds.

Results of the analyses of the 32 air-photographs (18 inside and 14 outside the Kuroshio Current) thus seem to be fairly consistent except for those near the Bayonnaise Rocks and Miyake Island [Fig. 6(c)] which may be affected by the islands. The predominant direction of waves outside the Kuroshio with a longer wavelength differs slightly from that of shorter wavelengths and is almost



(6c)



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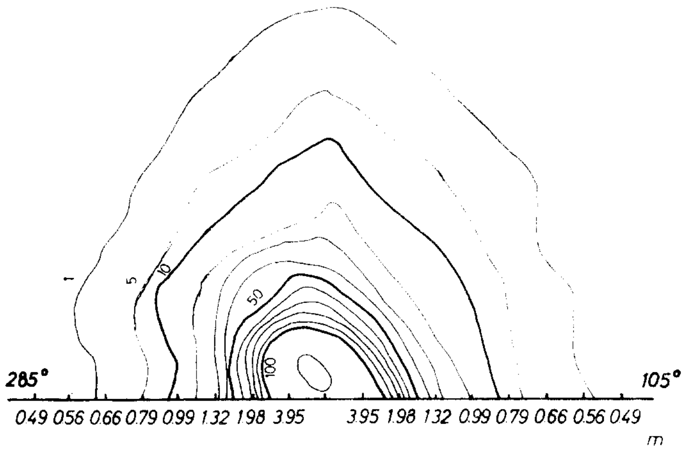


(6e)

due south (Fig. 7). It seems likely that the northward propagation of long waves had been previously caused by a typhoon (71–22) which passed near the Pacific coast of Japan on the previous day, as seen from the weather map of noon for August, 1971 (Fig. 8).

THEORETICAL CONSIDERATIONS

Gravity waves entering a non-uniform flow may be subject to the two different processes of interaction between the waves and the flow. One process is the linear interaction between the waves and the non-uniform flow. The other is the wave refraction at the boundary of a non-uniform flow which causes wave scattering. The first process is examined for waves of constant frequency representing all incident waves inside and outside the Kuroshio on the basis of the linear interaction between the incident waves and a non-uniform flow.

Let the phase velocities of the gravity waves outside the current be C_x and C_y for x-, y-coordinates, respectively. If the phase velocity of the gravity waves inside the Kuroshio is changed to C'_x by the wave current interaction, then C'_x may be expressed as follows:

$$C'_x = C_x + U, \tag{1}$$

where U is the stream velocity of the current which is assumed to flow only along the x-coordinate. The y-coordinate is then written

$$C'_y = C_y. \tag{2}$$

Hence the following relation will be consistent inside the current.

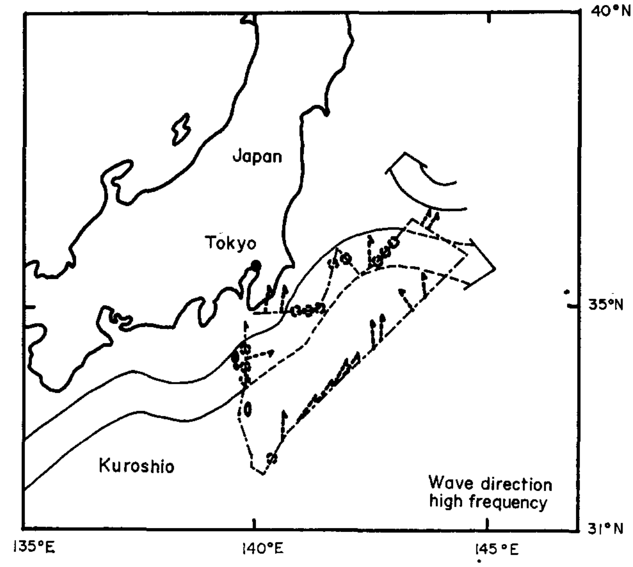
$$\begin{aligned} C' &= \{C_y^2 + (C_x + U)^2\}^{1/2} \\ &= \{C_x^2 + C_y^2 + 2C_xU(1 + U/2C_x)\}^{1/2} \\ &= \{C^2 + 2C_xU(1 + U/2C_x)\}^{1/2} \\ &\simeq C(1 + C_x/C \cdot U/C), \end{aligned} \tag{3}$$

as the relation $1 \gg U/C$ is usual in the Kuroshio. For the comparison with our observations, two assumptions are made for equation (3) on the condition of constant direction for the incident waves for all frequency ranges.

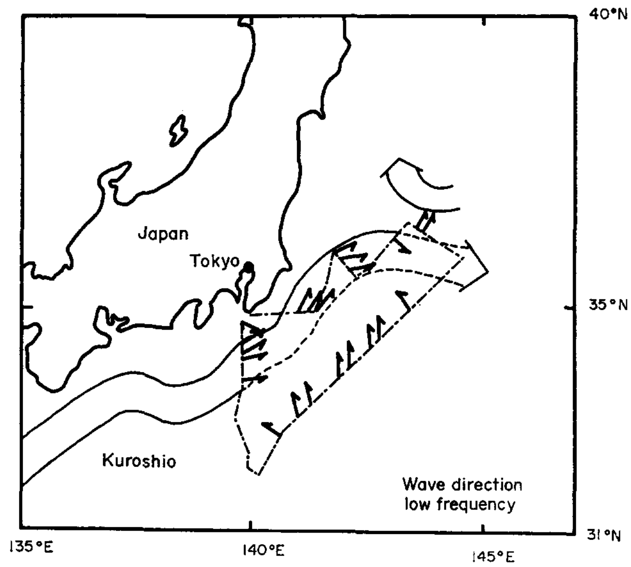
(1) $C_xU > 0$

This condition means that incident gravity waves turn in the direction of the averaged stream and the angle between the direction of the averaged stream and the incident waves is less than 90 degrees. Under this condition, we obtain the following relation from equation (3).

$$C' > C. \tag{4}$$



(a)



(b)

Fig. 7. General characteristics of short gravity waves on the whole flight course. (a) Dashed circles show the directional dispersion of the spectra of short gravity waves inside the Kuroshio Current and the dashed arrows show the directions of the waves with wavelength shorter than 1 m outside and inside the Kuroshio Current. (b) Arrows show the directions of the waves with wavelengths longer than 1 m inside and outside the Kuroshio Current.

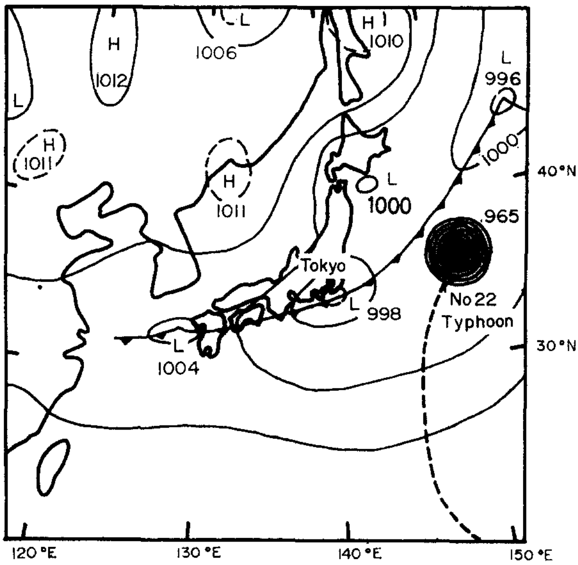


Fig. 8. Course of the 22nd typhoon of 1971 and the weather map at noon on 17 August, 1971.

Assuming that the wave frequency is equal to ω, all positions inside and outside the current, we denote the wave-number by K outside and by K' inside the Kuroshio Current. Then we have

$$KC = K'C' = \omega = \text{constant}. \tag{5}$$

Substituting equation (3) into equation (4) and applying the result to equation (4), we obtain the following.

$$K' < K. \tag{6}$$

This relation is contradictory with the observation of 17 August, 1971, because the wave number inside the Kuroshio Current K' was mostly larger than K everywhere outside.

(2) $C_x U < 0$

This condition indicates that the direction of the averaged Kuroshio stream reverses and is essentially the same as the incident waves. This angle exceeds 90 degrees. As in (1), we obtain the following relation:

$$C' < C, \text{ then } K' > K, \tag{7}$$

which is in agreement with our observation. It is, however, noteworthy that the second case $C_x U < 0$ does not agree with the field observations for 17 August, 1971, because the incident angle between the waves and the current was about 35 degrees at the Kuroshio front on the course from Bayonnaise Rocks to Tokyo Bay. Consequently, it is not reasonable to consider that the directional dispersion of a two-dimensional spectrum of short gravity waves is caused by the wave-current interaction if the incident waves including all frequency range have a constant direction.

The second process, as proposed by Kenyon (1971), is the wave refraction caused by various shear flows. Thus, gravity waves entering a strong stream would be refracted and bent in the direction of the current.

For an individual frequency ω with a certain band-width corresponding to frequencies of all incident waves, the approximation of geometrical optics for an inhomogeneous moving medium is applied to surface gravity waves propagating in non-uniform steady current. An approximate expression for the radius of curvature of the incident wave-trains R is

$$R = C_g/\zeta, \tag{8}$$

where $C_g(= 1/2 \cdot C)$ is the group velocity of the incident waves and ζ is a component of current vorticity in the positive z-coordinate. For deep water the dispersion relation of surface gravity waves is

$$\omega = (g \cdot K)^{1/2}. \tag{9}$$

From equation (8), we obtain

$$R = g/(2\omega\zeta). \tag{10}$$

Equation (10) shows that the radius of curvature decreases with increasing frequency relative to the current and increasing current vorticity. Hence, the refraction of surface gravity waves in deep water increases with increasing frequency and current vorticity.

The observed waves entering the Kuroshio Current have various frequencies. Thus, the minimum wavelength is about 50 cm and the maximum one is about 5 m. Therefore the radius of curvature of each wave-train differs with each frequency of the gravity waves according to equation (10).

Moreover, inside the Kuroshio, two types of shear flows may exist; one has various magnitudes of vorticity and the other has various directions. Therefore, even if the incident waves have a constant wave-number, the radius of curvature of refracted wave-trains will differ due to the various non-uniform flows. Many wave-trains with different radii of curvature would produce a directional divergence of the incident waves by the total reflection and entrapment inside the Kuroshio Current. These reflected waves with different radii of curvature may consequently cause the wave scattering inside the current and increase the directional dispersion of the surface waves.

CONCLUSION

If the duration of the wind is sufficiently long, the direction of the gravity waves coincides with that of the wind. Thus, the wind near Izu Islands (Fig. 2) was roughly southwesterly. Hence, the gravity waves would enter into the Kuroshio Current at an angle of 35 degrees. Then no dispersion of the directional spectrum of the short gravity waves at the boundary of the Kuroshio Current is produced by the interaction of waves and current since the interaction of equation (7) occurs when the direction of averaged stream opposes that of the waves. While the total reflection and entrapment of incident wave-trains may occur in our observed frequency range (short gravity waves) especially at the boundary of the Kuroshio Current, the current shear may be so strong that the higher frequencies are refracted to a greater extent than are the lower frequencies. Many wave-trains with different radii of curvature consequently produce the directional divergence of the incident waves.

The total spectral power of the short gravity waves inside the Kuroshio is larger than that outside the current (Fig. 5). The momentum flux corresponding to the increment of spectral power inside the current may be supplied from beneath the stream by the radiation stress of the interaction between the incident waves refracted from the Kuroshio boundary and the inside shear flows (LONGUET-HIGGINS and STEWART, 1961). Therefore the directional dispersion of the power spectrum would be caused by the total reflection and entrapment of wave-trains in the shear flows of the Kuroshio boundary and by the wave-current interaction especially for the short wavelength waves inside the current.

A detailed comparison of the theory with actual measurements is not attempted here because the velocity structure of the stream is little known. Thus, the velocity structure should be measured together with the incident waves by simultaneous aerial observation.

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